Hadronic *B* decays involving even-parity charmed mesons

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Hadronic B decays containing an even-parity charmed meson in the final state are studied. Specifically we focus on the Cabibbo-allowed decays $\overline{B}\to D^{**}\pi(\rho)$, $D^{**}\overline{D}_s^{(*)}$, $\overline{D}_s^{**}D^{(*)}$ and $\overline{B}_s\to D_s^{**}\pi(\rho)$, where D^{**} denotes generically a p-wave charmed meson. The $B\to D^{**}$ transition form factors are studied in the improved version of the Isgur-Scora-Grinstein-Wise quark model. We apply heavy quark effective theory and chiral symmetry to study the strong decays of p-wave charmed mesons and determine the magnitude of the $D_1^{1/2}$-$D_1^{3/2}$ mixing angle (the superscript standing for the total angular momentum of the light quark). Except for the decay to $D_1(2427)^0\pi^-$ the predictions of $\mathcal{B}(B^-\to D^{**0}\pi^-)$ agree with experiment. The sign of the $D_1^{1/2}$-$D_1^{3/2}$ mixing angle is found to be positive in order to avoid a severe suppression of the production of $D_1(2427)^0\pi^-$. The interference between color-allowed and color-suppressed tree amplitudes is expected to be destructive in the decay $B^-\to D_1(2427)^0\pi^-$. Hence, an observation of the ratio $D_1(2427)^0\pi^-/D_1(2427)^+\pi^-$ can be used to test the relative signs of various form factors as implied by heavy quark symmetry. Although the predicted $B^-\to D_1(2420)^0\rho^-$ at the level of 3×10^{-3} exceeds the present upper limit, it leads to the ratio $D_1(2420)\rho^-/D_1(2420)\pi^-\approx2.6$, as expected from the factorization approach and from the ratio $f_\rho/f_\pi\approx1.6$. Therefore, it is crucial to have a measurement of this mode to test the factorization hypothesis. For $\overline{B}\to\overline{D}_s^{**}D$ decays, it is expected that $\overline{D}_{s0}^{*}D\gtrsim\overline{D}_{s1}D$ as the decay constants of the multiplet (D_{s0}^*,D_{s1}) become the same in the heavy quark limit. The preliminary Belle observation of fairly less abundant production of $\overline{D}_{s0}^*D$ than $\overline{D}_{s1}D$ is thus a surprise. What is the cause for the discrepancy between theory and experiment remains unclear. Meanwhile, it is also important to measure the B decay into $\overline{D}_{s1}(2536)D^{(*)}$ to see if it is suppressed relative to $\overline{D}_{s1}(2463)D^{(*)}$ to test the heavy quark symmetry relation $f_{D_{s1}(2536)}\ll f_{D_{s1}(2463)}$. Under the factorization hypothesis, the production of $\overline{D}_{s2}^*D$ is prohibited as the tensor meson cannot be produced from the $V-A$ current. Nevertheless, it can be induced via final-state interactions or nonfactorizable contributions and hence an observation of $\overline{B}\to\overline{D}_{s2}^*D^{(*)}$ could imply the importance of final-state rescattering effects.

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I. INTRODUCTION

Interest in even-parity charmed mesons has been revived by the recent discovery of a new narrow resonance by BaBar [1]. This state, which can be identified with $J^P=0^+$, is lighter than most theoretical predictions for a 0^+ $c\bar{s}$ state [2]. Moreover, a renewed lattice calculation [3] yields a larger mass than what is observed. This unexpected and surprising disparity between theory and experiment has sparked a flurry of many theory papers. For example, it has been advocated that this new state is a four-quark bound state[1] [4,5] or a DK molecular [9] or even a $D\pi$ atom [10]. On the contrary, it has been put forward that, based on heavy quark effective theory and chiral perturbation theory, the newly observed $D_s(2317)$ is the 0^+ $c\bar{s}$ state and that there is a 1^+ chiral partner with the same mass splitting with respect to the 1^- state as that between the 0^+ and 0^- states [11,12]. The existence of a new narrow resonance with a mass near 2.46 GeV which can be identified with the 1^+ state was first hinted at by BaBar and has been observed and established by CLEO [13] and Belle [14].

Although the $D_{s0}^*(2317)$ and $D_{s1}(2463)$ states were discovered in charm fragmentation of $e^+e^-\to c\bar{c}$, it will be much more difficult to measure the counterpart of $D_{s0}^*(2317)$ and $D_{s1}(2463)$ in the nonstrange charm sector—namely, D_0^* and D_1—owing to their large widths. Indeed, the broad D_0^* and D_1 resonances were explored by Belle [14] in charged B

[1]The low-lying noncharm scalar mesons in the conventional $q\bar{q}'$ states are predicted by the quark potential model to lie in between 1 and 2 GeV, corresponding to the nonet states $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, and $f_0(1500)/f_0(1710)$. The light scalar nonet formed by $\sigma(600)$, $\kappa(800)$, $f_0(980)$, and $a_0(980)$ can be identified primarily as four-quark states [6]. It has been argued [7] that a strong attraction between $(qq)_{\mathbf{3^*}}$ and $(\bar{q}\bar{q})_{\mathbf{3}}$ [6,8], where **3*** and **3** here refer to color, and the absence of the orbital angular momentum barrier in the s-wave four-quark state may explain why the scalar nonet formed by four-quark bound states is lighter than the conventional $q\bar{q}$ nonet. By the same token, it is likely that a scalar $cn\bar{n}\bar{s}$ four-quark state, where $n=u,\ d$, will be lighter than the 0^+ p-wave $c\bar{s}$ state, where a typical potential model prediction gives 2487 MeV [2]. It has been suggested in [4] to search for exotic four-quark $cq\bar{q}\bar{q}$ charmed meson production in B decays. Particularly noteworthy are resonances in the doubly charged $D_s^+\pi^+$ (D^+K^+) and wrong pairing D^+K^- channels. However, contrary to the case of scalar resonances, the 1^+ $D_s(2463)$ state is unlikely a four-quark state as it is heavier than the axial-vector meson formed by $c\bar{s}$. A nonobservation of a heavier and broad 0^+ $c\bar{s}$ state will not support the four-quark interpretation of $D_s(2317)$.

to $D^+\pi^-\pi^-$ and $D^{*+}\pi^-\pi^-$ decays (see Table I). The study of even-parity charmed meson production in B decays, which is the main object of this paper, also provides an opportunity to test heavy quark effective theory.

This work is organized as follows. The masses and widths of p-wave charmed mesons are summarized in Sec. II. In order to determine the mixing angle of the axial-vector charmed mesons, we apply heavy quark effective theory and chiral symmetry to study their strong decays. The decay constants of p-wave charmed mesons and $B\to D^{**}$ form factors are studied in Sec. III within the Isgur-Scora-Grinstein-Wise quark model. The production of p-wave charmed mesons in B decays is studied in detail in Sec. IV. Conclusions are presented in Sec. V.

II. MASS SPECTRUM AND DECAY WIDTH

In the quark model, the even-parity mesons are conventionally classified according to the quantum numbers J,L,S: the scalar and tensor mesons correspond to ${}^{2S+1}L_J={}^3P_0$ and ${}^3P_2$, respectively, and there exit two different axial-vector meson states—namely, ${}^1P_1$ and ${}^3P_1$—which can undergo mixing if the two constituent quarks do not have the same masses. For heavy mesons, the heavy quark spin S_Q decouples from the other degrees of freedom in the heavy quark limit, so that S_Q and the total angular momentum of the light quark j are separately good quantum numbers. The total angular momentum J of the meson is given by $\vec{J}=\vec{j}+\vec{S}_Q$ with $\vec{S}=\vec{s}+\vec{S}_Q$ being the total spin angular momentum. Consequently, it is more natural to use $L_J^j=P_2^{3/2}$, $P_1^{3/2}$, $P_1^{1/2}$, and $P_0^{1/2}$ to classify the first excited heavy meson states where L here is the orbital angular momentum of the light quark. It is obvious that the first and last of these states are ${}^3P_2$ and ${}^3P_0$, while [16]

$$|P_1^{3/2}\rangle=\sqrt{\frac{2}{3}}|{}^1P_1\rangle+\sqrt{\frac{1}{3}}|{}^3P_1\rangle,$$

$$|P_1^{1/2}\rangle=-\sqrt{\frac{1}{3}}|{}^1P_1\rangle+\sqrt{\frac{2}{3}}|{}^3P_1\rangle. \qquad (2.1)$$

In the heavy quark limit, the physical eigenstates with $J^P=1^+$ are $P_1^{3/2}$ and $P_1^{1/2}$ rather than ${}^3P_1$ and ${}^1P_1$.

The masses and decay widths of even-parity[2] (or p-wave) charmed mesons D_J^* and D_{sJ}^* are summarized in Table I. We shall use 1^+ and $1'^+$ or D_1 and D_1' to distinguish between two different physical axial-vector charmed meson states. As we shall see below, the physical 1^+ state is primarily $P_1^{1/2}$, while $1'^+$ is predominately $P_1^{3/2}$. A similar broad D_1 state not listed in Table I was reported by CLEO [17] with $M=2461^{+42}_{-35}$ MeV and $\Gamma=290^{+104}_{-83}$ MeV. For the known narrow resonances, the Belle measurement of the D_2^{*0} width (see Table I) is substantially higher than the current world average of 23 ± 5 MeV [15].

TABLE I. The masses and decay widths of even-parity charmed mesons. We follow the naming scheme of the Particle Data Group [15] to add a superscript "*" to the states if the spin-parity is in the "normal" sense, $J^P=0^+,1^-,2^+,\ldots$. The four p-wave charmed meson states are thus denoted by D_0^*, D_1, D_1', and D_2^*. In the heavy quark limit, D_1 has $j=1/2$ and D_1' has $j=3/2$ with j being the total angular momentum of the light degrees of freedom.

State	Mass (MeV)	Width (MeV)	Ref.
$D_0^*(2308)^0$	$2308\pm17\pm15\pm28$	$276\pm21\pm18\pm60$	[14]
$D_1(2427)^0$	$2427\pm26\pm20\pm15$	$384^{+107}_{-75}\pm24\pm70$	[14]
$D_1'(2420)^0$	2422.2 ± 1.8	$18.9^{+4.6}_{-3.5}$	[15]
	$2421.4\pm1.5\pm0.4\pm0.8$	$23.7\pm2.70.2\pm4.0$	[14]
$D_1'(2420)^\pm$	2427 ± 5	28 ± 8	[15]
$D_2^*(2460)^0$	2458.9 ± 2.0	23 ± 5	[15]
	$2461.6\pm2.1\pm0.5\pm3.3$	$45.6\pm4.4\pm6.5\pm1.6$	[14]
$D_2^*(2460)^\pm$	2459 ± 4	25^{+8}_{-7}	[15]
$D_{s0}^*(2317)$	2317.3 ± 0.4	<7	[1,13, 14]
$D_{s1}(2463)$	$2463.6\pm1.7\pm1.0$	<7	[13, 14]
$D_{s1}'(2536)$	$2535.35\pm0.34\pm0.5$	<2.3	[15]
$D_{s2}^*(2573)$	2572.4 ± 1.5	15^{+5}_{-4}	[15]

In the heavy quark limit, the states within the chiral doublets $(0^+,1^+)$ with $j=1/2$ and $(1'^+,2^+)$ with $j=3/2$ are degenerate. After spontaneous chiral symmetry breaking, 0^+ states acquire masses while 0^- states become massless Goldstone bosons. As shown in [11], the fine splitting between 0^+ and 0^- is proportional to the constituent quark mass. The hyperfine mass splittings of the four p-wave charmed meson states arise from spin-orbit and tensor-force interactions [see Eq. (2.21) below], while the spin-spin interaction is solely responsible for the hyperfine splitting within the multiplet $(0^-,1^-)$.

From Table I and the given masses of pseudoscalar and vector charmed mesons in the PDG [15], it is found empirically that the hyperfine splittings within the chiral multiplets $(0^+,1^+)$, $(1'^+,2^+)$, and $(0^-,1^-)$ are independent of the flavor of the light quark:

$$m(D_2^*)-m(D_1')\approx m(D_{s2}^*)-m(D_{s1}')\approx 37 \text{ MeV},$$

$$m(D_{s1})-m(D_{s0}^*)\approx m(D_s^*)-m(D_s)=144 \text{ MeV},$$

$$m(D_1)-m(D_0^*)\approx m(D^*)-m(D)=143 \text{ MeV}. \qquad (2.2)$$

However, the fine splittings

$$m(D_{s0}^*)-m(D_s)\approx m(D_{s1})-m(D_s^*)\approx 350 \text{ MeV},$$

$$m(D_0^*)-m(D)\approx m(D_1)-m(D^*)\approx 430 \text{ MeV} \qquad (2.3)$$

[2]If the even-parity mesons are the bound states of four quarks, they are in an orbital s wave. In this case, one uses $J^P=0^+$ rather ${}^3P_0$ to denote scalar mesons, for example.

depend on the light quark flavor.[3] Since the fine splitting between 0^+ and 0^- or 1^+ and 1^- should be heavy flavor independent in the heavy quark limit, the experimental result (2.3) implies that the fine splitting is light quark mass dependent. Indeed, if the first line rather than the second line of Eq. (2.3) is employed as an input for the fine splittings of nonstrange charmed mesons, one will predict [11]

$$M(D_0^{*\pm})=2217 \text{ MeV}, \quad M(D_0^{*0})=2212 \text{ MeV},$$
$$M(D_1^{\pm})=2358 \text{ MeV}, \quad M(D_1^0)=2355 \text{ MeV}, \tag{2.6}$$

which are evidently smaller than what are measured by Belle [14].

It is interesting to note that the mass difference between strange and nonstrange charmed mesons is of order 100–110 MeV for 0^-, 1^-, $1'^+$, and 2^+ as expected from the quark model. As a consequence, the experimental fact that $m(D_{s0}^*)\approx m(D_0^*)$ and $m(D_{s1})\sim m(D_1)$ is very surprising.

In the heavy quark limit, the physical mass eigenstates D_1 and D_1' can be identified with $P_1^{1/2}$ and $P_1^{3/2}$, respectively. However, beyond the heavy quark limit, there is a mixing between $P_1^{1/2}$ and $P_1^{3/2}$, denoted by $D_1^{1/2}$ and $D_1^{3/2}$, respectively,

$$D_1(2427)=D_1^{1/2}\cos\theta+D_1^{3/2}\sin\theta,$$
$$D_1'(2420)=-D_1^{1/2}\sin\theta+D_1^{3/2}\cos\theta. \tag{2.7}$$

Likewise for strange axial-vector charmed mesons,

$$D_{s1}(2463)=D_{s1}^{1/2}\cos\theta_s+D_{s1}^{3/2}\sin\theta_s,$$
$$D_{s1}'(2536)=-D_{s1}^{1/2}\sin\theta_s+D_{s1}^{3/2}\cos\theta_s. \tag{2.8}$$

Since $D_1^{1/2}$ is much broader than $D_1^{3/2}$ as we shall see shortly, the decay width of $D_1'(2420)$ is sensitive to the mixing angle θ. Our task is to determine the $D_1^{1/2}$-$D_1^{3/2}$ mixing angle from the measured widths. In contrast, the present upper limits on the widths of $D_{s1}(2463)$ and $D_{s1}'(2536)$ do not allow us to get any constraints on the mixing angle θ_s. Hence, we will turn to the quark potential model to extract θ_s as will be shown below.

It is suitable and convenient to study the strong decays of heavy mesons within the framework of heavy quark effective theory in which heavy quark symmetry and chiral symmetry are combined [20]. It is straightforward to generalize the formalism to heavy mesons in p-wave excited states [21]. The decay D_0^* undergoes an s-wave hadronic decay to $D\pi$, while $D_1^{1/2}$ can decay into D^* by s-wave and d-wave pion emissions but only the former is allowed in the heavy quark limit $m_c\to\infty$:

$$\Gamma(D_0^*\to D\pi)=g^2_{D_0^*D\pi}\frac{p_c}{8\pi m^2_{D_0}},$$
$$\Gamma(D_1^{1/2}\to D^*\pi)=g^2_{D_1^{1/2}D^*\pi}\frac{p_c}{8\pi m^2_{D_1^{1/2}}}, \tag{2.9}$$

where p_c is the c.m. momentum of the final-state particles in the B rest frame. The tensor meson D_2^* decays into D^* or D via d-wave pion emission. In the heavy quark limit where the total angular momentum j of the light quark is conserved, $D_1^{3/2}\to D\pi$ is prohibited by heavy quark spin symmetry. The explicit expressions for the decay rates are [21]

$$\Gamma(D_1^{3/2}\to D^*\pi)=\frac{1}{6\pi}\frac{m_{D^*}}{m_{D_1^{3/2}}}\frac{h'^2}{\Lambda_\chi^2}\frac{p_c^5}{f_\pi^2},$$
$$\Gamma(D_2^*\to D^*\pi)=\frac{1}{10\pi}\frac{m_{D^*}}{m_{D_2}}\frac{h'^2}{\Lambda_\chi^2}\frac{p_c^5}{f_\pi^2},$$
$$\Gamma(D_2^*\to D\pi)=\frac{1}{15\pi}\frac{m_{D}}{m_{D_2}}\frac{h'^2}{\Lambda_\chi^2}\frac{p_c^5}{f_\pi^2}, \tag{2.10}$$

where Λ_χ is a chiral symmetry breaking scale, $f_\pi=132$ MeV, and h' is a heavy-flavor-independent coupling constant. The p_c^5 dependence of the decay rate indicates the d-wave nature of pion emission. From Eq. (2.10) we obtain

$$\frac{\Gamma(D_2^{*0}\to D^+\pi^-)}{\Gamma(D_2^{*0}\to D^{*+}\pi^-)}=\frac{2}{3}\frac{m_D}{m_{D^*}}\left(\frac{p_c(D_2^*\to D\pi)}{p_c(D_2^*\to D^*\pi)}\right)^5=2.3, \tag{2.11}$$

in excellent agreement with the measured value of 2.3 ± 0.6 [15].

Since the d-wave decay is severely phase-space suppressed, it is evident that D_0^* and D_1 are very broad, of order 250 MeV in their widths, whereas D_1' and D_2^* are narrow with widths of order 20 MeV.

The strong couplings appearing in Eq. (2.9) are given by

$$g_{D_0^*D\pi}=\sqrt{m_{D_0}m_D}\,\frac{m^2_{D_0}-m^2_D}{m_{D_0}}\,\frac{h}{f_\pi},$$
$$g_{D_1^{1/2}D^*\pi}=\sqrt{m_{D_1^{1/2}}m_{D^*}}\,\frac{m^2_{D_1^{1/2}}-m^2_{D^*}}{m_{D_1^{1/2}}}\,\frac{h}{f_\pi}, \tag{2.12}$$

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[3]Likewise, considering the spin-averaged masses of the doublets $(0^+,1^+)$ and $(1'^+,2^+)$,

$$\overline{m}_{01}(D)\equiv\frac{1}{4}m(D_0^*)+\frac{3}{4}m(D_1),\quad \overline{m}_{12}(D)\equiv\frac{3}{8}m(D_1')+\frac{5}{8}m(D_2^*), \tag{2.4}$$

the hyperfine mass splittings

$$\overline{m}_{12}(D)-\overline{m}_{01}(D)\approx48 \text{ MeV},\quad \overline{m}_{12}(D_s)-\overline{m}_{01}(D_s)\approx132 \text{ MeV} \tag{2.5}$$

also depend on the light quark flavor. Based on a quark-meson model, the spin-weighted masses $\overline{m}_{01}(D)=2165\pm50$ MeV [18] and $\overline{m}_{01}(D_s)=2411\pm25$ MeV [19] were predicted, while experimentally they are very similar (see Table I).

with h being another heavy-flavor-independent coupling constant in the effective Lagrangian [21]. It can be extracted from the measured width of $D_0^*(2308)$ (see Table I) to be

$$h=0.65\pm0.12. \tag{2.13}$$

From the averaged width 29.4±4.2 MeV measured for D_2^{*0} we obtain

$$\frac{h'}{\Lambda_\chi}=0.67\pm0.05\ \ \mathrm{GeV}^{-1}. \tag{2.14}$$

Substituting the couplings h and h' into Eqs. (2.9) and (2.10) leads to

$$\Gamma(D_1^{3/2}\to D^*\pi)=10.5\pm1.5\ \ \mathrm{MeV},$$
$$\Gamma(D_1^{1/2}\to D^*\pi)=248\pm92\ \ \mathrm{MeV}, \tag{2.15}$$

where we have assumed that $D_1^{3/2}$ has a mass close to $D_1'(2420)$ and $m(D_1^{1/2})\approx m(D_1(2427))$. Therefore, $D_1^{3/2}$ is much narrower than $D_1^{1/2}$ owing to the phase-space suppression for d waves. However, the physical state $D_1'(2420)$ can receive an s-wave contribution as there is a mixing between $D_1^{1/2}$ and $D_1^{3/2}$ beyond the heavy quark limit. The observed narrowness of $D_1'(2420)$ indicates that the mixing angle should be small; that is, $D_1'(2420)$ should be dominated by $D_1^{3/2}$ while $D_1(2427)$ is primarily the $D_1^{1/2}$ state. Using

$$\Gamma[D_1'(2420)]=\Gamma(D_1^{3/2}\to D^*\pi)\cos^2\theta + \Gamma(D_1^{1/2}\to D^*\pi)\sin^2\theta \tag{2.16}$$

and the averaged width 20.9±2.1 MeV for $D_1'(2420)^0$, it is found[4] that

$$\theta=\pm(12.1^{+6.6}_{-4.4})^\circ. \tag{2.17}$$

We shall see in Sec. IV that a positive mixing angle is preferred by a study of $D_1(2427)^0\pi^-$ production in B decays.

The scalar resonance $D_{s0}^*(2317)$ is below the threshold of DK and its only allowed strong decay $D_{s0}^*(2317)\to D_s\pi^0$ is isospin violating. Therefore, it is extremely narrow with a width of order 10 keV [4,11,22]. As for $D_{s1}(2463)$, it is below the D^*K threshold and its decay to DK is forbidden by parity and angular momentum conservation. Hence, the allowed strong decays are $D_s^*\pi^0$, $D_{s0}^*\pi^0$, $D_s\pi\pi$, and $D_s\pi\pi\pi$. The isospin-violating decay $D_{s1}(2463)\to D_s^*\pi^0$ has a rate similar to $D_{s0}^*(2317)\to D_s\pi^0$. At first sight, it is tempting to argue that $D_s\pi\pi$ could dominate over $D_s^*\pi^0$ as the former can proceed without violating isospin symmetry. However, a detailed analysis shows this is not the case. The decay $D_{s1}\to D_s\pi\pi$ arises from the weak transitions $D_{s1}(2463)\to D_s\{\sigma(600),f_0(980),\ldots\}$ followed by the strong decays $\{\sigma(600),f_0(980),\ldots\}\to\pi\pi$. Consider the dominant contributions from the intermediate states $\sigma(600)$, $f_0(980)$ and their mixing,

$$f_0=s\bar{s}\cos\phi+n\bar{n}\sin\phi,\quad \sigma=-s\bar{s}\sin\phi+n\bar{n}\cos\phi, \tag{2.18}$$

with $n\bar{n}\equiv(u\bar{u}+d\bar{d})/\sqrt{2}$. The σ-f_0 mixing angle can be inferred from various processes; see [23] for a summary. In general, the mixing angle is small so that $f_0(980)$ has a large $s\bar{s}$ component while σ is primarily $n\bar{n}$. The f_0 production is favored by the weak decay of D_{s1} into D_s, but its contribution to $\pi\pi$ is suppressed by the large off shellness of $f_0(980)$, recalling that the mass difference between $D_{s1}(2463)$ and D_s is only 494 MeV. In contrast, $\sigma(600)$ is favored by phase-space considerations and yet its contribution is suppressed by the small σ-f_0 mixing angle. As a net result, although the strong decay into $D_s\pi\pi$ is isospin conserving, its Okubo-Zweig-Iizuka (OZI) suppression is more severe than the isospin one for $D_s^*\pi^0$. This is confirmed by a recent measurement of CLEO [13]:

$$\frac{\mathcal{B}(D_{s1}(2463)\to D_s\pi^+\pi^-)}{\mathcal{B}(D_{s1}(2463)\to D_s^*\pi^0)}<0.08. \tag{2.19}$$

As for the electromagnetic decays of $D_{s1}(2463)$, CLEO and Belle found

$$\frac{\mathcal{B}(D_{s1}(2463)\to D_s\gamma)}{\mathcal{B}(D_{s1}(2463)\to D_s^*\pi^0)}=\begin{cases}<0.49 & \text{CLEO [13]},\\ 0.21\pm0.07\pm0.03 & \text{Belle [14]}.\end{cases} \tag{2.20}$$

Hence, just as its 0^+ partner $D_{s0}^*(2317)$, $D_{s1}(2463)$ is also extremely narrow. A theoretical estimation yields 38.2 keV for its width [11].

Equation (2.14) leads to $\Gamma(D_{s2}^*)=12.6$ MeV, in agreement with experiment (see Table I). Since $\Gamma(D_{s1}^{3/2})=280$ keV followed from Eqs. (2.10) and (2.14) and $D_{s1}^{1/2}$ is very narrow as its mass is close to $D_{s1}(2463)$ which is below D^*K threshold, the decay width of $D_{s1}'(2536)$ is thus at most of order 0.3 MeV and is consistent with the experimental limit 2.3 MeV [15]. In short, while D_1' and D_2^* are rather narrow, D_0^* and D_1 are quite broad as they are allowed to have s-wave hadronic decays. In sharp contrast, D_{s0}^* and D_{s1} are even much narrower than D_{s1}' as their allowed strong decays are isospin violating.

Since the width of $D_{s1}'(2536)$ has not been measured, we will appeal to the quark potential model to estimate the $D_{s1}^{1/2}$-$D_{s1}^{3/2}$ mixing angle [see Eq. (2.8)]. It is known that spin-orbital and tensor-force interactions are responsible for the mass splitting of the four p-wave charmed mesons. In the quark potential model the relevant mass operator has the form [24]

$$M=\lambda\boldsymbol{\ell}\cdot\mathbf{s}_1+4\tau\boldsymbol{\ell}\cdot\mathbf{s}_2+\tau S_{12}, \tag{2.21}$$

[4]The $D_1^{1/2}$-$D_1^{3/2}$ mixing angle was just reported to be $\theta=0.10\pm0.03\pm0.02\pm0.02$ rad$=(5.7\pm2.4)^\circ$ by Belle through a detailed $B\to D^*\pi\pi$ analysis [14]. This is consistent with the result of Eq. (2.17).

where $\mathbf{s}_1$ and $\mathbf{s}_2$ refer to the spin of the light and heavy quarks, respectively, and

$$\lambda = \frac{1}{2m_q^2}\left[\frac{V'}{r}\left(1+\frac{2m_q}{m_c}\right)-\frac{S'}{r}\right],$$

$$\tau = \frac{1}{4m_q m_c}\frac{V'}{r}, \tag{2.22}$$

where $V(r)$ is the zero component of a vector potential, $S(r)$ is a scalar potential responsible for confinement, and S_{12} is the tensor-force operator:

$$S_{12}=3\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2. \tag{2.23}$$

Note that the assumption of a Coulomb-like potential for $V(r)$ has been made in deriving Eq. (2.21) [24]. Under this hypothesis, the mass splitting is governed by the two parameters λ and τ.

Following Cahn and Jackson [24], the masses of the $J=2$ and $J=0$ states read

$$M_2=\frac{\lambda}{2}+\frac{8}{5}\tau+c, \quad M_0=-\lambda-8\tau+c, \tag{2.24}$$

while the masses of the two $J=1$ states obtained by diagonalizing the matrix in the $|J,j,m\rangle=|1,3/2,m\rangle$ and $|1,1/2,m\rangle$ bases are (up to a common mass c)

$$\begin{pmatrix} \frac{\lambda}{2}-\frac{8}{3}\tau & -2\sqrt{\frac{2}{3}}\tau \\ -2\sqrt{\frac{2}{3}}\tau & -\lambda+\frac{8}{3}\tau \end{pmatrix}. \tag{2.25}$$

It is clear that the mixing vanishes in the heavy quark limit $m_c\to\infty$. However, $1/m_c$ corrections will allow charm quark spin to flip and mix $D_1^{1/2}$ and $D_1^{3/2}$. The two eigenmasses for $J=1$ are then

$$M_{1\pm}=-\frac{\lambda}{4}+c\pm\sqrt{\frac{\lambda^2}{16}+\frac{1}{2}(\lambda-4\tau)^2}. \tag{2.26}$$

From Eqs. (2.8) and (2.25) we arrive at

$$\theta=\sin^{-1}\left(\frac{-R_+}{\sqrt{1+R_+^2}}\right) \quad \text{with} \quad R_+=\frac{\frac{\lambda}{2}-\frac{8}{3}\tau-M_{1+}}{2\sqrt{\frac{2}{3}}\tau}. \tag{2.27}$$

The parameters λ and τ are obtained by a global fit to the charm spectroscopy. For D_s^{**} mesons, it is found that

$$\tau\approx 12 \text{ MeV}, \quad \lambda\approx 104 \text{ MeV}, \quad \theta_s\approx 7°. \tag{2.28}$$

As pointed out in [24], a positive spin-orbit energy λ implies a less important scalar potential S. On the contrary, the existing potential model calculation such as the one by Di Pierro and Eichten [2] yields $\lambda<0$ [24] or a very strong confining potential S. This will cause a reversed splitting: namely, $j=1/2$ states lying above $j=3/2$ states. However, we will not address this issue here. For D^{**} mesons, the parameters λ and τ fall into some large regions because of large uncertainties associated with the measured masses of D_0^* and D_1. Hence, the magnitude and even the sign of the $D_1^{1/2}$-$D_1^{3/2}$ mixing angle θ at present cannot be fixed within this approach. Instead, we have used heavy quark effective theory together with the measured widths to extract $|\theta|$. As will be seen below, the sign of θ can be inferred from a study of the p-wave charmed meson production in B decays.

III. DECAY CONSTANTS AND FORM FACTORS

A. Decay constants

The decay constants of scalar and pseudoscalar mesons are defined by

$$\langle 0|A_\mu|P(q)\rangle=if_P q_\mu, \quad \langle 0|V_\mu|S(q)\rangle=f_S q_\mu. \tag{3.1}$$

It is known that the decay constants of noncharm light scalar mesons are smaller than that of pseudoscalar mesons as they vanish in the SU(3) limit. For the neutral scalars $\sigma(600)$, $f_0(980)$, and $a_0^0(980)$, the decay constant must be zero owing to charge conjugation invariance or conservation of vector current:

$$f_\sigma=f_{f_0}=f_{a_0^0}=0. \tag{3.2}$$

Applying the equation of motion, it is easily seen that the decay constant of K_0^{*+} (a_0^+) is proportional to the mass difference between the constituent s (d) and u quarks. Consequently, the decay constant of the charged $a_0(980)$ is very small, while the one for $K_0^*(1430)$ is less suppressed. A calculation based on the finite-energy sum rules [25] yields

$$f_{a_0^\pm}=1.1 \text{ MeV}, \quad f_{K_0^*}=42 \text{ MeV}. \tag{3.3}$$

Contrary to the noncharm scalar resonances, the decay constant of the scalar charmed meson is not expected to be suppressed because of charm and light quark mass imbalance. Applying the equation of motion again leads to

$$m_{K_0^*}^2 f_{K_0^*}=i(m_s-m_u)\langle K_0^*|\bar{s}u|0\rangle,$$

$$m_{D_0^*}^2 f_{D_0^*}=i(m_c-m_u)\langle D_0^*|\bar{c}u|0\rangle. \tag{3.4}$$

For a crude estimate, we assume $\langle D_0^*|\bar{c}u|0\rangle\approx\langle K_0^*|\bar{s}u|0\rangle$ and obtain

$$f_{D_0^*}\approx 160 \text{ MeV}. \tag{3.5}$$

This is comparable to $f_D\approx 200$ MeV, the decay constant of the pseudoscalar D meson.

The decay constants of the axial-vector charmed mesons are defined by

$$\langle 0|A_\mu|D_1^{1/2}(q,\varepsilon)\rangle = f_{D_1^{1/2}} m_{D_1^{1/2}} \varepsilon_\mu,$$
$$\langle 0|A_\mu|D_1^{3/2}(q,\varepsilon)\rangle = f_{D_1^{3/2}} m_{D_1^{3/2}} \varepsilon_\mu. \quad (3.6)$$

It has been shown that, in the heavy quark limit [26–28],

$$f_{D_1^{1/2}} = f_{D_0^*}, \quad f_{D_1^{3/2}} = 0. \quad (3.7)$$

Since the decay constant of D_2^* vanishes irrespective of heavy quark symmetry (see below), the charmed mesons within the multiplet $(0^+,1^+)$ or $(1'^+,2^+)$ thus have the same decay constant. This is opposite to the case of light p-wave mesons where the decay constant of ${}^1P_1$ meson vanishes in the SU(3) limit [29] based on the argument that for noncharm axial vector mesons, the ${}^3P_1$ and ${}^1P_1$ states transfer under charge conjunction as

$$M_a^b({}^3P_1) \to M_b^a({}^3P_1),$$
$$M_a^b({}^1P_1) \to -M_b^a({}^1P_1), \quad (a=1,2,3), \quad (3.8)$$

where the axial-vector mesons are represented by a 3×3 matrix. Since the weak axial-vector current transfers as $(A_\mu)_a^b \to (A_\mu)_b^a$ under charge conjugation, it is clear that the decay constant of the ${}^1P_1$ meson vanishes in the SU(3) limit [29].

The polarization tensor $\varepsilon_{\mu\nu}$ of a tensor meson satisfies the relations

$$\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu}, \quad \varepsilon^\mu{}_\mu = 0, \quad p_\mu \varepsilon^{\mu\nu} = p_\nu \varepsilon^{\mu\nu} = 0. \quad (3.9)$$

Therefore,

$$\langle 0|(V-A)_\mu|D_2^*(\varepsilon,p)\rangle = a\varepsilon_{\mu\nu}p^\nu + b\varepsilon^\nu{}_\nu p_\mu = 0. \quad (3.10)$$

The above relation in general follows from Lorentz covariance and parity considerations. Hence the decay constant of the tensor meson vanishes; that is, the tensor meson D_2^* cannot be produced from the $V-A$ current.

Beyond the heavy quark limit, the relations (3.7) receive large $1/m_c$ corrections which have been estimated in [28] using the relativistic quark model. In the present paper we shall use $f_\rho = 216$ MeV and (in units of MeV)

$$f_D = 200, \quad f_{D_s} = 230, \quad f_{D_s^*} = 230,$$
$$f_{D_0} = 160, \quad f_{D_1^{1/2}} = 120, \quad f_{D_1^{3/2}} = 40,$$
$$f_{D_{s0}} = 140, \quad f_{D_{s1}^{1/2}} = 170, \quad f_{D_{s1}^{3/2}} = 70. \quad (3.11)$$

Note that the measurements of $B\to D_s^{(*)}D^{(*)}$ [15,30] indicate that the decay constants of D_s^* and D_s are similar.

B. Form factors

Form factors for $B\to M$ transitions with M being a parity-odd meson are given by [31]

$$\langle P(p)|V_\mu|B(p_B)\rangle = \left((p_B+p)_\mu - \frac{m_B^2-m_P^2}{q^2}q_\mu\right)F_1^{BP}(q^2) + \frac{m_B^2-m_P^2}{q^2}q_\mu F_0^{BP}(q^2),$$

$$\langle V(p,\varepsilon)|V_\mu|B(p_B)\rangle = \frac{2}{m_P+m_V}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha p^\beta V(q^2),$$

$$\langle V(p,\varepsilon)|A_\mu|B(p_B)\rangle = i\left\{(m_P+m_V)\varepsilon_\mu^* A_1(q^2) - \frac{\varepsilon^*\cdot p_B}{m_P+m_V}(p_B+p)_\mu A_2(q^2) - 2m_V\frac{\varepsilon^*\cdot p_B}{q^2}q_\mu[A_3(q^2)-A_0(q^2)]\right\}, \quad (3.12)$$

where $q=p_B-p$, $F_1(0)=F_0(0)$, $A_3(0)=A_0(0)$, and

$$A_3(q^2) = \frac{m_P+m_V}{2m_V}A_1(q^2) - \frac{m_P-m_V}{2m_V}A_2(q^2). \quad (3.13)$$

For $B\to P$ and $B\to V$ form factors, we will use the Melikhov-Stech (MS) model [32] based on the constituent quark picture. Other form factor models give similar results.

The general expressions for $B\to D^{**}$ transitions (D^{**} being a p-wave charmed meson) are given by [33]

$$\langle D_0^*(p)|A_\mu|B(p_B)\rangle = i[u_+(q^2)(p_B+p)_\mu + u_-(q^2)(p_B-p)_\mu],$$

$$\langle D_1^{1/2}(p,\varepsilon)|V_\mu|B(p_B)\rangle = i[\ell_{1/2}(q^2)\varepsilon_\mu^* + c_+^{1/2}(q^2)(\varepsilon^*\cdot p_B)(p_B+p)_\mu + c_-^{1/2}(q^2)(\varepsilon^*\cdot p_B)(p_B-p)_\mu],$$

$$\langle D_1^{1/2}(p,\varepsilon)|A_\mu|B(p_B)\rangle = -q_{1/2}(q^2)\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}(p_B+p)^\rho(p_B-p)^\sigma,$$

$$\langle D_1^{3/2}(p,\varepsilon)|V_\mu|B(p_B)\rangle = i[\ell_{3/2}(q^2)\varepsilon_\mu^* + c_+^{3/2}(q^2)(\varepsilon^*\cdot p_B)(p_B+p)_\mu + c_-^{3/2}(q^2)(\varepsilon^*\cdot p_B)(p_B-p)_\mu],$$

$$\langle D_1^{3/2}(p,\varepsilon)|A_\mu|B(p_B)\rangle = -q_{3/2}(q^2)\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}(p_B+p)^\rho(p_B-p)^\sigma,$$

$$\langle D_2^*(p,\varepsilon)|V_\mu|B(p_B)\rangle = h(q^2)\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu\alpha}(p_B)_\alpha(p_B+p)^\rho(p_B-p)^\sigma,$$

$$\langle D_2^*(p,\varepsilon)|A_\mu|B(p_B)\rangle = -i[k(q^2)\varepsilon_{\mu\nu}^* p_B^\nu + b_+(q^2)\varepsilon_{\alpha\beta}^* p_B^\alpha p_B^\beta(p_B+p)_\mu + b_-(q^2)\varepsilon_{\alpha\beta}^* p_B^\alpha p_B^\beta(p_B-p)_\mu]. \tag{3.14}$$

In order to know the sign of various form factors appearing in Eq. (3.14), it is instructive to check the heavy quark limit behavior of $B\to D^{**}$ transitions which have the form [16]

$$\langle D_0^*(v')|A_\mu|B(v)\rangle = \sqrt{m_B m_{D_0}}\,2\,\tau_{1/2}(\omega)\,i(v'-v)_\mu,$$

$$\langle D_1^{1/2}(v',\varepsilon)|V_\mu|B(v)\rangle = \sqrt{m_B m_{D_1^{1/2}}}\,2\,\tau_{1/2}(\omega)\,i[(\omega-1)\varepsilon_\mu^* - (\varepsilon^*\cdot v)v'_\mu],$$

$$\langle D_1^{1/2}(v',\varepsilon)|A_\mu|B(v)\rangle = \sqrt{m_B m_{D_1^{1/2}}}\,2\,\tau_{1/2}(\omega)(-)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta,$$

$$\langle D_1^{3/2}(v',\varepsilon)|V_\mu|B(v)\rangle = \sqrt{\frac{1}{2}m_B m_{D_1^{3/2}}}\,\tau_{3/2}(\omega)\,i\{(1-\omega^2)\varepsilon_\mu^* - (\varepsilon^*\cdot v)[3v_\mu - (\omega-2)v'_\mu]\},$$

$$\langle D_1^{3/2}(v',\varepsilon)|A_\mu|B(v)\rangle = \sqrt{\frac{1}{2}m_B m_{D_1^{3/2}}}\,\tau_{3/2}(\omega)(\omega+1)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta,$$

$$\langle D_2^*(v',\varepsilon)|V_\mu|B(v)\rangle = \sqrt{3m_B m_{D_2}}\,\tau_{3/2}(\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu\gamma}v_\gamma v'^\alpha v^\beta,$$

$$\langle D_2^*(v',\varepsilon)|A_\mu|B(v)\rangle = \sqrt{3m_B m_{D_2}}\,\tau_{3/2}(\omega)(-i)\{(\omega+1)\varepsilon_{\mu\nu}^* v^\nu - \varepsilon_{\alpha\beta}^* v^\alpha v^\beta v'_\mu\}, \tag{3.15}$$

where $\omega \equiv v\cdot v'$ and there are two independent functions $\tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ first introduced in [16]. It is easily seen that the matrix elements of weak currents vanish at the zero recoil point $\omega=1$ owing to the orthogonality of the wave functions of B and D^{**}. The universal functions $\tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ are conventionally parametrized as

$$\tau_i(\omega) = \tau_i(1)[1-\rho_i^2(\omega-1)] \tag{3.16}$$

for $i=1/2$ and 3/2. The slope parameter ρ^2 can be related to $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ via the Bjorken sum rule. Comparing Eq. (3.14) with Eq. (3.15) we see that heavy quark symmetry requires that the form factors u_+, $\ell_{1/2}$, $q_{1/2}$, $c_-^{1/2}$, h, k, and b_- be positive, while u_-, $\ell_{3/2}$, $q_{3/2}$, $c_+^{1/2}$, $c_+^{3/2}$, $c_-^{3/2}$, and b_+ be negative. Heavy quark symmetry also demands the relations $c_+^{1/2}+c_-^{1/2}=0$ and $b_++b_-=0$. It is easily seen that these heavy quark symmetry requirements are satisfied in realistic model calculations shown below.

In the present paper, we shall use the improved version, the so-called ISGW2 model [34], of the nonrelativistic quark model by Isgur-Scora-Grinstein-Wise (ISGW) [33] to compute the $B\to D^{**}$ transition form factors.[5] In general, the form factors evaluated in the original version of the ISGW model are reliable only at $q^2=q_m^2$, the maximum momentum transfer. The reason is that the form-factor q^2 dependence in the ISGW model is proportional to $\exp[-(q_m^2-q^2)]$ and hence the form factor decreases exponentially as a function of $(q_m^2-q^2)$. This has been improved in the ISGW2 model in which the form factor has a more realistic behavior at large $(q_m^2-q^2)$ which is expressed in terms of a certain polynomial term. In addition to the form-factor momentum dependence, the ISGW2 model incorporates a number of improvements, such as the constraints imposed by heavy quark symmetry, hyperfine distortions of wave functions, etc. [34].

[5]Note that in the original version of the ISGW model [33], the form factors for B to axial-vector charmed meson transition are evaluated for $D_1(^1P_1)$ and $D_1(^3P_1)$. As a result, one has to apply Eq. (2.1) to obtain $B\to D_1^{1/2}$ and $B\to D_1^{3/2}$ form factors.

TABLE II. The form factors at various q^2 for $B\to D_0^*$ and $B_s\to D_{s0}^*$ transitions calculated in the ISGW2 model.

Transition	$u_+(m_\pi^2)$	$u_-(m_\pi^2)$	$u_+(m_\rho^2)$	$u_-(m_\rho^2)$	$u_+(m_{D_s}^2)$	$u_-(m_{D_s}^2)$
$B\to D_0^*$	0.175	-0.462	0.178	-0.471	0.198	-0.524
$B_s\to D_{s0}^*$	0.196	-0.515	0.200	-0.527	0.230	-0.605

The results of the ISGW2 model predictions for various form factors are shown in Tables II–IV. Evidently, the signs of various calculated form factors are consistent with what are expected from heavy quark symmetry.

In realistic calculations of decay amplitudes it is convenient to employ the dimensionless form factors defined by [31]

$$\langle D_0^*(p_{D_0})|A_\mu|B(p_B)\rangle = i\left[\left((p_B+p_{D_0})_\mu-\frac{m_B^2-m_{D_0}^2}{q^2}q_\mu\right)F_1^{BD_0}(q^2)+\frac{m_B^2-m_{D_0}^2}{q^2}q_\mu F_0^{BD_0}(q^2)\right],$$

$$\langle D_1(p_{D_1},\varepsilon)|V_\mu|B(p_B)\rangle = i\left\{(m_B+m_{D_1})\varepsilon_\mu^* V_1^{BD_1}(q^2)-\frac{\varepsilon^*\cdot p_B}{m_B+m_{D_1}}(p_B+p_{D_1})_\mu V_2^{BD_1}(q^2)-2m_{D_1}\frac{\varepsilon^*\cdot p_B}{q^2}(p_B-p_{D_1})_\mu[V_3^{BD_1}(q^2)-V_0^{BD_1}(q^2)]\right\},$$

$$\langle D_1(p_{D_1},\varepsilon)|A_\mu|B(p_B)\rangle = \frac{2}{m_B+m_{D_1}}\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_B^\rho p_{D_1}^\sigma A^{BD_1}(q^2), \tag{3.17}$$

with

$$V_3^{BD_1}(q^2)=\frac{m_B+m_{D_1}}{2m_{D_1}}V_1^{BD_1}(q^2)-\frac{m_B-m_{D_1}}{2m_{D_1}}V_2^{BD_1}(q^2) \tag{3.18}$$

TABLE IV. The $B\to D_2^*$ and $B_s\to D_{s2}^*$ form factors at $q^2=m_\pi^2$ calculated in the ISGW2 model, where k is dimensionless and h, b_+, b_- are in units of GeV^{-2}.

Transition	h	k	b_+	b_-
$B\to D_2^*$	0.011	0.60	-0.010	0.010
$B_s\to D_{s2}^*$	0.013	0.70	-0.011	0.012

and $V_3^{BD_1}(0)=V_0^{BD_1}(0)$. The form factors relevant for $B\to D_0^*P$ decays are $F_0^{BD_0}$ and F_0^{BP}. Note that only the form factor V_0^{BP} or $F_1^{BD_1}$ will contribute to the factorizable amplitude of $B\to D_1P$ as one can check the matrix elements $q^\mu\langle D_1(p_{D_1},\varepsilon)|V_\mu|B(p_B)\rangle$ and $\varepsilon^\mu\langle P|V_\mu|B\rangle$. The ISGW2 model predictions for the form factors $F_{0,1}$, $V_{0,1,2}$, and A are summarized in Tables V and VI. It is evident that the form factor $F_{0,1}(0)\approx 0.18$ for the $B\to D_0^*$ transition is much smaller than the typical value of 0.65–0.70 for the $B\to D$ transition form factor at $q^2=0$.

IV. ANALYSIS OF $\bar{B}\to D^{**}M$, $\bar{D}_s^{**}M$ DECAYS

A. Factorization

In the present work we focus on the Cabibbo-allowed decays $\bar{B}\to D^{**}\pi(\rho)$, $D^{**}\bar{D}_s^{(*)}$, $\bar{D}_s^{**}D^{(*)}$ and $\bar{B}_s\to D_s^{**}\pi(\rho)$, where D^{**} denotes generically a p-wave charmed meson. We will study these decays within the framework of generalized factorization in which the hadronic decay amplitude is expressed in terms of factorizable contributions multiplied by the *universal* (i.e., process-independent) effective parameters a_i that are renormalization scale and scheme independent. Since the aforementioned B decays either proceed through only via tree diagrams or are tree dominated, we will thus neglect the small penguin contributions and write the weak Hamiltonian in the form

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}\{V_{cb}V_{cs}^*[a_1(\bar{c}b)(\bar{s}c)+a_2(\bar{s}b)(\bar{c}c)]+V_{cb}V_{ud}^*[a_1(\bar{c}b)(\bar{d}u)+a_2(\bar{d}b)(\bar{c}u)]+\text{H.c.}, \tag{4.1}$$

with $(\bar{q}_1q_2)\equiv\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. For hadronic B decays, we shall use $a_1=1.15$ and $a_2=0.26$.

Under the factorization hypothesis, the decays $B^-\to D^{**0}\bar{D}_s^-$, $\bar{B}^0\to D^{**+}\bar{D}_s^-$, and $\bar{B}_s^0\to D_s^{**+}\pi^-$ receive

TABLE III. The form factors at $q^2=m_\pi^2$ for $B\to D_1^{1/2}$ and $B\to D_1^{3/2}$ transitions calculated in the ISGW2 model, where $\ell_{1/2}$ and $\ell_{3/2}$ are in units of GeV and all others are in units of GeV^{-1}.

Transition	$q_{1/2}$	$\ell_{1/2}$	$c_+^{1/2}$	$c_-^{1/2}$	$q_{3/2}$	$\ell_{3/2}$	$c_+^{3/2}$	$c_-^{3/2}$
$B\to D_1^{1/2}$	0.057	0.54	-0.064	0.068				
$B\to D_1^{3/2}$					-0.057	-1.15	-0.043	-0.018
$B_s\to D_{s1}^{1/2}$	0.063	0.66	-0.072	0.078				
$B_s\to D_{s1}^{3/2}$					-0.063	-1.31	-0.048	-0.023

TABLE V. The form factors F_0 and F_1 at various q^2 for $B \to D_0^*$ and $B_s \to D_{s0}^*$ transitions calculated in the ISGW2 model.

Transition	$F_1(m_\pi^2)$	$F_0(m_\pi^2)$	$F_1(m_\rho^2)$	$F_0(m_\rho^2)$	$F_1(m_{D_s}^2)$	$F_0(m_{D_s}^2)$
$B \to D_0^*$	0.175	0.175	0.178	0.166	0.198	0.108
$B_s \to D_{s0}^*$	0.196	0.196	0.200	0.187	0.230	0.130

contributions only from the external W-emission diagram. As stated before, the penguin contributions to the first two decay modes are negligible.

Apart from a common factor of $G_F V_{cb} V_{ud}^*/\sqrt{2}$, the factorizable amplitudes for $B^- \to D^{**0}\pi^-$ read

$$\begin{aligned} A(B^- \to D_0^*(2308)^0\pi^-) &= -a_1 f_\pi (m_B^2 - m_{D_0}^2) F_0^{BD_0}(m_\pi^2) \\ &\quad - a_2 f_{D_0}(m_B^2 - m_\pi^2) F_0^{B\pi}(m_{D_0}^2), \end{aligned}$$

$$\begin{aligned} A(B^- \to D_1(2427)^0\pi^-) &= -2(\varepsilon^* \cdot p_B)\{a_1 f_\pi [V_0^{BD_1^{3/2}}(m_\pi^2) m_{D_1^{3/2}} \sin\theta \\ &\quad + V_0^{BD_1^{1/2}}(m_\pi^2) m_{D_1^{1/2}} \cos\theta] \\ &\quad + a_2 [F_1^{B\pi}(m_{D_1^{3/2}}^2) m_{D_1^{3/2}} f_{D_1^{3/2}} \sin\theta \\ &\quad + F_1^{B\pi}(m_{D_1^{1/2}}^2) m_{D_1^{1/2}} f_{D_1^{1/2}} \cos\theta]\}, \end{aligned}$$

$$\begin{aligned} A(B^- \to D_1'(2420)^0\pi^-) &= -2(\varepsilon^* \cdot p_B)\{a_1 f_\pi [V_0^{BD_1^{3/2}}(m_\pi^2) m_{D_1^{3/2}} \cos\theta \\ &\quad - V_0^{BD_1^{1/2}}(m_\pi^2) m_{D_1^{1/2}} \sin\theta] \\ &\quad + a_2 [F_1^{B\pi}(m_{D_1^{3/2}}^2) m_{D_1^{3/2}} f_{D_1^{3/2}} \cos\theta \\ &\quad - F_1^{B\pi}(m_{D_1^{1/2}}^2) m_{D_1^{1/2}} f_{D_1^{1/2}} \sin\theta]\}, \end{aligned}$$

$$\begin{aligned} A(B^- \to D_2^*(2460)^0\pi^-) &= i a_1 f_\pi \varepsilon_{\mu\nu}^* p_B^\mu p_B^\nu [k(m_\pi^2) + b_+(m_\pi^2)(m_B^2 - m_{D_2}^2) \\ &\quad + b_-(m_\pi^2) m_\pi^2]. \end{aligned} \tag{4.2}$$

TABLE VI. The dimensionless form factors A and $V_{0,1,2}$ at $q^2 = m_\pi^2$ for $B \to D_1^{1/2}$ and $B \to D_1^{3/2}$ transitions calculated in the ISGW2 model.

Transition	A	V_0	V_1	V_2
$B \to D_1^{1/2}$	−0.43	−0.18	0.070	0.49
$B \to D_1^{3/2}$	0.44	−0.43	−0.15	0.33
$B_s \to D_{s1}^{1/2}$	−0.49	−0.20	0.085	0.57
$B_s \to D_{s1}^{3/2}$	0.50	−0.47	−0.17	0.38

Note that except $B^- \to D_2^{*0}\pi^-$ all other modes receive contributions from color-suppressed internal W emission. The decay rates are given by

$$\Gamma(B \to D_0^*\pi) = \frac{p_c}{8\pi m_B^2}|A(B \to D_0^*\pi)|^2,$$

$$\Gamma(B \to D_1\pi) = \frac{p_c^3}{8\pi m_{D_1}^2}|A(B \to D_1\pi)/(\varepsilon^* \cdot p_B)|^2,$$

$$\Gamma(B \to D_2^*\pi) = \frac{p_c^5}{12\pi m_{D_2}^2}\left(\frac{m_B}{m_{D_2}}\right)^2 |M(B \to D_2^*\pi)|^2, \tag{4.3}$$

where $A(B \to D_2^*\pi) = \varepsilon_{\mu\nu}^* p_B^\mu p_B^\nu M(B \to D_2^*\pi)$ and p_c is the c.m. momentum of the pion. The p_c^{2L+1} dependence in the decay rate indicates that only s, p, and d waves are allowed in $D_0^*\pi$, $D_1\pi$, and $D_2^*\pi$ systems, respectively. The factorizable decay amplitudes for $B^- \to D_0^{*0}\rho^-$ and $B^- \to D_2^{*0}\rho^-$ are (up to a common factor of $G_F V_{cb} V_{ud}^*/\sqrt{2}$)

$$\begin{aligned} A(B^- \to D_0^*(2308)^0\rho^-) &= 2(\varepsilon^* \cdot p_B)[a_1 f_\rho m_\rho F_1^{BD_0}(m_\rho^2) + a_2 f_{D_0} m_{D_0} A_0^{B\rho}(m_{D_0}^2)], \end{aligned}$$

$$\begin{aligned} A(B^- \to D_2^*(2460)^0\rho^-) &= a_1 f_\rho m_\rho^2 \varepsilon^{*\alpha\beta} \varepsilon_\mu^* (p_B - p_{D_2})_\lambda \\ &\quad \times [ih(m_\rho^2) \epsilon^{\mu\nu\lambda\sigma} g_{\alpha\nu}(p_\rho)_\beta (p_\rho)_\sigma + k(m_\rho^2)\delta_\alpha^\mu \delta_\beta^\lambda \\ &\quad + b_+(m_\rho^2)(p_\rho)_\alpha (p_\rho)_\beta g^{\mu\lambda}]. \end{aligned} \tag{4.4}$$

The expression for $\overline{B} \to D_1\rho$ is more complicated. In the absence of the $D_1^{1/2} - D_1^{3/2}$ mixing, one has

$$\begin{aligned} A(B^- \to D_1'(2420)^0\rho^-) &= -i a_1 f_\rho m_\rho \Bigg[(\varepsilon_\rho^* \cdot \varepsilon_{D_1}^*)(m_B + m_{D_1}) V_1^{BD_1}(m_\rho^2) \\ &\quad - (\varepsilon_\rho^* \cdot p_B)(\varepsilon_{D_1}^* \cdot p_B) \frac{2V_2^{BD_1}(m_\rho^2)}{m_B + m_{D_1}} \\ &\quad + i\epsilon_{\mu\nu\alpha\beta} \varepsilon_{D_1}^{*\mu} \varepsilon_\rho^{*\nu} p_B^\alpha p_\rho^\beta \frac{2A^{BD_1}(m_\rho^2)}{m_B + m_{D_1}} \Bigg] \\ &\quad - i a_2 f_{D_1} m_{D_1} \Bigg[(\varepsilon_\rho^* \cdot \varepsilon_{D_1}^*)(m_B + m_\rho) A_1^{B\rho}(m_{D_1}^2) \\ &\quad - (\varepsilon_\rho^* \cdot p_B)(\varepsilon_{D_1}^* \cdot p_B) \frac{2A_2^{B\rho}(m_{D_1}^2)}{m_B + m_\rho} \\ &\quad + i\epsilon_{\mu\nu\alpha\beta} \varepsilon_\rho^{*\mu} \varepsilon_{D_1}^{*\nu} p_B^\alpha p_{D_1}^\beta \frac{2V^{B\rho}(m_{D_1}^2)}{m_B + m_\rho} \Bigg]. \end{aligned} \tag{4.5}$$

In the presence of the $D_1^{1/2}$-$D_1^{3/2}$ mixing, it is more convenient to express the decay amplitude as

$$A[B^-\to D_1^0(\varepsilon_{D_1},p_{D_1})\rho^-(\varepsilon_\rho,p_\rho)] \propto \varepsilon_{D_1}^{*\mu}\varepsilon_\rho^{*\nu}[S_1 g_{\mu\nu}+S_2(p_B)_\mu(p_B)_\nu+iS_3\epsilon_{\mu\nu\alpha\beta}p_{D_1}^\alpha p_\rho^\beta], \tag{4.6}$$

where $\epsilon^{0123}=+1$ in our convention and the coefficient S_3 corresponds to the p-wave amplitude and S_1 and S_2 to the mixture of s- and d-wave amplitudes:

$$\begin{aligned}
S_1=&a_1 f_\rho m_\rho[(m_B+m_{D_1^{3/2}})V_1^{BD_1^{3/2}}(m_\rho^2)\cos\theta \\
&-(m_B+m_{D_1^{1/2}})V_1^{BD_1^{1/2}}(m_\rho^2)\sin\theta] \\
&+a_2(m_B+m_\rho)[m_{D_1^{3/2}}f_{D_1^{3/2}}A_1^{B\rho}(m_{D_1^{3/2}}^2)\cos\theta \\
&-m_{D_1^{1/2}}f_{D_1^{1/2}}A_1^{B\rho}(m_{D_1^{1/2}}^2)\sin\theta], \\
S_2=&a_1 f_\rho m_\rho\left[\frac{1}{m_B+m_{D_1^{3/2}}}V_2^{BD_1^{3/2}}(m_\rho^2)\cos\theta\right. \\
&\left.-\frac{1}{m_B+m_{D_1^{1/2}}}V_2^{BD_1^{1/2}}(m_\rho^2)\sin\theta\right] \\
&+a_2\frac{1}{m_B+m_\rho}[m_{D_1^{3/2}}f_{D_1^{3/2}}A_2^{B\rho}(m_{D_1^{3/2}}^2)\cos\theta \\
&-m_{D_1^{1/2}}f_{D_1^{1/2}}A_2^{B\rho}(m_{D_1^{1/2}}^2)\sin\theta], \\
S_3=&a_1 f_\rho m_\rho\left[\frac{1}{m_B+m_{D_1^{3/2}}}A^{BD_1^{3/2}}(m_\rho^2)\cos\theta\right. \\
&\left.-\frac{1}{m_B+m_{D_1^{1/2}}}A^{BD_1^{1/2}}(m_\rho^2)\sin\theta\right] \\
&+a_2\frac{1}{m_B+m_\rho}[m_{D_1^{3/2}}f_{D_1^{3/2}}V^{B\rho}(m_{D_1^{3/2}}^2)\cos\theta \\
&-m_{D_1^{1/2}}f_{D_1^{1/2}}V^{B\rho}(m_{D_1^{1/2}}^2)\sin\theta].
\end{aligned} \tag{4.7}$$

Then the helicity amplitudes H_0, H_+, and H_- can be constructed as

$$\begin{aligned}
H_0&=\frac{1}{2m_{D_1}m_\rho}[(m_B^2-m_{D_1}^2-m_\rho^2)S_1+2m_B^2p_c^2S_2], \\
H_\pm&=S_1\pm m_Bp_cS_3.
\end{aligned} \tag{4.8}$$

For $B^-\to D_1(2427)^0\rho^-$, the amplitudes $S_{1,2,3}$ are the same as in Eq. (4.7) except for the replacement of $\cos\theta\to\sin\theta$ and $\sin\theta\to-\cos\theta$. The decay rates read (up to the common factor of $G_F^2|V_{cb}V_{ud}^*|^2/2$)

$$\begin{aligned}
\Gamma(B\to D_0^*\rho)&=\frac{p_c^3}{8\pi m_{D_0}^2}|A(B\to D_0^*\rho)/(\varepsilon^*\cdot p_B)|^2, \\
\Gamma(B\to D_1\rho)&=\frac{p_c}{8\pi m_B^2}(|H_0|^2+|H_+|^2+|H_-|^2), \\
\Gamma(B\to D_2^*\rho)&=\frac{f_\rho^2}{24\pi m_{D_2}^4}(ap_c^7+bp_c^5+cp_c^3),
\end{aligned} \tag{4.9}$$

with

$$\begin{aligned}
&a=8m_B^4b_+^2, \quad c=5m_{D_2}^2m_\rho^2k^2, \\
&b=2m_B^2[6m_\rho^2m_{D_2}^2h^2+2(m_B^2-m_{D_2}^2-m_\rho^2)kb_++k^2].
\end{aligned} \tag{4.10}$$

B. Results and discussion

Given the decay constants and form factors discussed in Sec. III, we are ready to study the B decays into p-wave charmed mesons. The predicted branching ratios are shown in Tables VII and VIII. The experimental results are taken from PDG [15] and Belle [14]. For $B^-\to D_2^{*0}\pi^-$ we combine the Belle measurements [14]

$$\begin{aligned}
&\mathcal{B}(B^-\to D_2^{*0}\pi^-)\mathcal{B}(D_2^{*0}\to D^+\pi^-) \\
&\quad=(3.4\pm0.3\pm0.6\pm0.4)\times10^{-4}, \\
&\mathcal{B}(B^-\to D_2^{*0}\pi^-)\mathcal{B}(D_2^{*0}\to D^{*+}\pi^-) \\
&\quad=(1.8\pm0.3\pm0.3\pm0.2)\times10^{-4},
\end{aligned} \tag{4.11}$$

to arrive at

$$\mathcal{B}(B^-\to D_2^{*0}\pi^-)\mathcal{B}(D_2^{*0}\to D^+\pi^-,D^{*+}\pi^-) =(5.5\pm0.8)\times10^{-4}. \tag{4.12}$$

Using $\mathcal{B}(D_2^{*0}\to D^+\pi^-,D^{*+}\pi^-)=2/3$ following from the assumption that the D_2^{*0} width is saturated by $D\pi$ and $D^*\pi$, we are led to $\mathcal{B}(B^-\to D_2^{*0}\pi^-)=(7.8\pm1.4)\times10^{-4}$ as shown in Table VII.

From Table VII we see that except for $D_1(2427)^0\pi^-$ the predictions of $\mathcal{B}(B^-\to D^{**}\pi^-)$ agree with experiment. It is worth mentioning that the ratio

$$R=\frac{\mathcal{B}(B^-\to D_2^*(2463)^0\pi^-)}{\mathcal{B}(B^-\to D_1'(2420)^0\pi^-)} \tag{4.13}$$

is measured to be 0.77 ± 0.15 by Belle [14] and 1.8 ± 0.8 by CLEO [17]. The early prediction by Neubert [38] yields a value of 0.35. Our prediction of $R=0.61$ is in accordance with the data. However, the predicted rate for $D_1(2427)^0\pi^-$

TABLE VII. The predicted branching ratios for $B^-\to D^{**0}(\pi^-,\rho^-,\bar{D}_s)$ and $B^-\to \bar{D}_s^{**}D^{(*)}$ decays, where D^{**} denotes a generic p-wave charmed meson. Experimental results are taken from PDG [15] and Belle [14]. The axial-vector meson mixing angles are taken to be $\theta=17°$ for D_1 and D_1' systems and $\theta_s=7°$ for D_{s1} and D_{s1}' systems.

Decay	This work	KV [35]	CM [36]	KLO [37]	Expt.
$B^-\to D_0^*(2308)^0\pi^-$	7.7×10^{-4}	4.2×10^{-4}			$(9.2\pm2.9)\times10^{-4}$ [14]
$B^-\to D_1(2427)^0\pi^-$	3.6×10^{-4}	2.4×10^{-4}			$(7.5\pm1.7)\times10^{-4}$ [14]
$B^-\to D_1'(2420)^0\pi^-$	1.1×10^{-3}	2.1×10^{-3}			$(1.0\pm0.2)\times10^{-3}$ [14]
					$(1.5\pm0.6)\times10^{-3}$ [15]
$B^-\to D_2^*(2460)^0\pi^-$	6.7×10^{-4}	7.2×10^{-5}	4.1×10^{-4}	3.5×10^{-4}	$(7.8\pm1.4)\times10^{-4}$ [14]
$B^-\to D_0^*(2308)^0\rho^-$	1.3×10^{-3}				
$B^-\to D_1(2427)^0\rho^-$	1.1×10^{-3}				
$B^-\to D_1'(2420)^0\rho^-$	2.8×10^{-3}				$<1.4\times10^{-3}$ [15]
$B^-\to D_2^*(2460)^0\rho^-$	1.8×10^{-3}		1.1×10^{-3}	9.8×10^{-4}	$<4.7\times10^{-3}$ [15]
$B^-\to D_0^*(2308)^0\bar{D}_s^-$	8.0×10^{-4}	2.7×10^{-3}			
$B^-\to D_1(2427)^0\bar{D}_s^-$	9.6×10^{-4}	1.4×10^{-3}			
$B^-\to D_1'(2420)^0\bar{D}_s^-$	1.3×10^{-3}	5.0×10^{-3}			
$B^-\to D_2^*(2460)^0\bar{D}_s^-$	4.2×10^{-4}	1.0×10^{-4}	2.7×10^{-4}	4.9×10^{-4}	
$B^-\to D_0^*(2308)^0\bar{D}_s^{*-}$	3.5×10^{-4}				
$B^-\to D_1(2427)^0\bar{D}_s^{*-}$	6.0×10^{-4}				
$B^-\to D_1'(2420)^0\bar{D}_s^{*-}$	1.6×10^{-3}				
$B^-\to D_2^*(2460)^0\bar{D}_s^{*-}$	1.1×10^{-3}	1.0×10^{-4}	1.1×10^{-3}	1.2×10^{-3}	
$B^-\to \bar{D}_{s0}^*(2317)^-D^0$	5.1×10^{-3}	0			see text
$B^-\to \bar{D}_{s1}(2463)^-D^0$	4.3×10^{-3}	3.5×10^{-3}			see text
$B^-\to \bar{D}_{s1}'(2536)^-D^0$	3.1×10^{-4}	3.4×10^{-3}			
$B^-\to \bar{D}_{s2}^*(2572)^-D^0$	–	0			
$B^-\to \bar{D}_{s0}^*(2317)^-D^{*0}$	2.7×10^{-3}				
$B^-\to \bar{D}_{s1}(2463)^-D^{*0}$	1.6×10^{-2}				
$B^-\to \bar{D}_{s1}'(2536)^-D^{*0}$	1.2×10^{-3}				
$B^-\to \bar{D}_{s2}^*(2572)^-D^{*0}$	0				

is too small by a factor of 2. This is ascribed to a destructive interference between color-allowed and color-suppressed tree amplitudes because the form factors $V_0^{BD_1^{1/2}}$ and $V_0^{BD_1^{3/2}}$ have signs opposite to that of $F_1^{B\pi}$ as required by heavy quark symmetry [see Eq. (4.2) and Table VI]. In contrast, the production of $D_1(2427)^+\pi^-$ is larger than $D_1(2427)^0\pi^-$ by a factor of about 2 because the former does not receive a destructive contribution from internal W emission. Hence, a measurement of the ratio $D_1(2427)^0\pi^-/D_1(2427)^+\pi^-$ can be used to test the relative signs of various form factors as implied by heavy quark symmetry. Note that for the $D_1^{1/2}$-$D_1^{3/2}$ mixing angle we use $\theta=17°$ [see Eq. (2.7)]. If a negative value of $-17°$ is employed, the decay $B^-\to D_1(2427)^0\pi^-$ will be severely suppressed with a branching ratio of order 6×10^{-6}. This means that the $D_1^{1/2}$-$D_1^{3/2}$ mixing angle is preferred to be positive. In their study Katoch and Verma [35] obtained a small branching ratio for $B^-\to D_0^*(2308)^0\pi^-$ as they assumed a vanishing decay constant for D_0^*. As stressed before, this decay constant is comparable to f_D because of charm and light quark mass imbalance and a rough estimate yields $f_{D_0^*}\approx160$ MeV [cf. Eq. (3.5)]. Consequently, the contribution from internal W emission will account for the aforementioned discrepancy between theory and experiment. Moreover, the ratio $D_0^{*+}\pi^-/D_0^{*0}\pi^-$ is predicted to be 0.34 instead of unity because of the absence of the color-suppressed tree contribution to the former.

At first glance, it appears that the prediction $\mathcal{B}(B^-\to D_1'(2420)^0\rho^-)=2.8\times10^{-3}$ already exceeds the experimental limit 1.4×10^{-3} [15]. However, it should be noticed that the $D_1'^0\rho^-$ rate is about 3 times larger than that of $D_1'^0\pi^-$ as expected from the factorization approach and

TABLE VIII. Same as Table VII except for neutral B and B_s mesons.

Decay	This work	KV [35]	CM [36]	KLO [37]	Expt.
$\overline{B}^0\to D_0^*(2308)^+\pi^-$	2.6×10^{-4}	4.1×10^{-4}			
$\overline{B}^0\to D_1(2427)^+\pi^-$	6.8×10^{-4}	1.2×10^{-4}			
$\overline{B}^0\to D_1'(2420)^+\pi^-$	1.0×10^{-3}	2.4×10^{-3}			
$\overline{B}^0\to D_2^*(2460)^+\pi^-$	6.1×10^{-4}	7.1×10^{-5}	4.1×10^{-4}	3.3×10^{-4}	$<2.2\times10^{-3}$ [15]
$\overline{B}^0\to D_0^*(2308)^+\rho^-$	6.4×10^{-4}				
$\overline{B}^0\to D_1(2427)^+\rho^-$	1.6×10^{-3}				
$\overline{B}^0\to D_1'(2420)^+\rho^-$	2.6×10^{-3}				
$\overline{B}^0\to D_2^*(2460)^+\rho^-$	1.7×10^{-3}		1.1×10^{-3}	9.2×10^{-4}	$<4.9\times10^{-3}$ [15]
$\overline{B}^0\to D_0^*(2308)^+\overline{D}_s^-$	7.3×10^{-4}	2.6×10^{-3}			
$\overline{B}^0\to D_1(2427)^+\overline{D}_s^-$	8.8×10^{-4}	1.3×10^{-3}			
$\overline{B}^0\to D_1'(2420)^+\overline{D}_s^-$	1.2×10^{-3}	4.9×10^{-3}			
$\overline{B}^0\to D_2^*(2460)^+\overline{D}_s^-$	3.8×10^{-4}	1.0×10^{-4}	2.7×10^{-4}	4.6×10^{-4}	
$\overline{B}^0\to D_0^*(2308)^+\overline{D}_s^{*-}$	3.2×10^{-4}				
$\overline{B}^0\to D_1(2427)^+\overline{D}_s^{*-}$	5.5×10^{-4}				
$\overline{B}^0\to D_1'(2420)^+\overline{D}_s^{*-}$	1.5×10^{-3}				
$\overline{B}^0\to D_2^*(2460)^+\overline{D}_s^{*-}$	1.0×10^{-3}	1.0×10^{-4}	1.1×10^{-3}	1.1×10^{-3}	
$\overline{B}^0\to \overline{D}_{s0}^*(2317)^-D^+$	4.7×10^{-3}	0			see text
$\overline{B}^0\to \overline{D}_{s1}(2463)^-D^+$	3.9×10^{-3}	3.4×10^{-3}			see text
$\overline{B}^0\to \overline{D}_{s1}'(2536)^-D^+$	2.8×10^{-4}	3.3×10^{-3}			
$\overline{B}^0\to \overline{D}_{s2}^*(2572)^-D^+$	–	0			
$\overline{B}^0\to \overline{D}_{s0}^*(2317)^-D^{*+}$	2.5×10^{-3}				
$\overline{B}^0\to \overline{D}_{s1}(2463)^-D^{*+}$	1.5×10^{-2}				
$\overline{B}^0\to \overline{D}_{s1}'(2536)^-D^{*+}$	1.1×10^{-3}				
$\overline{B}^0\to \overline{D}_{s2}^*(2572)^-D^{*+}$	–				
$\overline{B}_s^0\to D_{s0}^*(2317)^+\pi^-$	3.3×10^{-4}				
$\overline{B}_s^0\to D_{s1}(2463)^+\pi^-$	5.2×10^{-4}				
$\overline{B}_s^0\to D_{s1}'(2536)^+\pi^-$	1.5×10^{-3}				
$\overline{B}_s^0\to D_{s2}^*(2572)^+\pi^-$	7.1×10^{-4}				
$\overline{B}_s^0\to D_{s0}^*(2317)^+\rho^-$	8.3×10^{-4}				
$\overline{B}_s^0\to D_{s1}(2463)^+\rho^-$	1.3×10^{-3}				
$\overline{B}_s^0\to D_{s1}'(2536)^+\rho^-$	3.8×10^{-3}				
$\overline{B}_s^0\to D_{s2}^*(2572)^+\rho^-$	1.9×10^{-3}				

from the ratio $f_\rho/f_\pi\approx1.6$ (see Table IX below). Hence, it appears that the present limit on $D_1'\rho^-$ is not consistent with the observed rate of $D_1'\pi^-$. Of course, it is crucial to measure $B\to D_1'(2420)\rho$ in order to clarify the issue.

Apart from the external W-emission diagram, the $D^{**}\pi(\rho)$ productions in neutral B decays also receive W-exchange contributions which are neglected in the present work. This will constitute a main theoretical uncertainty for $\overline{B}^0\to D^{**+}\pi^-(\rho^-)$.

For $B\to\overline{D}D_s^{**}$ decays, the Belle data are given by [39]

$$\begin{aligned}
&\mathcal{B}(B\to\overline{D}D_{s0}^*(2317))\mathcal{B}(D_{s0}^*(2317)\to D_s\pi^0)\\
&\quad=(8.5^{+2.1}_{-1.9}\pm2.6)\times10^{-4},\\
&\mathcal{B}(B\to\overline{D}D_{s1}(2463))\mathcal{B}(D_{s1}(2463)\to D_s^*\pi^0)\\
&\quad=(17.8^{+4.5}_{-3.9}\pm5.3)\times10^{-4},\\
&\mathcal{B}(B\to\overline{D}D_{s1}(2463))\mathcal{B}(D_{s1}(2463)\to D_s\gamma)\\
&\quad=(6.7^{+1.3}_{-1.2}\pm2.0)\times10^{-4}.
\end{aligned}\tag{4.14}$$

TABLE IX. The ratios $\Gamma(\bar{B}\to D^{**}V)/\Gamma(\bar{B}\to D^{**}P)$ and $\Gamma(\bar{B}\to D_s^{**}V)/\Gamma(\bar{B}\to D_s^{**}P)$. The last column is for the ratio $\Gamma(\bar{B}_s\to D_s^{**}\rho^-)/\Gamma(\bar{B}_s\to D_s^{**}\pi^-)$.

$D^{**}(D_s^{**})$	$\frac{D^{**0}\rho^-}{D^{**0}\pi^-}$	$\frac{D^{**+}\rho^-}{D^{**+}\pi^-}$	$\frac{D^{**}\bar{D}_s^*}{D^{**}\bar{D}_s}$	$\frac{\bar{D}_s^{**}D^*}{\bar{D}_s^{**}D}$	$\frac{D_s^{**}\rho^-}{D_s^{**}\pi^-}$
$D_0^*(D_{s0}^*)$	1.7	2.4	0.43	0.54	2.5
$D_1(D_{s1})$	3.1	2.4	0.63	3.8	2.5
$D_1'(D_{s1}')$	2.6	2.6	1.2	4.0	2.6
$D_2^*(D_{s2}^*)$	2.7	2.7	2.7	–	2.7

Since $D_{s0}^*(2317)$ is dominated by its hadronic decay to $D_s\pi^0$,[6] the branching ratio of $B\to DD_{s0}^*(2317)$ is of order 1×10^{-3}. This means that the production rate of $\bar{D}^0D_{s0}^*$ in B decays is smaller than that of $\bar{D}^0D_s$ by one order of magnitude. Since this decay proceeds only via external W emission, it can be used to determine the decay constant of D_{s0}^*. It is found that $f_{D_{s0}^*}\sim60$ MeV,[7] which appears too small, recalling that $f_{D_0^*}\sim160$ MeV is needed to account for the production of $D_0^*\pi^-$. For the branching ratios of $D_s^*\pi^0$ and $D_s\gamma$ in $D_{s1}(2463)$ decay, we can apply the theoretical estimates made in [11]: namely, 0.56 and 0.13, respectively. [The predicted ratio $D_s\gamma/D_s^*\pi^0=0.24$ agrees with experiment; see Eq. (2.20).] Therefore, our prediction of 4.7×10^{-3} is consistent with the measurement $\mathcal{B}(B\to\bar{D}D_{s1}(2463))\sim4\times10^{-3}$. The Belle measurement of $B\to\bar{D}D_{s1}(2463)$ implies $f_{D_{s1}}\sim172$ MeV. This is consistent with the theoretical estimate

$$f_{D_{s1}}=f_{D_{s1}^{1/2}}\cos\theta_s+f_{D_{s1}^{3/2}}\sin\theta_s\sim177\text{ MeV},\qquad(4.15)$$

where use of Eqs. (2.8) and (3.11) has been made. Since $f_{D_{s0}^*}$ and $f_{D_{s1}^{1/2}}$ become identical in the heavy quark limit, this reinforces the previous statement that a decay constant of order 60 MeV for D_{s0}^* is probably too small. At any rate, it is crucial to check experimentally if the $\bar{D}_{s0}D$ production is less abundant than $\bar{D}_{s1}D$ to test heavy quark symmetry.

Since the tensor meson cannot be produced from the $V-A$ current, the B decay into $\bar{D}_{s2}^*D^{(*)}$ is prohibited under the factorization hypothesis. However, it can be induced via final-state interactions (FSIs) and/or nonfactorizable contributions. For example, it can be generated via the color-allowed decay $B^-\to\bar{D}_sD_2^{*0}$ followed by the rescattering process $\bar{D}_sD_2^{*0}\to\bar{D}_{s2}^*D^0$. Since the nonfactorizable term is of order c_2/N_c with Wilson coefficient $c_2(m_b)\approx-0.20$, it is likely to be suppressed relative to the FSIs. Hence, an observation of $B\to\bar{D}_{s2}^*D^{(*)}$ could imply the importance of final-state rescattering effects.

Since heavy quark symmetry implies $f_{D_{s1}'(2536)}\ll f_{D_{s1}(2463)}$, it is important to measure the B decay into $\bar{D}_{s1}'(2536)D^{(*)}$ to see if it is suppressed relative to $\bar{D}_{s1}(2463)D^{(*)}$ to test heavy quark symmetry. From Tables VII and VIII we see that the latter mode has the largest branching ratio of order 10^{-2} in two-body hadronic B decays involving a p-wave charmed meson in the final state. It is essential to test all these anticipations in the near future. A recent CLEO measurement yields [40]

$$\mathcal{B}(B^-\to(\bar{D}_s+\bar{D}_s^*)(D_1^0+D_1'^0+D_2^{*0}))=(2.73\pm0.78\pm0.48\pm0.68)\%.\qquad(4.16)$$

Our prediction of 6×10^{-3} is slightly small.

It is interesting to consider the ratios $\Gamma(\bar{B}\to D^{**}V)/\Gamma(\bar{B}\to D^{**}P)$ and $\Gamma(\bar{B}\to D_s^{**}V)/\Gamma(\bar{B}\to D_s^{**}P)$. The calculated results are shown in Table IX. Several remarks are in order: (i) The ratios $D^{**0}\rho^-/D^{**0}\pi^-$ and $D^{**+}\rho^-/D^{**+}\pi^-$ for $D^{**}=D_0^*$ and D_1 are not the same as the former receives an additional color-suppressed internal W-emission contribution. (ii) Whether the ratio $D^{**}V/D^{**}P$ is greater than unity or not depends essentially on the ratio of the decay constants. For example, $D_0^*\bar{D}_s^*/D_0^*\bar{D}_s=0.43$ if the decay constants of D_s^* and D_s are similar, while $D_0^{*+}\rho^-/D_0^{*+}\pi^-=2.5$ for $f_\rho/f_\pi=1.6$. It should be stressed that the proximity of the ratio $D^{**+}\rho^-/D^{**+}\pi^-$ to 2.5 has less to do with the three degrees of freedom of ρ; rather, it is mainly related to the decay constant ratio of f_ρ/f_π. (iii) The ratio of $D_s^{**}\rho^-/D_s^{**}\pi^-$ in B_s decay is the same as $D^{**+}\rho^-/D^{**+}\pi^-$ in B decay as they proceed via external W emission.

Because the scalar resonances D_0^* and D_1 have widths of order 300 MeV, we have checked the finite-width effects on their production in B decays and found that the conventional narrow-width approximation is accurate enough to describe the production of broad resonances owing to the large energy released in hadronic two-body decays of B mesons.

C. Comparison with other works

The decays $B\to D^{**}(\pi,D_s)$ and $B\to D_s^{**}D$ have been studied previously by Katoch and Verma (KV) [35]. López Castro and Muñoz (CM) [36] and Kim, Lee, and Oh (KLO) [37] also have a similar study with focus on the tensor charmed meson production. We shall comment their works separately.

In the paper of KLO, the $B\to D_2^*$ form factors are evaluated using the ISGW2 model. However, the predicted rates for $D_2^*\pi$ and $D_2^*\rho$ by KLO are smaller than ours by a factor of 2 (see Tables VII and VIII); that is, their $B\to D_2^*$ form factor differs from ours roughly by a factor of $\sqrt{2}$. In contrast, the results for $D_2^*D_s^{(*)}$ are similar owing to the fact that

[6] The upper limit on the ratio $\Gamma(D_{s0}^*\to D_s^*\gamma)/\Gamma(D_{s0}^*\to D_s\pi^0)<0.059$ was set recently by CLEO [13].

[7] Interestingly, this happens to be the original estimate of $f_{D_{s0}^*}$ made in [4] for D_{s0}^* in the four-quark state.

KLO employ $f_{D_s}=280$ MeV and $f_{D_s^*}=270$ MeV which are larger than ours [see Eq. (3.11)].

The work of KV and CM is based on the original version of the ISGW model. However, as stressed by KLO [37], the exponentially decreasing behavior of the form factors in the ISGW model is not realistic and justified. This has been improved in the ISGW2 model which provides a more realistic description of the form-factor behavior at large $(q_m^2-q^2)$. The values of form factors at small q^2 in the ISGW2 model can be a few times larger than that obtained in the ISGW model as the maximum momentum transfer q_m^2 in B decays is large. However, the expected form-factor suppression does not appear in the calculations of KV as they calculated the form factors at q_m^2 and then employed them even at low q^2. In contrast, CM did compute the form factors at proper q^2. The fact that CM and KLO have similar results for $B\to D_2^*\pi(\rho)$ and $B\to D_2^*\overline{D}_s^{(*)}$ (see Tables VII and VIII) is surprising as the $B\to D_2^*$ transition is evaluated in two different versions of the ISGW model. To check this, we find that $\eta(q^2)\equiv k(q^2)+b_+(q^2)(m_B^2-m_{D_2}^2)+b_-(q^2)q^2=0.286$ and 0.386 for $q^2=m_\pi^2$ in the ISGW and ISGW2 models, respectively, which in turn imply the respective branching ratios 6.7×10^{-4} and 3.8×10^{-4} for $B^-\to D_2^{*0}\pi^-$. Therefore, our result obtained in the ISGW model is consistent with that of CM and the estimate of $\eta(q^2)$ within the ISGW2 model by KLO is likely too small, as noted in passing.

The axial-vector charmed meson production considered by KV is for $D_1(^1P_1)$ and $D_1(^3P_1)$ rather than $D_1^{1/2}$ and $D_1^{3/2}$. Therefore, the expressions of the decay amplitudes involving D_1 or D_{s1} by KV should be modified by taking into account a proper wave function combination, Eq. (2.1). For the decay constants, KV assumed that $f_{D_0^*}=0$ and $f_{D_1(^1P_1)}=0$. As a consequence, $B^-\to D_0^{*0}\pi^-$ is too small compared to experiment and the decay into $\overline{D}_{s0}^*D$ is not allowed. This is not consistent with the heavy quark symmetry relation $f_{D_{s1}^{1/2}}=f_{D_{s0}^*}$. Finally, the predicted rate of $B^-\to D_2^{*0}\pi^-$ by KV is too small by one order of magnitude compared to experiment. This is ascribed to a missing factor of $(m_B/m_T)^2$ in their calculation of decay rates.

V. CONCLUSIONS

The hadronic decays of B mesons to a p-wave charmed meson in the final state are studied. Specifically we focus on the Cabibbo-allowed decays $\overline{B}\to D^{**}\pi(\rho)$, $D^{**}\overline{D}_s^{(*)}$, $\overline{D}_s^{**}D^{(*)}$, and $\overline{B}_s\to D_s^{**}\pi(\rho)$. The main conclusions are as follows.

(i) We apply heavy quark effective theory in which heavy quark symmetry and chiral symmetry are unified to study the strong decays of p-wave charmed mesons and determine the magnitude of the $D_1^{1/2}$-$D_1^{3/2}$ mixing angle. In contrast, the present upper limits on the widths of $D_{s1}(2463)$ and $D_{s1}'(2536)$ do not provide any constraints on the $D_{s1}^{1/2}$-$D_{s1}^{3/2}$ mixing angle θ_s. Therefore, we appeal to the quark potential model to extract θ_s.

(ii) Various form factors for $B\to D^{**}$ transitions and their q^2 dependence are studied using the improved version of the Isgur-Scora-Grinstein-Wise quark model. Heavy quark symmetry constraints are respected in this model calculation.

(iii) The predicted branching ratios for $B^-\to D^{**}\pi^-$ agree with experiment except $D_1(2427)^0\pi^-$. The $D_1^{1/2}$-$D_1^{3/2}$ mixing angle is preferred to be positive in order to avoid a severe suppression on the production of $D_1(2427)^0\pi^-$. The decay $B^-\to D_1'(2420)^0\rho^-$ is predicted at the level of 3×10^{-3}. Although it exceeds the present experimental limit of 1.4×10^{-3}, it leads to the ratio $D_1'(2420)\rho^-/D_1'(2420)\pi^-\approx2.6$ as expected from the factorization approach and from the ratio $f_\rho/f_\pi\approx1.6$. Therefore, it is crucial to have a measurement of this mode to test the factorization hypothesis.

(iv) The predicted rate for $B^-\to D_1(2427)^0\pi^-$ is too small by a factor of 2 owing to a destructive interference between color-allowed and color-suppressed tree amplitudes as the relevant form factors for $B\to D_1^{1/2}$ and $B\to D_1^{3/2}$ transitions are negative. It is crucial to measure the production of $D_1(2427)^+\pi^-$ to see if it is larger than $D_1(2427)^0\pi^-$ by a factor of about 2 because the former does not receive the internal W-emission contribution.

(v) Under the factorization hypothesis, the production of $\overline{D}_{s2}^*D^{(*)}$ in B decays is prohibited as the tensor meson cannot be produced from the $V-A$ current. Nevertheless, the decays $B\to\overline{D}_{s2}^*D^{(*)}$ can be induced via final-state interactions and/or nonfactorizable contributions. Since the latter are suppressed by the order of c_2/N_c, an observation of $B\to\overline{D}_{s2}^*D^{(*)}$ could imply the importance of final-state rescattering effects.

(vi) For $\overline{B}\to\overline{D}_s^{**}D$ decays, it is expected that $\overline{D}_{s0}^*D\gtrsim\overline{D}_{s1}D$ as the decay constants of the multiplet (D_{s0}^*,D_{s1}) become identical in the heavy quark limit. The preliminary Belle measurements of these two modes imply $\overline{D}_{s0}^*D/\overline{D}_{s1}D\sim1/4$ and $f_{D_{s1}}\sim170$ MeV, $f_{D_{s0}^*}\sim60$ MeV. The fairly less abundant production of $\overline{D}_{s0}^*D$ than $\overline{D}_{s1}D$ and the large disparity between $f_{D_{s1}}$ and $f_{D_{s0}^*}$ are surprising. The reason for the discrepancy between theory and experiment remains unclear. In the meantime, it is also important to measure the decay to $\overline{D}_{s1}'(2536)D^{(*)}$ to see if it is suppressed relative to $\overline{D}_{s1}(2463)D^{(*)}$ to test the heavy quark symmetry relation $f_{D_{s1}(2536)}\ll f_{D_{s1}(2463)}$.

Note added. After this work was completed, we noticed the appearance of the related works on $B\to\overline{D}D_s^{**}$ decays by Chen and Li [41], Datta and O'Donnell [42], and Suzuki [43].

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