Cosmological effects of a class of fluid dark energy models

Daniela Carturan
Department of Astronomy, University of Bologna, Via Ranzani 1, I-40127 Bologna, Italy and IASF-CNR/Bologna, Via Gobetti 101, I-40129, Bologna, Italy

Fabio Finelli
IASF-CNR/Bologna, Via Gobetti 101, I-40129, Bologna, Italy and INFN, Sezione di Bologna, via Irnerio 46, I-40126 Bologna, Italy

(Received 6 December 2002; published 7 November 2003)

We study the impact of a generalized Chaplygin gas as a candidate for dark energy on density perturbations and on cosmic microwave background (CMB) anisotropies. The generalized Chaplygin gas is a fluid component with an exotic equation of state $p=-A/\rho^{\alpha}$ (a polytropic gas with negative constant and exponent). Such a component interpolates in time between dust and a cosmological constant, with an intermediate behavior as $p=A^{1/(1+\alpha)}+\alpha\rho$. Perturbations of this fluid are stable on small scales but behave in a very different way with respect to standard quintessence. Moreover, a generalized Chaplygin gas could also represent an archetypal example of the phenomenological unified models of dark energy and dark matter. The results presented here show how CMB anisotropies and density perturbations in this class of models differ from those of a cold dark matter model with a cosmological constant.

DOI: 10.1103/PhysRevD.68.103501 PACS number(s): 98.80.Cq, 95.35.+d

According to the present recipe to explain observational data, two dark components seem to fill the Universe up to 95% of the total content. The baryon density is indeed very small, $\Omega_b h^2 \sim 0.02$ [1], assuming a flat universe. This situation has been slowly reached over decades. In addition to cold dark matter (CDM), in the 1990s a cosmological constant term Λ was called into play to explain the recent acceleration of the Universe indicated by the supernova data [2], but then confirmed by other observations.

Unable to explain on theoretical grounds the embarrassing smallness of the cosmological constant Λ constrained by observations, a scalar field Q [3], dubbed quintessence, was suggested in order to explain the accelerating Universe [4]. Because of the different background evolution and of the presence of fluctuations, the challenge is to distinguish Λ from quintessence, for instance through the cosmic microwave background (CMB) anisotropies [5].

An alternative to quintessence for modeling a dark energy component could be a perfect fluid (PF) with a generic pressure $p=p(\rho)$ (not linear in energy density ρ) whose energy momentum tensor is

$$T_{\mu\nu}=pg_{\mu\nu}+(\rho+p)u_{\mu}u_{\nu}, \tag{1}$$

where $g_{\mu\nu}$ is the metric and u_{μ} is the fluid velocity ($u_{\mu}u^{\mu}=-1$). From a theoretical point of view a scalar field description would be logically preferable to relate an accelerating universe with a fundamental quantum. From a phenomenological point of view the reasons to prefer one over the other are less obvious. An exotic fluid capable of developing a negative pressure at late times may also represent the effective degree of freedom that drives the present acceleration of the universe. In particular, a PF model with this property would allow one to explore the possibility that dark energy clusters on small scales. Indeed, it is important to note that, by parametrizing dark energy with an uncoupled scalar field with an ordinary kinetic term, one does implicitly assume no clustering of dark energy on scales smaller than the Hubble radius.

Among the class of PF models that could work as a dark energy component—in principle one could design suitable pressure profiles $p(\rho)$ instead of potentials $V(\phi)$ for a quintessence field ϕ—the Chaplygin gas [6] has recently received a lot of attention [7]. A Chaplygin gas is characterized by a pressure p_X related to the energy density ρ_X in the following way:

$$p_X=-\frac{A}{\rho_X^{\alpha}} \tag{2}$$

with $\alpha=1$. Even though this exotic fluid was proposed in the context of aerodynamics [6], there are interesting connections with particle physics and D-branes [8]. A Chaplygin gas is also equivalent to a tachyon field [9] with a constant potential [10] and, at the homogeneous level, to a complex scalar field [11] or to a quantum scalar field [12] at the bottom of a potential (called the Thomas-Fermi approximation in [11]). In this paper, we study a generalized version of the Chaplygin gas (GCG) [13] by considering $0<\alpha\leqslant 1$ in Eq. (2).

In a Robertson-Walker metric

$$ds^2=-dt^2+a(t)^2\left(\frac{dr^2}{1-Kr^2}+r^2 d\Theta^2\right), \tag{3}$$

where $K=0,\pm 1$ is the curvature of the spatial sections and Θ is the solid angle, the energy conservation equation for a GCG

$$\dot{\rho}_X+3H(\rho_X+p_X)=0 \tag{4}$$

can be immediately integrated [7,13]:

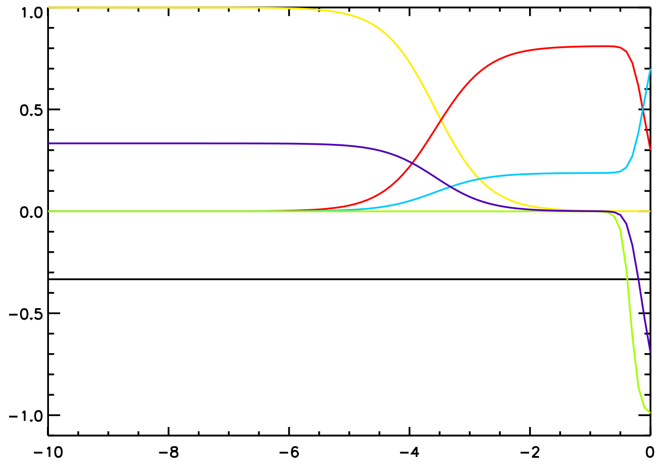

FIG. 1. Evolution of background quantities for a flat universe versus the scale factor: from top to bottom at the present time we have Ω_X, Ω_m, Ω_r, $w_{\rm tot}$ and w_X. The horizontal black line is $-1/3$, which denotes the threshold for w below which the universe expands accelerating. The parameters are $\Omega_{r0}=10^{-4}$, $\Omega_{m0}\simeq 0.3$, $\Omega_{X0}=0.7$, $\alpha=1$, $B/A=0.01$, and $h=0.7$ (where $h=H_0/100$ with H_0 the Hubble parameter today measured in units of km/s/Mpc).

$$\rho_X=\left(A+\frac{B}{a^{3(1+\alpha)}}\right)^{1/(1+\alpha)}, \tag{5}$$

where A,B are constants with dimensions $[M^{4(1+\alpha)}]$. We note that a GCG reduces to a CDM model with a cosmological constant (ΛCDM) for $\alpha=0$ and to a SCDM model for $A=0$. We see that this fluid with an exotic equation of state behaves like dust for small a (when $B/A \gg a^{3(1+\alpha)}$, assuming $a=1$ at the present time) and a cosmological constant given by $A^{1/(1+\alpha)}$ in the opposite limit ($B/A \ll a^{3(1+\alpha)}$). By Taylor expanding in this limit [7,13] we obtain from Eqs. (5) and (2)

$$\rho_X \simeq A^{1/(1+\alpha)}\left[1+\frac{B}{(1+\alpha)Aa^{3(1+\alpha)}}+\mathcal{O}\left(\frac{B^2}{A^2}\right)\right],$$

$$p_X \simeq A^{1/(1+\alpha)}\left[-1+\frac{\alpha B}{(1+\alpha)Aa^{3(1+\alpha)}}+\mathcal{O}\left(\frac{B^2}{A^2}\right)\right]. \tag{6}$$

Therefore, in this limit a GCG behaves as the sum of a cosmological constant and a perfect fluid characterized by $p=\alpha\rho$, shedding light upon the physical meaning of the parameter α. We note that the solution (5) to Eq. (4) is valid for *any* $\alpha>-1$, but also for $\alpha<-1$ (a standard polytropic gas). In this latter case the behavior of such a PF interpolates between a cosmological constant and dust. For $\alpha=-1$, Eq. (2) describes the usual barotropic perfect fluid. According to Eq. (2), the ratio between pressure and energy w_X defined as

$$w_X=\frac{p_X}{\rho_X}=-\frac{A}{\rho_X^{1+\alpha}} \tag{7}$$

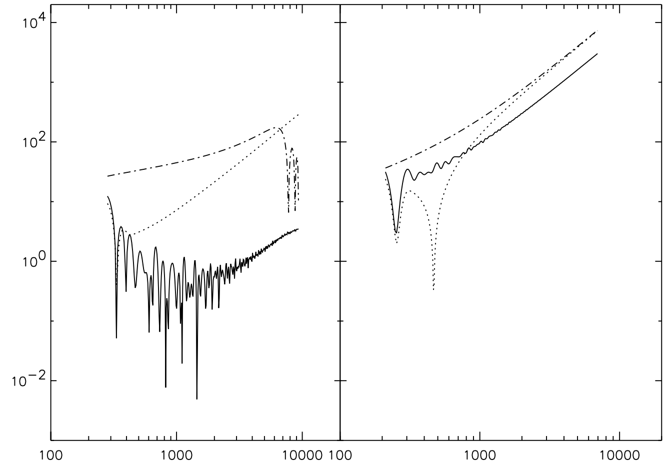

FIG. 2. Density contrast versus conformal time of photons (solid line), baryons (dotted line), GCG (dot-dashed, on the left), and CDM (dot-dashed, on the right) in a GCG model (left) and a ΛCDM model (right).

decreases from the value 0 to -1. An example of the background evolution for a GCG as dark energy is given in Fig. 1.

The presence of a GCG as a dark energy component could be distinguishable from a quintessence component because of the parametric form of the pressure $p=p(\rho)$. The time derivative of the equation of state of the dark energy component is important in the program of reconstructing the total equation of state [14]. Indeed, the time derivative of w_X for a GCG is

$$\dot{w}_X=3H(\alpha+1)w_X(1+w_X), \tag{8}$$

while for a scalar field ϕ with potential $V=V(\phi)$ it is

$$\dot{w}_\phi=3H(1+w_\phi)(w_\phi-1)-2\frac{\dot{V}}{\rho_\phi}. \tag{9}$$

Supernova (SN) Ia observations can constrain the GCG as a candidate for dark energy, as recently studied by different authors [15]. The purpose of this article is to show how CMB data could be more selective.

The behavior of perturbations is a very interesting aspect of the model. When trying to build a model for an accelerating universe with a barotropic (constant w) perfect fluid, one runs into the problem of instabilities on short scales because of a negative sound speed for the perturbations. In fact, the sound speed is equal to w and this must be negative ($w<-1/3$) to explain the acceleration. This is the usual problem for a fluid description of domain walls and cosmic strings. Quintessence models with scalar fields with a standard kinetic term do not have this problem. The sound speed for a scalar field is equal to 1. *K*-essence models [16] based on scalar fields with a nonstandard kinetic term are different in this respect [17], but still have positive sound speed (even if it can exceed the speed of light). For a GCG the sound speed c_X^2 for perturbations is

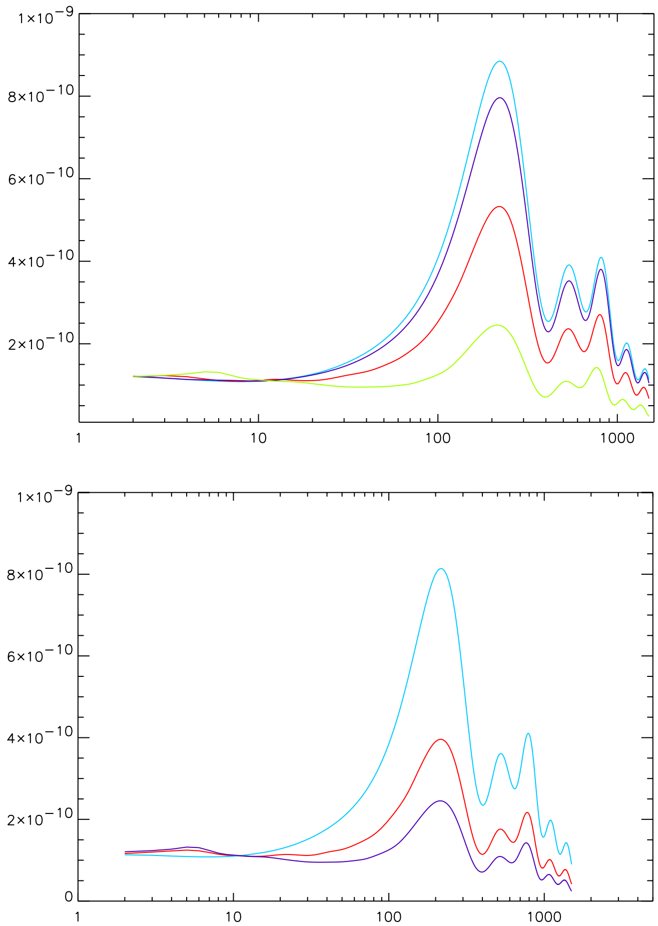

FIG. 3. C_ℓ spectra of temperature anisotropies $\delta T/T$ versus ℓ for CGC varying the ratio B/A (on the left) and α (on the right). The parameters are the same as in Fig. 1 except $B/A=0.1,0.01,0.001$, and ΛCDM (from bottom to top, on the left) and $\alpha=1,0.5,0$ (from bottom to top, on the right) for $B/A=0.1$.

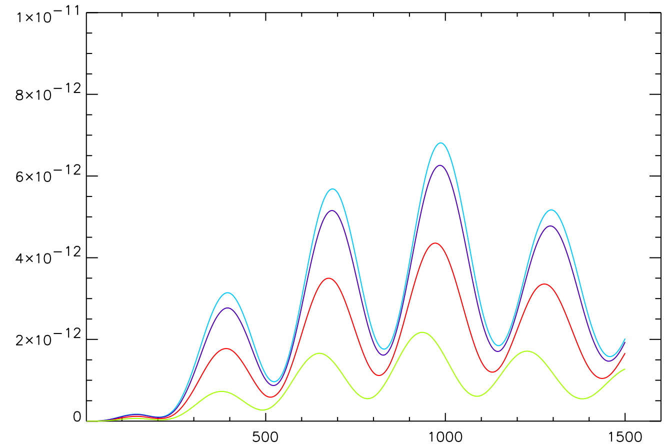

FIG. 4. E-mode polarization anisotropy spectra when the ratio B/A is varied (for the same set of parameters as in the left panel of Fig. 3).

$$c_X^2=\frac{\partial p}{\partial \rho}=\alpha\frac{A}{\rho_X^{1+\alpha}}=-\alpha w_X. \qquad (10)$$

Therefore, because of its nonbarotropic nature, perturbations of a GCG are stable on small scales even in an accelerating phase, and behave similarly to dust perturbations when the gas is in the dust regime. When the behavior of the background Chaplygin gas is of Λ type, the sound speed is α. In order to avoid causality issues, we shall consider $\alpha\leqslant 1$. This last constraint, with the requirement of positive sound speed $(\alpha>0)$, marks the interesting physical range of α. We note that the possibility of having a non-negative sound speed and a negative equation of state, as happens for a GCG, could open new developments in modeling a domain wall or cosmic string network which is not plagued by short scale instability for perturbations.

The equations for the energy density contrast $\delta_X=\delta\rho_X/\rho_X$ and the velocity potential θ_X in the synchronous gauge are, according to Ref. [18] and by using Eqs. (8),(10),

$$\delta_X'=-(1+w_X)\left(\theta_X+\frac{h'}{2}\right)+3\mathcal{H}(w_X-c_X^2)\delta_X,$$

$$\theta_X'=-\mathcal{H}(1-3c_X^2)\theta_X+\frac{c_X^2}{1+w_X}k^2\delta_X, \qquad (11)$$

where h is the trace of the metric perturbations in the synchronous gauge [18]. This set of equations agrees with those used in [19]. In order to study the Jeans instability for a GCG it is useful to study the equation for the (gauge invariant) comoving density contrast $\Delta_X=\delta_X+3(1+w_X)\mathcal{H}\theta_X/k^2$ in the approximation in which the GCG is the only component of the universe:

$$\Delta_X''+\mathcal{H}(1+3c_X^2-6w_X)\Delta_X'+c_X^2k^2\Delta_X-\frac{3}{2}\mathcal{H}^2(1+8w_X-3w_X^2-6c_X^2)\Delta_X=0. \qquad (12)$$

We see that the Jeans instability of a GCG is very similar to CDM in the dust limit (when $w_X\sim c_X^2\sim 0$). However, because of the time dependence of w_X, the Jeans instability is progressively removed since the quantity $1+8w_X-3w_X^2-6c_X^2$ changes sign as w_X departs from 0. At the same time, GCG perturbations start to oscillate as soon as c_X^2 draws away from 0. We confirm this behavior numerically when other fluids are present also. In Fig. 2 we show a comparison of the evolution of cosmological perturbations in GCG models (without CDM) and ΛCDM models.

We have implemented Eqs. (5) and (11) in a modified version of CMBFAST [20]. We tested the code against the SCDM model obtained by setting $A=0$ in Eq. (5) and considered an initial adiabatic scale invariant spectrum for perturbations. By setting $B=0$ in Eq. (5) instead, one obtains a ΛCDM model in the background, but with nontrivial perturbations in the dark energy sector [see Eq. (12)].

In Fig. 3 we show the dependence of the spectrum of temperature anisotropies on the ratio B/A and α. In Fig. 4 we show the E-mode polarization spectrum when B/A is varied. The three parameters A,B,α are in direct relation to the physical quantities of dark energy at the present time, as

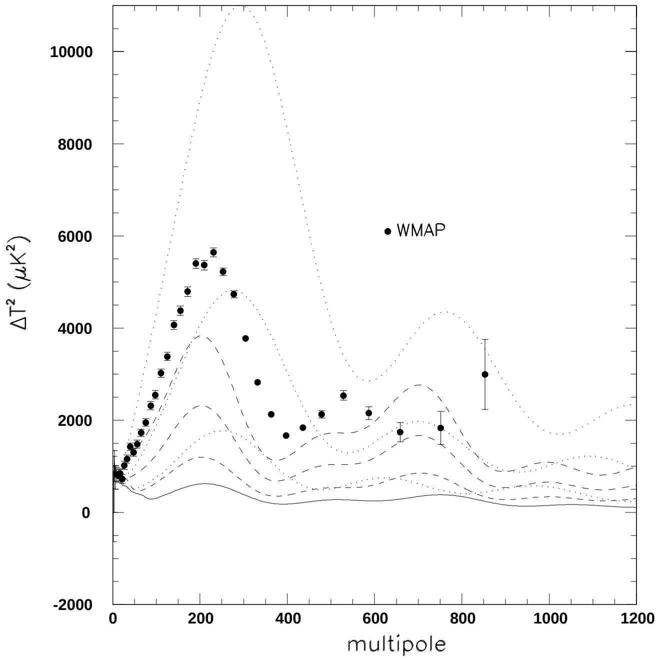

FIG. 5. Spectra of temperature anisotropies δT for a unified model ($\Omega_{X0}=0.95$ and $h=0.7$) when B/A is varied, compared with the WMAP data. The solid line at the bottom is $B/A=1$. The dashed lines correspond to 10, 50, and SCDM, respectively, from bottom to top. The dotted lines correspond to 0.01, 0.001, and a Λ-baryon model from bottom to top.

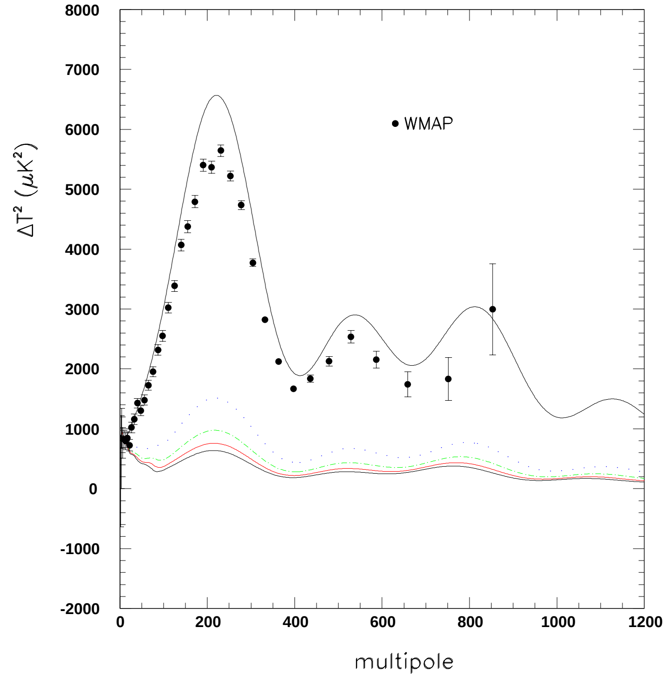

FIG. 6. Spectra of temperature anisotropies δT for a unified model ($\Omega_{X0}=0.95$, $h=0.7$, $B/A=0.36$) when α is varied (1, 0.75, 0.5, 0.25, 0, from bottom to top) compared with the WMAP data.

$\Omega_{X0}, w_{X0}, \dot{w}_{X0}$. Therefore these parameters can be constrained by maximizing the likelihood function with the present data [21]. In particular, from the left panel of Fig. 3 one can see how spectra can be noticeably different for a GCG model which differs from a ΛCDM model by less than 10% in the background evolution.

Because of its early dust behavior, a GCG may also represent a prototypical unified model of dark matter and dark energy (see [22] for a similar proposal, but with a scale dependent equation of state). In Figs. 5 and 6 we compare GCG models without CDM ($h=0.7,\Omega_{X0}=0.95$) with the WMAP [23] data, when B/A and α are varied.

In particular, in Fig. 5 we see how all the models lie below the limiting case of SCDM ($A=0$) and a Λ-baryon model (very similar, but *not* equivalent to $B=0$). From these plots, we see how the CMB data constrain the parameter of these GCG models [21] much more than the SN Ia data [15].

As this paper was being written, two related projects on a unified model of dark matter and dark energy based on the GCG appeared [24,25]. Bento *et al.* [24] studied the location of the CMB peaks in the presence of a unified GCG model within an analytic approximation (see also the resulting constraints on the parameter space of a unified GCG model arising from the most recent data [26]). We have checked their analytical results with our code and found a systematic overestimation of the peak positions (in particular for the third peak) with respect to [24]. We present this comparison in Table I, recalling that the numerical accuracy of the CMBFAST code can be as low as 0.1% [27]. Sandvik *et al.* [25] addressed the issue of the matter power spectrum in this class of unified models and claimed that a GCG is ruled out as a UDM candidate at 99.999%. The analysis in [25] does not take into account baryons (which are taken into account in our code) and is linear. Baryons keep on clustering at all times after decoupling, even after the end of the Jeans instability for the GCG component [21]. The inclusion of baryons affects the total linear matter power spectrum, smoothing out the oscillations in the spectrum of the GCG component and improving the agreement with observations [21]. Moreover,

TABLE I. Comparison between the analytic evaluation by Bento *et al.* and numerical estimate by our code for the first (upper part) and third (lower part) Doppler peaks for $\alpha=1$. $A_s \equiv A/(A+B)$ as in the definition by Bento *et al.* All the other parameters are the same as in Figs. 1 and 2 of the paper by Bento *et al.* [24].

A_s	Bento *et al.*	Numerical estimate
	First Doppler peak	
0.4	207	209
0.5	209	211
0.6	211	213
0.7	214	216
0.8	218	220
0.9	226	228
	Third Doppler peak	
0.7	765	770
0.75	775	781
0.8	785	794
0.85	800	810
0.9	825	836

the analysis in [25] is based on a linear treatment of perturbations until the present time, neglecting any nonlinear effects, which may be important and unexpected because of the time dependence of the GCG Jeans length. Such nonlinear effects should be more important for the present matter power spectrum than for the CMB spectrum. Therefore, we think that more study is necessary to compare the matter power spectrum in unified models with observations.

We have studied the implications for the evolution of cosmological perturbations and for CMB anisotropies of a GCG as a candidate for dark energy. This GCG covers all the interesting possible cases of a dark energy model from a polytropic gas. A GCG is more clearly distinguishable from a ΛCDM model than a QCDM model [5] since both GCG background and perturbations are important, not only at late redshift, when the GCG behaves like a cosmological constant. In fact the GCG plays the role of a dust component before turning into a cosmological constant, modifying not only the positions of the peaks, but also the overall shape of C_ℓ, because of a big integrated Sachs-Wolfe effect. In this sense, the QCDM models with standard scalar fields [3,4] are the most economical way to modify a ΛCDM model into a dark energy model. The dark component sector(s) may be much more obscure and less simple [28], and the GCG models are an example of this. The next CMB experiments [29] and LSS data will be very helpful in constraining the physical properties of the dark component sector(s).

We would like to thank R. Abramo, S. Matarrese, L. Popa, and M. Sandri for useful suggestions.

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