$t \to bW$ in the noncommutative standard model

Namit Mahajan*
Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211019, India
(Received 19 May 2003; revised manuscript received 25 August 2003; published 13 November 2003)

We study the top quark decay to a *b* quark and *W* boson in the noncommutative standard model. The lowest contribution to the decay comes from the terms quadratic in the matrix describing the noncommutative (NC) effects while the linear term is seen to identically vanish because of symmetry. The NC effects are found to be significant only for low values of the NC characteristic scale.

DOI: 10.1103/PhysRevD.68.095001 PACS number(s): 14.65.Ha, 11.10.Nx, 12.90.+b

I. INTRODUCTION

The belief behind the existence of the top quark, even before it was discovered [1], had its reasons buried deep in the consistency of the standard model (SM), whether it was due to the requirements of anomaly cancellation or used to explain the precision electroweak data. The experimental evidence for the top quark strengthened the three-family structure of the SM and opened a whole new world of top physics. The top quark is the heaviest of all the fundamental particles known, having a mass 174.3 ± 5.1 GeV, and contributes significantly to radiative processes. Such a large mass leads to a very short lifetime for the top quark ($\tau_{top} \sim 4 \times 10^{-25}$ s). This number is roughly one order of magnitude smaller than the typical QCD hadronization time scale (30×10^{-25} s). The top quark therefore decays before it can hadronize, unlike the other quarks. This feature offers the possibility of looking at a quark and its properties almost free of QCD confinement. This makes the top quark a wonderful laboratory for QCD studies. To complete the picture consistently with the SM, various properties of the top quark need to be confirmed. For a review of the top quark and related issues, see [2] and further references therein.

Within the SM, the top quark decays almost completely into a *b* quark and a *W* boson with the branching ratio $BR(t \to bW) \sim 0.998$. Like the other particles, the charged interactions of the top quark in the SM are of $V-A$ nature and the strengths for the top quark going into a *W* boson and a lighter quark are directly proportional to the Cabibbo-Kobayashi-Maskawa (CKM) mixing angles. The top quark, in contrast to any other particle, say a *b* quark, decays into a lighter quark (mostly a *b* quark) and an on-shell *W*. This makes the helicity of the emitted *W* boson an important tool for determining many properties and testing the universal nature of the $V-A$ interactions. In the limit of a massless *b* quark, the *W* cannot be right handed (or at least such a helicity state will be highly suppressed). The *W* helicity can be measured from the angular correlations of its decay products and thus offers a window to look for new physics beyond the SM as well. Due to the smallness of other CKM elements, other top quark decays fall into the category of rare decays and the branching ratios are very small. Top quark decays beyond the SM have been extensively studied. For example, the top quark decay to a charged Higgs boson and the experimental signatures has been discussed in detail in [3], while the neutral current decay into a charm and two vector particles has been considered in [4]. In [5] the decay into a top squark and a neutralino has been studied while the authors of [6] consider the technicolor models and discuss related issues.

The top quark properties play an important role in the electroweak physics. The large mass of the top quark makes its role in the electroweak precision data fits much more pronounced than for any other quark. Also the spin configuration of the top quark in any process is very sensitive to new physics beyond the SM, particularly any anomalous coupling other than allowed by the SM. The anomalous top quark couplings have been a very important and interesting area of activity. These couplings can be probed directly at colliders and indirectly via the rare decays of mesons [7,8]. It is expected that about 10^8 top quark pairs per year at the LHC will make a detailed study of the top quark couplings possible and with great accuracy. Since the dominant decay channel for the top quark is decay into a *b* quark and a *W* boson, it is not wrong to expect that the *Wtb* coupling will be measured with high precision. This coupling is proportional to the CKM element V_{tb} and enters the expressions for the flavor changing neutral current *B* meson decays [9]. Therefore, any anomalous contribution to the coupling should show up in the *B* decays and should be measurable even before LHC begins. Using the CLEO and LEP data the top couplings have been constrained [10] and future experiments are expected to improve these constraints.

In the present note we study the dominant decay mode of the top quark, $t \to bW$, in the context of the noncommutative standard model (NCSM). The simple picture of space-time that we have in our minds is based on the notion of space-time being described by a suitable manifold with the points on that manifold being labeled by a countable number of real coordinates. For most practical purposes the space-time acts as a static background on top of which the processes occur. However, it is believed that this naive picture must undergo a drastic change when one probes very small distances. At those energy scales the classical notions of space-time cease to be the correct description of the world and modifications have to be incorporated to yield correct results. There is no clear-cut and unique solution to this puzzle and the kind of modifications required, but a possible way to approach this

*Email address: nmahajan@mri.ernet.in

problem is to formulate physical theories on noncommutative (NC) space-times. This idea dates back to the work of Snyder [11] where it was shown that the usual continuum space-time is not the only solution to the assumption that the spectrum of the coordinates describing the space-time is invariant under Lorentz transformations and there exists a Lorentz invariant space-time with an inherent unit of length. In such cases, the notion of a point is not well defined, and the usual commutation relations between the coordinates and momenta get an extra piece proportional to the momentum value. Therefore, for small enough momenta and energies, they just approach the usual quantum mechanical relations. A very strong motivation to formulate quantum theory on a noncommutative space-time was to render the quantum field theory calculations finite and free of infinities. But the success of the renormalization theory abandoned this approach altogether. The idea has been remotivated because of some string theory results [12]. Apart from being boosted by string theory arguments, field theories formulated on noncommutative space-times have very interesting features of their own. For a review of noncommutative field theories (NCFTs), see [13].

In order to describe a noncommutative space-time, the usual commutation property between the coordinates is abandoned and replaced with the following commutator for the Hermitian operators $\hat{x}^\mu$ [14]:

$$[\hat{x}^\mu,\hat{x}^\nu]=i\Theta^{\mu\nu}, \tag{1}$$

where $\Theta^{\mu\nu}$ is a constant, real, and antisymmetric matrix and describes the noncommutativity. This constant matrix can also be thought of as some background field relative to which the various space-time directions are distinguished. *A priori* there is no reason to assume that $\Theta^{\mu\nu}$ is a constant matrix but we consider it to be so in our study. Also, most of the studies involving field theories on such spaces have been restricted to constant $\Theta^{\mu\nu}$. The theories on such a space-time clearly violate Lorentz invariance explicitly. Following the Weyl-Moyal correspondence [15], the ordinary product of two functions is replaced by the Moyal product (sometimes also called the star product and denoted by *), which takes the form

$$f(x)*g(x)=\left[\exp\left(\frac{i}{2}\Theta^{\mu\nu}\partial_{\eta\mu}\partial_{\zeta\nu}\right)f(x+\eta)g(x+\zeta)\right]_{\eta=\zeta=0} \tag{2}$$

$$=\sum_{n=0}^{\infty}\left(\frac{i}{2}\right)^n\frac{1}{n!}[\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_n}f(x)]$$
$$\times\Theta^{\mu_1\nu_1}\Theta^{\mu_2\nu_2}\ldots\Theta^{\mu_n\nu_n}[\partial_{\nu_1}\partial_{\nu_2}\cdots\partial_{\nu_n}g(x)].$$

Thus the recipe for formulating the noncommutative version of the field theories is to replace all the ordinary products by Moyal products. As can be seen, such theories are highly nonlocal and bring with them several new features like UV/IR mixing [16] and unitarity problems [17]. However, in spite of these there has been considerable activity in this field [18]. The noncommutative version of quantum electrodynamics has been examined in [19]. A method for formulating the non-Abelian noncommutative field theories has been discussed in [20], and using these ideas a noncommutative version of the standard model has been proposed [21]. In the noncommutative version, there are several new features and interactions, like triple gauge boson vertices, that appear, and some of the related phenomenological aspects of these have been investigated [22].

II. NONCOMMUTATIVE CORRECTIONS TO $t\to bW$

We take as our starting point the action given in [21] for the quark sector. We assume here that the fields have been redefined in terms of the physical fields and as far as this work is concerned, we concentrate only on the charged current interactions. The action with the ordinary products replaced by Moyal products looks like

$$S_{charged\ current}=\frac{g}{2\sqrt{2}}\int d^4x\sum_{families}[\overline{\nu}*W\!\!\!/*(1-\gamma_5)e$$
$$+V_{ud}\overline{u}*W\!\!\!/*(1-\gamma_5)d], \tag{3}$$

where we generically denote the leptons as e and their corresponding neutrinos as ν and for the quarks, we use u and d to denote the up and the down type quarks. W is the W boson field and V_{ud} are the relevant CKM elements. The fields are expanded in powers of Θ, and here we do not concern ourselves about the terms that have more than one gauge field coupling to fermions. The fermion fields when expanded to $\mathcal{O}(\Theta^2)$ have the following structure:

$$\psi=\left(1-\frac{1}{2}g\Theta^{\mu\nu}A_\mu\partial_\nu-\frac{i}{8}g\Theta^{\mu\nu}\Theta^{\rho\sigma}\partial_\mu A_\rho\partial_\nu\partial_\sigma\right)\psi_0$$
$$+\mathcal{O}(\Theta^3), \tag{4}$$

where ψ_0 refers to the usual fermionic field, A is the associated gauge field, and g is the gauge coupling. Plugging this expression into the action, we get the matrix element for the process $t(p_t)\to b(p_b)+W(p_W)$

$$\mathcal{M}=\left(\frac{ig}{2\sqrt{2}}V_{tb}\right)\overline{b}(p_b)\bigg[\gamma^\beta-\frac{1}{2}(\Theta^{\mu\beta}p\!\!\!/_t p_{W\mu}+\Theta^{\beta\alpha}p\!\!\!/_W p_{t\alpha}$$
$$-\Theta^{\mu\alpha}\gamma^\beta p_{W\mu}p_{t\alpha})+\frac{i}{8}(\Theta^{\mu\beta}\gamma^\alpha+\Theta^{\beta\alpha}\gamma^\mu$$
$$-\Theta^{\mu\alpha}\gamma^\beta)\Theta_{\rho\sigma}p_{W\alpha}p_{t\mu}p_W^\rho p_t^\sigma\bigg](1-\gamma_5)t(p_t)\epsilon_\beta^*(p_W). \tag{5}$$

The charged current contribution due to noncommutativity in our case is the same as that obtained by Iltan [22] in the context of W decay into a lepton and antineutrino modulo an overall sign factor arising due to a difference in the convention for momentum flow. The noncommutative correction to the matrix element explicitly reads (after some algebra)

$$\mathcal{M}_{NC}=\left(\frac{ig}{2\sqrt{2}}V_{tb}\right)\epsilon_\beta^*\bigg[\overline{b}(p_b)\gamma^\beta(1-\gamma_5)t(p_t)\left(\frac{1}{2}\Theta^{\mu\alpha}p_{W\mu}p_{t\alpha}+\frac{i}{8}\Theta^{\rho\sigma}p_{W\rho}p_{t\sigma}\right)-\frac{m_t}{2}\overline{b}(p_b)(1+\gamma_5)t(p_t)\Theta^{\beta\lambda}p_{b\lambda}+\frac{m_b}{2}\overline{b}(p_b)(1-\gamma_5)t(p_t)\Theta^{\beta\delta}p_{t\delta}\bigg]. \qquad (6)$$

Using the expression for the matrix element above, the decay rate can easily be evaluated. The decay rate, including the noncommutative effects, can be expressed in the following form:

$$\Gamma_{total}=\frac{|V_{tb}|^2}{16\pi p_t^0}\left(\frac{g}{2\sqrt{2}m_W}\right)^2\lambda^{1/2}\left(1,\frac{m_W^2}{m_t^2},\frac{m_b^2}{m_t^2}\right)[\mathcal{C}_{SM}+\mathcal{C}_{NC_1}+\mathcal{C}_{NC_2}], \qquad (7)$$

where

$$\mathcal{C}_{SM}=2[m_W^2(m_t^2+m_b^2-m_W^2)+(m_t^2-m_b^2-m_W^2)(m_t^2-m_b^2+m_W^2)], \qquad (8)$$

$$\mathcal{C}_{NC_1}=\Theta_\beta^{\alpha}\Theta^{\beta\sigma}p_{t\alpha}p_{t\sigma}\left(\frac{1}{12m_t^2}\right)[2m_b^8+2m_t^8-3m_t^6m_W^2+3m_t^4m_W^4+m_t^2m_W^6-3m_W^8-5m_b^6(m_t^2-m_W^2)+m_b^4(6m_t^4-7m_t^2m_W^2-19m_W^4)+m_b^2(-5m_t^6+5m_t^4m_W^2+3m_t^2m_W^4+15m_W^6)], \qquad (9)$$

$$\mathcal{C}_{NC_2}=-\Theta_{\alpha\beta}\Theta^{\beta\alpha}\left(\frac{m_W^2}{24}\right)\{(5m_b^2+m_t^2-m_W^2)[m_b^4+(m_t^2-m_W^2)^2-2m_b^2(m_t^2+m_W^2)]\}. \qquad (10)$$

It is worthwhile to point out that the term linear in Θ that would arise due to the interference between the SM and the noncommutative contributions does not show up in the decay rate. This is not hard to see because the phase space integrals would yield terms proportional to the metric tensor or the top quark momentum and such terms would identically vanish because of the antisymmetry of $\Theta^{\mu\nu}$.[1] Thus, the decay rate simply splits into pure SM and NC contributions. It is well known that QCD corrections lower the tree level decay rate by roughly 10%. From the above expressions we see that the contribution $\mathcal{C}_{\mathrm{NC}_2}$ has a negative sign, and it is interesting to see the effects of noncommutativity on the decay rate. We consider three cases: (a) when $\Theta^{0i}=0$, (b) when $\Theta^{ij}=0$, and (c) both Θ^{0i} and Θ^{ij} nonzero. Noncommutativity in the time direction is expected to lead to nonunitary behavior of the theory, but it has been pointed out by Liao and Sibold [18] that careful handling of time ordered products and the time derivatives arising due to the Moyal/star product avoids any such problem. We thus retain the piece with noncommutativity in the time direction as well. We define the following:

$$\Theta^{0i}=\theta_{ts}^i, \qquad \theta_{ss}^i=\frac{1}{2}\epsilon^{ijk}\Theta^{jk}. \qquad (11)$$

Also for the sake of simplicity we assume that $|\theta_{ts}^i|=|\theta_{ss}^i|$ and denote it by Λ_{NC}^{-2}, where Λ_{NC} (in units of GeV) is the characteristic scale of noncommutative interactions.

[1]This is typical of a particle decaying and is not expected in a scattering process where the linear term will in general be present and give the leading contribution.

In Fig. 1, we plot the decay rate as a function of Λ_{NC} for all three cases. Only for the case when the noncommutativity in the time direction is put to zero do we get a lower rate when the noncommutative effects are included. But this effect is seen only for low values of Λ_{NC} and as it is increased the rates in all three cases approach the SM value. The ratio of the decay rate for a longitudinally polarized W (with polarization vector $\epsilon_L^\mu\sim p_W^\mu/m_W$) to the unpolarized decay rate has been measured by the CDF Collaboration [23] and it comes out to be

$$F_0=\frac{\Gamma(t\to bW_L)}{\Gamma^{total}(t\to bW)}=0.91\pm0.37\pm0.13 \qquad (12)$$

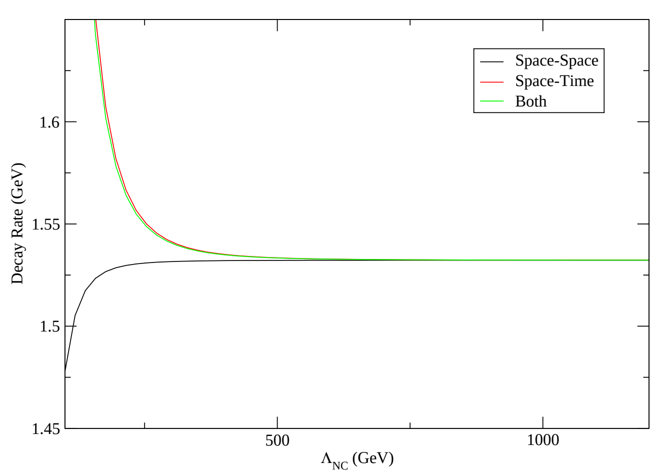

FIG. 1. The decay rate (in GeV) as a function of the noncommutativity scale (in GeV) for space-space ($\Theta^{0i}=0$), space-time ($\Theta^{ij}=0$), and both nonzero.

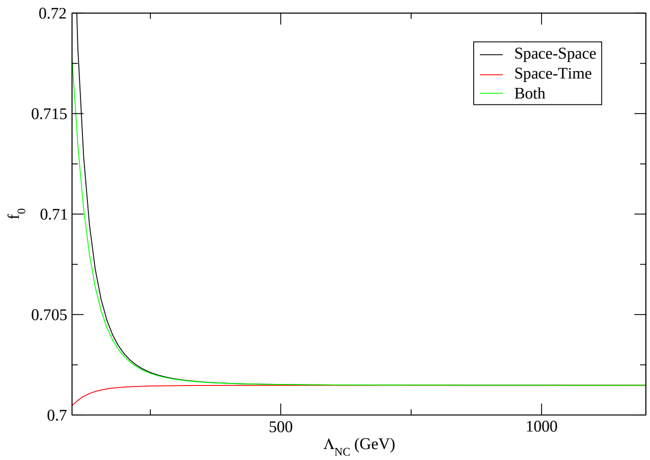

FIG. 2. The quantity F_0 as a function of the noncommutativity scale (in GeV) for space-space ($\Theta^{0i}=0$), space-time ($\Theta^{ij}=0$), and both nonzero.

while the SM value is $F_0^{SM}=0.701$. It is straightforward algebra to get the noncommutative correction to the polarized rate:

$$\Gamma_{NC}^{long}=\frac{|V_{tb}|^2}{16\pi p_t^0}\left(\frac{g}{2\sqrt{2}m_W}\right)^2\lambda^{1/2}\left(1,\frac{m_W^2}{m_t^2},\frac{m_b^2}{m_t^2}\right)\Delta_{long}, \tag{13}$$

where

$$\Delta_{long}=\frac{1}{12}[m_b^4+(m_t^2-m_W^2)^2-2m_b^2(m_t^2+m_W^2)] \times[2m_b^4+2m_t^2(m_t^2-m_W^2)-m_b^2(m_t^2+2m_W^2)]|\Theta^{0i}|^2. \tag{14}$$

One finds that the polarized rate contains no terms proportional to Θ^{ij}, i.e., the polarized rate depends only on Θ^{0i}. Therefore in the absence of space-time noncommutativity, there are no noncommutative corrections to the longitudinal polarized rate and the sole contribution comes from the SM. Thus the quantity F_0 stands a good chance of determining the kind of noncommutativity that is present in the theory. In Fig. 2 we plot F_0 against Λ_{NC} and find that, for lower values of the noncommutativity scale, there is a significant deviation from the SM result for the case of zero timelike noncommutativity or when both are present. But as before, with the increase in Λ_{NC}, all the curves approach the SM value. We can try to get some lower bound on the noncommutative scale by imposing the condition Eq. (6). If that is done, we get a bound $\Lambda_{NC}\geqslant\mathcal{O}(100\text{ GeV})$ for space-space noncommutativity while no physically meaningful values are obtained in either of the other two cases. In fact, one gets imaginary values for the noncommutativity scale. This may be an indication that a more careful and thorough treatment is required at the level of the action itself, and it is hoped that such a treatment should bypass any such troubles.

The transverse plus rate Γ_+ is also of considerable interest because simple helicity arguments lead to the conclusion that at the tree level this rate vanishes for vanishing m_b. Therefore, a nonvanishing Γ_+ at the Born amplitude level can arise from $m_b\neq 0$ effects or can arise because of higher order corrections beyond the Born amplitude. Also, such an effect can be an artifact of departure from the $V-A$ current structure. From the matrix element Eq. (12) it is clear that there is a departure from the $V-A$ structure of the weak current, and this fact is expected to show up in corrections to Γ_+. We evaluate the noncommutative corrections to Γ_+. The total rate is

$$\Gamma_+=\Gamma_+^{SM}+\Gamma_+^{NC} \tag{15}$$

where the SM expression is well known, and the second term is

$$\Gamma_+^{NC}=\frac{|V_{tb}|^2}{16\pi p_t^0}\left(\frac{g}{2\sqrt{2}}\right)^2\lambda^{1/2}\left(1,\frac{m_W^2}{m_t^2},\frac{m_b^2}{m_t^2}\right)\Delta_+, \tag{16}$$

where the quantity $\Delta+$ is

$$\Delta_+=\frac{m_t^4}{24}(m_t^2+m_b^2-m_W^2)\sum_{i=1}^{3}\Theta^{0i}\Theta_{i0}-\frac{m_t^4}{48}(m_t^2+m_b^2-m_W^2)\times\sum_{a=1}^{2}\Theta^{a\lambda}\Theta_\lambda^a+\frac{m_b^2m_t^2}{4}(m_t^2+m_b^2-m_W^2)\sum_{a=1}^{2}|\Theta^{a0}|^2+\frac{m_b^2m_t^4}{2}\sum_{a=1}^{2}|\Theta^{a0}|^2. \tag{17}$$

From this expression it is clear that even in the limit of vanishing m_b the Born level noncommutative corrections to the SM transverse plus rate are nonzero. Quite clearly, this stands out as another test of possible existence of any noncommutativity. Also note that this rate picks out terms having noncommutativity in selected space directions as well (the terms with index a). This fact can thus be used to distinguish between any inhomogeneity in the noncommutativity parameter. However, for the present analysis, we have assumed equal and constant values along all the directions, but in principle a more general analysis can be carried out using these expressions. The CDF measurement for the quantity Γ_+/Γ is [23]

$$\frac{\Gamma_+}{\Gamma}=0.11\pm 0.15.$$

For the sake of illustration we consider the case of only space-space noncommutativity. Imposing the experimental results, we get a central value for the noncommutativity scale of $\sim$50 GeV. It is considered safe to say that, if the top quark decay reveals a deviation from the SM predictions at the 1% level, then the origin of this deviation can be attributed to some non-SM physics. For the noncommutativity scale $\mathcal{O}(100)$ GeV, the estimated value for Γ_+/Γ is approximately (slightly less than) 0.01, which satisfies the 1% criterion and therefore should be probed in future measurements. However, for higher values of the scale, the noncommutative corrections get smaller and smaller.

Consider the radiative decay of the top quark, namely, $t\rightarrow bW\gamma$. The branching ratio for this decay channel (for a photon of energy 10 GeV) is 3.5×10^{-3}. The noncommutative corrections to this process come in the form of modification of the SM vertices plus a completely new vertex where the radiated photon is attached to the *tbW* vertex. For the sake of illustration we consider this piece of the total amplitude. The relevant interaction Lagrangian can be written in the following form:

$$\mathcal{L}_{tbWA}=\frac{eg}{2\sqrt{2}}\bar{b}(p_b)\Bigg[\left(\frac{1}{2}\Theta^{\mu\nu}\gamma^{\alpha}+\Theta^{\nu\alpha}\gamma^{\mu}\right)\Bigg(-\frac{3}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)W_\alpha-(A_\mu W_\nu-A_\nu W_\mu)\partial_\alpha+\frac{1}{6}(\partial_\mu W_\nu-\partial_\nu W_\mu)A_\alpha\Bigg)\Bigg](1-\gamma_5)t(p_t). \quad (18)$$

The contribution of this piece alone for the bounds obtained above fits the experimental values and, as expected, for higher scales the noncommutative contribution diminishes and the SM piece surfaces as the sole contributor.

The bound obtained is a very low bound and should be directly verifiable at the collider experiments. This value is of the same order (141 GeV) as was obtained for noncommutative QED (NCQED) for the process $e^+e^-\rightarrow\gamma\gamma$ at LEP by the OPAL Collaboration [24]. There is no reason that the NCQED and NCSM bounds should be very similar but one can expect them to be roughly of the same order. However, it is known that the collider bounds that one gets in the case of NCQED are much lower than the bounds obtained from Lamb shift measurements (Chaichian *et al.* [19]). Also, the sidereal variation effects on atomic clocks give very strong limits [25]. Thus, we expect that more data and bounds from other sources should supplement such preliminary results and give a better picture. Some more hints can be obtained by looking at the charged lepton (arising from the *W* decay) energy and the angular spectrum. This sector contains information about the top polarization as well, which implies more independent data to confirm the theoretical results. However, in this case it becomes even more complicated because even the *W* decay element will pick up additional noncommutative corrections and the whole picture gets very messy. Also, if one wants to use the results obtained directly in a complete analysis, i.e., consider top quark production at a collider and then its decay, etc., then these results have to be suitably convoluted with the SM expressions for other subprocesses to have a rough estimate of the noncommutative corrections. A complete treatment would require a detailed analysis of noncommutative corrections for each subprocess and a careful convolution to extract the leading terms. Therefore, presently it appears that there is a very narrow window to probe noncommutative effects by directly looking at the decay of the top quark, as the SM contribution conceals the extra effect completely for almost the whole region of the parameter space.

III. CONCLUSIONS

In conclusion, noncommutative effects seem to be completely hidden under the shadow of the SM results for most of the parameter space, and it is only in a very small range, for very low values of the noncommutativity scale, that there are any significant deviations from the SM values, for both the unpolarized rate and F_0 and the transverse plus rate. The suppression due to the noncommutativity scale is rather large (Λ^{-4}) to be easily overcome in this case. Thus it may seem that there is not much hope of determining possible noncommutative effects directly from the top quark decay. However, there is still hope, as angular asymmetries and correlations between the *W* decay products, say a lepton and an antineutrino, may still be able to reveal even the feeble presence of any terms not dictated by the SM. Therefore, a detailed analysis of the top quark production process followed by cascade decay studies can possibly probe such effects. Moreover, the *tbW* vertex now contains terms that essentially give rise to right handed weak currents, and it should be possible to detect the presence of such terms either in hadronic scattering reactions (possibly at the LHC) or indirectly through their contribution to rare meson decays, where we expect even the terms linear in Θ to contribute and give the dominant effect. Also, from a theoretical point of view, more careful treatment at the level of the action is required so as to avoid possible problems like encountering physically meaningless bounds.

[1] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **74**, 2626 (1995).

[2] S. Willenbrock, Rev. Mod. Phys. **72**, 1141 (2000); D. Chakraborty, J. Konigsberg, and D. Rainwater, hep-ph/0303092; S. Dawson, hep-ph/0303191.

[3] J. F. Gunion, H. E. Haber, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990).

[4] J.L. Diaz-Cruz, M.A. Perez, G. Tavares-Velasco, and J.J. Toscano, Phys. Rev. D **60**, 115014 (1999).

[5] G. Mahlon and G.L. Kane, Phys. Rev. D **55**, 2779 (1997); S. Mrenna and C.P. Yuan, Phys. Lett. B **367**, 188 (1996); J.D. Wells and G.L. Kane, Phys. Rev. Lett. **76**, 869 (1996).

[6] C.T. Hill, Phys. Lett. B **345**, 483 (1995); B. Balaji, *ibid.* **393**, 89 (1997).

[7] D. Espriu and J. Manzano, Phys. Rev. D **66**, 114009 (2002); F. del Aguila and J.A. Aguilar-Saaverda, *ibid.* **67**, 014009 (2003); T. Han, Y.J. Kim, A. Likhoded, and G. Valencia, Nucl. Phys. **B593**, 415 (2001); S. Atag, O. Cakir, and B. Dilec, Phys. Lett. B **522**, 76 (2001); J.J. Cao, J.X. Wang, J.M. Yang, B.L. Young, and X.M. Zhang, Phys. Rev. D **58**, 094004 (1998); J.L. Hewett, Int. J. Mod. Phys. A **13**, 2389 (1998); E. Boos, M. Dubinin, M. Sachwitz, and H.J. Schreiber, Eur. Phys. J. C **16**, 269 (2000); K. Hikasa *et al.*, Phys. Rev. D **58**, 114003 (1998); G.L. Kane, G.A. Ladinsky, and C.P. Yuan, *ibid.* **45**, 124

(1992); S. Dawson, *ibid.* **53**, 1721 (1996); S. Rindani, Pramana **54**, 791 (2000); P. Poulose and S. Rindani, Phys. Lett. B **383**, 212 (1996); Phys. Rev. D **54**, 4326 (1996); R.D. Peccei and X. Zhang, Nucl. Phys. **B337**, 269 (1990); A. Datta and X. Zhang, Phys. Rev. D **55**, 2530 (1997).

[8] K.Y. Lee and W.Y. Song, Phys. Rev. D **66**, 057901 (2002); A. Abd El-Hady and G. Valencia, Phys. Lett. B **414**, 173 (1997); K. Fujikawa and Y. Yamada, Phys. Rev. D **49**, 5890 (1994); J.L. Hewett and T. Rizzo, *ibid.* **49**, 319 (1994); R.D. Peccei, S. Peris, and X. Zhang, Nucl. Phys. **B349**, 305 (1991).

[9] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[10] F. Larios, M.A. Perez, and C.P. Yuan, Phys. Lett. B **457**, 334 (1999).

[11] H.S. Snyder, Phys. Rev. **71**, 38 (1947).

[12] M.R. Douglas and C. Hull, J. High Energy Phys. **02**, 008 (1998); N. Seiberg and E. Witten, *ibid.* **09**, 032 (1998).

[13] M.R. Douglas and N.A. Nekrasov, Rev. Mod. Phys. **73**, 977 (2001).

[14] T. Filk, Phys. Lett. B **376**, 53 (1996).

[15] L. Alvarez-Gaume and S.R. Wadia, Phys. Lett. B **501**, 319 (2001).

[16] S. Minwalla, M.V. Raamsdonk, and N. Seiberg, J. High Energy Phys. **02**, 020 (2000).

[17] J. Gomis and T. Mehen, Nucl. Phys. **B591**, 265 (2000).

[18] M. Chachian, A. Demichev, and P. Presnajder, Nucl. Phys. **B567**, 360 (2000); M.M. Sheikh-Jabbari, J. High Energy Phys. **06**, 015 (1999); C.P. Martin and D. Sanchez-Ruiz, Phys. Rev. Lett. **83**, 476 (1999); T. Krajewski and R. Wulkenhaar, Int. J. Mod. Phys. A **15**, 1011 (2000); A. Armoni, Nucl. Phys. **B593**, 229 (2001); O. Aharony, J. Gomis, and T. Mehen, J. High Energy Phys. **09**, 023 (2000); Y. Liao and K. Sibold, Eur. Phys. J. C **25**, 469 (2002); D. Bahns, S. Doplicher, K. Fredenhagen, and G. Piacitelli, Phys. Lett. B **533**, 178 (2002); M.M. Sheikh-Jabbari, Phys. Rev. Lett. **84**, 5265 (2000); A. Micu and M.M. Sheikh-Jabbari, J. High Energy Phys. **01**, 025 (2001); F.J. Petriello, Nucl. Phys. **B601**, 169 (2001); M. Chaichian, K. Nishijima, and A. Tureanu, Phys. Lett. B **565**, 146 (2003); Y. Liao and K. Sibold, *ibid.* **549**, 352 (2002).

[19] M. Hayakawa, hep-th/9912167; I.F. Riad and M.M. Sheikh-Jabbari, J. High Energy Phys. **08**, 045 (2000); N. Chair and M.M. Sheikh-Jabbari, Phys. Lett. B **504**, 141 (2001); M. Chaichian, M.M. Sheikh-Jabbari, and A. Tureanu, Phys. Rev. Lett. **86**, 2716 (2001); H. Arfaei and M.H. Yavartanoo, hep-th/0010244; N.t. Binh, hep-th/0301084; F.T. Brandt and A. Das, Phys. Rev. D **65**, 085017 (2002); X.J. Wang and M.L. Yan, J. High Energy Phys. **03**, 047 (2002); K.B. Eom, S.S. Kang, B.H. Lee, and C. Park, hep-th/0205093; T. Mariz, C.A. de S. Pires, and R.F. Ribeiro, hep-ph/0211416; J.L. Hewett, F.J. Petriello, and T.G. Rizzo, Phys. Rev. D **64**, 075012 (2001); S. Baek, D.K. Ghosh, X.G. He, and W.Y. Hwang, *ibid.* **64**, 056001 (2001); N. Mahajan, hep-ph/0110148; J.L. Hewett, F.J. Petriello, and T.G. Rizzo, Phys. Rev. D **66**, 036001 (2002); C.E. Carlson, C.D. Carone, and R.F. Lebed, Phys. Lett. B **518**, 201 (2001); **549**, 337 (2002).

[20] J. Madore, S. Schraml, P. Schupp, and J. Wess, Eur. Phys. J. C **16**, 161 (2000); B. Jurco, S. Schraml, P. Schupp, and J. Wess, *ibid.* **17**, 521 (2000); B. Jurco, P. Schupp, and J. Wess, Nucl. Phys. **B604**, 148 (2001); B. Jurco, L. Moller, S. Schraml, P. Schupp, and J. Wess, Eur. Phys. J. C **21**, 383 (2001).

[21] X. Calmet, B. Jurco, P. Schupp, J. Wess, and M. Wohlgenannt, Eur. Phys. J. C **23**, 363 (2002).

[22] X.G. He, Eur. Phys. J. C **28**, 557 (2003); W. Behr, N.G. Deshpande, G. Duplancic, P. Schupp, J. Trampetic, and J. Wess, *ibid.* **29**, 441 (2003); E. Iltan, J. High Energy Phys. **11**, 029 (2002); New J. Phys. **4**, 54 (2002); Phys. Rev. D **66**, 034011 (2002); Z. Chang and Z. Xing, *ibid.* **66**, 056009 (2002).

[23] T. Affolder *et al.*, Phys. Rev. Lett. **84**, 216 (2000).

[24] OPAL Collaboration, G. Abbiendi *et al.*, Phys. Lett. B **568**, 181 (2003).

[25] I. Mocioiu, M. Pospelov, and R. Roiban, Phys. Lett. B **489**, 390 (2000).