# $N=2$ super Yang-Mills action as a Becchi-Rouet-Stora-Tyutin term, topological Yang-Mills action, and instantons

K. Ülker*
*Feza Gürsey Institute, Çengelköy, 81220 İstanbul, Turkey*
(Received 28 May 2003; published 16 October 2003)

By constructing a nilpotent extended Becchi-Rouet-Stora-Tyutin (BRST) operator $\overline{s}$ that involves the $N=2$ global supersymmetry transformations of one chirality, we show that the standard $N=2$ off-shell super Yang-Mills action can be represented as an exact BRST term $\overline{s}\Psi$, if the gauge fermion $\Psi$ is allowed to depend on the inverse powers of supersymmetry (SUSY) ghosts. By using this nonanalytical structure of the gauge fermion (via inverse powers of supersymmetry ghosts), we give field redefinitions in terms of composite fields of SUSY ghosts and $N=2$ fields and we show that Witten's topological Yang-Mills (TYM) theory can be obtained from the ordinary Euclidean $N=2$ super Yang-Mills (SYM) theory directly by using such field redefinitions. In other words, TYM theory is obtained as a change of variables (without twisting). As a consequence it is found that physical and topological interpretations of $N=2$ SYM theory are intertwined together due to the requirement of analyticity of global SUSY ghosts. Moreover, after an instanton-inspired truncation of the model is used, we show that the given field redefinitions yield the Baulieu-Singer formulation of topological Yang-Mills theory.

DOI: 10.1103/PhysRevD.68.085005 PACS number(s): 11.30.Pb, 11.27.+d, 12.60.Jv

## I. INTRODUCTION

$N=2$ super Yang-Mills (SYM) theory has been extensively studied in recent years after the work of Seiberg and Witten [1], in which a self-consistent nonperturbative effective action was calculated by using certain *Ansätze* dictated by physical intuition. This solution is unique [2].

After this seminal paper [1], one of the main areas of research in $N=2$ SYM theory has been to calculate directly the multi-instanton contributions to the holomorphic prepotential and consequently to check the correctness of the results of [1]. These multi-instanton contributions are calculated in a pioneering work [3] for one and two instantons by using a semiclassical expansion around *approximate* saddle points of the action [3].[1] The results are found to be in agreement with those of [1]. (For a self-contained review, see Ref. [6].) However, a natural question was posed by several researchers: how could it be possible that an approximate approach gives an exact result and what was the mechanism behind it [7]?

In order to get an answer to the above questions, in Ref. [8] the instanton calculus was performed in the framework of topological Yang-Mills (TYM) theory and it was found that the results are the same as that of the *traditional* instanton calculus of [3]. As noted in [8], the underlying fact of this result is that the action of TYM theory, which can be obtained as a twist of ordinary $N=2$ SYM theory [9], can be written as a Becchi-Rouet-Stora-Tyutin (BRST) exact term [10] and the functional integration over the antifields of the topological theory gives the same field configurations of the constrained instantons of the ordinary theory without any approximation. Therefore, the authors of Ref. [8] have noted that the twisting procedure can be thought of as variable redefinitions in flat space-time.

However, up to now no explicit variable redefinitions have been given in order to obtain the topological theory. For instance, even if the twist can be considered as a linear change of variables, after twisting, the physical interpretation of some fields changes, i.e., some become ghosts or antighosts. The dimensions of the topological fields are also different from their untwisted counterparts.

Our main aim in this paper is to derive explicit field redefinitions in order to obtain the topological fields with correct dimensionality and ghost number. The strategy that we follow is to use the extension of the BRST formalism (also called the BV or field-antifield formalism [14,15]) to include global supersymmetry (SUSY) [16–18].[2] By using this formalism we show that the $N=2$ SYM action can be written as an exact term and both of the formulations of TYM theory given by Witten [9] and Baulieu and Singer [10] can be obtained as a change of variables without twisting.

The paper is organized as follows. In Sec. II, we review briefly how to include SUSY in an extended BRST operator. Since the actions of SYM theories can be represented as chiral (or antichiral) multiple supervariations of lower dimensional gauge invariant field polynomials [11,12],[3] in Sec. III we construct a nilpotent extended BRST operator $\overline{s}$ on the field space that only contains the chiral part of the $N=2$ SUSY as a global symmetry in addition to gauge symme-

*Email address: kulker@gursey.gov.tr

[1]One instanton contribution is also calculated in [4,5].

[2]Such an extension of BRST transformations that includes rigid symmetries is first introduced in [19]. The problem of how to extend the BRST formalism to include arbitrary global symmetries can be found in Ref. [20].

[3]For a similar approach of constructing $N=1$ globally and locally supersymmetric actions and also for the discussion of anomalies, see [13].

 

tries. It is then straightforward to find an expression as a *gauge fermion* of which the full off-shell $N=2$ action is its BRST ($\bar{s}$) variation under some conditions in Minkowski space. In Sec. IV, we give a Euclidean formulation of the result given in Sec. III in order to derive field redefinitions by comparing the Euclidean $N=2$ SYM action written as an exact term and the topological theory. We demonstrate explicitly that Witten's topological Yang-Mills theory can be obtained from the ordinary Euclidean $N=2$ SYM theory directly by using such field redefinitions. In Sec. V, by using an instanton- inspired truncation of the model given by Zumino [25], we also show how to obtain the approach of Baulieu-Singer of TYM theory with the help of field redefinitions. Finally, Sec. VI is devoted to concluding remarks.

## II. N=2 SYM THEORY AND EXTENDED BRST TRANSFORMATIONS

$N=2$ SYM theory is a rather old and well studied theory. In this work, we will study the off-shell formulation of the theory given in [26,27] by using the conventions of [28]. The action of the theory is given in Minkowski space as

$$S_{N=2}=\frac{1}{g^2}\mathrm{Tr}\int d^4x\Bigg(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-i\lambda^i \not{D}\overline{\lambda}_i+\phi D_\mu D^\mu\phi^\dagger -\frac{i\sqrt{2}}{2}(\lambda_i[\lambda^i,\phi^\dagger]+\overline{\lambda}^i[\overline{\lambda}_i,\phi]) -\frac{1}{2}[\phi,\phi^\dagger]^2+\frac{1}{2}\vec{D}.\vec{D}\Bigg), \tag{1}$$

where the gauge field $A_\mu$ and the scalar fields $\phi$, $\phi^\dagger$ are singlets, the Weyl spinors $\lambda_{i\alpha}$ $\overline{\lambda}^i_{\dot\alpha}$ are doublets, and the auxiliary field $\vec{D}$ is a triplet under the $SU(2)_R$ symmetry group. These fields are members of N=2 vector multiplet $V=(A_\mu,\phi,\phi^\dagger,\lambda_{i\alpha},\overline{\lambda}^i_{\dot\alpha},\vec{D})$ [26,27]. The $SU(2)_R$ indices of the spinors are raised and lowered due to

$$\lambda^i=\mathcal{E}^{ij}\lambda_j,\qquad \lambda_i=\lambda^j\mathcal{E}_{ji}, \tag{2}$$

$$\overline{\lambda}_i=\mathcal{E}_{ij}\overline{\lambda}^j,\qquad \overline{\lambda}^i=\overline{\lambda}_j\mathcal{E}^{ji}, \tag{3}$$

where the antisymmetric tensor $\mathcal{E}^{ij}$ is given as[4]

$$\mathcal{E}_{12}=\mathcal{E}^{12}=-\mathcal{E}_{21}=-\mathcal{E}^{21}=1.$$

The extension of the BRST transformations with the global $N=2$ SUSY and translation symmetry was first given in Ref. [17],

$$s=s_0-i\xi^iQ_i-i\overline{\xi}_i\overline{Q}^i-i\eta^\mu\partial_\mu, \tag{4}$$

where $s_0$ is the ordinary BRST transformations, $Q_i\overline{Q}^i$ are chiral and antichiral parts of $N=2$ SUSY transformations, and $\xi^{i\alpha},\overline{\xi}_{i\dot\alpha}$, and $\eta_\mu$ are the constant commuting chiral- antichiral SUSY ghosts and constant imaginary anticommuting translation ghost, respectively. Note that the parameters of the global transformations are promoted to the status of constant ghosts, and their Grassmann parity is changed so that the extended transformation $s$ is a homogeneous transformation.

The extended BRST transformations on the elements of the $N=2$ vector multiplet read as[5]

$$sA_\mu=D_\mu c+\xi_i\sigma_\mu\overline{\lambda}^i+\overline{\xi}^i\overline{\sigma}_\mu\lambda_i-i\eta^\nu\partial_\nu A_\mu, \tag{5}$$

$$s\lambda_i=i\{c,\lambda_i\}-i\sigma^{\mu\nu}\xi_iF_{\mu\nu}+\xi_i[\phi,\phi^\dagger] -\sqrt{2}\sigma^\mu\overline{\xi}_iD_\mu\phi+\vec{\tau}_i^{\,j}\xi_j\cdot\vec{D}-i\eta^\mu\partial_\mu\lambda_i, \tag{6}$$

$$s\overline{\lambda}^i=i\{c,\overline{\lambda}^i\}-i\overline{\sigma}^{\mu\nu}\overline{\xi}^iF_{\mu\nu}-\overline{\xi}^i[\phi,\phi^\dagger] -\sqrt{2}\overline{\sigma}^\mu\xi^iD_\mu\phi^\dagger-\overline{\xi}^j\vec{\tau}_j^{\,i}\cdot\vec{D}-i\eta^\mu\partial_\mu\overline{\lambda}^i, \tag{7}$$

$$s\phi=i[c,\phi]-i\sqrt{2}\xi_i\lambda^i-i\eta^\mu\partial_\mu\phi, \tag{8}$$

$$s\phi^\dagger=i[c,\phi^\dagger]-i\sqrt{2}\overline{\xi}^i\overline{\lambda}_i-i\eta^\mu\partial_\mu\phi^\dagger, \tag{9}$$

$$s\vec{D}=i[c,\vec{D}]+i\vec{\tau}_i^{\,j}(\xi_j\not{D}\overline{\lambda}^i-\overline{\xi}^i\not{D}\lambda_j+\sqrt{2}\xi^i[\lambda_j,\phi^\dagger] -\sqrt{2}\overline{\xi}_j[\overline{\lambda}^i,\phi])-i\eta^\mu\partial_\mu\vec{D}. \tag{10}$$

On the other hand, in order to get a nilpotent $s$,

$$s^2=0. \tag{11}$$

The Fadeev-Popov ghost field and the global ghosts are required to transform as

$$sc=\frac{i}{2}\{c,c\}-2i\xi_i\sigma^\mu\overline{\xi}^iA_\mu-\sqrt{2}\xi_i\xi^i\phi^\dagger -\sqrt{2}\overline{\xi}^i\overline{\xi}_i\phi-i\eta^\mu\partial_\mu c, \tag{12}$$

$$s\eta_\mu=-2\xi^i\sigma_\mu\overline{\xi}_i, \tag{13}$$

$$s\xi_i=s\overline{\xi}_i=0. \tag{14}$$

Note that with the help of extra terms in $sc$, the characteristic complication that SUSY algebra is modified by field-dependent gauge transformations is solved, whereas the closure on translations disappears due to inclusion of translation

[4]Note that in our convention the $\mathcal{E}^{ij}$ is different than the one, $\epsilon_{\alpha\beta}$, used for spinor indices.

[5]Here, $\vec{\tau}'$s are Pauli spin matrices.

TABLE I. Dimensions $D$, Grassmann parity GP, ghost number Gh, and $R$ weights of the fields and ghosts.

| | $A_\mu$ | $\phi$ | $\phi^\dagger$ | $\lambda^i$ | $\overline{\lambda_i}$ | $\vec{D}$ | $c$ | $\xi^i$ | $\overline{\xi_i}$ | $\eta_\mu$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $D$ | 1 | 1 | 1 | 3/2 | 3/2 | 2 | 0 | $-1/2$ | $-1/2$ | $-1$ |
| Gh | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| $R$ | 0 | $-2$ | 2 | $-1$ | 1 | 0 | 0 | $-1$ | 1 | 0 |
| GP | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

ghosts. We summarize the dimension, ghost number, and the $R$ charges of the fields and ghosts in Table I.

## III. $N=2$ SYM ACTION AS AN EXACT TERM

It is known from cohomological arguments that the actions of SYM theories can be represented as chiral (or antichiral) multiple supervariations of lower dimensional gauge invariant field polynomials [11,12]. For instance, $N=2$ SYM action can be written as a fourfold chiral SUSY transformation of Tr $\phi^2$ in component formalism of SUSY.

On the other hand, from the definition of $s$ it is still possible to derive another nilpotent operator by using a suitable filtration of global ghosts. We choose this filtration to be

$$\mathcal{N}=\overline{\xi}_{i\dot{\alpha}}\frac{\delta}{\delta\overline{\xi}_{i\dot{\alpha}}}+\eta_\mu\frac{\delta}{\delta\eta_\mu},$$

$$s=\sum s^{(n)},\quad [\mathcal{N},s^{(n)}]=ns^{(n)}, \tag{15}$$

so that the zeroth order in the above expansion is an operator that includes ordinary BRST and chiral SUSY on the space of the fields of the $N=2$ vector multiplet,

$$\overline{s}\coloneqq s^{(0)}=s_0-i\xi^iQ_i, \tag{16}$$

which is also nilpotent,

$$\overline{s}^2=0. \tag{17}$$

The $\overline{s}$ transformations of the fields are now given as

$$\overline{s}A_\mu=D_\mu c+\xi_i\sigma_\mu\overline{\lambda}^i, \tag{18}$$

$$\overline{s}\lambda_i=i\{c,\lambda_i\}-i\sigma^{\mu\nu}\xi_iF_{\mu\nu}+\xi_i[\phi,\phi^\dagger]+\vec{\tau}_i^j\xi_j\cdot\vec{D}, \tag{19}$$

$$\overline{s}\overline{\lambda}^i=i\{c,\overline{\lambda}^i\}-\sqrt{2}\overline{\sigma}^\mu\xi^iD_\mu\phi^\dagger, \tag{20}$$

$$\overline{s}\phi=i[c,\phi]-i\sqrt{2}\xi_i\lambda^i, \tag{21}$$

$$\overline{s}\phi^\dagger=i[c,\phi^\dagger], \tag{22}$$

$$\overline{s}\vec{D}=i[c,\vec{D}]+i\vec{\tau}_i^j(\xi_j\not{D}\overline{\lambda}^i+\sqrt{2}\xi^i[\lambda_j,\phi^\dagger]), \tag{23}$$

$$\overline{s}c=\frac{i}{2}\{c,c\}-\sqrt{2}\xi_i\xi^i\phi^\dagger, \tag{24}$$

$$\overline{s}\eta_\mu=\overline{s}\xi_i=\overline{s}\overline{\xi}_i=0. \tag{25}$$

Since $\overline{s}$ contains gauge transformations and chiral SUSY and the action is gauge invariant, it is straightforward to assume that the action can be written also as an $\overline{s}$ exact term of a gauge invariant field polynomial which is independent of Fadeev-Popov ghost fields,[6]

$$I=\overline{s}\Psi. \tag{26}$$

It is clear that $\Psi$, the so-called gauge fermion in the BV formalism, has a negative ghost number, Gh$(\Psi)=-1$. However, since no fields with negative ghost number have been introduced and since we have chosen the gauge fermion to be free of Fadeev-Popov ghosts, the only way to assign a negative ghost number to $\Psi$ is to choose $\Psi$ to depend on the negative powers of the global SUSY ghosts. In other words, the action can be written as an exact $\overline{s}$ term only if the extended BRST operator $\overline{s}$ is defined on the space of field polynomials that are not necessarily analytic in constant ghosts.

Therefore, a further assumption to assign a negative ghost number to $\Psi$ is to choose a gauge fermion that has the following form:

$$\Psi=\frac{1}{\xi_k\xi^k}\xi^i\int d^4x\,\psi_i, \tag{27}$$

where $\psi_i^\alpha$ is a dimension 7/2 fermion that is made from the fields of the $N=2$ vector multiplet. The most general such gauge fermion that is covariant in its Lorentz, spinor, and $SU(2)_R$ indices is easy to find,

$$\Psi=\frac{1}{\xi_k\xi^k}\mathrm{Tr}\int d^4x\{(k_1\xi^i\lambda_i[\phi,\phi^\dagger]+k_2\xi^i\vec{\tau}_i^j\lambda_j\cdot\vec{D}$$
$$+k_3\xi^i\sigma^{\mu\nu}\lambda_iF_{\mu\nu}+k_4\phi\xi^i\not{D}\overline{\lambda}_i)\}. \tag{28}$$

Here $k'$s are constants. In order that the $\overline{s}$ variation of $\Psi$ to be free of chiral ghosts, after some algebra it is seen that constants $k$ should be fixed and as a result

[6] In other words, we assume that the action can be chosen to be a trivial element of equivariant cohomology of $\overline{s}$. See, for instance, Refs. [21–23] and references therein.

$$\overline{s}\Psi = \overline{s}\frac{1}{\xi_k \xi^k}\mathrm{Tr}\int d^4x\left\{\left(\frac{1}{2}\xi^i\lambda_i[\phi,\phi^\dagger] - \frac{1}{2}\xi^i\vec{\tau}_i^j\lambda_j\cdot\vec{D} - \frac{i}{2}\xi^i\sigma^{\mu\nu}\lambda_i F_{\mu\nu} + \frac{\sqrt{2}}{2}\phi\xi^i \not{D}\overline{\lambda}_i\right)\right\}, \tag{29}$$

$$= \mathrm{Tr}\int d^4x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{i}{8}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} - i\lambda^i\not{D}\overline{\lambda}_i + \phi D_\mu D^\mu\phi^\dagger - \frac{i\sqrt{2}}{2}(\lambda_i[\lambda^i,\phi^\dagger] + \overline{\lambda}^i[\overline{\lambda}_i,\phi]) - \frac{1}{2}[\phi,\phi^\dagger]^2 + \frac{1}{2}\vec{D}\cdot\vec{D}\right) \tag{30}$$

is found. This expression is exactly the action (1) of $N=2$ SYM theory with a topological term $\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$,

$$I_{N=2} = \frac{1}{4\pi}\mathrm{Im}[\Upsilon(\overline{s}\Psi)] = S_{N=2} - \frac{\theta}{16\pi}\mathrm{Tr}\int d^4x\,\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}, \tag{31}$$

where $\Upsilon$ is the complex coupling constant, $\Upsilon = (i4\pi/g^2) + (\theta/2\pi)$.

One can argue that the action written in this form is obvious since the action can be obtained by applying all four supersymmetry transformations to the prepotential Tr $\phi^2$. However, it has important consequences.

First of all, the operator $\overline{s}$ is strictly nilpotent. It is a well known fact that the cohomology of the complete operator $s$, given in Eq. (4), is isomorphic to a subspace of the cohomology of $\overline{s}$, since $\overline{s}$ is obtained after using a suitable filtration of $s$ (see, for instance, [24,21]). If the functional space is defined to be the polynomials of the fields that are not necessarily analytic in the constant ghosts $\xi_i$, as is shown above, the action belongs to the trivial cohomology of $\overline{s}$ and therefore to that of the complete operator $s$.

Second, the above nonanalyticity argument plays an important role in obtaining the topological Yang-Mills theory from Euclidean $N=2$ SYM theory by identifying the topological fields with certain functions of fields and SUSY ghosts of $N=2$ SYM theory (which are also partly nonanalytic $\xi_i'$s). These points will be clarified in the next sections.

## IV. EUCLIDEAN $N=2$ SYM AND TYM THEORY AS A VARIABLE REDEFINITION

As is well known, TYM theory can be obtained by twisting $N=2$ SYM theory in Euclidean space [9]. In summary, the twisting procedure is simply identifying the $SU(2)_R$ index $i$ with the spinor index of one chirality, i.e., $\alpha$[7] and $R$ charges of the fields with ghost number. It is then possible to show that TYM action can be written as a BRST-exact term up to some field redefinitions [10]. Therefore, TYM theory is called a topological theory of cohomological type and it is natural to look for an analogy between the results of the previous section and TYM theory by rewriting the results of the previous section in Euclidean space.

However, before formulating the results of Sec. III in Euclidean space, we find it useful to clarify our approach to Euclidean $N=2$ SYM theory. First of all, obviously in Euclidean space the chiral and antichiral spinors are not related with each other. Nevertheless, it is still possible to find consistent reality conditions for the spinors of extended supersymmetry [30]. We will take these reality conditions in our conventions as

$$(\lambda_i^\alpha)^\dagger = i\epsilon_{\alpha\beta}\mathcal{E}^{ij}\lambda_j^\beta, \quad (\overline{\lambda}^{i\dot{\alpha}})^\dagger = i\epsilon_{\dot{\alpha}\dot{\beta}}\mathcal{E}_{ij}\overline{\lambda}^{j\dot{\beta}}. \tag{32}$$

On the other hand, since the spinors of different chirality are independent of each other and since supersymmetry is manifest, it is clear that one should also consider the complex scalar field $\phi$ and its Hermitean conjugate $\phi^\dagger$ defined in Minkowski space as independent fields from each other in Euclidean space. Indeed, this somehow unusual treatment appears naturally if one defines a continuous Wick rotations to Euclidean space [31]: pseudoscalar field $B$ where $\phi = A + iB$ goes over into $iB_E$ in Euclidean space, i.e., Euclidean scalar fields become $\phi_E = A_E + B_E$ and $\phi_E^\dagger = A_E - B_E$. By applying the above-mentioned continuous Wick rotation to $N=2$ SYM theory in Minkowski space, one gets [31] the $N=2$ supersymmetric Euclidean theory that is constructed by Zumino [25]. Note that corresponding action in Euclidean space is Hermitean.

Following the above remarks, we perform a (continuous) Wick rotation to formulate the results of the previous section in Euclidean space, i.e., Minkowskian vector quantities $v^\mu = (v^0,\vec{v})$, $\mu = 0,1,2,3$ become Euclidean ones $v_\mu = (\vec{v}, iv^0)$, $\mu = 1,2,3,4$ and Euclidean sigma matrices are taken as $e_{\mu\alpha\dot{\alpha}} = (i\vec{\tau},1)$ and $\overline{e}_\mu^{\dot{\alpha},\alpha} = (-i\vec{\tau},1)$. We will also take the gauge field anti-Hermitean rather than Hermitean in order to follow the instanton literature,[8] and to avoid confusion we will denote Euclidean scalar fields as $M := \phi_E$, $N := \phi_E^\dagger$.

The Euclidean $\overline{s}$ transformations of the fields now read

[7] In $\mathbb{R}^4$, the symmetry group of $N=2$ SYM theory is $SU(2)_L \otimes SU(2)_R \otimes SU(2)_R \otimes U(1)_R$. The twist $(i \equiv \alpha)$ consists of replacing the rotation group $SU(2)_L \otimes SU(2)_R$ with $SU(2)_L' \otimes SU(2)_R$, where $SU(2)_L'$ is the diagonal sum of $SU(2)_L \otimes SU(2)_R$. For a detailed analysis of topological theories, see, for instance, [29].

[8] We use the Euclidean conventions of Ref. [6].

$$\bar{s}A_\mu = D_\mu c - \xi_i e_\mu \bar{\lambda}^i, \tag{33}$$

$$\bar{s}\lambda_i = -\{c,\lambda_i\} - e_{\mu\nu}\xi_i F_{\mu\nu} + \xi_i [M,N] + \vec{\tau}_i^j \xi_j \cdot \vec{D}, \tag{34}$$

$$\bar{s}\bar{\lambda}^i = -\{c,\bar{\lambda}^i\} + i\sqrt{2}\bar{e}_\mu \xi^i D_\mu N, \tag{35}$$

$$\bar{s}M = -[c,M] + i\sqrt{2}\xi^i \lambda_i, \tag{36}$$

$$\bar{s}N = -[c,N], \tag{37}$$

$$\bar{s}\vec{D} = -[c,\vec{D}] + \vec{\tau}_i^j(\xi_j e_\mu D_\mu \bar{\lambda}^i + i\sqrt{2}\xi^i[\lambda_j, N]), \tag{38}$$

$$\bar{s}c = -\frac{1}{2}\{c,c\} + i\sqrt{2}\xi_i\xi^i N \tag{39}$$

and consequently the gauge fermion (28) in Euclidean space is given as

$$\Psi_E = \frac{1}{\xi_k \xi^k}\mathrm{Tr}\int d^4x \left(\frac{1}{2}\xi^i\lambda_i[M,N] - \frac{1}{2}\xi^i\vec{\tau}_i^j\lambda_j\cdot\vec{D} - \frac{1}{2}\xi^i e_{\mu\nu}\lambda_i F_{\mu\nu} - \frac{i\sqrt{2}}{2}M\xi^i e_\mu D_\mu \bar{\lambda}_i\right). \tag{40}$$

The $N=2$ supersymmetric Euclidean action, that is constructed by Zumino [25], can now be written as the $\bar{s}$ variation of $\Psi_E$,

$$I_E = \bar{s}\Psi_E = \mathrm{Tr}\int d^4x \left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{8}\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} - \lambda^i \not{D}\bar{\lambda}_i + MD_\mu D_\mu N - \frac{i\sqrt{2}}{2}(\lambda_i[\lambda^i,N] + \bar{\lambda}^i[\bar{\lambda}_i, M]) \right.$$
$$\left. - \frac{1}{2}[M,N]^2 + \frac{1}{2}\vec{D}\cdot\vec{D}\right) \tag{41}$$

up to the topological term $\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$ and the auxiliary term $\frac{1}{2}\vec{D}\cdot\vec{D}$, since in our approach the inclusion of the auxiliary fields, i.e., off-shell formulation is mandatory.

As noted before, the only way to write the action as an $\bar{s}$-exact term is to allow the gauge fermion to depend on the negative powers of SUSY ghost $\xi_i$. In other words $\bar{s}$ has to be defined on the space of field polynomials that are not necessarily analytic in these constant ghosts.

On the other hand, the above-mentioned nonanalyticity argument can be used to derive a relation between the above expressions and topological theory. Note that after twisting, the physical nature of some fields are interpreted differently, i.e., some fields become ghosts while some others become antighosts [9,29]. We summarize the dimensions and ghost numbers of the topological fields $(A_\mu, \Phi, \bar{\Phi}, \psi_\mu, \eta, \mathcal{X}_{\mu\nu}, B_{\mu\nu})$ in Table II.

In order to get the topological fields that are given in Table II that have the correct dimensions and ghost numbers we note that the SUSY ghosts $\xi_i$ have ghost number 1 and dimension 1/2. Therefore, it is natural to think that the topological fields can be written as certain functions of ordinary $N=2$ fields and SUSY ghosts which are also partly nonanalytic in these constant ghosts. These relations can be found by using the nonanalytic structure of the gauge fermion $\Psi_E$ given in Eq. (40). The only consistent field redefinitions that assign the correct dimensionality and ghost number to the topological fields are found to be[9]

$$A_\mu = A_\mu, \tag{42}$$

$$\psi_\mu = -\xi_i e_\mu \bar{\lambda}^i, \tag{43}$$

$$\Phi = i\sqrt{2}\xi_i\xi^i N, \quad \bar{\Phi} = \frac{i}{\sqrt{2}\xi_i\xi^i}M, \tag{44}$$

$$\eta = \frac{1}{\xi_k\xi^k}\xi_i\lambda^i, \quad \mathcal{X}_{\mu\nu} = \frac{-2}{\xi_k\xi^k}\xi^i e_{\mu\nu}\lambda_i, \tag{45}$$

$$B_{\mu\nu} = \frac{-2}{\xi_k\xi^k}\xi^i e_{\mu\nu}\vec{\tau}_i^j\xi_j\cdot\vec{D}. \tag{46}$$

It is straightforward to show that when the above variable redefinitions are inserted in the ordinary Euclidean action (41) and in the transformations (33)–(39), the action and corresponding BRST transformations that are found are exactly the TYM action of Witten [9] with an auxiliary term and the (extended) BRST transformations defined in TYM

TABLE II. Dimensions $D$, Grassmann parity GP, and ghost numbers Gh of the topological fields.

| | $A_\mu$ | $\Phi$ | $\bar{\Phi}$ | $\psi_\mu$ | $\eta$ | $\mathcal{X}_{\mu\nu}$ | $B_{\mu\nu}$ | $c$ |
|---|---|---|---|---|---|---|---|---|
| $D$ | 1 | 0 | 2 | 1 | 2 | 2 | 2 | 0 |
| Gh | 0 | 2 | $-2$ | 1 | $-1$ | $-1$ | 0 | 1 |
| GP | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

[9]We have chosen the coefficients in the definitions of the topological fields in order to get the conventions of [29].

theory which contain Witten's scalar SUSY. In other words, as mentioned by several authors (but not shown explicitly to the best of our knowledge), TYM theory in flat Euclidean space can be obtained directly as variable redefinitions from the ordinary $N=2$ SYM theory. As is obvious from the above definitions of the topological fields, the ghost numbers and the dimensions that are assigned to the fields in the twisting procedure by hand appear here naturally due to the composite structure of the topological fields in terms of global ghosts $\xi_i$ and the original fields, i.e., with respect to the power of $\xi_i'$s in the definitions.

To be clear (and hoping not to be too tedious), we demonstrate the above points explicitly. By using the field redefinitions (33)–(39), $\bar{s}$ transformations can be rewritten as[10]

$$\bar{s}A_\mu = D_\mu c + \Psi_\mu, \tag{47}$$

$$\bar{s}\psi_\mu = -\{c,\Psi_\mu\} - D_\mu\Phi, \tag{48}$$

$$\bar{s}\Phi = -[c,\Phi], \tag{49}$$

$$\bar{s}c = -\frac{1}{2}\{c,c\} + \Phi, \tag{50}$$

and

$$\bar{s}\bar{\Phi} = -[c,\bar{\Phi}] + \eta, \tag{51}$$

$$\bar{s}\eta = -\{c,\eta\} + [\Phi,\bar{\Phi}], \tag{52}$$

$$\bar{s}\mathcal{X}_{\mu\nu} = -[c,\mathcal{X}_{\mu\nu}] + F^+_{\mu\nu} + \mathcal{B}_{\mu\nu}, \tag{53}$$

$$\bar{s}B_{\mu\nu} = -[c,B_{\mu\nu}] + [\Phi,\mathcal{X}_{\mu\nu}] - (D_\mu\psi_\nu - D_\nu\psi_\mu)^+, \tag{54}$$

where $F^+_{\mu\nu} = F_{\mu\nu} + \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}$ is the self-dual part of the field strength $F_{\mu\nu}$. If one decomposes $\bar{s}$ on the fields $(A_\mu,\Phi,\bar{\Phi},\psi_\mu,\eta,\mathcal{X}_{\mu\nu})$ as

$$\bar{s} = s_o + \delta, \tag{55}$$

where $s_0$ is the ordinary BRST transformation, one can see that $\delta$ transformations are exactly the same with the scalar supersymmetry transformations introduced by Witten [9]. Note that this scalar SUSY generator can also be written as a composite generator,

$$\delta = -i\xi^i Q_i,$$

where $Q_i$ are the chiral SUSY generators.

In terms of topological fields given in Eqs. (42)–(46), the gauge fermion (40) now reads

$$\Psi_{top} = \mathrm{Tr}\int d^4x\left(-\frac{1}{2}\eta[\Phi,\bar{\Phi}] + \frac{1}{8}\mathcal{X}_{\mu\nu}F^+_{\mu\nu} - \frac{1}{8}\mathcal{X}_{\mu\nu}B_{\mu\nu} + \bar{\Phi}D_\mu\psi_\mu\right) \tag{56}$$

and the corresponding action is found as

$$I_{top} = \bar{s}\Psi_{top} \tag{57}$$

$$= \mathrm{Tr}\int d^4x\left(\frac{1}{8}F^+_{\mu\nu}F^+_{\mu\nu} + \eta D_\mu\psi_\mu - \frac{1}{4}\mathcal{X}_{\mu\nu}(D_\mu\psi_\nu - D_\nu\psi_\mu)^+ - \bar{\Phi}D^2\Phi - \frac{1}{2}\Phi\{\eta,\eta\} - \frac{1}{8}\Phi\{\mathcal{X}_{\mu\nu},\mathcal{X}_{\mu\nu}\} + \bar{\Phi}\{\psi_\mu,\psi_\mu\} - \frac{1}{2}[\Phi,\bar{\Phi}]^2 - \frac{1}{8}B_{\mu\nu}B_{\mu\nu}\right). \tag{58}$$

The above given action $I_{top}$ is exactly the topological Yang-Mills action [9,29] with an auxiliary field term. We remark once more that the inclusion of the auxiliary field is crucial in order to write the action as an exact term.[11]

We should stress here that above results do not mean that $N=2$ SYM theory is just a topological theory. It has its own physical degrees of freedom. However, it becomes a topological theory (of cohomological type) if the analyticity requirement of the SUSY ghosts is relaxed. This fact has also been pointed out in Refs. [21–23], for twisted $N=2$ SYM theory that the twist of the $N=2$ theory can be interpreted as a topological theory only if the analyticity is lost in (scalar) SUSY ghosts. Note that, in many cases the cohomology of $s$ can be understood by studying a simpler operator that is found by using a suitable filtration of $s$ [24]. In our case we take $\bar{s}$ as the filtered operator. The cohomology of $\bar{s}$ is empty only if when $\bar{s}$ is allowed to act on the field polynomials that are not necessarily analytic in the parameters $\xi_i$. Since the cohomology of complete operator $s$ is isomorphic to a subspace of the filtrated operator $\bar{s}$ [24], the cohomology of $s$ is

[10]Here we note that the derivation of the above results depends crucially on the commuting nature of the global ghost $\xi_i$. For example, it is easy to verify that $\bar{s}$ transformation of $\lambda_i$ [Eq. (34)] decomposes into Eq. (52),(53) by using $\xi^i e_{\mu\nu}\xi_i = 0$ and $\xi^i\vec{\tau}_i^j\xi_j = 0$.

[11]The reason why the action could not be written as an exact term in the original paper [9] is that the twisted theory was obtained from the on-shell SYM theory. Note that, since $\Psi_{top}$ given in Eq. (56) is gauge invariant, we have $I_{top} = \bar{s}\Psi_{top} = \delta\Psi_{top}$.

also empty when the analyticity requirement is relaxed and the theory can be interpreted as a topological theory.

## V. INSTANTONS AND BAULIEU-SINGER APPROACH TO TYM THEORY

As is well known, instantons are finite action solutions of the Euclidean field theories. Aiming to incorporate the instantons into supersymmetric theories, Zumino constructed a supersymmetric field theory directly in the Euclidean space [25]. This theory has $N=2$ supersymmetry with a Hermitean action. (Recently, this theory has been derived by defining a continuous Wick rotation [31] and also by using dimensional reduction via time direction from six dimensional $N=1$ SYM theory [30].) It is observed by Zumino that when one imposes for instance an anti-self-dual field strength,

$$F^{+}_{\mu\nu}=F_{\mu\nu}+\frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}=0 \tag{59}$$

with the following restrictions:

$$M=\lambda_i=0 \tag{60}$$

the equations of motion from Eq. (41) reduce to a simple form [25],

$$F^{+}_{\mu\nu}=F_{\mu\nu}+\frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}=0, \tag{61}$$

$$D^2N=\frac{i\sqrt{2}}{2}\{\overline{\lambda^i},\overline{\lambda_i}\}, \tag{62}$$

$$e_\mu D_\mu\overline{\lambda^i}=0. \tag{63}$$

These restrictions (59),(60) and Eq. (61)–(63) are covariant under the supersymmetry transformations found by applying the above restrictions [25].

Equations (61)–(63) are also the ones whose solutions are used as approximate solutions of the saddle point equations in the context of constraint instanton method [3,6]. On the other hand, similar equations are obtained in TYM theory without any approximation [8] from an action functional that can be written as a BRST transformation of a gauge fermion given by Baulieu-Singer [10]. Both of the approaches to the instanton calculations give the same result [8].

Therefore, since Euclidean $N=2$ SYM action can be written as a BRST exact term (41) and Wittens TYM theory [9] can be obtained by using simple field redefinitions (42)–(46), we look for another analogy between the above instanton-inspired truncation of Euclidean $N=2$ SYM theory and the Baulieu-Singer approach to TYM theory.

The first step towards this purpose is to define a truncated $\tilde{s}$,

$$\tilde{s}=\bar{s}|_{F^{+}_{\mu\nu}=\not{D}\overline{\lambda^i}=M=\lambda_i=0}, \tag{64}$$

such that,

$$\tilde{s}A_\mu=D_\mu c-\xi_i e_\mu\overline{\lambda^i}, \tag{65}$$

$$\tilde{s}\overline{\lambda^i}=-\{c,\overline{\lambda^i}\}+i\sqrt{2}\overline{e_\mu}\xi^i D_\mu N, \tag{66}$$

$$\tilde{s}N=-[c,N], \tag{67}$$

$$\tilde{s}c=-\frac{1}{2}\{c,c\}+i\sqrt{2}\xi_i\xi^i N, \tag{68}$$

and

$$\tilde{s}M=i\sqrt{2}\xi^i\lambda_i, \tag{69}$$

$$\tilde{s}\lambda_i=\vec{\tau}_i^{\,j}\xi_j\cdot\vec{D}, \tag{70}$$

$$\tilde{s}\vec{D}=0. \tag{71}$$

The reason why we do not set $\lambda_i=\vec{D}=0$ in Eq. (69),(70) is that the pairs $(M,\xi^i\lambda_i)$ and $(\xi^i\vec{\tau}_i^{\,j}\lambda_j,\vec{D})$ behave like trivial pairs (sometimes called BRST doublets), i.e., like $(\bar{c},b)$, such that $s\bar{c}=b,\ sb=0$, where $s$ is a nilpotent operator. It is known that the cohomology of an operator does not depend on inclusion of such trivial pairs (see, for instance, [15,24]).

It is straightforward to derive that $\tilde{s}$ is also nilpotent,

$$\tilde{s}^2=0,$$

and after performing the field redefinition given in Eqs. (42)–(46), $\tilde{s}$ transformations are found to be exactly that of Baulieu-Singer [10,29].

On the other hand, the gauge fermion that is compatible with the restrictions of Zumino [25] has to be chosen slightly different from the one given for the Euclidean case (40),

$$\Psi_{inst}=\frac{1}{\xi_k\xi^k}\mathrm{Tr}\int d^4x\left(-\frac{\alpha}{2}\xi^i\vec{\tau}_i^{\,j}\lambda_j\cdot\vec{D}-\frac{1}{2}\xi^i e_{\mu\nu}\lambda_i F^{+}_{\mu\nu}+\frac{i\sqrt{2}}{2}\xi^i e_\mu\overline{\lambda_i}D_\mu M\right). \tag{72}$$

The reason for this modification becomes apparent when the corresponding action is driven,

$$I_{inst}^{(\alpha)}=\tilde{s}\Psi_{inst}=\mathrm{Tr}\int d^4x\bigg[-\frac{\alpha}{8}B_{\mu\nu}B_{\mu\nu}+\frac{1}{4}B_{\mu\nu}F^+_{\mu\nu}-\lambda^i e_\mu D_\mu\overline{\lambda}_i+M\bigg(D_\mu D_\mu N-\frac{i\sqrt{2}}{2}\{\overline{\lambda^i},\overline{\lambda}_i\}\bigg)+\frac{1}{\xi_k\xi^k}\bigg(-\frac{1}{2}\xi^i e_{\mu\nu}\lambda_i[c,F^+_{\mu\nu}]+\frac{i\sqrt{2}}{2}M\{c,\xi^i e_\mu D_\mu\overline{\lambda}_i\}\bigg)\bigg]+\frac{1}{\xi_k\xi^k}\mathrm{Tr}\int d^4x\,\partial_\mu\bigg(\tilde{s}\,\frac{i\sqrt{2}}{2}M\xi^i e_\mu\overline{\lambda}_i\bigg), \tag{73}$$

where we have used the definition of $B_{\mu\nu}$ in order to have notational simplification.

First of all, the gauge fermion $\Psi_{inst}$ (72) and the above action $I_{inst}$ are exactly the ones given in Baulieu-Singer approach [10] up to ordinary gauge fixing.[12] It is straightforward to rewrite the above given gauge fermion $\Psi_{inst}$ and the action $I_{inst}$ by using the field redefinitions given in Eqs. (42)–(46) in order to get the results of [10]. However, if the above relations are considered on their own, to be able to derive the instanton equations (61)–(63) from the action functional without having any dependence on the constant ghosts, the Euclidean $\Psi_E$ has to be modified. For instance when the restrictions of [25] are used the $\xi_i$ dependence that comes from the $\tilde{s}$ variation of $\mathrm{Tr}\xi^i\lambda_i[M,N]$ cannot be eliminated. Therefore, the coefficient of this term has to be chosen to vanish. The coefficient of $\mathrm{Tr}\,\tilde{s}\xi^i\vec{\tau}_i^j\lambda_j\cdot\vec{D}$ can be left arbitrary since after performing the Gaussian integration over the auxiliary field $B_{\mu\nu}$ the action is

$$\begin{aligned}I_{inst}^{(\alpha)}&=\tilde{s}\Psi_{inst}\\&=\mathrm{Tr}\int d^4x\bigg[\frac{1}{8\alpha}F^+_{\mu\nu}F^+_{\mu\nu}-\lambda^i e_\mu D_\mu\overline{\lambda}_i+M\bigg(D_\mu D_\mu N-\frac{i\sqrt{2}}{2}\{\overline{\lambda^i},\overline{\lambda}_i\}\bigg)\\&\quad+\frac{1}{\xi_k\xi^k}\bigg(-\frac{1}{2}\xi^i e_{\mu\nu}\lambda_i[c,F^+_{\mu\nu}]+\frac{i\sqrt{2}}{2}M\{c,\xi^i e_\mu D_\mu\overline{\lambda}_i\}\bigg)\bigg]\\&=\mathrm{Tr}\int d^4x\bigg[\frac{1}{8\alpha}F^+_{\mu\nu}F^+_{\mu\nu}-\lambda^i e_\mu D_\mu\overline{\lambda}_i+M\bigg(D_\mu D_\mu N-\frac{i\sqrt{2}}{2}\{\overline{\lambda^i},\overline{\lambda}_i\}\bigg)\\&\quad+\frac{1}{\xi_k\xi^k}\bigg(-\frac{1}{2}\xi^i e_{\mu\nu}\lambda_i[c,F^+_{\mu\nu}]+\frac{i\sqrt{2}}{2}M\{c,\xi^i e_\mu D_\mu\overline{\lambda}_i\}\bigg)\bigg]+\frac{1}{\xi_k\xi^k}\mathrm{Tr}\int d^4x\,\partial_\mu\bigg(\tilde{s}\,\frac{i\sqrt{2}}{2}M\xi^i e_\mu\overline{\lambda}_i\bigg)\end{aligned} \tag{74}$$

and the Fadeev-Popov ghost field, $c$, independent part of the action is also SUSY ghost $\xi_i$ free. The form of the last term in $\Psi_{inst}$ is inspired from Ref. [8] in order to get a surface contribution, since

$$\mathrm{Tr}\,\tilde{s}\xi^i e_\mu\overline{\lambda}_i D_\mu M=\mathrm{Tr}\,\partial^\mu\tilde{s}\xi^i e_\mu\overline{\lambda}_i M-\mathrm{Tr}\,\tilde{s}M\xi^i e_\mu D_\mu\overline{\lambda}_i$$

if the scalar field has nontrivial boundary conditions. Therefore, the gauge fermion $\Psi_{inst}$ is the only consistent choice up to total derivatives that gives the right action to derive the exact instanton equations, when the truncated transformations (65)–(71) are used.

On the other hand, the free parameter $\alpha$ can be thought of as a gauge parameter, since it is so in the Baulieu-Singer approach [10]. By choosing directly $\alpha=0$, the action (71) takes the form

$$\begin{aligned}I_{inst}=\tilde{s}\Psi_{inst}&=\mathrm{Tr}\int d^4x\bigg[\frac{1}{4}B_{\mu\nu}F^+_{\mu\nu}-\lambda^i e_\mu D_\mu\overline{\lambda}_i+M\bigg(D_\mu D_\mu N-\frac{i\sqrt{2}}{2}\{\overline{\lambda^i},\overline{\lambda}_i\}\bigg)\\&\quad+\frac{1}{\xi_k\xi^k}\bigg(-\frac{1}{2}\xi^i e_{\mu\nu}\lambda_i[c,F^+_{\mu\nu}]+\frac{i\sqrt{2}}{2}M\{c,\xi^i e_\mu D_\mu\overline{\lambda}_i\}\bigg)\bigg]+\frac{1}{\xi_k\xi^k}\mathrm{Tr}\int d^4x\,\partial_\mu\bigg(\tilde{s}\,\frac{i\sqrt{2}}{2}M\xi^i e_\mu\overline{\lambda}_i\bigg)\end{aligned} \tag{75}$$

and by performing a functional integration over the fields $\lambda_i, M$, and $\mathcal{B}_{\mu\nu}$, the configurations of the constraint instanton method (61)–(63) are obtained without using any approximation procedure. In other words, as is demonstrated above, the Baulieu-Singer approach can also be obtained by using field redefinitions given in Eqs. (42)–(46) when an instanton-inspired truncation (59),(60) of the Euclidean model is used.

[12] For instance, adding $\tilde{s}\int d^4x\;\overline{c}\partial^\mu A_\mu$ to $\Psi_{inst}$ such that $\tilde{s}\overline{c}=b$, $\tilde{s}b=0$.

## VI. CONCLUSION AND DISCUSSION

In this paper, we have shown how to write the off-shell $N=2$ SYM action as an exact term by using a nilpotent extended BRST operator $\overline{s}$ that includes supersymmetry transformations of one chirality. The corresponding gauge-fixing fermion is found to be nonanalytic in global SUSY ghosts. In other words, it is shown that the action belongs to trivial cohomology of the extended BRST operator $\overline{s}$, if this operator is allowed to act on the field polynomials that are not necessarily analytic in these global ghosts.

Due to this nonanalytical structure, we have found field redefinitions such that Witten's TYM theory [9] can be obtained from the Euclidean $N=2$ SYM theory by identifying the fields of TYM theory with composite fields of an $N=2$ vector multiplet and the chiral SUSY ghosts $\xi_i$ in Euclidean space. These field redefinitions are also partly nonanalytic in global SUSY ghosts. The ghost numbers and the dimensions of the topological fields, which are assigned by hand when it is formulated by twisting, appear in our approach naturally according to this composite structure. In other words, we have shown explicitly that TYM theory can be found from $N=2$ SYM theory exactly as a change of variables (i.e., without twisting).

The above mentioned analyticity requirement also plays a decisive role in understanding when $N=2$ SYM theory can be interpreted as a topological theory. Note that the topological theory is obtained via a change of variables only when the requirement of analyticity of the constant ghosts $\xi_i$ is relaxed. Therefore, physical and topological interpretations of $N=2$ SYM theory are intertwined together. However, in order to have a better understanding of implications of the results presented above, it would be interesting to investigate the perturbative regime of the theory by using the standard techniques of algebraic renormalization framework [24].

On the other hand, when the restrictions on the fields are used in order to get supersymmetric instanton configurations [25], we show that with the help of a truncated BRST operator $\tilde{s}$, an action can be written as an $\tilde{s}$-exact term. After using the given variable redefinitions, it is seen that this formulation is exactly TYM theory in the approach of Baulieu-Singer [10]. The instanton equations, which are used in the *traditional* instanton computations [3,6], can be derived from this action (73) without using any approximation. Moreover, it is known that Witten's action [9] can be obtained from the one given in the Baulieu-Singer approach by a continuous deformation of gauge fixing [10]. As a consequence, a similar relation also occurs between the Euclidean $N=2$ SYM action (41) and the truncated (instanton) action (73) since both of the formulations of TYM theory can be obtained by using the variable redefinitions.

Finally, it is worthwhile to mention that the instanton calculations performed in the Baulieu-Singer approach of TYM theory [8] give exactly the same result as the one performed in $N=2$ SYM theory [3,6]. Since both $N=2$ SYM and TYM theories are shown to be equivalent by simple variable redefinitions, it would also be interesting to reinvestigate the equivalence of the instanton calculations of $N=2$ SYM and TYM theory and to find out whether the instantons localize in the topological sector of the theory where the functional space of field polynomials is not necessarily analytic in global SUSY ghosts.

## ACKNOWLEDGMENTS

I am indebted to R. Flume for an initiation into the subject. I also gratefully acknowledge the numerous enlightening discussions with Ö. F. Dayı and M. Hortaçsu.

[1] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); **B431**, 19 (1994).
[2] R. Flume, M. Magro, L. O'Raifeartaigh, I. Sachs, and O. Schnetz, Nucl. Phys. **B494**, 331 (1997).
[3] N. Dorey, V. V. Khoze, and M. P. Mattis, Phys. Rev. D **54**, 2921 (1996).
[4] D. Finnell and P. Pouliot, Nucl. Phys. **B453**, 225 (1995).
[5] K. Ito and N. Sasakura, Phys. Lett. B **382**, 95 (1996).
[6] N. Dorey, T. J. Hollowood, V. V. Khoze, and M. P. Mattis, Phys. Rep. **371**, 231 (2002).
[7] R. Flume, L. O'Raifeartaigh, and I. Sachs, talk presented at the Inaugural Conference of the Asia Pacific Center for Theoretical Physics, Seoul, 1996, the XXIst Conference on Group Theor. Methods in Physics, Goslar, Germany, 1996, and 2nd SIMI Conference, Tbilisi, 1996, hep-th/9611118; R. Flume, talk presented at the 9th Max Born Symposium, Karpacz, 1996, hep-th/9702192; D. Bellisai, F. Fucito, A. Tanzini, and G. Travaglini, hep-th/9812145.
[8] D. Bellisai, F. Fucito, A. Tanzini, and G. Travaglini, Phys. Lett. B **480**, 365 (2000); J. High Energy Phys. **07**, 017 (2000).
[9] E. Witten, Commun. Math. Phys. **117**, 353 (1988).
[10] L. Baulieu and I. M. Singer, Nucl. Phys. B (Proc. Suppl.) **5**, 12 (1985).
[11] K. Ulker, Mod. Phys. Lett. A **16**, 881 (2001).
[12] K. Ulker, Mod. Phys. Lett. A **17**, 739 (2002).
[13] F. Brandt, Nucl. Phys. **B392**, 428 (1993); Class. Quantum Grav. **11**, 849 (1994); Ann. Phys. (N.Y.) **259**, 253 (1997).
[14] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. **69B**, 309 (1983); Phys. Rev. D **28**, 2567 (1983); Phys. Lett. **102B**, 27 (1981).
[15] J. Gomis, J. Paris, and S. Samuel, Phys. Rep. **259**, 1 (1995) for a self-contained review.
[16] P. L. White, Class. Quantum Grav. **9**, 413 (1992); **9**, 1663 (1992).
[17] N. Maggiore, Int. J. Mod. Phys. A **10**, 3937 (1995); **10**, 3781 (1995).
[18] N. Maggiore, O. Piguet, and S. Wolf, Nucl. Phys. **B458**, 403 (1996); **B476**, 329 (1996).
[19] A. Blasi and R. Collina, Nucl. Phys. **B285**, 204 (1987); C. Becchi *et al.*, Commun. Math. Phys. **120**, 121 (1988).
[20] F. Brandt, M. Henneaux, and A. Wilch, Phys. Lett. B **387**, 320 (1996); Nucl. Phys. **B510**, 640 (1998).
[21] F. Fucito, A. Tanzini, L. C. Vilar, O. S. Ventura, C. A. Sasaki,

and S. P. Sorella, Algebraic Renormalization: Perturbative twisted considerations on topological Yang-Mills theory and on N=2 supersymmetric gauge theories, lectures given at the First School on Field Theory and Gravitation, Victoria, Esprinto, Brazil, 1997, hep-th/9707209.

[22] A. Blasi, V. E. Lemes, N. Maggiore, S. P. Sorella, A. Tanzini, O. S. Ventura, and L. C. Vilar, J. High Energy Phys. **05**, 039 (2000).

[23] V. E. Lemes, M. S. Sarandy, S. P. Sorella, O. S. Ventura, and L. C. Vilar, J. Phys. A **34**, 9485 (2001).

[24] O. Piguet and S. P. Sorella, *Algebraic Renormalization*, Lecture Notes in Physics Vol. 28 (Springer-Verlag, Berlin, 1995).

[25] B. Zumino, Phys. Lett. **69B**, 369 (1977).

[26] R. Grimm, M. F. Sohnius, and J. Wess, Nucl. Phys. **B133**, 275 (1978).

[27] M. Sohnius, Phys. Rep. **128**, 39 (1985).

[28] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, NJ, 1992).

[29] D. Birmingham, M. Blau, M. Rakowski, and G. Thompson, Phys. Rep. **209**, 129 (1991).

[30] A. V. Belitsky, S. Vandoren, and P. van Nieuwenhuizen, Phys. Lett. B **477**, 335 (2000).

[31] P. van Nieuwenhuizen and A. Waldron, Phys. Lett. B **389**, 29 (1996).