T-odd correlation in the $K^+ \to \pi l \nu \gamma$ decays beyond the standard model

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The dependence of the T-odd correlation on the effective Lagrangian parameters in the $K^+ \to \pi l \nu \gamma$, $l = e, \mu$, decays is analyzed. It is shown that the observable introduced is a perspective in the search for new physics in the vector and pseudovector sectors of the Lagrangian. As for the scalar and pseudoscalar sectors, T-odd correlation studies will not allow one to improve current restrictions on the parameters of models beyond the standard model.

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I. INTRODUCTION

T invariance is one of the fundamental symmetries in physics; therefore, many experimental groups carry out their studies in this area. The search for new physics is extremely promising in processes where the standard model contribution to experimental observables is suppressed. One such experimental observable, for instance, is the muon transverse polarization in $K^+ \to \pi^0 \mu \nu, K^+ \to \mu \nu \gamma$ decays [1,2]. In these processes the standard model (SM) contribution to P_T vanishes at the tree level. A nonvanishing contribution appears only at the one-loop level and is caused by the electromagnetic final state interaction; therefore, it is significantly suppressed. In the $K^+ \to \pi^0 \mu \nu$ decay the lepton transverse polarization is equal to 5×10^{-6}; [3,4], and in $K^+ \to \mu \nu \gamma$ it is equal to 6×10^{-4} [5,6].

In contrast with the SM, in some extensions the transverse polarization appears already at the tree level of perturbation theory [7,8].

At the moment the E246 experiment at KEK is performing the analysis of the data on the $K^+ \to \pi^0 \mu \nu$ process to put bounds on the T-violating muon transverse polarization, and the following result has been obtained [1]:

$$P_T = [-1.12 \pm 2.17(\mathrm{stat}) \pm 0.90(\mathrm{syst})] \times 10^{-3}. \tag{1}$$

Unfortunately, there is no experimental result for the $K^+ \to \mu \nu \gamma$ process, as the experimental data are being still processed. The transverse polarization is expected to be of the order of 1.5×10^{-2} [2].

Another important experimental observable used in CP violation searches is the T-odd correlation in the $K^+ \to \pi^0 l \nu \gamma$ decay defined as $\xi = \mathbf{q} \cdot [\mathbf{p}_\pi \times \mathbf{p}_l]/m_K^3$.

In this case the T-violating signal is the asymmetry of the differential distribution of the decay width relative to the $\xi = 0$ line. As in the case of lepton transverse polarization, T-odd correlation vanishes at the tree level of the SM and appears due to the electromagnetic final state interaction. In the framework of the SM this effect was examined earlier in [9]. However, it would be interesting to compare that result with the value of the asymmetry induced by some of the SM extensions. Our work is devoted to this problem.

New perspectives on T-odd correlation studies are connected with the OKA experiment [10], where the measurement of this observable is planned. For this reason, the problem under discussion is of particular importance. Event samples of 10^6–10^7 for the $K^+ \to \pi^0 e \nu_e \gamma$ decay and 10^5–10^6 for $K^+ \to \pi^0 \mu \nu_\mu \gamma$ decay are expected to be accumulated.

This paper is organized as follows. In Sec. II we present the model independent Lagrangian and the expression for the asymmetry in terms of the Lagrangian, and discuss the SM contribution to T correlation. In Secs. III and IV the contributions of the $SU(2)_L \times SU(2)_R \times U(1)$ model and scalar models are examined. The last section summarizes the results and conclusions.

II. MODEL INDEPENDENT APPROACH IN T-ODD CORRELATION STUDY

The model independent Lagrangian of the four-fermion interaction is as follows:

$$\begin{aligned} L = \frac{G_f}{\sqrt{2}} \sin\theta_c [& \bar{s}\gamma^\alpha (1-\gamma_5) u \bar{\nu} \gamma_\alpha (1-\gamma_5) l + g_s \bar{s} u \bar{\nu} (1+\gamma_5) l \\ & + g_p \bar{s}\gamma_5 u \bar{\nu}(1+\gamma_5) l + g_v \bar{s}\gamma^\alpha u \bar{\nu}\gamma_\alpha (1-\gamma_5) l \\ & + g_a \bar{s}\gamma^\alpha \gamma_5 u \bar{\nu} \gamma_\alpha (1-\gamma_5) l], \end{aligned} \tag{2}$$

where G_f is the Fermi constant, θ_c is the Cabibbo angle, and g_s, g_p, g_v, g_a are the scalar, pseudoscalar, vector, and pseudovector constants. Using this Lagrangian, the matrix element for $K(p) \to \pi^0(p') l(p_l) \nu(p_\nu) \gamma(q)$ decay can be written as

$$\begin{aligned} T = \frac{G_f}{\sqrt{2}} V_{us}^* e \epsilon_\alpha^* \Bigg[& [(1+g_v) V^{\alpha\beta} - (1-g_a) A^{\alpha\beta}] \\ & \times \bar{\nu}(1+\gamma_5)\gamma_\beta l + (1+g_v) F_\beta \bar{\nu}(1+\gamma_5)\gamma^\beta \\ & \times \left(\frac{p^\alpha}{pq} - \frac{p_l^\alpha}{p_l q} - \frac{\widehat{q\gamma^\alpha}}{2(p_l q)} \right) l + (g_s F_s^\alpha + g_p F_p^\alpha) \bar{\nu}(1+\gamma_5) l \\ & + g_s f \bar{\nu}(1+\gamma_5) \left(\frac{p^\alpha}{pq} - \frac{p_l^\alpha}{p_l q} - \frac{\hat{q}\gamma^\alpha}{2(p_l q)} \right) l \Bigg], \end{aligned} \tag{3}$$

where ϵ_α is the photon polarization vector, and the tensors $V^{\alpha\beta}, A^{\alpha\beta}, F^{\beta}, F_s^\alpha, F_p^\alpha, f$ can be presented in the following way:

$$V^{\alpha\beta}+\frac{p^\alpha}{pq}F^\beta=i\int d^4x e^{iqx}\langle\pi^0(p')|TJ^\alpha(x)(\overline{s}\gamma^\alpha u)\times(0)|K(p)\rangle, \tag{4}$$

$$A^{\alpha\beta}=i\int d^4x e^{iqx}\langle\pi^0(p')|TJ^\alpha(x)(\overline{s}\gamma^\alpha\gamma_5 u)\times(0)|K(p)\rangle,$$

$$F_s^\alpha+\frac{p^\alpha}{pq}f=i\int d^4x e^{iqx}\langle\pi^0(p')|TJ^\alpha(x)(\overline{s}u)(0)\times|K(p)\rangle,$$

$$F_p^\alpha=i\int d^4x e^{iqx}\langle\pi^0(p')|TJ^\alpha(x)(\overline{s}\gamma_5 u)\times(0)|K(p)\rangle,$$

$$F^\beta=\langle\pi^0(p')|(\overline{s}\gamma^\beta u)(0)|K\rangle,$$

$$f=\langle\pi^0(p')|(\overline{s}u)(0)|K(p)\rangle,$$

where J^α is the electromagnetic current. From the Ward identity [11] we have the following relations among the tensors (4):

$$q_\alpha V^{\alpha\beta}=0, \tag{5}$$

$$q_\alpha A^{\alpha\beta}=0,$$

$$q_\alpha F_s^\alpha=0,$$

$$q_\alpha F_p^\alpha=0.$$

Using these relations, we introduce the following parametrization of the tensors:

$$V_{\alpha\beta}=V_1\left(g_{\alpha\beta}-\frac{W_\alpha q_\beta}{qW}\right)+V_2\left(p'_\alpha q_\beta-\frac{p'q}{qW}W_\alpha q_\beta\right)+V_3\left(p'_\alpha W_\beta-\frac{p'q}{qW}W_\alpha W_\beta\right)+V_4\left(p'_\alpha p'_\beta-\frac{p'q}{qW}W_\alpha p'_\beta\right), \tag{6}$$

$$A_{\alpha\beta}=i\epsilon_{\alpha\beta\rho\sigma}(A_1 p'^\rho q^\sigma+A_2 q^\rho W^\sigma)+i\epsilon_{\alpha\lambda\rho\sigma}p'^\lambda q^\rho W^\sigma\times(A_3 W_\beta+A_4 p'_\beta),$$

$$F^\beta=C_1 p'_\beta+C_2(p-p')^\beta,$$

$$F_s^\alpha=S\left(p^\alpha-\frac{pq}{p'q}p'^\alpha\right),$$

$$F_p^\alpha=iP\epsilon^{\alpha\lambda\rho\sigma}p_\lambda p'_\rho q_\sigma,$$

$$W=p_l+p_{\nu_l}.$$

We use the values of the V_i, A_i, C_i form factors calculated in the framework of chiral perturbation theory (χPT) up to $O(p^4)$ [11]. The values of the S, f form factors can be found from those for V_i, C_i using the Ward identities. The derivation of these relations is given in the Appendix, where the value of the P form factor is calculated as well.

In searches for possible *CP*-violating effects, we are interested in the distribution of the partial $K^+(p)\to\pi^0(p')l(p_l)\nu(p_\nu)\gamma(q)$ decay width over the kinematical variable $\boldsymbol{\xi}=\mathbf{q}\cdot[\mathbf{p}_\pi\times\mathbf{p}_l]/m_K^3$ in the K^+ meson rest frame:

$$\rho(\xi)=\frac{d\Gamma}{d\xi}. \tag{7}$$

Obviously, the $\rho(\xi)$ function can be written as

$$\rho=f_{\mathrm{even}}(\xi)+f_{\mathrm{odd}}(\xi),$$

where $f_{\mathrm{even}}(\xi)$ and $f_{\mathrm{odd}}(\xi)$ are even and odd functions of ξ, respectively. $f_{\mathrm{odd}}(\xi)$ can be rewritten as follows:

$$f_{\mathrm{odd}}=g(\xi^2)\xi. \tag{8}$$

Integrating $\rho(\xi)$ over the kinematical region, one can see that the contribution of $f_{\mathrm{odd}}(\xi)$ vanishes.

To analyze $K^+\to\pi^0 l^+\nu_l\gamma$ decay data we introduce the following observable:

$$A_\xi=\frac{N_+-N_-}{N_++N_-}, \tag{9}$$

where N_+ and N_- are the numbers of events with $\xi>0$ and $\xi<0$, respectively. Obviously, the numerator of A_ξ depends only on $f_{\mathrm{odd}}(\xi)$.

Because the V_i, A_i, C_i form factors are real at the tree level of the SM, the distribution of $\rho(\xi)$ is symmetrical with respect to the $\xi=0$ line, i.e., the numbers of events with $\xi>0$ and $\xi<0$ in $K^+\to\pi^0 l^+\nu_l\gamma$ decay are equal and $A_\xi=0$. This can be easily explained: at the SM tree level the form factors V_i, A_i, C_i are real, the matrix element squared depends only on the scalar products of momenta of π, μ, ν, γ, and the terms linear in ξ vanish. Therefore $\rho(\xi)$ is an even function of ξ. The ξ-odd SM terms appear due to the electromagnetic final state interaction. This leads to the appearance of nonzero imaginary parts of the form factors, which, in turn, gives the nonvanishing contribution to the $f_{\mathrm{odd}}(\xi)$ function and asymmetry A_ξ.

The contribution of the one-loop final state interaction to A_ξ was considered earlier in [9]. A calculation using *S*-matrix unitarity led to the following result:

$$A_\xi=1.14\times10^{-4}\,K^+\to\pi^0\mu^+\nu_\mu\gamma, \tag{10}$$

$$A_\xi=-0.59\times10^{-4}\,K^+\to\pi^0 e^+\nu_e\gamma.$$

This result indicates that the SM contribution to $K^+\to\pi^0 l^+\nu_l\gamma$ decay asymmetry is suppressed, thus making the A_ξ observable a good prospect to search for new physics.

Let us consider the value of the asymmetry A_ξ induced by the Lagrangian (2). In the K^+ meson rest frame the squared decay amplitude (3) can be presented as

$$|T|^2=|T_{\rm even}|^2+[\mathrm{Im}(g_v)C_v+\mathrm{Im}(g_a)C_a+\mathrm{Im}(g_s)C_s+\mathrm{Im}(g_p)C_p]m_K^4\xi. \tag{11}$$

The first term is a ξ-even part of the matrix element squared, and the second one is a ξ-odd part; C_a,C_v,C_s,C_p are the kinematical factors, which depend only on the scalar products of the final particle momenta. We do not present here the explicit expressions for C_a,C_v,C_s,C_p, as they seem to be quite cumbersome. It follows from Eq. (11) that the asymmetry has a nonzero value only when there are nonzero imaginary parts of the parameters of the Lagrangian (2).

From the expression for matrix element (3) one can obtain the relation between the C_a,C_v kinematical factors. Let us suppose that we use the model with $\mathrm{Im}(g_v)=-\mathrm{Im}(g_a)$ and $\mathrm{Im}(g_s)=\mathrm{Im}(g_p)=0$. The matrix element in this model differs from the SM one only by a total phase, which cannot lead to nonzero asymmetry. So in this model $C_v-C_a=0$, and since the inner structure of the model does not affect the kinematical factors C_v,C_a, it would be correct to state that $C_v=C_a$ in any model.

Integrating the squared amplitude (11) over the phase space, one can obtain the A_ξ value. The value of this asymmetry, averaged over the kinematical region $E_\gamma>30$ MeV, $\theta_{\gamma l}>20°$ in the kaon rest frame, is given as follows:

$$K^+\to\pi e\nu_e\gamma:$$

$$A_\xi=-[2.9\times10^{-6}\mathrm{Im}(g_s)+3.7\times10^{-5}\mathrm{Im}(g_p)+3.0\times10^{-3}\mathrm{Im}(g_v+g_a)];$$

$$K^+\to\pi\mu\nu_\mu\gamma:$$

$$A_\xi=-[3.6\times10^{-3}\mathrm{Im}(g_s)+1.2\times10^{-2}\mathrm{Im}(g_p)+1.0\times10^{-2}\mathrm{Im}(g_v+g_a)].$$

It should be noticed that in the formula for the asymmetry of the $K^+\to\pi e\nu_e\gamma$ decay amplitude the contributions of the $\mathrm{Im}(g_s),\mathrm{Im}(g_p)$ parameters are suppressed, in contrast with the case of $K^+\to\pi\mu\nu_\mu\gamma$ decay. This suppression is due to kinematical factors in front of these parameters, which are proportional to the masses of the leptons in the final state.

III. $SU(2)_L\times SU(2)_R\times U(1)$ MODELS

In this section extensions of the SM based on the $SU(2)_L\times SU(2)_R\times U(1)$ [12] gauge group are considered. In these models each generation of fermions is formed in $SU(2)_L$ and $SU(2)_R$ doublets. At least one Higgs multiplet $\Phi(2,2,0)$ is introduced to generate the fermion masses:

$$\Phi=\begin{pmatrix}\phi_1^0 & \phi_1^+\\ \phi_2^- & \phi_2^0\end{pmatrix}, \tag{12}$$

whose vacuum expectation value can be presented as follows:

$$\Phi=\begin{pmatrix}k & 0\\ 0 & k'\end{pmatrix}. \tag{13}$$

Generally, the vacuum expectation values k,k', are complex. Additional Higgs multiplets are required to break the $SU(2)_L\times SU(2)_R\times U(1)$ symmetry to $U(1)$. In the simplest case two doublets $\delta_L(2,1,1)$ and $\delta_R(1,2,1)$, are introduced

$$\delta_L=\begin{pmatrix}\delta_L^+\\ \delta_L^0\end{pmatrix},\quad \delta_R=\begin{pmatrix}\delta_R^+\\ \delta_R^0\end{pmatrix}. \tag{14}$$

For a large mass scale M_R it is necessary to have the vacuum expectation value $\langle\delta_R^0\rangle=v_R$ greater than $k,k',\langle\delta_L^0\rangle=v_L$.

Another scenario of spontaneous $SU(2)_L\times SU(2)_R\times U(1)$ symmetry violation is also possible. In this case two triplets $\Delta_L(1,3,2)$ and $\Delta_L(3,1,2)$ are introduced:

$$\Delta_{L,R}=\begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}_{L,R}, \tag{15}$$

whose vacuum expectation values can be expressed as

$$\Delta_{L,R}=\begin{pmatrix}0 & 0\\ v_{L,R} & 0\end{pmatrix}. \tag{16}$$

As in the model with two Higgs doublets, the condition $v_R\gg k,k',v_L$ should be valid for large mass scale appearance.

One can also require the Lagrangian of the model under investigation to be invariant under the following transformations:

$$\Psi_L\leftrightarrow\Psi_R,\quad \delta_R\leftrightarrow\delta_L,\quad \Delta_R\leftrightarrow\Delta_L,\quad \Phi\leftrightarrow\Phi^+, \tag{17}$$

which leads to the fermions $SU(2)_L$, $SU(2)_R$ bosons coupling constants being equal.

In this case, *CP* violation appears due to the Cabibbo-Kobayashi-Maskawa (CKM) matrices in left (K^L) and right (K^R) sectors of the model. The effects of *CP* violation are deeper than in the SM, as the analogue of the CKM matrix for the right sector of the theory contains $N(N+1)/2$ phases, where N is the number of fermion generations. Notice that in our calculations we neglect the neutrino masses, which allows us to neglect the lepton mixing matrices and consider them as diagonal. Depending on the model parameters there are two possible mechanisms of *CP* violation. The first one is spontaneous *CP* violation, due to the complexity of k,k',v_R,v_L, where the matrix of the Yukawa couplings of Φ with the fermions is real. The second scenario assumes the complexity of the Yukawa constants when the vacuum expectation values are real. The latter mechanism applies in the SM. In the general case, both variants are possible.

The interaction of charged gauge bosons with quarks may be written as [13]

$$L=\frac{g_L}{\sqrt{2}}W_L^\mu\bar{U}\gamma_\mu K^L P_L D+\frac{g_R}{\sqrt{2}}W_R^\mu\bar{U}\gamma_\mu K^R P_R D+\text{H.c.},\tag{18}$$

where g_R,g_L are the coupling constants of the right and left sectors of the model, $U^T=(u,c,t)$ and $D^T=(d,s,b)$ are the quark physical states, and $P_{L,R}=(1\mp\gamma_5)/2$. The states W_R,W_L are nonphysical states, but it is possible to make them physical by performing the unitary transformation

$$\begin{pmatrix} W_L\\ W_R\end{pmatrix}=\begin{pmatrix}\cos\eta & -\sin\eta\\ e^{i\omega}\sin\eta & e^{i\omega}\cos\eta\end{pmatrix}\begin{pmatrix}W_1\\ W_2\end{pmatrix},\tag{19}$$

where η is the mixing angle and ω is the phase. In the following calculations the phase factor will be included in the K^R matrix. From the formulas (18), (19) the effective Lagrangian of the $s\to u\mu\nu_\mu$ process can be written as

$$L=-\frac{G_F}{\sqrt{2}}\frac{g_R}{g_L}K_{su}^{R*}\eta[\bar{s}(1-\gamma_5)\gamma_\alpha u][\bar{\nu}(1+\gamma_5)\gamma^\alpha l].\tag{20}$$

Comparing Eq. (20) with the Lagrangian (2), one can obtain the expression for the g_a,g_v parameters:

$$g_v=g_a=-\frac{g_R}{g_L}\frac{K_{su}^{R*}}{\sin\theta_c}\eta.\tag{21}$$

Further, we suppose that the model Lagrangian is invariant under the transformation (17), which gives us the following identity: $g_R=g_L$. Moreover, we suppose that the K_R,K_L matrices are related as follows: $|(K_R)_{ij}|=|(K_L)_{ij}|$. This condition applies when the vacuum expectation values of the Higgs fields are real, i.e., *CP* violation appears due to the complexity of the Yukawa couplings. In this case $K_L=K_R$ [14]. The identity $|(K_R)_{ij}|=|(K_L)_{ij}|$ is also valid in the models with spontaneous *CP* violation. Here, the Yukawa matrix is real and symmetric, but the vacuum expectation values are complex, which leads to the identity $K_L=(K_R)^*$ [15]. Using this relation one can rewrite K_R as follows:

$$K^R=e^{i\gamma}\begin{pmatrix}e^{-i\delta_2}\cos\theta_c & e^{-i\delta_1}\sin\theta_c\\ -e^{-i\delta_1}\sin\theta_c & e^{i\delta_2}\cos\theta_c\end{pmatrix}.\tag{22}$$

From the explicit expression for the K^R matrix the imaginary parts of g_a,g_v can be written as

$$\text{Im}(g_a)=\text{Im}(g_v)=-\eta\sin(\gamma-\delta_1).\tag{23}$$

Bounds on the model parameters, $M_R>715$ GeV, $\eta<0.013$, have been derived from the low-energy data [16].

Using these bounds and the inequality $|\text{Im}(g_a)|=|\text{Im}(g_v)|<\eta$, one can derive the following upper bounds on the A_ξ value:

$$|A_\xi|<2.6\times10^{-4},\quad K^+\to\pi^0\mu\nu_\mu\gamma,$$
$$|A_\xi|<0.8\times10^{-4},\quad K^+\to\pi^0 e\nu_e\gamma.\tag{24}$$

These values of the asymmetry can be experimentally observed only if the experimental statistics collected is about $\sim10^7$ for $K^+\to\pi\mu\nu_\mu\gamma$ decay and $\sim10^8$ for $K^+\to\pi e\nu_e\gamma$ decay.

Let us now estimate the potential for *T*-odd correlation in connection with the search for new physics. Obviously, if *T*-odd correlation coming from new physics is of the same order of magnitude as the SM background (10), an extraction of new physics signal becomes problematic. However, in this case one may put further constraints on the parameters of the Lagrangian under study:

$$|\text{Im}(g_a)|=|\text{Im}(g_v)|<5.7\times10^{-3},\quad K^+\to\pi^0\mu\nu_\mu\gamma,$$
$$|\text{Im}(g_a)|=|\text{Im}(g_v)|<9.8\times10^{-3},\quad K^+\to\pi^0 e\nu_e\gamma.\tag{25}$$

It is instructive to compare these values with the bounds obtained in [16], $|\text{Im}(g_a)|=|\text{Im}(g_v)|<0.013$. Although the improvement of these bounds is not very large, the experiment measuring the asymmetry could give us model independent restrictions on the vector and pseudovector parameters.

IV. MODELS WITH SCALAR INTERACTION

In this section we consider models with $\text{Im}(g_a)=\text{Im}(g_v)=0$. For this case nonzero asymmetry appears due only to nonvanishing values of the $\text{Im}(g_s),\text{Im}(g_p)$ parameters. Among these models there are some leptoquark and multi-Higgs-boson SM extensions [7,8,17].

Note that the $K^+\to\pi e\nu_e\gamma$ decay is not useful to probe such models. This follows from the proportionality of the kinematical factors C_s,C_p entering the formula for asymmetry (12) to the lepton mass, which leads to the suppression of scalar and pseudoscalar contributions to this asymmetry. Moreover, in multi-Higgs-boson models, additional suppression appears due to the fact that the Yukawa couplings are proportional to the fermion mass.

So, of the two decays considered, only $K^+\to\pi\mu\nu_\mu\gamma$ seems useful. To set upper limits on the possible asymmetry value in this decay, let us consider the muon transverse polarization in $K^+\to\pi\mu\nu_\mu$. A model independent investigation of the muon transverse polarization in this decay [7] allows one to claim that P_T is not sensitive to the g_v,g_a,g_p constants.

In order to set bounds on the $\text{Im}(g_s)$ constant, one needs to write down the matrix element of $K(p)^+\to\pi^0(p')\mu(p_\mu)\nu_\mu(p_\nu)$ decay:

$$M = \frac{G_f}{2}\sin\theta_c[f_+(p+p')^\lambda + f_-(p-p')^\lambda]\overline{u(p_\nu)} \times(1+\gamma_5)\gamma_\lambda v(p_\mu). \tag{26}$$

The data from the KEK E246 experiment on transverse polarization measurement give the following result for the value of $\mathrm{Im}(\chi)=\mathrm{Im}(f_-/f_+)$ [1]:

$$\mathrm{Im}(\chi)=[-0.28\pm0.69(\mathrm{stat})\pm0.30(\mathrm{syst})]\times10^{-2}. \tag{27}$$

Using the expression for the effective Lagrangian (2), one can relate the values of $\mathrm{Im}(\chi)$ and $\mathrm{Im}(g_s)$:

$$\mathrm{Im}(\chi)=\mathrm{Im}(g_s)\frac{m_K^2}{m_\mu m_s}. \tag{28}$$

From Eqs. (27), (28) it is easy to obtain the following upper limit: $|\mathrm{Im}(g_s)|<6.7\times10^{-4}$. Further, we assume that $\mathrm{Im}(g_p)\sim-\mathrm{Im}(g_s)$. This assumption is valid in any model if one neglects the mass of the *u* quark. Obviously, within such an approach it is not necessary to consider the inner structure of the models. Using the upper limit on the model parameters one can derive the bounds on the A_ξ asymmetry of the $K^+\to\pi^0\mu\nu_\mu\gamma$ decay:

$$|A_\xi|<6.0\times10^{-6}. \tag{29}$$

Having this upper limit on the A_ξ value, one can suppose that for reliable observation of this asymmetry it is necessary to have more than $\sim10^{10}$ events.

From a comparison of the bound (29) with the SM background we may conclude that there is no possibility of improving the restrictions for scalar and pseudoscalar parameters.

V. CONCLUSION

The asymmetry A_ξ in $K^+\to\pi l\nu\gamma$ decays was investigated in the framework of models corresponding to the effective Lagrangian (2) up to $O(p^4)$ terms of χPT.

It was shown that the scalar and pseudoscalar sectors of the Lagrangian contribute to the asymmetry A_ξ. However, since the kinematical factors in Eq. (11) are proportional to the lepton mass, A_ξ is strongly suppressed in the $K^+\to\pi e\nu_e\gamma$ decay. As for the decay with muons in the final state, the dependence of the asymmetry on the scalar interaction effects is quite strong. The KEK E246 data allow one to obtain strict bounds on the coupling constant, which significantly narrows the search for the scalar and pseudoscalar interaction contributions to asymmetry. It is necessary to have at least $\sim10^{10}$ events to observe this asymmetry experimentally, and the anticipated value of this observable could be

$$|A_\xi|<6.0\times10^{-6}, \tag{30}$$

which is two orders of magnitude less than the SM contribution to A_ξ.

The KEK E246 experiment allows one to set strict enough constraints only for the case of the pseudoscalar and scalar constants, while the vector and pseudovector sectors of the Lagrangian remains obscure. The bounds on these parameters can probably be obtained in the OKA experiment, which will come into operation in the very near future. Our results reveal a high sensitivity of the asymmetry A_ξ to vector and pseudovector interactions of the effective Lagrangian. In addition, in order to search for *CP*-violating effects, one can consider the $K^+_{l3\gamma}$ meson decays with electrons and muons in the final state. Taking into account the bounds on the parameters of the $SU(2)_L\times SU(2)_R\times U(1)$ model, one can get the upper limit on the A_ξ value:

$$|A_\xi|<2.6\times10^{-4},\quad K^+\to\pi^0\mu\nu_\mu\gamma,$$
$$|A_\xi|<0.8\times10^{-4},\quad K^+\to\pi^0 e\nu_e\gamma. \tag{31}$$

Therefore, provided that the statistic in the OKA experiment are increased by an order of magnitude, we claim that it may provide further information about the vector and pseudovector sectors of the studied Lagrangian. It follows from our study that the asymmetry A_ξ may serve as a quite effective observable in the search for new physics.

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APPENDIX

In the framework of χPT the following QCD Lagrangian with external sources is introduced [18]:

$$L=L_{\mathrm{QCD}}+\overline{q}\gamma_\mu(v^\mu+\gamma_5a^\mu)q-\overline{q}(s-i\gamma_5p)q, \tag{A1}$$

where L_{QCD} is the massless QCD Lagrangian, $q^T=(u,d,s)$ are the quark fields, and v_μ,a_μ,s,p are 3×3 Hermitian matrices. It is easy to see that the Lagrangian (A1) is invariant under local transformations $SU(3)_L\times SU(3)_R$:

$$q_L\to g_Lq_L,\quad q_R\to g_Rq_R,\quad s+ip\to g_R(s+ip)g_L^+, \tag{A2}$$
$$l_\mu=g_Ll_\mu g_L^+ + ig_L\partial_\mu g_L^+,\quad r_\mu=g_Rr_\mu g_R^+ + ig_R\partial_\mu g_R^+,$$
$$l_\mu=v_\mu-a_\mu,\quad r_\mu=v_\mu+a_\mu.$$

The effective χPT Lagrangian is constructed [with the symmetry (A2) taken into account] as an expansion in a series of external momenta:

$$L_{\mathrm{eff}}=L_2+L_4+\cdots, \tag{A3}$$

where L_2,L_4 are the effective Lagrangian terms up to $O(p^4),O(p^6)$, respectively. Notice that L_2 is invariant under (A2) transformations, while L_4 invariance is broken due to the chiral anomaly [11,18]. Nevertheless, the effective Lagrangian is invariant under the transformation

$$v_\mu \pm a_\mu \to g(v_\mu \pm a_\mu)g^+ + ig\partial_\mu g^+, \qquad \text{(A4)}$$
$$s+ip \to g(s+ip)g^+$$
$$g \in SU(3).$$

Taking into account the fact that the generating functional is invariant under (A4) transformations,

$$Z[v',a',s',p'] = Z[v,a,s,p], \qquad \text{(A5)}$$

one can transform $g = 1 + i\alpha + O(\alpha^2) \in SU(3)$, and obtain the Ward identities in χPT [11]:

$$\left\langle \alpha \partial_\mu \frac{\delta Z}{\delta v_\mu} \right\rangle = i \left\langle \sum_{I=v,a,s,p} [\alpha, I] \frac{\delta Z}{\delta I} \right\rangle, \qquad \text{(A6)}$$

where $\langle\rangle$ means performing the trace operation.

Let us consider the marix element $\langle 0|Ta^3_\mu(x)a^{4+i5}_\nu(y)V^{4-i5}_\alpha(z)V^{\text{e.m.}}_\beta(w)|0\rangle$, where $a^3_\mu(x)$, $a^{4+i5}_\nu(y)$ are the axial currents corresponding to π^0 and K^+ mesons, $V^{4-i5}(z)$ is the vector current of the $\overline{s} \to \overline{u}$ transition, and $V^{\text{e.m.}}_\beta(w)$ is the electromagnetic current. The divergence ∂^α_z of this matrix element can be obtained using the Ward identities. Therefore, one must replace α in Eq. (A6) by $\lambda^4 - i\lambda^5$ and act upon the matrix element by the operator

$$\hat{A} = \frac{\delta}{\delta a^3_\mu(x)} \frac{\delta}{\delta a^{4+i5}_\nu(y)} \frac{\delta}{\delta V^{\text{e.m.}}_\alpha(z)} \qquad \text{(A7)}$$

At the point $v_\mu = a_\mu = p = 0, s = M$, where M is the quark mass matrix, this expression takes the form

$$\begin{aligned}
&\partial^\alpha_z \langle 0|Ta^3_\mu(x)a^{4+i5}_\nu(y)V^{4-i5}_\alpha(z)V^{\text{e.m.}}_\beta(w)|0\rangle \\
&\quad = i(m_u - m_s)\langle 0|Ta^3_\mu(x)a^{4+i5}_\nu(y)s^{4-i5}(z)V^{\text{e.m.}}_\beta(w)|0\rangle \\
&\qquad + \langle 0|Ta^3_\mu(x)a^{4+i5}_\nu(y)V^{4-i5}_\beta(z)|0\rangle \delta(w-z) \\
&\qquad + \langle 0|Ta^{4+i5}_\nu(y)a^{4-i5}_\mu(z)V^{\text{e.m.}}_\beta(w)|0\rangle \delta(z-x) \\
&\qquad - \frac{1}{2}\langle 0|Ta^3_\mu(x)a^3_\nu(z)V^{\text{e.m.}}_\beta(w)|0\rangle \delta(z-y) \\
&\qquad - \frac{\sqrt{3}}{2}\langle 0|Ta^3_\mu(x)a^8_\nu(z)V^{\text{e.m.}}_\beta(w)|0\rangle \delta(z-y). \qquad \text{(A8)}
\end{aligned}$$

Futher, we use the reduction formulas to relate the vacuum matrix elements of (A8) with those of the $K^+ \to \pi^0$ transition. Obviously, the last three terms in expression (A8) do not contribute to the final result, since they do not contain any pole terms on the π^0 and K^+ meson mass scales simultaneously. Using this fact one can rewrite this expression as follows:

$$\begin{aligned}
&\partial^y_\nu \langle \pi^0|TV^{\text{e.m.}}_\mu(x)V^{4-i5}_\nu(y)|K^+\rangle \\
&\quad = \langle \pi^0|V^{4-i5}_\nu(y)|K^+\rangle \delta(x-y) + i(m_u - m_s) \\
&\qquad \times \langle \pi^0|TV^{\text{e.m.}}_\mu(x)S^{4-i5}(y)|K^+\rangle. \qquad \text{(A9)}
\end{aligned}$$

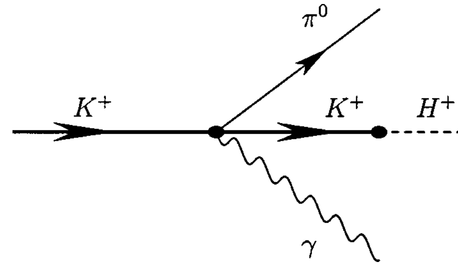

FIG. 1. Feynman diagram that gives a nonzero contribution to the form factor P.

The relation between the scalar and vector form factors in terms of Eq. (4) has the form

$$V^{\mu\nu}W_\nu + \left(F^\mu - \frac{F^\nu q_\nu}{pq} p^\mu \right) = (m_u - m_s)F^\mu_s. \qquad \text{(A10)}$$

Similarly, one can obtain an expression for f:

$$F^\nu(p_\nu - p'_\nu) = (m_u - m_s)f. \qquad \text{(A11)}$$

A nonzero contribution to the form factor P can appear only due to the anomalous term of the effective χPT Lagrangian. It has the following form [11]:

$$L_{\text{anom}}(\Phi^3\gamma) = -i\frac{e\sqrt{2}}{4\pi^2 f^3_\pi}\epsilon^{\mu\nu\rho\sigma}A_\sigma\langle Q\partial_\mu\Phi\partial_\nu\Phi\partial_\rho\Phi\rangle, \qquad \text{(A12)}$$

where Φ is the pseudoscalar meson octet matrix, $Q = 1/3 \times \text{diag}(2,-1,-1)$, and $f_\pi = 93.2$ MeV. The Feynman diagram contributing to the form factor P is shown in Fig. 1. Taking into account this diagram one can rewrite the expression for the form factor in the following form:

$$P = \frac{\sqrt{2}}{4\pi^2 f^2}\frac{1}{W^2 - M^2_K}\frac{M^2_K}{m_s + m_u}, \qquad \text{(A13)}$$

where $W = p - p' - q$.

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