Analytic estimates for penguin operators in quenched QCD

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Strong penguin operators are singlets under the right-handed flavor symmetry group $SU(3)_R$. However, they do not remain singlets when the operator is embedded in (partially) quenched QCD, but instead they become linear combinations of two operators with different transformation properties under the (partially) quenched symmetry group. This is an artifact of the quenched approximation. Each of these two operators is represented by a different set of low-energy constants in the chiral effective theory. In this paper, we give analytic estimates for the leading low-energy constants, in quenched and partially quenched QCD. We conclude that the effects of quenching on Q_6 are large.

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Even at present, the number of light dynamical (or sea) quarks in lattice QCD computations is often not equal to that of the real world. In particular, in quite a few of the more difficult computations, no sea quarks have been taken into account at all—in other words, they have been done in the quenched approximation.

The adaptation of chiral perturbation theory (ChPT) to the quenched theory has been useful for gaining insight into the effects of quenching. However, ChPT itself does not give any insight into the values of the parameters of the effective theory, the so-called low-energy constants (LECs). It follows that ChPT also does not tell us anything about the effects of quenching on the values of the LECs. Within lattice QCD, therefore, we do in principle not know anything systematic about the errors due to quenching, until computations are done with dynamical quarks, and the results are compared with those of quenched QCD.

At the same time, analytic approaches are being explored to obtain estimates for, in particular, electroweak-interaction LECs. Some of these approaches make sophisticated use of analytic knowledge available about QCD, such as its chiral behavior, the operator-product expansion, and large-N_c techniques. For the purpose of this paper, a relevant reference is Ref. [1], to which we also refer for references to other work. While some assumptions have to be made in such approaches because a nonperturbative analytic solution to QCD is not available, they are often so tightly constrained that it is quite likely that the results obtained from them will help us with our quantitative understanding of hadron phenomenology. Moreover, they have the advantage of exhibiting the underlying reasons for the size of specific contributions.

It is therefore natural to adapt these analytic techniques to include the effects of (partial) quenching. This is useful, because it gives us *quantitative* information about the effects of quenching. Results from this approach can thus serve as a guide to the merits and pitfalls of the quenched approximation.

In this paper, we apply the large-N_c expansion to the calculation of the leading LECs associated with the strong penguin operator Q_6 [2]. It was recently pointed out that even the definition of the quenched version of this operator is ambiguous [3], and that as a consequence, the quenched theory has more LECs associated with this operator than the unquenched theory. It turns out that the leading-order LECs can be estimated analytically if one ignores order $1/N_c^2$ corrections. These estimates tell us that the effects of quenching on the contribution of Q_6 to nonleptonic kaon decays are likely to be large.

The operator Q_6 is defined as

$$Q_6=4(\overline{s_L^\alpha}\gamma_\mu d_L^\beta)\sum_{q=u,d,s}(\overline{q_R^\beta}\gamma_\mu q_R^\alpha), \tag{1}$$

where $q_{R,L}=P_{R,L}q$ with $P_{R,L}=\frac{1}{2}(1\pm\gamma_5)$, and α and β are color indices. This operator transforms in the (8,1) representation of $SU(3)_L\times SU(3)_R$.

In order to "embed" this operator in the quenched theory, let us briefly recall how one may define quenched QCD as a field theory [4,5]. For each quark q one introduces a ghost quark $\tilde{q}$ with the same mass, spin, and color, but opposite statistics. The opposite statistics cause the path integral over the ghost quarks to cancel the quark determinant (for each gauge field configuration), effectively replacing the quark determinant by one. This is precisely the definition of the quenched approximation. It follows that the flavor symmetry of the quenched theory is not described by SU(3), but by the larger, graded group $SU(3|3)$. Q_6 is not a singlet under $SU(3|3)_R$, but instead can be decomposed as [3]

$$Q_6=\frac{1}{2}Q_6^{QS}+Q_6^{QNS}, \tag{2}$$

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$$Q_6^{QS}=4(\overline{s}_L^{\alpha}\gamma_\mu d_L^{\beta})\sum_\psi (\overline{\psi}_R^{\beta}\gamma_\mu\psi_R^{\alpha}),$$

$$Q_6^{QNS}=4(\overline{s}_L^{\alpha}\gamma_\mu d_L^{\beta})\sum_\psi (\overline{\psi}_R^{\beta}\gamma_\mu\hat{N}\psi_R^{\alpha}),$$

$$\hat{N}=\frac{1}{2}\mathrm{diag}(1,1,1,-1,-1,-1),$$

where $\psi=(q,\tilde{q})$. Q_6^{QS} clearly is a singlet under $SU(3|3)_R$, but Q_6^{QNS} is not.

To leading order in quenched ChPT, these operators are bosonized by (we work in Euclidean space)

$$\begin{aligned}Q_6^{QS}\to&-\alpha_{q1}^{(8,1)}\mathrm{str}(\Lambda L_\mu L_\mu)\\&+\alpha_{q2}^{(8,1)}\mathrm{str}(2B_0\Lambda(\Sigma M+M\Sigma^\dagger)),\\Q_6^{QNS}\to& f^2\alpha_q^{NS}\mathrm{str}(\Lambda\Sigma\hat{N}\Sigma^\dagger).\end{aligned}\tag{3}$$

In these expressions $\Sigma=\exp(2i\Phi/f)$ is the nonlinear field describing the quenched Goldstone-meson multiplet, M is the quark mass matrix, B_0 is defined in Ref. [6], and f is the chiral limit of the pion decay constant normalized such that $f_\pi=132$ MeV. Λ is the tensor picking out the octet operator $\overline{s}d=\overline{q}\Lambda q$, and $L_\mu=i\Sigma\partial_\mu\Sigma^\dagger$ is the left-handed current to leading order in ChPT. $\alpha_{q1,2}^{(8,1)}$ are the LECs associated with the order p^2 weak octet kinetic and mass terms [7], while α_q^{NS} is the order p^0 LEC associated with the nonsinglet operator Q_6^{QNS} [3]. Note that, while this operator is enhanced in ChPT relative to the singlet operator, its contributions to matrix elements with only physical quarks on the external lines start only at order p^2 [3,8]. The subscript q is there as a reminder that the α's refer to LECs of the quenched theory.

We begin with estimating the magnitude of α_q^{NS}. A simple way of doing this was outlined in Ref. [8]. One first rotates Q_6^{QNS} by an $SU(3|3)_L$ rotation into

$$\begin{aligned}\tilde{Q}_6^{QNS}&=4(\overline{s}_L^{\alpha}\gamma_\mu\tilde{d}_L^{\beta})\sum_\psi(\overline{\psi}_R^{\beta}\gamma_\mu\hat{N}\psi_R^{\alpha}),\\&=-4((\overline{s}P_Rq)(\overline{q}P_L\tilde{d})+(\overline{s}P_R\tilde{q})(\overline{\tilde{q}}P_L\tilde{d}))\end{aligned}\tag{4}$$

where in the second line we Fierz transformed the operator, paying careful attention to the fact that the ghost-quark fields are commuting. One then considers the $\tilde{K}^0\to0$ matrix element of this operator, with $\tilde{K}^0$ a hybrid kaon made of a ghost-d quark and a physical anti-s quark. The advantage of considering this matrix element is that it is of order p^0 in ChPT [8]:

$$\langle 0|\tilde{Q}_6^{QNS}|\tilde{K}^0\rangle=2if\alpha_q^{NS}+O(p^2).\tag{5}$$

It turns out to be quite simple to find an expression for α_q^{NS}. By carrying out Wick contractions, we may write the $\tilde{K}^0\to0$ matrix element as

$$\begin{aligned}\langle 0|\tilde{Q}_6^{QNS}|\tilde{K}^0\rangle=&(\langle\overline{s}s\rangle-\langle\overline{\tilde{d}}\tilde{d}\rangle)\langle 0|\overline{s}\gamma_5\tilde{d}|\tilde{K}^0\rangle\left(1+O\left(\frac{1}{N_c^2}\right)\right)\\&-4\langle 0|(\overline{s}P_R\underline{q\overline{q}}P_L\tilde{d}+\overline{s}P_R\underline{\tilde{q}\overline{\tilde{q}}}P_L\tilde{d})|\tilde{K}^0\rangle,\end{aligned}\tag{6}$$

where $\underline{q\overline{q}}$ denotes the contraction of q with $\overline{q}$, and likewise for $\underline{\tilde{q}\overline{\tilde{q}}}$. The first line contains terms with two quark loops, while the second term has only one quark loop. In order to connect the two loops on the first line, at least two gluon lines are needed to obtain a nonzero contribution, making such connected contributions suppressed by $1/N_c^2$. To leading order in $1/N_c$ we thus obtain the factorized contributions shown explicitly.

The one-loop contribution may be dealt with as follows. First, we observe that ghost and physical quark propagators are equal, flavor by flavor, by construction. Then, we may, in this term, rotate the $\tilde{K}^0$ back to a K^0, and correspondingly the $\tilde{d}_L$ quark to d_L. For the second term we find

$$\begin{aligned}&-4\langle 0|(\overline{s}P_R\underline{q\overline{q}}P_L\tilde{d}+\overline{s}P_R\underline{\tilde{q}\overline{\tilde{q}}}P_L\tilde{d})|\tilde{K}^0\rangle\\&=-8\langle 0|\overline{s}P_R\underline{q\overline{q}}P_Ld|K^0\rangle\\&=-8\langle 0|(\overline{s}P_Rq)(\overline{q}P_Ld)|K^0\rangle.\end{aligned}\tag{7}$$

The last step follows because, as one can show, the Wick contractions leading to contributions with two quark loops cancel each other in the chiral limit. The last expression in Eq. (7) is just the $K^0\to0$ matrix element of Q_6,[1] which is of order p^2. This is also true in the quenched theory [3]. Hence, this term does not contribute to α_q^{NS}.

Using that, in the chiral limit, $\langle\overline{\tilde{d}}\tilde{d}\rangle=-\langle\overline{s}s\rangle=\frac{1}{2}f^2B_0$, we thus find for the matrix element to order p^0 that

$$\begin{aligned}\langle 0|\tilde{Q}_6^{QNS}|\tilde{K}^0\rangle&=2\langle\overline{s}s\rangle\langle 0|\overline{s}\gamma_5\tilde{d}|\tilde{K}^0\rangle\\&=2\langle\overline{s}s\rangle\ \ if\frac{M_K^2}{m_s+m_d}=-if^3B_0^2.\end{aligned}\tag{8}$$

Comparing with Eq. (5), we obtain an estimate for α_q^{NS} correct to order $1/N_c^2$:

$$\alpha_q^{NS}=-\frac{1}{2}f^2B_0^2.\tag{9}$$

In order to get an idea about the value of α_q^{NS}, we may compare it to the value of $\alpha_{q1}^{(8,1)}$. Fortunately, it turns out to be remarkably simple to obtain an estimate for $\alpha_{q1}^{(8,1)}$ in the quenched theory. It turns out that in the quenched case, the

[1]Through the weak mass term [7].

unfactorized contribution vanishes. This can be seen as follows. The Fierz transformed form of the singlet operator Q_6^{QS} is, from Eq. (2),

$$Q_6^{QS} = -8((\overline{s}P_R q)(\overline{q}P_L d) - (\overline{s}P_R \tilde{q})(\overline{\tilde{q}}P_L d)). \quad (10)$$

Following the analysis of Ref. [1],[2] again there is a contribution with two quark loops, and a contribution with one quark loop. The one-loop contribution again corresponds to the terms in which q_R and $\overline{q}_R$ or $\tilde{q}_R$ and $\overline{\tilde{q}}_R$ are contracted, and thus vanishes because of the relative minus sign in Eq. (10). This leaves us with the two-loop contribution, which factorizes to order $1/N_c^2$, yielding [9]

$$\alpha_{q1}^{(8,1)} = -8L_5 f^2 B_0^2 \quad (11)$$

(equivalent to $g_8 = -16L_5 B_0^2/F_0^2$, $F_0 = f/\sqrt{2}$ in the notation of Ref. [1]).[3] Note that quenched L_5 does not run.

Putting things together, we find that

$$\frac{\alpha_q^{NS}}{\alpha_{q1}^{(8,1)}} = \frac{1}{16L_5}\left(1 + O\left(\frac{1}{N_c^2}\right)\right). \quad (12)$$

Our first conclusion based on these results is that α_q^{NS} is likely to be large compared to the singlet LECs. The value of L_5 is of order 10^{-3} both in the quenched [10] and unquenched [6] theories, making this ratio of order 60. This is not small,[4] and casts doubt on the tentative conclusion of Ref. [11] (concluding section) on the size of α_q^{NS}. If α_q^{NS} is not small, this could have a dramatic effect on the extraction of $\alpha_{q2}^{(8,1)}$ from $K^0 \to 0$, which in turn is needed for the extraction of $\alpha_{q1}^{(8,1)}$ from $K \to \pi$ [7], because it appears at the same order as $\alpha_{q2}^{(8,1)}$ in ChPT in the $K^0 \to 0$ matrix element [3].

Another interesting observation is that the value of $\alpha_{q1}^{(8,1)}$ may be significantly smaller (in absolute value) than that of the unquenched theory. This is because of the absence of unfactorized contributions in the quenched theory, which, in the unquenched theory, have been estimated to be sizeable compared to the factorized contribution, and of the same sign [1].[5] Even if one would decide that one should define Q_6 in the quenched theory to be Q_6^{QS} only, omitting Q_6^{QNS} altogether, as an alternative possibility [3,8],[6] this smaller value of $\alpha_{q1}^{(8,1)}$ would lead to a reduction of the value of ε'/ε in quenched QCD, relative to its unquenched value.

It is rather easy to extend these estimates to the partially quenched situation, in which N (massless) sea quarks are added to the quenched theory [15]. First, the nonsinglet part of Q_6 is now represented by the order p^0 LEC $\alpha^{(8,8)}$ [3], because in the partially quenched case the nonsinglet operator is in the same irreducible representation as Q_8 [3] (and thus it corresponds to g_{ew} of Ref. [1]). One finds that the same expression as given for α_q^{NS} in Eq. (9) is also valid in the partially quenched theory for $\alpha^{(8,8)}$. For $\alpha_1^{(8,1)}$, a naive estimate would be to interpolate linearly in the number of sea-quark flavors N between the quenched and unquenched theories. Using the results obtained in Ref. [1], which considers the unfactorized contribution proportional to the number of light flavors (which is three in the real world), this would lead to the estimate

$$\alpha_1^{(8,1)} = -8L_5 f^2 B_0^2 + \frac{N}{3} \times (\text{unfactorized contribution of Ref. [1]}), \quad (13)$$

where of course now L_5, f, B_0, etc. take their values in the partially quenched theory with N sea quarks.

Finally, let us comment on the fact that, since our aim was to extract values for the leading order LEC for each of the operators we considered, all calculations were done in the chiral limit. It is well known that the chiral limit of the quenched theory is hampered by severe infrared divergences, and, in fact, the chiral limit of the quenched theory may not exist [5,16,17]. However, we believe that our results for LECs are not affected by this issue. If the quenched effective theory makes any sense, its parameters, which are the LECs, should be well defined and finite in the chiral limit. All the sicknesses associated with the quenched infrared behavior should be correctly reproduced if the appropriate fields, in particular the η', are kept in the effective theory.

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[2] As in Ref. [1], we will work in the leading-log approximation, in which Q_6^{QS} does not mix with any other operator in the quenched theory.

[3] L_5 is one of the Gasser-Leutwyler constants [6].

[4] In particular, "strategy 3" of Ref. [8] would yield results significantly different from the other strategies.

[5] The unfactorized contribution is found to be about twice the factorized one, at the scale of the ρ mass.

[6] This change is known to have a large effect [12–14].

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