Study of color suppressed modes $B^0 \to \overline{D}^{(*)0} \eta^{(\prime)}$

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The color suppressed modes $B^0 \to \overline{D}^{(*)0} \eta^{(\prime)}$ are analyzed in the perturbative QCD approach. We find that the dominant contribution is from the nonfactorizable diagrams. The branching ratios calculated in our approach for $B^0 \to \overline{D}^{(*)0} \eta$ agree with current experiments. By neglecting the gluonic contribution, we predict that the branching ratios of $B^0 \to \overline{D}^{(*)0} \eta'$ are comparable in size to those for $B^0 \to \overline{D}^{(*)0} \pi^0$, but smaller than those for $B^0 \to \overline{D}^{(*)0} \eta$.

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The hadronic decays $B^0 \to \overline{D}^{(*)0} \eta^{(\prime)}$ are color suppressed modes, which are class II decays in the factorization approach (FA) [1]. The relevant effective weak Hamiltonian for these decays is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)], \quad (1)$$

where the four-quark operators are

$$O_1 = (\overline{d} b)_{V-A} (\overline{c} u)_{V-A}, \quad O_2 = (\overline{c} b)_{V-A} (\overline{d} u)_{V-A}, \quad (2)$$

with the definition $(\overline{q}_1 q_2)_{V-A} \equiv \overline{q}_1 \gamma_\mu (1-\gamma_5) q_2$. The Wilson coefficients $C_1 \sim -0.2$ and $C_2 \sim 1$ are calculated at the m_b scale. The main contribution of these decays in the FA is proportional to the Wilson coefficient $a_2 = C_1 + C_2/3$, which is a small number. That is the reason why class II decays usually have small branching ratios. A theoretical study of $B^0 \to \overline{D}^{(*)0} \eta^{(\prime)}$ decays gives a branching ratio of 10^{-5} [2]. However, recent experiments by the Belle and BABAR Collaborations show that the branching ratios of class II decays are not so small [3,4]. The branching ratios of $B^0 \to \overline{D}^{(*)0} \eta$ are of the order of 10^{-4}. Although the gluonic mechanism can enhance the $B^0 \to \overline{D}^0 \eta'$ decay branching ratio to 10^{-4} [5], it may be difficult to explain the large branching ratio of $B^0 \to \overline{D}^{(*)0} \eta$. This means that the nonfactorizable contributions in these decays are very important. This is confirmed in a recent theoretical study on charmed final state B meson decays in the perturbative QCD approach [6].

The perturbative QCD (PQCD) approach for exclusive hadronic B decays was developed some time ago [7,8] and applied to semileptonic [9] and nonleptonic decays [10–12] successfully. In this formalism, factorizable, nonfactorizable, and annihilation contributions are all calculable. By including the k_T dependence of the wave functions and the Sudakov form factor, this approach is free of the end point singularity. Recent study shows that the PQCD approach works well for charmless B decays [10–12], as well as for channels with one charmed meson in the final states [6,13]. In this Brief, we will show the PQCD calculation of $B^0 \to \overline{D}^{(*)0} \eta^{(\prime)}$ decays and discuss the numerical results.

In two-body hadronic B decays, the two outgoing mesons are energetic. Each of the valence quarks inside these mesons carries large momentum. Most of the energy comes from the heavy b quark decay at the quark level. The light quark (d quark) inside the B^0 meson, which is usually called the spectator quark, carries small momentum of the order of Λ_{QCD}. This quark also goes into the final state meson in spectator diagrams. Therefore, we need an energetic gluon to connect this quark to the four-quark operator involved in the b quark decay, such that the spectator quark gets energy from the four-quark operator to form a fast moving light meson. The hard four-quark dynamic together with the spectator quark becomes a six-quark effective interaction. Since the six-quark interaction is hard dynamics, it is perturbatively calculable. The nonperturbative dynamics in this process is described by the wave functions of mesons consisting of quark and antiquark pairs. The decay amplitude is then expressed as

$$\text{amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr}[C(t) \Phi_B(k_1) \Phi_{D^{(*)}}(k_2) \Phi_{\eta^{(\prime)}} \times (k_3) H(k_1, k_2, k_3, t) e^{-S(t)}]. \quad (3)$$

Here $C(t)$ is the QCD corrected Wilson coefficient of the relevant four-quark operator at scale t. Although next-to-leading order results have been given [14], we will use the leading order here [11]. Φ_i are the meson wave functions, which include the nonperturbative contributions in these decays. The nonperturbative wave functions are not calculable in principle. But they are universal for all the hadronic decays. We will use the ones determined from other measured decay channels [6,10–12]. The exponential $S(t)$ is the so-called Sudakov form factor, which includes the double logarithm resulting from the resummation of the soft and collinear divergence. This form factor is also calculated to next-to-leading order in the literature [15]. The Sudakov factor effectively suppresses the soft contributions in the process [6,10,11]; thus it makes the perturbative calculation of the hard part reliable.

Now the only remaining part of the decay amplitude is the hard part $H(t)$. Since it involves the four-quark operator and the spectator quark connected by a hard gluon, it is channel dependent, but perturbatively calculable. There are altogether eight kinds of diagrams in our $B^0 \to \overline{D}^{(*)0} \eta^{(\prime)}$ decays, which are shown in Fig. 1 for the spectator diagrams and Fig. 2 for

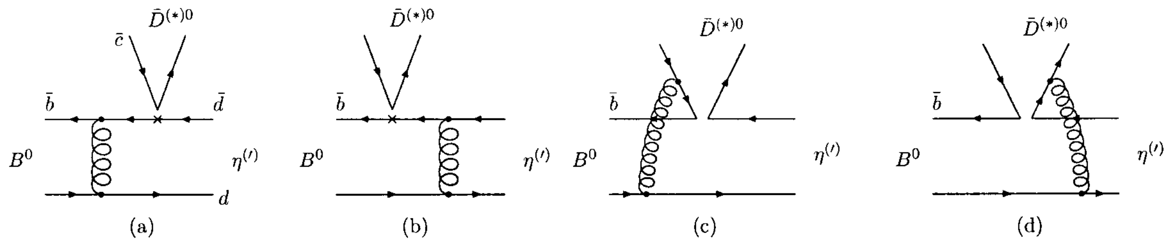

FIG. 1. Color suppressed emission diagrams contributing to the $B^0\to\overline{D}^{(*)0}\eta^{(\prime)}$ decays.

the annihilation type diagrams. Notice that in Fig. 1 the $\eta^{(\prime)}$ meson consists of $d\bar{d}$ content, while in Fig. 2, it is a $u\bar{u}$ pair making the $\eta^{(\prime)}$ meson. Since the $\eta^{(\prime)}$ meson is an isospin singlet $(u\bar{u}+d\bar{d})$, these two sets of diagrams give relatively positive contributions. On the other hand, in the case of $B^0\to\overline{D}^{(*)0}\pi^0$ decays [6], where π^0 is an isospin triplet $(u\bar{u}\text{-}d\bar{d})$, these two sets of diagrams give destructive contributions there. Fortunately, as we will see later, the annihilation type diagrams are suppressed comparing to the spectator diagrams. Therefore, the branching ratios of these two kinds of decays are still comparable.

The structures of the meson wave functions are

$$B_{\rm in}(P):\quad [\not P+m_B]\gamma_5\phi_B(x),\tag{4}$$

$$D_{\rm out}(P):\quad \gamma_5[\not P+m_D]\phi_D(x),\tag{5}$$

$$D^*_{\rm out}(P):\quad [\not P+m_{D^*}]\phi_{D^*}(x),\tag{6}$$

$$\eta^{(\prime)}_{\rm out}(P):\quad \gamma_5[\not P\phi_A(x)+m_0\phi_P(x) + \zeta m_0(\not n_-\not n_+-1)\phi_T(x)],\tag{7}$$

with $m_0\equiv m_\pi^2/(m_u+m_d)=1.4\,\text{GeV}$, utilizing isospin symmetry. The lightlike vectors are defined as $n_+=(1,0,\mathbf{0}_T)$ and $n_-=(0,1,\mathbf{0}_T)$. As shown in Ref. [9], ϕ_B is identified as ϕ_+; the contribution of the other B meson wave function $\bar{\phi}_B\propto\phi_+-\phi_-$ is smaller in the PQCD calculations and therefore we neglect it. For heavy quark symmetry, there is only one applicable independent distribution amplitude $\phi_{D^{(*)}}$ in the heavy $D^{(*)}$ meson wave function [6,13]. However, there are three distribution amplitudes for the light $\eta^{(\prime)}$ meson wave functions [12], like the π meson wave function. The coefficients $\zeta=+1$ are for $\eta^{(\prime)}_{\rm out}$ with $\bar{u}$ $(\bar{d})$ carrying the momentum x_3P_3, while $\zeta=-1$ for $\eta^{(\prime)}_{\rm out}$ with u (d) carrying the momentum x_3P_3.

The gluonic mechanism of η' may make sizable contributions in the $B\to D^{(*)}\eta'$ decays, but they are usually model dependent [5]. Thus we will not consider it here. The η and η' mesons are mixtures of flavor SU(3) octet (η_8) and singlet (η_0) states in a two-mixing-angle formalism [16],

$$\eta=\cos\theta_8|\eta_8\rangle-\sin\theta_0|\eta_0\rangle,$$
$$\eta'=\sin\theta_8|\eta_8\rangle+\cos\theta_0|\eta_0\rangle.\tag{8}$$

The definitions of the decay constants of η and η' are

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u|\eta^{(\prime)}(p)\rangle=if^u_{\eta^{(\prime)}}p_\mu,$$
$$\langle 0|\bar{d}\gamma_\mu\gamma_5 d|\eta^{(\prime)}(p)\rangle=if^d_{\eta^{(\prime)}}p_\mu.\tag{9}$$

The $\bar{s}s$ components of η and η' are not relevant in our decay channels. Therefore we did not show them. The decay constants in the two-angle-mixing formalism are

$$f^u_\eta=f^d_\eta=\frac{f_8}{\sqrt{6}}\cos\theta_8-\frac{f_0}{\sqrt{3}}\sin\theta_0,\tag{10}$$

$$f^u_{\eta'}=f^d_{\eta'}=\frac{f_8}{\sqrt{6}}\sin\theta_8+\frac{f_0}{\sqrt{3}}\cos\theta_0.\tag{11}$$

The parameters are determined to be [16]

$$\theta_8=-22^\circ\ \text{to}\ -21^\circ,\quad f_8=1.28f_\pi,$$
$$\theta_0=-9^\circ\ \text{to}\ -4^\circ,\quad f_0=(1.20\text{–}1.25)f_\pi.\tag{12}$$

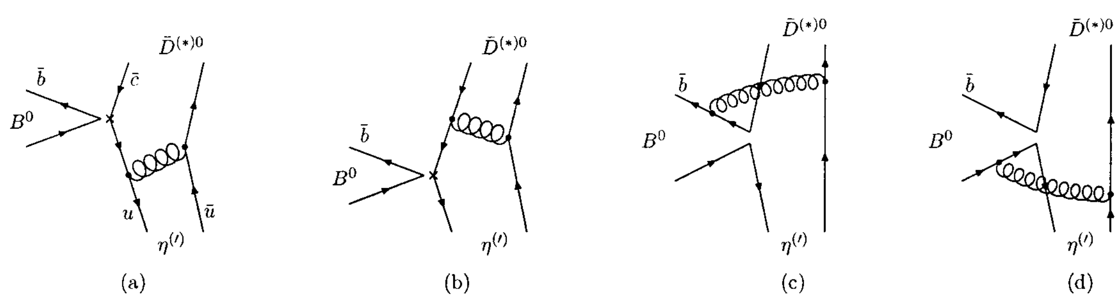

FIG. 2. Annihilation diagrams contributing to the $B^0\to\overline{D}^{(*)0}\eta^{(\prime)}$ decays.

TABLE I. PQCD predictions with one-II (I) and two-angle-mixing formalism (II) and experimental data (in units of 10^{-4}) of the $B^0\to\overline{D}^{(*)0}\eta^{(\prime)}$ branching ratios.

Decay mode	PQCD (I)	PQCD (II)	Belle	BABAR	PDG
$B^0\to\overline{D}^0\eta'$	1.7–2.3	2.2–2.6	—	—	<9.4
$B^0\to\overline{D}^0\eta$	2.4–3.0	2.6–3.2	$1.4^{+0.6}_{-0.5}$	2.41 ± 0.50	
$B^0\to\overline{D}^0\pi^0$	2.3 ± 0.1		3.1 ± 0.6	2.89 ± 0.48	
$B^0\to\overline{D}^{*0}\eta'$	2.0–2.7	2.6–3.2		—	<14
$B^0\to\overline{D}^{*0}\eta$	2.8–3.5	3.1–3.8	$2.0^{+1.0}_{-0.9}$	—	
$B^0\to\overline{D}^{*0}\pi^0$	2.8 ± 0.1		2.7 ± 0.9	—	

The η and η' mesons can also be expressed as a mixing in the quark flavor basis [17]:

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}(u\bar{u}+d\bar{d})/\sqrt{2}\\ s\bar{s}\end{pmatrix}, \qquad (13)$$

where $\alpha=\pi+\theta-\arctan(1/\sqrt{2})$ describes the deviation from ideal mixing. The angle θ is the one-angle-mixing parameter. From Eq. (13), applying isospin symmetry, we have

$$f_\eta^u=f_\eta^d=f_\pi\cos\alpha/\sqrt{2}, \qquad (14)$$

$$f_{\eta'}^u=f_{\eta'}^d=f_\pi\sin\alpha/\sqrt{2}. \qquad (15)$$

The range of mixing parameters is determined to be $\theta=-17°$ to $-11°$ [17].

The $B^0\to\overline{D}^{(*)0}\eta^{(\prime)}$ decay rate has the expression

$$\Gamma=\frac{1}{128\pi}G_F^2|V_{cb}|^2|V_{ud}|^2m_B^3|\mathcal{M}|^2. \qquad (16)$$

Including the hard part and the meson wave functions, the $B^0\to\overline{D}^{(*)0}\eta^{(\prime)}$ decay amplitude is written as

$$\mathcal{M}(B^0-\overline{D}^{(*)0}\eta^{(\prime)})=f_{D^{(*)}}\xi_{\rm int}+f_B\xi_{\rm exc}+\mathcal{M}_{\rm int}+\mathcal{M}_{\rm exc}, \qquad (17)$$

where $f_B=190$ MeV, $f_D=f_{D^*}=240$ MeV are the B and $D^{(*)}$ meson decay constants, respectively. The functions $\xi_{\rm int}$ and $\xi_{\rm exc}$ denote the internal W-emission, and W-exchange contributions, which come from Figs. 1(a) and 1(b), Figs. 2(a) and 2(b), respectively. The functions $\mathcal{M}_{\rm int}$ and $\mathcal{M}_{\rm exc}$ represent the internal W-emission, and W-exchange contributions, which come from Figs. 1(c) and 1(d), Figs. 2(c) and 2(d), respectively. The expressions of the four functions are already shown in the Appendix of Ref. [6] for $B\to D\pi$ decays. One need only replace the pion wave function by the $\eta^{(\prime)}$ wave function in those expressions.

In the FA, only the factorizable contribution of $\xi_{\rm int}$ [Figs. 1(a) and 1(b)] has been considered. Since $\xi_{\rm int}$ is proportional to the small Wilson coefficient $a_2=C_1+C_2/3$, the branching ratio predicted in the FA is smaller than in the experiments. Now in the PQCD approach all the topologies, including both factorizable and nonfactorizable ones, and also annihilation type ones, have been taken into account. In fact the nonfactorizable contribution $M_{\rm int}$, which is proportional to the large Wilson coefficient $C_2/3$, is the dominant contribution in the $B^0\to\overline{D}^{(*)0}\eta^{(\prime)}$ decays. The reason is that the two nonfactorizable diagrams in Figs. 1(c) and 1(d) do not cancel each other as in the decays of B to two light mesons, where the distribution amplitudes of the wave function are symmetric [10,11]. The large difference of the $\bar{c}$ and u quark masses makes the contribution of $M_{\rm int}$ large. Very recently, the soft collinear effective theory also confirmed that the nonfactorizable $M_{\rm int}$ dominates over the contribution of $f_{D^{(*)}}\xi_{\rm int}$ [18].

As stated earlier, we need various wave functions in our numerical calculations. Considering the previous calculations of other decay channels [6,9–12], the B meson wave function has been determined as

$$\phi_B(x,b)=N_Bx^2(1-x)^2\exp\left[-\frac{1}{2}\left(\frac{xM_B}{\omega_B}\right)^2-\frac{\omega_B^2b^2}{2}\right], \qquad (18)$$

where the shape parameter is chosen as $\omega_B=0.4$ GeV. The normalization constant N_B is related to the decay constant f_B through

$$\int dx\,\phi_B(x,0)=\frac{f_B}{2\sqrt{6}}. \qquad (19)$$

The $D^{(*)}$ meson distribution amplitude is given by

$$\phi_{D^{(*)}}(x)=\frac{3}{\sqrt{6}}f_{D^{(*)}}x(1-x)[1+C_{D^{(*)}}(1-2x)], \qquad (20)$$

with the shape parameter $C_D=C_{D^*}=0.8\pm0.2$ [6]. The range of $C_{D^{(*)}}$ was extracted from the $B\to D^{(*)}l\bar{\nu}$ decay spectrum at large recoil assuming $\omega_B=0.4$ GeV for the B meson wave function [9]. We do not consider the variation of $\phi_{D^{(*)}}$ with the impact parameter b, since the current data are not yet sufficient to control this dependence. The light $\eta^{(\prime)}$ meson wave functions are chosen to be the same as the pion wave function according to isospin symmetry, since the relevant valence quarks here are mainly $u\bar{u}$ and $d\bar{d}$.

In the numerical analysis we adopt $\Lambda_{\overline{\rm MS}}^{(f=4)}=250$ MeV, $M_B=5.2792$ GeV, $M_W=80.41$ GeV. Choosing $|V_{cb}|=0.043$ and $|V_{ud}|=0.974$, we obtain the PQCD predictions for the $B^0\to\overline{D}^{(*)0}\eta^{(\prime)}$ branching ratios shown in Table I. For comparison, we also list the $B^0\to\overline{D}^{(*)0}\pi^0$ decay branching ratios in this table. The theoretical uncertainty comes from the variation of the shape parameter for the $D^{(*)0}$ meson distribution amplitude, $0.6 < C_{D(*)} < 1.0$, and the η and η' mixing parameter $\theta = -17°$ to $-11°$ for the one-angle-mixing formalism (I). The range of parameters of the two-angle formalism (II) are shown in Eq. (12). From numerical study, we note that the branching ratios do not vary much upon the variation of $C_{D(*)}$. This can be seen from the numbers for $B^0 \to \overline{D}^{(*)0}\pi^0$ in Table I, since it depends only on this parameter. Most of the uncertainty in the $B^0 \to \overline{D}^{(*)0}\eta^{(\prime)}$ decays is from the mixing parameter θ or θ_8 and θ_0. The branching ratios of $B^0 \to \overline{D}^{(*)0}\eta^{(\prime)}$ increase while those for $B^0 \to \overline{D}^{(*)0}\eta$ decrease as the angle θ gets larger. Other input parameters, such as the parameters of B meson wave function also affect the branching ratios, but they are mostly constrained by other well measured decay channels, like $B \to \pi\pi$ [11] and $B \to K\pi$ [10] decays, etc. The uncertainties of the PQCD approach itself mainly come from the unknown higher twist contributions and higher order calculations of α_s corrections. No numerical estimation of the higher twist contribution exists, although it is expected to be suppressed. A higher order QCD calculation for $B \to \phi K$ decay shows that the next-to-leading order α_s correction may not be small in certain channels [19].

The recently observed class-II decay $B^0 \to \overline{D}^{(*)0}\eta$ branching ratios are also listed in Table I [3,4,20]. It is easy to see that our results agree with the experimental measurements within errors. Since we do not consider the extra gluon fusion contribution to $B^0 \to \overline{D}^{(*)0}\eta'$ decay, the not yet measured $B^0 \to \overline{D}^{(*)0}\eta'$ branching ratios are a little smaller than the $B^0 \to \overline{D}^{(*)0}\eta$ branching ratios. But they are still comparable with the $B^0 \to \overline{D}^{(*)0}\pi^0$ branching ratios. The reason for this comparable result is that we apply the assumption of exact isospin symmetry. We use the same wave function for the $\eta^{(\prime)}$ and π mesons where the only difference is the decay constant. The difference in the dynamics is the constructive or destructive contribution from annihilation type diagrams. This destructive contribution makes the $B^0 \to \overline{D}^{(*)0}\pi^0$ branching ratios smaller than those of $B^0 \to \overline{D}^{(*)0}\eta$ decays. The numerical results also show that the dominant contribution comes from the nonfactorizable contribution $M_{\rm int}$. The factorizable contribution $f_D\xi_{\rm int}$ and annihilation contribution $M_{\rm exc}$ are only 20–30% of $M_{\rm int}$. The factorizable annihilation contribution $f_B\xi_{\rm exc}$ is negligible.

In this work, we calculate the branching ratios of $B^0 \to \overline{D}^{(*)0}\eta^{(\prime)}$ decays in the perturbative QCD approach with k_T factorization, which is free of the end point singularity. Being class-II decays in the FA, these decays receive dominant contributions from the nonfactorizable diagrams. Naive factorization breaks down in these color suppressed modes. The branching ratios calculated in our approach for $B^0 \to \overline{D}^{(*)0}\eta$ agree with current experiments. We predict that the branching ratios of $B^0 \to \overline{D}^{(*)0}\eta'$ without gluconic contributions are of a comparable size with those for $B^0 \to \overline{D}^{(*)0}\pi^0$, but smaller than those for $B^0 \to \overline{D}^{(*)0}\eta$. They may be measured soon in the B factories.

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