# Higher dimensional Bell-Szekeres metric

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The collision of pure electromagnetic plane waves with collinear polarization in $N$-dimensional ($N=2+n$) Einstein-Maxwell theory is considered. A class of exact solutions for the higher dimensional Bell-Szekeres metric is obtained and its singularity structure is examined.

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## I. INTRODUCTION

One of the main fields of interest in general relativity is the collision of gravitational plane waves. Colliding plane wave space-times have been investigated in detail in general relativity [1]. The first exact solution of the Einstein-Maxwell equations representing colliding plane shock electromagnetic waves with collinear polarizations was obtained by Bell and Szekeres (BS) [2]. This solution is conformally flat in the interaction region and its singularity structure has been considered by Matzner and Tipler [3], Clarke and Hayward [4], and Helliwell and Konkowski [5]. Later Halil [6], Gürses and Halilsoy [7], Griffiths [8], and Chandrasekhar and Xanthopoulos [9] studied exact solutions of the Einstein-Maxwell equations describing the collision of gravitational and electromagnetic waves. Furthermore, Gürses and Sermutlu [10], and more recently Halilsoy and Sakallı [11], have obtained the extensions of the BS solution in the Einstein-Maxwell-dilaton and Einstein-Maxwell-axion theories, respectively.

One of the purposes of this work is to observe which of the relevant physical properties of BS metric are conveyed to higher dimensions. Another motivation is that the BS metric has attracted many researchers working in the context of string theory. Plane wave metrics in various dimensions provide exact solutions in string theory [12]. It is interesting to study the collision of plane waves at least in the low energy limit of string theory. There have been some attempts in this direction [10,13–19]. In addition, the collision of plane waves may shed some light on string cosmology (see [16] and references therein).

In this work we give a higher dimensional generalization of the BS metric. We present an exact solution generalizing the BS solution and examine the singularity structure of the corresponding space-times in the context of curvature and Maxwell invariants. We show that this space-time, unlike the BS metric, is not conformally flat.

In Sec. II we give a brief review of the BS solution and in Sec. III we formulate the $N$-dimensional Einstein-Maxwell equations. In Sec. IV we present the $N$-dimensional colliding exact plane wave solutions describing the collision of shock electromagnetic waves. We also examine the singularity structure of the corresponding space-times and show that interaction region of our solution admits curvature singularities.

## II. THE BELL-SZEKERES METRIC

The BS metric is given by

$$ds^2=2dudv+e^{-U}(e^{V}dx^2+e^{-V}dy^2) \tag{1}$$

where the metric functions $U$ and $V$ depend on the null coordinates $u$ and $v$. The electromagnetic vector potential has a single nonzero component $A=(0,0,0,A)$, where $A$ is functions of $u$ and $v$. The complete solution of the Einstein-Maxwell equations is

$$U=-\log[f(u)+g(v)],\quad A=\gamma(pw-rq),$$
$$V=\log(rw-pq)-\log(rw+pq), \tag{2}$$

where

$$r=\left(\frac{1}{2}+f\right)^{1/2},\quad p=\left(\frac{1}{2}-f\right)^{1/2},$$
$$w=\left(\frac{1}{2}+g\right)^{1/2},\quad q=\left(\frac{1}{2}-g\right)^{1/2} \tag{3}$$

with

$$f=\frac{1}{2}-\sin^2 P,\quad g=\frac{1}{2}-\sin^2 Q. \tag{4}$$

Here $P=au\Theta(u)$, $Q=bv\Theta(v)$, where $\Theta$ is the Heaviside unit step function, $a$ and $b$ are arbitrary constants, and $\gamma^2=8\pi/\kappa$ with $\kappa$ being the gravitational constant. The nature of the space-time singularity of the BS solution has been considered by Matzner and Tipler [3], Clarke and Hayward [4], and more recently by Helliwell and Konkowski [5]. To investigate the singularity structure of a space-time one needs all curvature invariants. Due to the simplicity of BS

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metric in the collision region the invariant $I=R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ is constant everywhere in the interaction region. It is given by

$$I_{BS}=\frac{8}{w^2p^2q^2r^2(f+g)^2}\Big\{fgf_u^2g_v^2+w^2p^2q^2r^2f_{uu}g_{vv}+\frac{1}{4}[(3g-f)r^2p^2f_{uu}g_v^2+(3f-g)w^2q^2g_{vv}f_u^2]\Big\}=32a^2b^2. \tag{5}$$

In the BS solution, the singularity that occurs when $f+g=0$ corresponds to a Cauchy horizon. This solution is conformally flat in the interaction region; as an example one of the components of the Weyl tensor is given by

$$C_{0202}=-\frac{wq}{2rp(rw+pq)^2(f+g)}\left[\frac{f}{r^2p^2}f_u^2+f_{uu}\right]. \tag{6}$$

The global structure of the BS solution has been analyzed in detail by Clarke and Hayward [4]. They have shown that this solution possesses quasiregular singularities at the null boundaries. Finally, the invariant

$$F_{\alpha\beta}F^{\alpha\beta}=-2\gamma^2ab \tag{7}$$

which is also a constant quantity in the interaction region. The BS solution in the interaction region is isometric to the Bertotti-Robinson space-time [1,4].

### III. *N*-DIMENSIONAL EINSTEIN-MAXWELL EQUATIONS

Let $M$ be an $N=2+n$ dimensional manifold with a metric

$$ds^2=g_{\alpha\beta}\ dx^\alpha\ dx^\beta=g_{ab}(x^c)dx^adx^b+H_{AB}(x^c)dy^Ady^B \tag{8}$$

where $x^\alpha=(x^a,\, y^A)$, $x^a$ denote the local coordinates on a two-dimensional manifold, and $y^A$ denote the local coordinates on a $n$-dimensional manifold, thus $a,b=1,2$, $A,B=1,2,\dots,n$. The Christoffel symbols of the metric $g_{\alpha\beta}$ can be calculated to give

$$\Gamma^A_{Ba}=\frac{1}{2}H^{AD}\quad H_{DB,a}, \quad \Gamma^a_{AB}=-\frac{1}{2}g^{ab}\quad H_{AB,b}, \tag{9}$$

$$\Gamma^A_{BD}=\Gamma^A_{ab}=\Gamma^a_{Ab}=0,\overline{\Gamma}^a_{bc}=\Gamma^a_{bc} \tag{10}$$

where the $\Gamma^a_{bc}$ are the Christoffel symbols of the two-dimensional metric $g_{ab}$.

The components of the Ricci tensor are given by

$$\mathcal{R}_{ab}=R^\alpha_{a\alpha b}=R_{ab}+\frac{1}{4}tr(\partial_aH^{-1}\partial_bH)-\nabla_a\nabla_b\log\sqrt{\det H}, \tag{11}$$

$$\mathcal{R}_{AB}=-\frac{1}{2}(g^{ab}H_{AB,b})_{,a}-\frac{1}{2}g^{ab}H_{AB,b}\left[\frac{(\sqrt{\det g})_{,a}}{\sqrt{\det g}}+\frac{(\sqrt{\det H})_{,a}}{\sqrt{\det H}}\right]+\frac{1}{2}g^{ab}H_{EA,b}H^{ED}H_{DB,a}, \tag{12}$$

$$\mathcal{R}_{aA}=0, \tag{13}$$

where $R_{ab}$ is the Ricci tensor of the two-dimensional metric $g_{ab}$. The Maxwell potential 1-form $\mathcal{A}$ is

$$\mathcal{A}=A_Ady^A. \tag{14}$$

The components of the electromagnetic field

$$\mathcal{F}=\frac{1}{2}F_{\alpha\beta}dx^\alpha\wedge dx^\beta \tag{15}$$

are

$$F_{aA}=A_{A,a}, \quad F_{ab}=0, \quad F_{AB}=0. \tag{16}$$

The components of the energy-momentum tensor

$$T_{\mu\nu}=\frac{1}{4\pi}\left[g^{\alpha\beta}F_{\mu\beta}F_{\nu\alpha}-\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right] \tag{17}$$

are

$$T_{ab}=\frac{1}{4\pi}\left[H^{AB}F_{aA}F_{bB}-\frac{1}{2}g_{ab}F^2\right],\quad T_{AB}=\frac{1}{4\pi}\left[g^{ab}F_{aA}F_{bB}-\frac{1}{2}H_{AB}F^2\right],\quad T_{aA}=0, \tag{18}$$

where $F^2=F_{aD}F^{aD}$. Then the Einstein field equations are

$$R_{\mu\nu}=\kappa\left[T_{\mu\nu}+\frac{1}{2-N}g_{\mu\nu}T\right] \tag{19}$$

where the trace of the energy momentum tensor $T$ is

$$T=\frac{1}{8\pi}(2-n)F^2. \tag{20}$$

The Einstein-Maxwell equations are

$$R_{ab}+\frac{1}{4}\mathrm{tr}(\partial_aH^{-1}\partial_bH)-\nabla_a\nabla_b\log\sqrt{\det H}=\frac{\kappa}{4\pi}H^{AB}F_{aA}F_{bB}-\frac{\kappa}{4\pi n}g_{ab}F^2, \tag{21}$$

$$\partial_a[\sqrt{\det Hg}\; g^{ab} H^{AS} \partial_b H_{AB}]$$
$$= -\frac{\kappa}{2\pi}\sqrt{\det Hg}\left[ H^{AS} g^{ab} F_{Aa} F_{Bb} - \frac{\delta_B^S}{n} F^2 \right], \tag{22}$$

and

$$\partial_a[\sqrt{\det H\, g}\;\; F^{Aa}] = 0, \tag{23}$$

where $\nabla$ is the covariant differentiation with respect to the connection $\Gamma^a_{bc}$ (or with respect to metric $g_{ab}$). We may rewrite the two-dimensional metric as

$$g_{ab} = e^{-M}\;\; \eta_{ab}, \tag{24}$$

where $\eta$ is the metric of flat 2-geometry with arbitrary signature $(0$ or $\pm 2)$the function $M$ depends on the local coordinates $x^a$. The corresponding Ricci tensor and the Christoffel symbols are

$$R_{ab} = \frac{1}{2}(\nabla_\eta^2 M)\;\; \eta_{ab},$$

$$\Gamma^c_{ab} = \frac{1}{2}[-M_{,b}\delta^c_a - M_{,a}\delta^c_b + M_{,d}\eta^{cd}\eta_{ab}]. \tag{25}$$

## IV. HIGHER DIMENSIONAL BELL-SZEKERES METRIC

In this section we give the higher dimensional colliding exact plane wave metric generalizing the BS metric. For this purpose let $H$ be a diagonal matrix

$$H = e^{-U} h \tag{26}$$

where

$$h = \begin{pmatrix} e^{V_1} & & \\ & \ddots & \bigcirc \\ \bigcirc & \ddots & \\ & & e^{V_n} \end{pmatrix}$$

with $\det h = 1$, i.e., $\sum_{k=1}^{n-1} V_k + V_n = 0$.

Now taking the signature of the flat-space metric with null coordinates

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad x^1 = u,\; x^2 = v,$$

and

$$A_A = (0, \ldots, A)$$

the Einstein-Maxwell equations become

$$2U_{uv} - nU_u U_v = 0, \tag{27}$$

$$-\frac{n}{2}U_u V_{kv} - \frac{n}{2}U_v V_{ku} + 2V_{kuv} = \frac{\kappa}{\pi n} e^{U - V_n} A_u A_v, \tag{28}$$

$$-\frac{n}{2}U_u V_{nv} - \frac{n}{2}U_v V_{nu} + 2V_{nuv} = \frac{\kappa(1-n)}{\pi n} e^{U - V_n} A_u A_v, \tag{29}$$

$$\left(\frac{n-2}{2}\right)(U_u A_v + U_v A_u) + V_{nu} A_v + V_{nv} A_u = 2A_{uv}, \tag{30}$$

$$-\frac{n}{2}U_u^2 - \frac{1}{2}\sum_{k=1}^{n-1}(V_{ku})^2 - \frac{1}{2}(V_{nu})^2 + nU_{uu} + nM_u U_u$$
$$= \frac{\kappa}{2\pi} e^{U - V_n}(A_u)^2, \tag{31}$$

$$-\frac{n}{2}U_v^2 - \frac{1}{2}\sum_{k=1}^{n-1}(V_{kv})^2 - \frac{1}{2}(V_{nv})^2 + nU_{vv} + nM_v U_v$$
$$= \frac{\kappa}{2\pi} e^{U - V_n}(A_v)^2, \tag{32}$$

$$2M_{uv} - \frac{n}{2}U_u U_v - \frac{1}{2}\sum_{k=1}^{n-1} V_{kv} V_{ku} - \frac{1}{2}V_{nv} V_{nu} + nU_{uv}$$
$$= \frac{\kappa}{2\pi n}(2-n) e^{U - V_n} A_u A_v, \tag{33}$$

where $k = 1, \ldots, n-1$. Note that the last equation is not independent. It can be obtained from the other equations. The most general solution to Eq. (27) is given by

$$U = -\frac{2}{n}\log[f(u) + g(v)] \tag{34}$$

in terms of two arbitrary functions $f$ and $g$. Now changing variables $(u,v)$ to $(f,g)$ the remaining field equations become

$$-\frac{n}{2}U_f V_{kg} - \frac{n}{2}U_g V_{kf} + 2V_{kfg} = \frac{\kappa}{\pi n} e^{U - V_n} A_f A_g, \tag{35}$$

$$-\frac{n}{2}U_f V_{ng} - \frac{n}{2}U_g V_{nf} + 2V_{nfg} = \frac{\kappa(1-n)}{\pi n} e^{U - V_n} A_f A_g, \tag{36}$$

$$\left(\frac{n-2}{2}\right)(U_f A_g + U_g A_f) + V_{nf} A_g + V_{ng} A_f = 2A_{fg}, \tag{37}$$

$$M_u = -\frac{f_{uu}}{f_u} + \frac{(n-1)}{n}\frac{f_u}{f+g} - \frac{(f+g)}{4f_u}\Bigg[\sum_{k=1}^{n-1}(V_{ku})^2 + (V_{nu})^2$$
$$+ \frac{\kappa}{\pi} e^{U - V_n}(A_u)^2\Bigg], \tag{38}$$

$$M_v = -\frac{g_{vv}}{g_v} + \frac{(n-1)}{n}\frac{g_v}{f+g} - \frac{(f+g)}{4g_v}\Bigg[\sum_{k=1}^{n-1}(V_{kv})^2 + (V_{nv})^2$$
$$+ \frac{\kappa}{\pi} e^{U - V_n}(A_v)^2\Bigg]. \tag{39}$$

Equations (35), (36), and (37) are integrability conditions for Eqs. (38) and (39). An exact solution to the above Eqs. (35), (36), and (37) is

$$V_k=\alpha_k \log(rw-pq)+\beta_k \log(rw+pq), \tag{40}$$

$$V_n=-\alpha \log(rw-pq)-\beta \log(rw+pq), \tag{41}$$

$$A=\gamma(pw-rq) \tag{42}$$

with

$$\alpha_k-\beta_k=\frac{\kappa\gamma^2}{2\pi n} \tag{43}$$

for all $k=1,\ldots,n-1$, and

$$\sum_{k=1}^{n-1} \alpha_k=\alpha=\frac{2}{n}, \quad \sum_{k=1}^{n-1} \beta_k=\beta=\frac{2}{n}-2. \tag{44}$$

Then we may obtain the value of $\gamma$ as $\gamma^2=4n\pi/\kappa(n-1)$.

It is convenient to put Eqs. (38) and (39) in the following form [1]:

$$e^{-M}=\frac{f_u g_v}{(f+g)^{(n-1)/n}}e^{-S} \tag{45}$$

where $S$ satisfies

$$S_f=-\frac{(f+g)}{4}\left[\sum_{k=1}^{n-1}(V_{kf})^2+(V_{nf})^2+\frac{\kappa}{\pi}e^{U-V_n}(A_f)^2\right], \tag{46}$$

$$S_g=-\frac{(f+g)}{4}\left[\sum_{k=1}^{n-1}(V_{kg})^2+(V_{ng})^2+\frac{\kappa}{\pi}e^{U-V_n}(A_g)^2\right]. \tag{47}$$

Therefore we may write the metric function $M$ as

$$\begin{aligned} M=&-\log(cf_u g_v)+\left[\frac{n-1}{n}-\frac{4+n^2m_1}{4n^2}\right]\log(f+g)\\ &+\left(\frac{n}{4(n-1)}\right)\log\left(\frac{1}{2}-f\right)+\left(\frac{n}{4(n-1)}\right)\log\left(\frac{1}{2}+f\right)\\ &+\left(\frac{n}{4(n-1)}\right)\log\left(\frac{1}{2}-g\right)+\left(\frac{n}{4(n-1)}\right)\log\left(\frac{1}{2}+g\right)\\ &+\frac{1}{8n}[8-4n+n(m_1-m_2)]\log(1+4fg+4prwq) \end{aligned} \tag{48}$$

where $c$ is a constant and

$$\sum_{k=1}^{n-1}\alpha_k^2=m_1, \quad \sum_{k=1}^{n-1}\beta_k^2=m_2. \tag{49}$$

$m_1$ and $m_2$, using Eq. (43), satisfy

$$m_1+m_2-2m_3=\frac{4}{n-1}, \quad m_1-m_2=\frac{4(2-n)}{n(n-1)} \tag{50}$$

with

$$\sum_{k=1}^{n-1}\alpha_k\beta_k=m_3.$$

The metric function $e^{-M}$ must be continuous across the null boundaries. To make it so we assume that the functions $f$ and $g$ take the form

$$f=\frac{1}{2}-\sin^{n_1}P, \quad g=\frac{1}{2}-\sin^{n_2}Q. \tag{51}$$

Then the metric function $e^{-M}$ is continuous across the boundaries if

$$n_1=n_2=\frac{4(n-1)}{3n-4}. \tag{52}$$

Therefore, the metric function $e^{-M}$ reads

$$e^{-M}=\frac{(1+4fg+4pqrw)^{k_1}\left[1-\left(\frac{1}{2}-f\right)^{2/n_1}\right]^{1/2}\left[1-\left(\frac{1}{2}-g\right)^{2/n_1}\right]^{1/2}}{\left(\frac{1}{2}+f\right)^{1-(1/n_1)}\left(\frac{1}{2}+g\right)^{1-(1/n_1)}(f+g)^{k_2}}, \tag{53}$$

where

$$k_1=\frac{n-2}{2(n-1)}, \quad k_2=\frac{n-1}{n}-\frac{1}{4n^2}(4+n^2m_1). \tag{54}$$

It may thus be observed that the constant $n_1$ $(=n_2)$ is restricted to the range satisfying

$$2\geqslant n_1=n_2>\frac{4}{3}. \tag{55}$$

It is also appropriate to choose $c=1/n_1^2ab$.

The space-time line element generalizing the BS metric in $N=2+n$ dimensions is

$$ds^2=2e^{-M}du\,dv+e^{-U}(e^{V_1}dx_1^2+\cdots+e^{V_n}dx_n^2), \tag{56}$$

where the metric functions are given in Eqs. (40), (41), and (53). Because of Eq. (55) the metric we have found is $C^1$ for $n>2$ across the null boundaries. In spite of this fact, the Ricci tensor is regular across the null boundaries due to the Einstein field equations. The above solution reduces to the well known BS solution for $n=2$.

We now discuss the nature of the space-time singularities. We study the behavior of the metric functions $U$, $V_k$, $V_n$, and $M$ as $f+g$ tends to zero. In the BS solution the collision of the two shock electromagnetic plane waves generates impulsive gravitational waves along the null boundaries. It is shown that, apart from the impulsive waves themselves, the BS solution has no curvature singularities and the only singularites are of the quasiregular type [4]. In general the analysis of space-time singularities requires all invariants. In a higher dimensional case it is not feasible to study all invariants of the space-time geometry. For this reason we consider only the quadratic Riemann invariant $I$ which is constant for the BS solution Eq. (5). However, for $n>2$ this invariant can be shown to have the behavior

$$I \sim e^{2M} \frac{(f_u g_v)^2}{(f+g)^4} \tag{57}$$

as $f+g \to 0$. Using $M$ from Eq. (53) we find

$$I \sim (f_u\, g_v)^2\, (f+g)^{2k_2-4} \tag{58}$$

as $f+g \to 0$. It is obvious that space-times possess curvature singularities when $k_2<2$ and their strength depend on $n$ and $m_1$. It is also of great importance to further analyze the global singularity structure of these space-times. At present the global structure of only a few solutions (colliding plane waves geometries) are known in the four-dimensional case [3,4].

We also investigate the singularity structure of space-times in the context of the Maxwell invariants; one of the invariants is

$$F_{\alpha\beta}F^{\alpha\beta} = -\frac{\gamma^2 n_1^2}{2^{k_1+1}} (rwpq)^{1-(2/n_1)} (rw+pq)^{-2k_1} \times (f+g)^{k_2} P_u Q_v \tag{59}$$

which has singularities for $n>2$ for the negative values of $k_2$.

We finally examine the Weyl tensor to see whether our space-time is conformally flat. One of the components of the Weyl tensor in region II for our space-times is

$$C_{0n0n} = \frac{f_u^2}{8\left(\frac{1}{2}+f\right)} \left[ -m_1 + \frac{2}{n(n-1)} + \frac{2n}{(n-1)}(\alpha+\beta)^3 + \frac{(1-n)}{n}(\alpha+\beta) + 2(\alpha+\beta) \times \frac{\left(\frac{1}{2}-f\right)^{-1+2/n_1}\left(\frac{1}{2}+f\right)}{1-\left(\frac{1}{2}-f\right)^{2/n_1}} \right] + \frac{(\alpha+\beta)}{4} f_{uu}. \tag{60}$$

It can be seen that it vanishes only for $n=2$. Therefore, the higher dimensional extensions of the BS metric are not conformally flat.

## V. CONCLUSION

In this paper, we give a higher dimensional generalization of the BS metric which describes the collision of pure electromagnetic plane waves with collinear polarization in all space-time dimensions. The solution has two free parameters; the space-time dimension $N(=2+n)$ and an arbitrary real number $m_1$. We show that these space-times, unlike the BS metric, are not conformally flat. We find that, even though purely electromagnetic plane wave collision in four-dimensional space-time possesses no curvature singularities, in higher dimensions there exist curvature singularities whose nature depend on the real number $m_1$ and the space-time dimension.

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