Improved lattice gauge field Hamiltonian: The three-dimensional case

Jun-Qin Jiang*

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China
and Department of Physics, Guangdong Institute of Education, Guangzhou 510303, People's Republic of China†

Jie-Ming Li

Department of Physics, Zhongshan University, Guangzhou 510275, People's Republic of China

(Received 13 May 2003; revised manuscript received 4 August 2003; published 5 November 2003)

Using the improved lattice gauge field Hamiltonian and the truncated eigenvalue equation method, we compute the vacuum wave function and mass gap of three-dimensional SU(2) gauge field theory. Our results show that the improved theory leads to a significant reduction of violation of scaling, that is, using the improved lattice gauge field Hamiltonian, the calculations can be carried out up to a much weaker coupling region than using the unimproved one, with good scaling behavior.

DOI: 10.1103/PhysRevD.68.094502 PACS number(s): 12.38.Gc, 11.15.Ha, 11.30.Qc

I. INTRODUCTION

Lattice gauge field theory (LGT) based on first principles, with two equivalent forms of the action and Hamiltonian, is the most reliable and powerful nonperturbative approach to QCD, but its progress has been hampered by systematical errors mainly due to the finite value of the lattice spacing a.

The standard Wilson gluonic (bosonic) action (or Kogut-Susskind Hamiltonian) differs from the continuum Yang-Mills action (or corresponding Hamiltonian) by the order of $O(a^2)$, while the error of the standard Wilson quark (fermionic) action (or Hamiltonian) is bigger, being of the order of $O(a)$. In the continuum limit $a\rightarrow 0$ or equivalently $1/g^2\rightarrow\infty$ in an asymptotic theory, these differences in principle disappear and the action or Hamiltonian becomes the continuum one. If the practical lattice calculations could be carried out up to a weak enough coupling region, the finite lattice errors would be negligible. Unfortunately, the standard Hamiltonian method can only be carried out up to the intermediate coupling region. For example, in the standard Kogut-Susskind Hamiltonian method, the calculations of the vacuum wave function and glueball mass of three-dimensional SU(N_C) can only be carried out up to $1/g^2\approx 2.0$ [1–6]. For such a lattice parameter, violation of scaling is still obvious and extrapolation of the results to the $1/g^2\rightarrow\infty$ limit induces unknown systematic uncertainties when extracting continuum physics.

One possible way to tackle these problems is to improving lattice action (or Hamiltonian), so as to the finite a errors become higher order in a. In recent years, one has been studying the problem of improvement of lattice Hamiltonian:

(i) For the quark sector, Hamber and Wu proposed the first improved lattice action [7], reducing the errors from $O(a)$ to $O(a^2)$. There have been some numerical simulations [8–10] of hadron spectroscopy using the Hamber-Wu action. In 1994, we constructed an improved Hamiltonian [11], which had been tested successfully in the two-dimensional QCD by Jiang (one author of the present paper) and Luo *et al.* [12].

(ii) For the gluonic sector, Lepage proposed an improved lattice action [13], while Luo, Guo, Kröger, and Schütte constructed a simpler improved Hamiltonian [14], reducing the errors from $O(a^2)$ to $O(a^4)$.

It is difficult in the Lagrangian formulation to compute the wave function. The advantage of the Hamiltonian formulation is that one can compute not only the mass spectrum but also the wave function. In past years, Guo *et al.* have done a lot of work [1–6,11,12] on Hamiltonian gauge field theory. The purpose of this paper is to show that the improved lattice Hamiltonian [14] can give better results than the unimproved one by computing the vacuum wave function and mass gap of three-dimensional SU(2) gauge field theory.

The remaining part of the paper is organized as follows. In Sec. II, we educe the truncated eigenvalue equation for the improved Hamiltonian. In Sec. III, the vacuum wave function and mass gap of three-dimensional SU(2) LGT is computed. A simple discussion is presented in Sec. IV.

II. IMPROVED HAMILTONIAN FOR GLUONS AND ITS TRUNCATED EIGENVALUE EQUATION

For simplicity, we study the three-dimensional SU(2) LGT in this paper. In SU(2) LGT, $\mathrm{Tr}U_p^+=\mathrm{Tr}U_p$, so that all loops with crossing can be transformed into loops without crossing. According to the improved gluonic (bosonic) Hamiltonian [14], we can obtain

$$H=\frac{g^2}{2a}\mathrm{Tr}\sum_{x,i}\left\{\frac{1+C'^2}{(1-C')^2}E_i^\alpha(x)E_i^\alpha(x)-\frac{2C'}{(1-C')^2}U_i^+E_i^\alpha(x)U_i(x)E_i^\alpha(x+i)\right\}-\frac{2}{g^2a}\left(C_1'\sum_p\mathrm{Tr}U_p+C_2'\sum_{x,i<j}R_{ij}\right), \tag{1}$$

*Email address: jjq203@21cn.com

†Permanent address and mailing address.

 ©2003 The American Physical Society

where g is the dimensionless coupling constant which is related to the lattice spacing a and the invariant charge e by $g^2=e^2a$, $E_i^{\alpha}(x)$ is the color-electric field. C', C_1', C_2' are constants, according to Ref. [14], $C_1'=5/3$, $C_2'=-1/6$, and C' satisfying

$$C'^3+11C'^2+11C'+1=0, \tag{2}$$

the root closest to zero is $C'=-5+2\sqrt{6}$ which makes the series $\Sigma_{n=1}^{\infty}C'^n$ constringe. $\Sigma_p \mathrm{Tr} U_p$ is the square loop □, $\Sigma_{x,i<j}R_{ij}$ is the rectangular loop ▭.

When $C'=0$, $C_1'=1_1$, $C_2'=0$, Eq. (1) reduces to the standard Hamiltonian [3,4]:

$$H_0=\frac{g^2}{2a}\mathrm{Tr}\sum_{x,i}\{E_i^{\alpha}(x)E_i^{\alpha}(x)\}-\frac{2}{g^2a}\sum_p \mathrm{Tr}U_p. \tag{3}$$

The vacuum wave function in exponential form is written as [3]

$$|\Omega\rangle=\mathrm{e}^R|0\rangle, \tag{4}$$

where R contains closed loops and the state $|0\rangle$ is defined as $E_i^{\alpha}(x)|0\rangle=0$.

Substituting H and $|\Omega\rangle$ in the eigenvalue equation (namely Schrödinger equation) $H|\Omega\rangle=E_{\Omega}|\Omega\rangle$, using the relation

$$\mathrm{e}^{-R}H\mathrm{e}^R=H-[R,H]+\frac{1}{2!}[R,[R,H]]-\frac{1}{3!}[R,[R,[R,H]]]+\cdots, \tag{5}$$

and the commutation relations

$$[U_i(x),E_j^{\alpha}(y)]=\lambda^{\alpha}U_i(x)\delta_{x,y}\delta_{i,j},$$
$$[U_i^{+}(x),E_j^{\alpha}(y)]=-U_i^{+}(x)\lambda^{\alpha}\delta_{x,y}\delta_{i,j}, \tag{6}$$

we can obtain the eigenvalue equation of the vacuum state for the improved Hamiltonian:

$$B_1'\left\{\sum_{x,i}[E_i^{\alpha}(x),[E_i^{\alpha}(x),R]]+\sum_{x,i}[E_i^{\alpha}(x),R][E_i^{\alpha}(x),R]\right\}+B_2'\left\{\sum_{x,i}[E_i^{\alpha}(x),[E_i^{\alpha}(x+i),R]]+\sum_{x,i}[E_i^{\alpha}(x+i),R]\times[E_i^{\alpha}(x),R]\right\}-\frac{4}{g^4}\left(C_1'\sum_p \mathrm{Tr}U_p+C_2'\sum_{x,i<j}R_{ij}\right)=\frac{2a}{g^2}E_{\Omega}, \tag{7}$$

where $B_1'=(1+C'^2)/[(1-C')^2]=5/6$, $B_2'=-4C'/(1-C')^2=1/3$, $C_1'=5/3$, $C_2'=-1/6$.

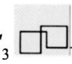
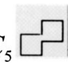
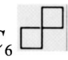
Defining the order of a loop graph as the number of plaquettes involved (overlapping plaquettes are also counted), we expand R in the order of graphs:

$$R=R_1+R_2+R_3+\cdots. \tag{8}$$

The lowest order loop graph (i.e. the first order graph) is

$$R_1=C_1\,□.$$

Let $R=R_1$, from Eq. (7), we can obtain the 2nd order loop graphs:

$R_2=C_2$  $+C_3$  $+C_4$  $+C_5$  $+C_6$ , (9)

where the coefficients C_i will be given by solving the algebraic equations (see Sec. III).

Let $R=R_1+R_2$, from Eq. (7), we can obtain not only the whole of the 3rd order loop graphs, but also a part of the 4th order loop graphs.

In general

$$\sum_{x,i}[E_i^{\alpha}(x),[E_i^{\alpha}(x),R_n]]\in R_n,$$

$$\sum_{x,i}[E_i^{\alpha}(x),[E_i^{\alpha}(x+i),R_n]]\in R_n,$$

$$\sum_{x,i}[E_i^{\alpha}(x),R_{n_1}][E_i^{\alpha}(x),R_{n_2}]\in R_{n_1+n_2}+\text{lower order graphs},$$

$$\sum_{x,i}[E_i^{\alpha}(x+i),R_{n_1}][E_i^{\alpha}(x),R_{n_2}]\in R_{n_1+n_2}+\text{lower order graphs}. \tag{10}$$

We now give a recipe for truncating the eigenvalue equation. Let R contain up to the Nth order graphs:

$$R=R_1+R_2+\cdots+R_N. \tag{11}$$

We must truncate the term $\Sigma_{x,i}[E_i^{\alpha}(x),R][E_i^{\alpha}(x),R]$ and term $\Sigma_{x,i}[E_i^{\alpha}(x+i),R][E_i^{\alpha}(x),R]$ because they create higher order graphs. The simplest way is just preserving these terms

$$\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^{\alpha}(x),R_{n_1}][E_i^{\alpha}(x),R_{n_2}],$$

$$\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^{\alpha}(x+i),R_{n_1}][E_i^{\alpha}(\chi),R_{n_2}].$$

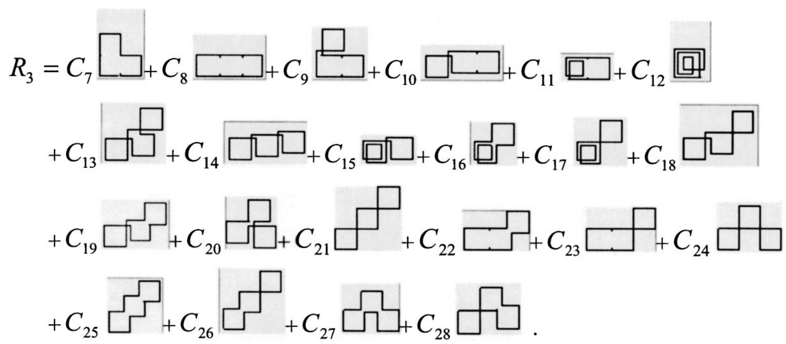
Thus we obtain the truncated eigenvalue equation of the vacuum state for the improved gluonic Hamiltonian at the Nth order:

$$B_1'\left\{\sum_{x,i}[E_i^\alpha(x),[E_i^\alpha(x),R]]+\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^\alpha(x),R_{n_1}]\times[E_i^\alpha(x),R_{n_2}]\right\}+B_2'\left\{\sum_{x,i}[E_i^\alpha(x),[E_i^\alpha(x+i),R]]+\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^\alpha(x+i),R_{n_1}][E_i^\alpha(x),R_{n_2}]\right\}-\frac{4}{g^4}\left(C_1'\sum_p \mathrm{Tr}U_p+C_2'\sum_{x,i<j}R_{ij}\right)=\frac{2a}{g^2}E_\Omega, \qquad (12)$$

where $B_1'=5/6$, $B_2'=1/3$, $C_1'=5/3$, $C_2'=-1/6$. When $B_1'=1$, $B_2'=0$, $\dot{C}_1'=1$, $C_2'=0$, Eq. (12) reduces to the standard truncated eigenvalue equation [3,4]:

$$\sum_{x,i}[E_i^\alpha(x),[E_i^\alpha(x),R]]+\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^\alpha(x),R_{n_1}][E_i^\alpha(x),R_{n_2}]-\frac{4}{g^4}\sum_p \mathrm{Tr}U_p=\frac{2a}{g^2}E_\Omega. \qquad (13)$$

From the terms $\Sigma_{x,i}[E_i^\alpha(x),R_1][E_i^\alpha(x),R_2]$, $\Sigma_{x,i}[E_i^\alpha(x+i),R_1][E_i^\alpha(x),R_2]$ and $\Sigma_{x,i}[E_i^\alpha(x+i),R_2][E_i^\alpha(x),R_1]$, we can obtain the 3rd order loop graphs:



(14)

Analogically, from R_1, R_2 and R_3, we can obtain $R_4, \ldots$.

Because the improvement term

$$\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^\alpha(x+i),R_{n_1}][E_i^\alpha(x),R_{n_2}]$$

creates new graphs, for the same value of N, Eq. (12) contains more loop graphs than Eq. (13). For example, for $N=3$, Eq. (12) contains 28 graphs, but Eq. (13) contains 13 graphs [3].

Now, we turn to the 0^{++} glueball mass. The glueball wave function in exponential form is written as

$$|\Psi\rangle=G|\Omega\rangle=G\mathrm{e}^R|0\rangle, \qquad (15)$$

where G consist of closed loops, which is also expanded up to the Nth order graphs:

$$G=G_1+G_2+\cdots+G_N, \qquad (16)$$

where G_N is the Nth order loop graphs, according to the same rules as R_N.

In a similar way of obtaining Eq. (12), we can obtain the truncated eigenvalue equation of the glueball state for the improved Hamiltonian at the Nth order:

$$\frac{5}{6}\left\{\sum_{x,i}[E_i^\alpha(x),[E_i^\alpha(x),G]]+2\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^\alpha(x),R_{n_1}]\times[E_i^\alpha(x),G_{n_2}]\right\}+\frac{1}{3}\left\{\sum_{x,i}[E_i^\alpha(x+i),[E_i^\alpha(x),G]]+\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^\alpha(x+i),R_{n_1}][E_i^\alpha(x),G_{n_2}]+\sum_{\substack{x,i\\ n_1+n_2\leqslant N}}[E_i^\alpha(x+i),G_{n_1}][E_i^\alpha(x),R_{n_2}]\right\}=\frac{2a\Delta m}{g^2}G, \qquad (17)$$

where Δm is the 0^{++} glueball mass.

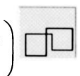
III. CALCULATIONS OF THE VACUUM WAVE FUNCTION AND MASS GAP

In order to study the low energy physics of hadrons, one needs more detailed information on the structure of the vacuum wave function. Thus the first work is to calculate the vacuum wave function.

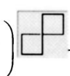
At the 2nd order, the truncated eigenvalue equation of the vacuum state for the improved gluonic Hamiltonian is

$$\frac{5}{6}\left\{\sum_{x,i}\,[E_i^\alpha(x),[E_i^\alpha(x),R_1+R_2]]+\sum_{x,i}\,[E_i^\alpha(x),R_1]\right.$$
$$\left.\times[E_i^\alpha(x),R_1]\right\}+\frac{1}{3}\left\{\sum_{x,i}\,[E_i^\alpha(x),[E_i^\alpha(x+i),R_1+R_2]]\right.$$
$$\left.+\sum_{x,i}\,[E_i^\alpha(x+i),R_1][E_i^\alpha(x),R_1]\right\}$$
$$-\frac{4}{g^4}\left(\frac{5}{3}\sum_p\,\mathrm{Tr}U_p-\frac{1}{6}\sum_{x,i<j}R_{ij}\right)=\frac{2a}{g^2}E_\Omega\,. \tag{18}$$

Substituting R_1 and R_2 in Eq. (18), we obtain

$$\left(\frac{10}{6}C_1-\frac{20}{3g^4}\right)$$  $$+\frac{5}{6}\left(2C_1^2+8C_2\right)$$  $$+\left(\frac{23}{6}C_3+C_1^2\right)$$ 

$$+\left(\frac{17}{6}C_4-C_1^2+\frac{2}{3g^4}\right)$$  $$+\left(3C_5-\frac{2}{3}C_1^2\right)$$ 

$$+\left(\frac{11}{3}C_6+\frac{2}{3}C_1^2\right)$$  $$+\ \mathrm{const}\ =\ 0. \tag{19}$$

Equating the coefficients of each loop graph to zero, the result is

$$C_1=\frac{4}{g^4},\ \ C_2=-\frac{1}{4}C_1^2,\ \ C_3=-\frac{6}{23}C_1^2,$$
$$C_4=\frac{6}{17}\left(C_1^2-\frac{2}{3g^4}\right),\quad C_5=\frac{2}{9}C_1^2,\ \ C_6=-\frac{2}{11}C_1^2. \tag{20}$$

In the continuum limit, the long wavelength vacuum wave function of three-dimensional SU(2) gauge field theory is [3,4,6]

$$|\Omega\rangle=N\exp\left\{-\frac{\mu_0}{e^2}\int\int\mathrm{Tr}F^2(x,y)dxdy\right.$$
$$-\frac{\mu_2}{e^6}\int\int\mathrm{Tr}[D_iF(x,y)]^2dxdy$$
$$\left.+\text{higher order terms}\right\}, \tag{21}$$

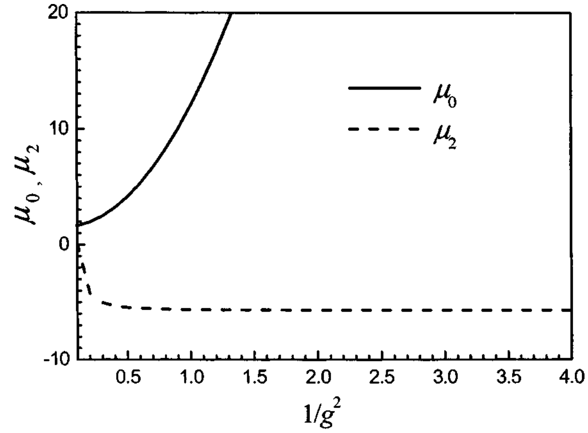


FIG. 1. μ_0 and μ_2 versus $1/g^2$ for the 2nd order when using the improved Hamiltonian.

where μ_0 and μ_2 are linear combination of the coefficients C_i, from expanding $\Sigma_p\mathrm{Tr}U_p$ in order of a. At the 2nd order, we obtain

$$\mu_0=\frac{g^4}{2}(C_1+4C_2+4C_4+4C_5),$$
$$\mu_2=\frac{g^8}{2}\left(\frac{1}{2}C_3-\frac{1}{2}C_4-C_5+C_6\right). \tag{22}$$

Substituting Eq. (20) in Eq. (22), we obtain μ_0 and μ_2 as the functions of $1/g^2$. Figure 1 gives the 2nd order results.

It is well known that SU(2) LGT in three dimensions is super-renormalizable, and possesses a simple scaling property, that is, when the lattice spacing a goes to zero,

$$\mu_0\sim\text{const},\ \ \mu_2\sim\text{const}\quad\text{as}\quad a\to 0. \tag{23}$$

From Fig. 1, one can see that when truncating up to the 2nd order the vacuum wave function does not have the correct scaling behavior. Higher order approximations are needed to obtain the correct scaling behavior. We now turn to the 3rd order calculation.

At the 3rd order, the truncated eigenvalue equation of the vacuum state for the improved gluonic Hamiltonian is

$$\frac{5}{6}\left\{\sum_{x,i}\,[E_i^\alpha(x),[E_i^\alpha(x),R_1+R_2+R_3]]+\sum_{x,i}\,[E_i^\alpha(x),R_1][E_i^\alpha(x),R_1]+2\sum_{x,i}\,[E_i^\alpha(x),R_1][E_i^\alpha(x),R_2]\right\}$$
$$+\frac{1}{3}\left\{\sum_{x,i}\,[E_i^\alpha(x),[E_i^\alpha(x+i),R_1+R_2+R_3]]+\sum_{x,i}\,[E_i^\alpha(x+i),R_1][E_i^\alpha(x),R_1]\right.$$
$$\left.+\sum_{x,i}\,[E_i^\alpha(x+i),R_1][E_i^\alpha(x),R_2]+\sum_{x,i}\,[E_i^\alpha(x+i),R_2][E_i^\alpha(x),R_1]\right\}$$
$$-\frac{4}{g^4}\left(\frac{5}{3}\sum_p\,\mathrm{Tr}U_p-\frac{1}{6}\sum_{x,i<j}R_{ij}\right)=\frac{2a}{g^2}E_\Omega\,. \tag{24}$$

Substituting R_1, R_2 and R_3 in Eq. (24), we obtain the nonlinear equations for the coefficients C_i ($i=1,2,3,...,28$):

$$-2C_1^2+2C_3+12C_4+(1/g^2)^2=0,$$

$$C_1^2+8C_2=0,$$

$$C_1^2+8C_3=0,$$

$$-2C_1^2+59C_5=0,$$

$$2C_1^2-2C_5+63C_6=0,$$

$$4C_1C_2+15C_{12}=0,$$

$$6C_2C_3+49C_{14}=0,$$

$$C_1C_5+44C_{25}=0,$$

$$-6C_1C_3+4C_1C_4+41C_{10}+8C_{14}=0,$$

$$-4C_1C_4+33C_8+4C_{10}=0,$$

$$-C_1C_5-C_1C_6-2C_{25}+46C_{26}=0,$$

$$C_1C_6+48C_{21}-C_{26}=0,$$

$$-C_1C_5+22C_{27}=0,$$

$$C_1C_5-C_1C_6-C_{27}+C_{28}=0,$$

$$2C_1C_6+48C_{24}-C_{28}=0,$$

$$16C_1C_2+24C_1C_3+59C_{15}=0,$$

$$-16C_1C_2+16C_1C_4+47C_{11}+4C_{15}=0,$$

$$-4C_1C_2+19C_1C_5+54C_{16}=0,$$

$$4C_1C_2+21C_1C_6-C_{16}+57C_{17}=0,$$

$$-4C_1C_3+6C_1C_5+93C_{19}=0,$$

$$4C_1C_3+6C_1C_6+97C_{18}-2C_{19}=0,$$

$$-8C_1C_3+20C_1C_5-20C_1C_6+101C_{20}=0,$$

$$5C_1C_3-C_1C_5+5C_1C_6+25C_{13}-C_{20}=0,$$

$$-12C_1C_3+20C_1C_4+83C_9+16C_{13}-4C_{20}=0,$$

$$-20C_1C_4-16C_1C_5+63C_7+10C_9+18C_{20}=0,$$

$$-4C_1C_4-16C_1C_5+4C_1C_6+8C_{19}+77C_{22}=0,$$

$$4C_1C_4-10C_1C_6+8C_{18}-2C_{22}+81C_{23}=0,$$

$$15C_1-20C_1C_2-24C_1C_3-16C_1C_4-19C_1C_5-21C_1C_6+16C_{11}+60C_{12}+24C_{15}+19C_{16}+21C_{17}-40(1/g^2)^2=0. \quad (25)$$

Solving Eq. (25) numerically, we can obtain C_i ($i=1,2,3,...,28$) as the functions of $1/g^2$.

Evaluating the long wavelength limit of all graphs up to the 3rd order, we obtain

$$\mu_0=\frac{g^4}{2}(C_1+4C_2+4C_4+4C_5+9C_7+9C_8+C_9+C_{10}+9C_{11}+9C_{12}+C_{13}+C_{14}+C_{15}+9C_{16}+C_{17}+C_{18}+C_{19}+C_{20}+C_{21}+9C_{22}+C_{23}+C_{24}+9C_{25}+C_{26}+9C_{27}+C_{28}), \quad (26a)$$

$$\mu_2=\frac{g^8}{2}(0.5C_3-0.5C_4-C_5+C_6-2C_7-3C_8+C_9+2C_{10}-C_{11}-C_{14}+C_{15}-2C_{16}+2C_{17}-C_{18}+2C_{19}-2C_{21}-4C_{22}+3C_{23}-6C_{25}+4C_{26}-4C_{27}+2C_{28}). \quad (26b)$$

Substituting C_i ($i=1,2,3,...,28$) in Eqs. (26a) and (26b), we obtain μ_0 and μ_2 as the functions of $1/g^2$. Figure 2 gives the 3rd order results for the improved Hamiltonian.

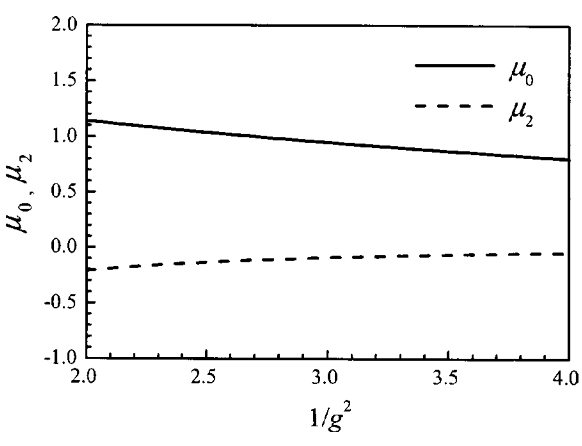


FIG. 2. μ_0 and μ_2 versus $1/g^2$ for the 3rd order when using the improved Hamiltonian.

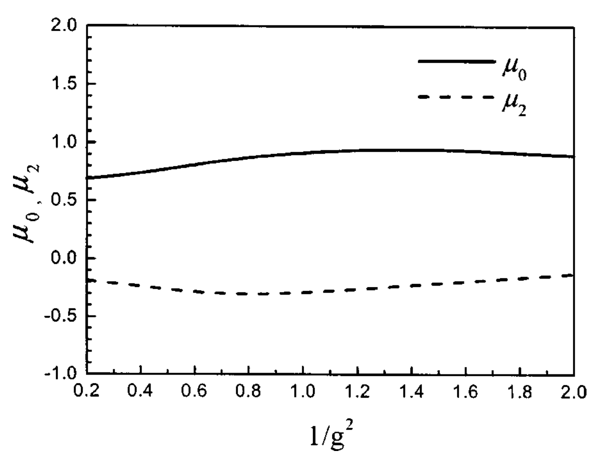


FIG. 3. μ_0 and μ_2 versus $1/g^2$ for the 3rd order when using the standard Hamiltonian.

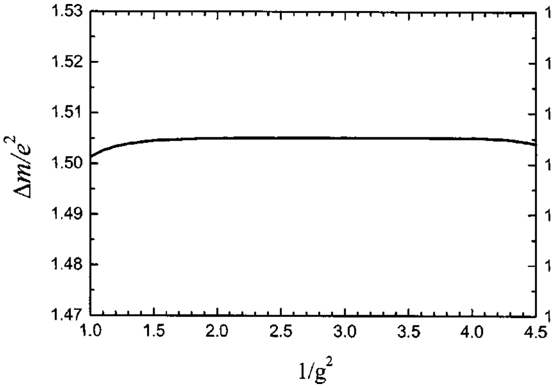


FIG. 4. $\Delta m/e^2$ vs $1/g^2$ for the 3rd order when using the improved Hamiltonian.

To compare our results with ones from Ref. [3], we give also the 3rd order results from the standard Hamiltonian. This is shown in Fig. 3.

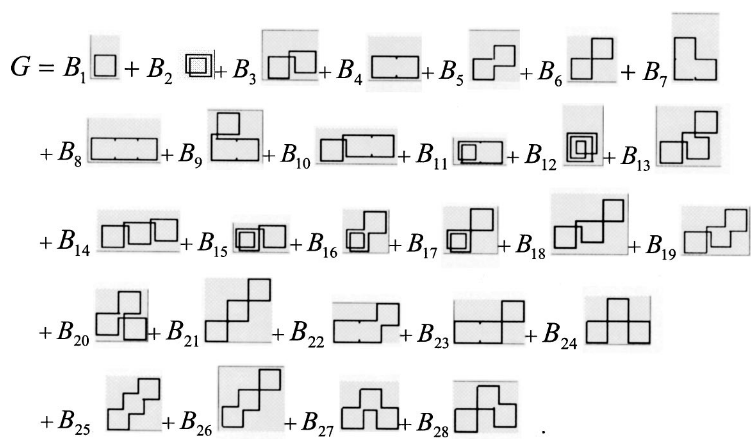
As can be seen, the 3rd order results from both the improved Hamiltonian (Fig. 2) and the standard Hamiltonian (Fig. 3 or see Refs. [3,4]) have a good scaling behavior. Significantly, the calculations for the improved Hamiltonian can be carried out up to $1/g^2=4.0$, i.e., for $1/g^2>4.0$, the physical quantities would not have good scaling behavior [by solving Eq. (25), we obtain the various C_i coefficients as the functions of g^{-2} and then get the g^{-2} dependence of the physical quantities: μ_0, μ_2 and glueball mass which is computed in the next three paragraphs]; while using the standard Hamiltonian, the calculation can only be carried out up to $1/g^2=2.0$, i.e., for $1/g^2>2.0$, the physical quantities would not have good scaling behavior.

TABLE I. Comparison of the improved Hamiltonian with the standard Hamiltonian about the 0^{++} glueball mass of $(2+1)$D SU(2) gauge theory. $\beta=4/g^2$.

Method	Order	Loop graphs	Scaling window	Glueball mass
Improved Hamiltonian	3	28	$\beta\in[6.0,16.0]$	1.505
Standard Hamiltonian [4]	3	13	$\beta\in[3.0,8.0]$	1.59
	4	70	$\beta\in[5.0,8.0]$	1.84

One is interested in computing the mass gap (namely 0^{++} glueball mass). We now compute the mass gap Δm.

The calculation of the mass gap is dependent upon the structure of the vacuum wave function, while truncating up to the 2nd order the vacuum wave function does not have the correct scaling behavior. Thus we compute directly the mass gap up to the 3rd order:



(27)

Substituting R and G in Eq. (17), we can obtain the nonlinear equations for the mass gap Δm and the coefficients B_i $(i=1,2,3,...,28)$, similar to Eq. (25). Solving these equations numerically [and using the g^{-2} dependence of C_i, from Eq. (25)], we can obtain Δm as a function of $1/g^2$. Figure 4 gives the numerical result.

It is well known that the SU(2) LGT in three dimensions possesses a simple scaling property for the mass gap:

$$\Delta m/e^2 \sim \text{const} \quad \text{as} \quad a\rightarrow 0. \tag{28}$$

From Fig. 4, one can see that our result of the mass gap is almost a constant (1.505) in a wide scaling window $1/g^2 \in[1.5,4.0]$.

To get a more explicit comparison of the glueball mass, we give the results of this paper and Ref. [4] in Table I.

From Table I, one can see that the scaling window ($\beta\in[6.0,16.0]$) is much wider and the coupling region is much weaker in this paper than in Ref. [4].

On the other hand, the 0^{++} glueball mass of $(2+1)$D SU(2) gauge theory is also studied in the Monte Carlo calculation. According to the results of Ref. [15], for the case of $(2+1)$D SU(2), the scaling window is $\beta\in[5.50,6.55]$, which is much narrower than the one of this paper. A part of

TABLE II. The Monte Carlo results of the $(2+1)$D SU(2) 0^{++} glueball mass near the scaling window, from Ref. [15]. $\beta=4/g^2$, $g^2=ae^2$; L is the loop length.

β	5	6	6	6	9	9
L	16	16	24	32	24	32
$\Delta m\times a$	1.478(24)	1.193(18)	1.191(18)	1.170(23)	0.7643(60)	0.7552(67)
$\Delta m/e^2$	1.847(30)	1.789(27)	1.782(27)	1.755(34)	1.720(13)	1.699(15)

the Monte Carlo results for the 0^{++} glueball mass, from Ref. [15], is given in Table II. From Table I and Table II, one also can see that the glueball masses from both the standard and the improved Hamiltonian are in agreement with Monte Carlo results.

IV. CONCLUSIONS

In this work we have computed the vacuum wave function and mass gap of three-dimensional SU(2) LGT using the improved lattice gauge field Hamiltonian [14]. Our calculations can be carried out up to a weaker coupling region, i.e., $1/g^2=4.0$, with a very well scaling behavior, while using the unimproved Hamiltonian, the calculations can only be carried out up to the intermediate coupling region, i.e., $1/g^2=2.0$ (see Refs. [3,4,6]), which would result in large uncertainty. In particular, our result of the mass gap is almost a constant (1.505) in a wide scaling window $1/g^2\in[1.5,4.0]$ (namely $\beta\in[6.0,16.0]$), which is in agreement with the Monte Carlo measurement [15].

To summarize, from the test of three-dimensional SU(2) LGT, we see that the improved Hamiltonian indeed leads to much better results. We believe that application of the improved theory [14] to the four-dimensional SU(N_c) LGT will be very promising.

ACKNOWLEDGMENTS

We are grateful to S. H. Guo and X. Q. Luo for useful discussions. Jun-Qin Jiang is grateful for support by Natural Science Foundation of the Education Department of Guangdong Province of China (Z02083).

[1] X. Q. Luo, B. P. He, Q. Z. Chen, and S. H. Guo, Z. Phys. C **51**, 423 (1991).
[2] X. Q. Luo and Q. Z. Chen, Phys. Rev. D **46**, 814 (1992).
[3] S. H. Guo *et al.*, Phys. Rev. D **49**, 507 (1994).
[4] Q. Z. Chen *et al.*, Phys. Rev. D **50**, 3564 (1994).
[5] Q. Z. Chen, X. Q. Luo, and S. H. Guo, Phys. Lett. B **348**, 560 (1995).
[6] Q. Z. Chen, P. F. Cai, X. Y. Fang, and S. H. Guo, Commun. Theor. Phys. **27**, 337 (1997).
[7] H. W. Hamber and C. M. Wu, Phys. Lett. **133B**, 251 (1983).
[8] M. P. Lombardo, G. Parisi, and A. Vladikas, Nucl. Phys. **B395**, 388 (1993).
[9] A. Borici and P. de Forcrand, Nucl. Phys. B (Proc. Suppl.) **55**, 873 (1997).
[10] H. Fiebig and R. Woloshyn, Nucl. Phys. B (Proc. Suppl.) **55**, 886 (1997).
[11] X. Q. Luo, Q. Z. Chen, G. C. Xu, and J. Q. Jiang, Phys. Rev. D **50**, 501 (1994).
[12] Jun-Qin Jiang, X. Q. Luo, Z. H. Mei, H. Jirari, H. Kröger, and C. M. Wu, Phys. Rev. D **60**, 014501 (1999).
[13] P. Lepage, in Proceedings of Schladming Winter School, 1996, hep-lat/9607076.
[14] X. Q. Luo, S. H. Guo, H. Kröger, and D. Schötte, Phys. Rev. D **59**, 034503 (1999).
[15] M. J. Teper, Phys. Rev. D **59**, 014512 (1999).