Disappearing dark matter in brane world cosmology: New limits on noncompact extra dimensions

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We explore cosmological implications of dark matter as massive particles trapped on a brane embedded in a Randall-Sundrum noncompact higher dimension AdS_5 space. It is an unavoidable consequence of this cosmology that massive particles are metastable and can disappear into the bulk dimension. Here, we show that a massive dark matter particle (e.g. the lightest supersymmetric particle) is likely to have the shortest lifetime for disappearing into the bulk. We examine cosmological constraints on this new paradigm and show that disappearing dark matter is consistent (at the 95% confidence level) with all cosmological constraints, i.e., present observations of type Ia supernovae at the highest redshift, trends in the mass-to-light ratios of galaxy clusters with redshift, the fraction of x-ray emitting gas in rich clusters, and the spectrum of power fluctuations in the cosmic microwave background. A best 2σ concordance region is identified corresponding to a mean lifetime for dark matter disappearance of $15 \leqslant \Gamma^{-1} \leqslant 80$ Gyr. The implication of these results for brane-world physics is discussed.

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INTRODUCTION

There is currently considerable interest in the possibility that our Universe could be a submanifold embedded in a higher-dimensional spacetime. This brane-world paradigm is motivated by the D-brane solution found in ten-dimensional superstring theory. Technically, in type IIB superstrings, an $AdS_5 \times S^5$ geometry is formed near the stacked D3-branes [1–4]. In simple terms this means that a model can be proposed [5] whereby our Universe is represented as a thin three-brane embedded in an infinite five-dimensional bulk anti–de Sitter (AdS_5) space. In such Randall-Sundrum (RSII) models, physical particles are trapped on a three-dimensional brane via curvature in the bulk dimension. Gravitons can reside as fluctuations in the background gravitational field living in both the brane and bulk dimension. This representation of large extra dimensions is an alternative to the standard Kaluza-Klein (KK) compactification.

Although massive particles can indeed be trapped on the brane, they are also, however, expected to be metastable [6]. That is, for both scalar and fermion fields, the quasi-normal modes are metastable states that can decay into continuum KK modes in the higher dimension. From the viewpoint of an observer on the three-brane, massive particles will appear to propagate for some time and then literally disappear into the bulk fifth dimension.

In the RSII model, curvature in the bulk dimension is introduced as a means to suppress the interaction of massless particles with the bulk continuum of KK states. However, introducing a mass term into the higher-dimensional action leads to nonzero coupling to that KK continuum. The mathematical realization of this decay is simply that the eigenvalues for the mass modes of the field theory are complex.

The simplest model to illustrate this is the case of a free scalar field to which a bulk mass term μ has been added [6]. In this case, the imposition of radiation (outgoing-wave) boundary conditions on the solution to the five-dimensional Klein-Gordon equation leads to complex eigenvalues of the form

$$m = m_0 - i\Gamma, \tag{1}$$

where, quasidiscrete four-dimensional masses are given by

$$m_0^2 = \mu^2/2, \tag{2}$$

with μ being the bulk mass term in the AdS_5 field equation. The width Γ is given by

$$\Gamma = (\pi/16)(m_0^3/L^2), \tag{3}$$

where L is the metric curvature parameter of the bulk dimension. That is, we write the five-dimensional metric,

$$ds^2 = \exp^{-2|z|L} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2, \tag{4}$$

where z is the bulk dimension and the bulk curvature parameter is

$$L = \sqrt{-\Lambda_5/6}, \tag{5}$$

where Λ_5 is the negative bulk cosmological constant. A construction of the propagator for particles on the brane then has a pole at complex p^2 which corresponds to an unstable particle with mass m_0 and width Γ. Thus, the comoving density of massive scalar particles can be expected to decay over time with a rate, $(\rho a^3)\exp[-\Gamma t]$, where a is the scale factor.

It is well known [7] that fermion fields cannot be localized on a brane with positive tension by gravitational interactions only. One must invoke a localization mechanism. A simple example [6] is to form a domain wall by introducing a scalar field χ with two degenerate vacua $\chi = \pm v$ separated by a domain wall at the brane. A fermion field is then introduced with a Yukawa coupling to the scalar field, $g\chi\bar{\psi}\psi$,

which confines fermions to the brane. Similar to the treatment of scalar particles, solving the Dirac equation for fermions with a bulk mass term μ, leads to complex mass eigenvalues. In the limit that the bulk mass is much less than the curvature scale, $\mu \ll L$, the width for decay into the bulk dimension becomes

$$\Gamma_{\rm fermion} = (m_0/2L)^{2gv/L}\{2\pi L/[\Gamma(gv/L+1/2)]^2\}, \quad (6)$$

where Γ on the right-hand side (rhs) is the normal gamma function. In the limit, $\mu \gg L$ one similarly obtains

$$\Gamma_{\rm fermion} = M(m_0/2M)^{2M/L}\exp\{2M/L\}, \quad (7)$$

where $M = \sqrt{(gv)^2+\mu^2}$.

Clearly, in each of these expressions, the largest width for tunneling into the bulk dimension is for the heaviest particle. In this case we argue that a heavy ($\gtrsim$TeV) dark matter particle [e.g. the lightest supersymmetric particle (LSP)] may have the shortest lifetime to tunnel into the bulk. In this paper, therefore, we consider the possibility that cold dark matter (CDM) disappears into the extra dimension. The comoving density of the CDM will then diminish over time as $(\rho_{CDM}a^3)\exp[-\Gamma t]$.

In principle, normal standard-model particles (e.g. baryons) would decay in this way as well. This would have many far reaching consequences in astrophysics and cosmology. However, the decay width of such light particles is likely to be suppressed relative to that of a heavy dark-matter particle by some power of the ratio of their masses [e.g. by $(m_{baryon}/M_{LSP})^{2gv/L} \sim (0.001)^{2gv/L}$ for a TeV fermion (e.g. neutralino) LSP]. We also note that even a light (axion-like) scalar dark matter particle could also be made to have a short disappearance time relative to normal fermionic matter [by Eq. (6)] as long as $(m_0/2L) < 1$, and gv/L is sufficiently large to suppress the disappearance of normal fermionic matter.

In what follows, we analyze cosmological constraints on such disappearing dark-matter particles and show that this hypothesis is consistent with and even slightly preferred by all cosmological constraints, including primordial nucleosynthesis, the present observations of type Ia supernovae at high redshift, the mass-to-light ratio vs redshift relation of galaxy clusters, the fraction of x-ray emitting gas in rich galactic clusters and the cosmic microwave background (CMB).

Cosmological constraints on decaying matter have been considered in many papers, particularly with regard to the effects of such decays on big-bang nucleosynthesis (cf. [8,9] and references therein). The present discussion differs from the previous considerations in that the decaying particles do not produce photons, hadronic showers, or residual annihilations in our four-dimensional spacetime. To distinguish the disappearance of dark matter in the present application from the previous decay applications, we shall refer to it here as *disappearing dark matter*.

In the present application, however, there are some complications. One is that an energy flow into the bulk can induce a back reaction from the background gravitational field. This leads to residual gravity waves in the 3-brane from the exiting particles [10]. Another effect is an enhanced electric part of the bulk Weyl tensor [11]. Together these effects will comprise the so-called ''dark radiation'' as analyzed below. Another consideration is that particles which enter the bulk can still interact gravitationally with particles on the brane. The strength of this interaction, however, is greatly diminished [12] by a factor of (R/z), where z is the distance between the bulk and the brane, and $R = 1/L$ is the ''radius'' of the bulk dimension. For a typical value of $L = 10^4$ GeV, we have $R \sim 10^{-4}$ fm. So, even though gravity can reside in the bulk, the residual gravity between particles in the bulk and brane is strongly suppressed.

COSMOLOGICAL MODEL

The five-dimensional Einstein equation for the brane world can be reduced to an effective set of four-dimensional equations on the brane [13–15] by decomposing the five-dimensional Riemann tensor into a Ricci tensor plus the five-dimensional Weyl tensor. The four-dimensional effective energy-momentum tensor contains the usual $T_{\mu\nu}$ term of ordinary and dark matter plus a new term quadratic in $T_{\mu\nu}$, and a residual term containing the five-dimensional Weyl tensor with two of its indices projected along a direction normal to the brane. The (0,0) component of the effective four-dimensional Einstein equation can then be reduced to a new generalized Friedmann equation [16–21] for the Hubble expansion as detected by an observer on the three-brane,

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G_{\rm N}}{3}(\rho+\rho_{DR}) - \frac{k}{a^2} + \frac{\Lambda_4}{3} + \frac{\kappa_5^4}{36}\rho^2. \quad (8)$$

Here, $a(t)$ is the scale factor at cosmic time t, and $\rho = \rho_B + \rho_\gamma + \rho_{DM}$, with ρ_B and ρ_γ the usual contributions from nonrelativistic (mostly baryons) and relativistic particles, respectively. In the present application we presume that only the dark matter can decay into the extra dimension. Hence, we write $\rho_{DM} = Ce^{-\Gamma t}/a^3$, where Γ is the decay width into the extra dimension.

In Eq. (8), several identifications of cosmological parameters were required in order to recover standard big-bang cosmology. For one, the first term on the right-hand side is obtained by relating the four-dimensional gravitational constant $G_{\rm N}$ to the five-dimensional gravitational constant, κ_5. Specifically,

$$G_{\rm N} = M_4^{-2} = \kappa_5^4\tau/48\pi, \quad (9)$$

where τ is the brane tension and

$$\kappa_5^2 = M_5^{-3}, \quad (10)$$

where M_5 is the five-dimensional Planck mass. Secondly, the four-dimensional cosmological constant Λ_4 is related to its five-dimensional counterpart Λ_5,

$$\Lambda_4 = \kappa_5^4\tau^2/36 + \Lambda_5/6. \quad (11)$$

A negative Λ_5 (and $\kappa_5^4\tau^2/36 \approx |\Lambda_5/6|$) is required for Λ_4 to obtain its presently observed small value.

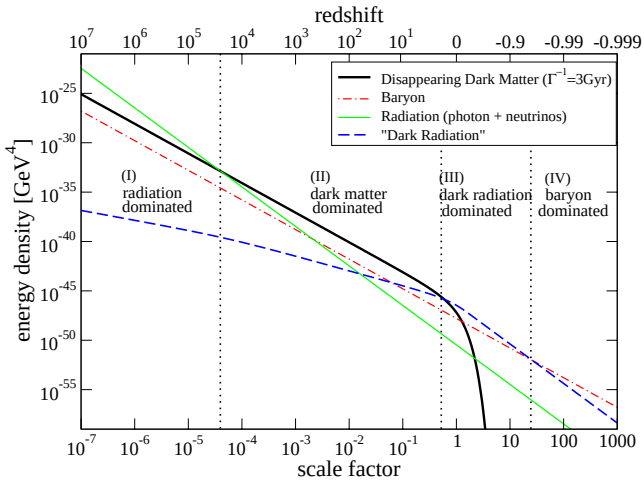

FIG. 1. Illustration of the energy densities with scale factor in models with dark-matter decay into the extra dimension.

Standard big-bang cosmology does not contain the ρ_{DR} and ρ^2 terms of Eq. (8). The ρ^2 term arises from the imposition of a junction condition for the scale factor on the surface of the brane. Physically, it derives from the fact that matter fields are initially confined to the brane. This term decays rapidly as a^{-8} in the early radiation dominated universe and is not of interest here.

In the present formulation, ρ_{DR} includes two contributions, $\rho_{DR}=\rho_E+\rho_{GW}$. One is the ρ_E term which derives from the electric part of the bulk Weyl tensor. The second (ρ_{GW}) arises from residual gravity waves left on the brane [10]. Since these gravity waves are associated with the disappearing particles, their dynamics can be formally absorbed together with ρ_E into a Bianchi identity for the effective four-dimensional Einstein equation. This leads to

$$\dot{\rho}_{DR}+4H\rho_{DR}=\Gamma\rho_{DM}. \tag{12}$$

When $\Gamma=0$, ρ_{DR} scales as a^{-4} like normal radiation even though it has nothing whatsoever to do with electromagnetic radiation. Hence, the name ''dark radiation.'' Upper and lower limits on such dark radiation can be deduced from big-bang nucleosynthesis [22]. In the present paper we will keep the same name, even though in this more general context ρ_{DR} no longer scales as a^{-4}.

The introduction of the dark radiation term into Eq. (8) leads to new cosmological paradigms. For example, Fig. 1 illustrates the evolution of a simple flat, $\Lambda_4=k=0$, disappearing dark matter cosmology with a negligible ρ^2 term. This cosmology separates into four characteristic regimes identified on Fig. 1. These are (I) the usual early radiation dominated era ($z>10^5$); (II) a dark-matter dominated era ($t\ll 2\Gamma^{-1}$, $10<z<10^5$); (III) a late dark radiation dominated era ($t\gg 2\Gamma^{-1}$, $0<z<0.2$); and (IV) eventually, a baryon-dominated regime also exists.

Early on the contribution from the dark radiation component evolves [from Eq. (12)] as $\rho_{DR}\propto a^{-1}$ or $\rho_{DR}\propto a^{-3/2}$ during regimes I and II, respectively, and can be neglected. Thus, the dark radiation does not affect (nor is it constrained by) primordial nucleosynthesis. Similarly, the dark radiation does not contribute much mass energy during the epoch of CMB photon decoupling (at $z\sim 10^3$), though it can become comparable to and even in excess of the dark matter contribution in epoch III and therefore affects the look-back time to the CMB epoch.

The most interesting region for our purpose is during the transition from epoch II to epoch III. This occurs at intermediate times $t\sim 2\Gamma^{-1}$ and redshifts of $0<z<2$ as indicated on Fig. 1. Here, the fact that there is both more dark matter and more dark energy at higher redshifts means that the universe decelerates faster during the redshift regime $1<z<2$ than during the more recent epoch $0<z<1$. As far as cosmological constraints are concerned, the most important effect is from the changing dark matter contribution. This is because the dark radiation does not become significant until the most recent ($z\lesssim 0.05$) epoch even for this extreme cosmology. The changing dark matter contribution in particular, can nevertheless have important observable consequences, for example on the luminosity-redshift relation, galaxy mass-to-light ratios, and the cosmic look-back time. Hence, this model is constrainable by the observations of supernovae and galaxy mass-to-light ratios at high redshift, and the power spectrum of the cosmic microwave background as we now show.

SUPERNOVA CONSTRAINT

The apparent brightness of the type Ia supernova standard candle with redshift is given [23] by a simple relation which we slightly modify to incorporate the brane-world cosmology given in Eq. (8). The luminosity distance becomes

$$D_L=\frac{c(1+z)}{H_0\sqrt{\Omega_k}}\sin n\left\{\sqrt{\Omega_k}\int_0^z dz'[\Omega_\gamma(1+z')^4+(\Omega_{DM}(z')+\Omega_B)(1+z')^3+\Omega_k(1+z')^2+\Omega_\Lambda+\Omega_{DR}(z')]^{-1/2}\right\}, \tag{13}$$

where H_0 is the present value of the Hubble constant, and $\sin n(x)=\sinh x$ for $\Omega_k>0$, $\sin n(x)=x$ for $\Omega_k=0$ and $\sin n(x)=\sin x$ for $\Omega_k<0$. The Ω_i are the usual closure quantities, i.e. the contribution from all relativistic particles is $\Omega_\gamma=8\pi G\rho_\gamma/3H_0^2$, the baryonic contribution is $\Omega_B=8\pi G\rho_B/3H_0^2=0.039$ [22] (for $H_0=71\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$). The curvature contribution is $\Omega_k=-k/a_0^2H_0^2$, and $\Omega_\Lambda=\Lambda/3H_0^2$ is the vacuum energy contribution. In the present context, we have added a redshift-dependent contribution from the dark radiation, $\Omega_{DR}=8\pi G\rho_{DR}(z)/3H_0^2$. The dark matter contribution Ω_{DM} becomes a function of redshift through $\Omega_{DM}\mapsto\Omega_{DM}^0\exp\{\Gamma(t_0-t)\}$, where $\Omega_{DM}^0=8\pi G\rho_{DM}^0/3H_0^2$ is the present dark-matter content, and the look-back time t_0-t is a function of redshift,

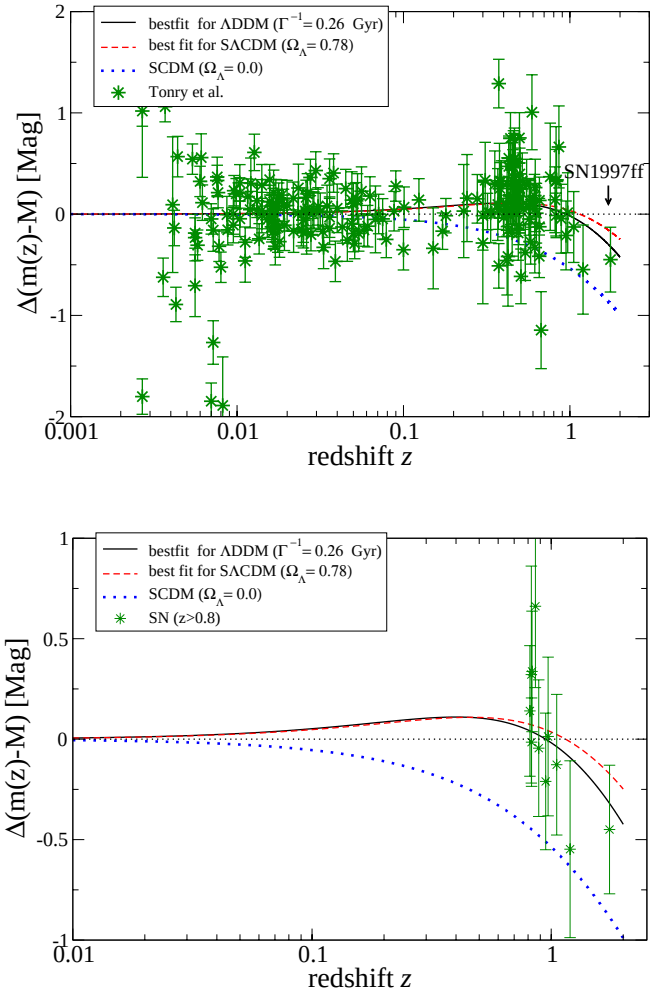

FIG. 2. Illustration of the supernova magnitude redshift relation for various cosmological models with and without disappearing dark matter as labeled. The upper figure shows the full data set of [25]. The lower figure highlights the points with $z>0.8$ most relevant to this paper.

$$t_0-t=H_0^{-1}\left\{\int_0^z(1+z')^{-1}[\Omega_R(1+z')^4+(\Omega_B+\Omega_{DM})\times(1+z')^3+\Omega_k(1+z')^2+\Omega_\Lambda+\Omega_{DR}]^{-1/2}dz'\right\}. \tag{14}$$

Figure 2 compares various cosmological models with some of the recent combined data from the High-Z Supernova Search Team [24,25] and the Supernova Cosmology Project [26]. The lower figure highlights the crucial data points at the highest redshift which are most relevant to this study. Shown are the *K*-corrected magnitudes $m=M+5\log D_L+25$ vs redshift. Curves are plotted relative to an open $\Omega_{DM},\Omega_B,\Omega_\Lambda,\Omega_{DR}=0$, $\Omega_k=1$ cosmology. Of particular interest are the highest redshift points (e.g. SN1997ff [27,25] at $z=1.7$). These points constrain the redshift evolution during the important dark-matter dominated decelerating phase relevant to this paper.

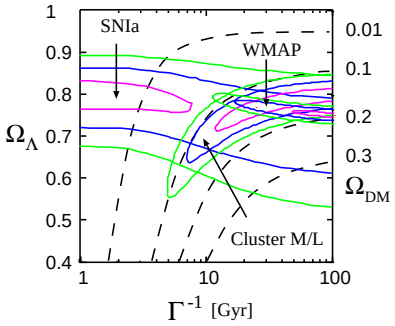

FIG. 3. Contours of constant χ^2 in the Γ^{-1} vs Ω_Λ plane. Lines drawn correspond to 1, 2, and 3σ confidence limits for fits to the magnitude-redshift relation for type Ia supernovae, the mass-to-light ratios of galaxy clusters, and constraints from the CMB. The dashed lines indicate contours of constant Ω_{DM} as labeled. The dark radiation contribution can be deduced from the figure, via $\Omega_{DR}=1-\Omega_\Lambda-\Omega_{DM}-\Omega_B$.

It is noteworthy that an optimum standard flat $\Omega_M=0.3$, $\Omega_\Lambda=0.7$ (SΛCDM) cosmology passes somewhat above the five points with $z\geqslant 0.9$. Indeed, the newest ''Fall 1999'' data [25] (shown in the lower box of Fig. 2) are consistently brighter than the best-fit standard flat SΛCDM cosmology in the epoch at $z>0.9$. This is made more relevant in view of the fact that dust around SN1997ff would cause that inferred data point to be even lower on this plot [27]. Thus, we find that the data all slightly favor the disappearing dark matter (ΛDCDM) cosmology.

The contours labeled SNIa of Fig. 3 show 1σ, 2σ, and 3σ confidence limit regions of constant goodness of fit to the $z>0.1$ data of [25] in the parameter space of disappearance lifetime Γ^{-1} versus Ω_Λ plane. For these data we use a simple χ^2 measure of the goodness of fit as in [25],

$$\chi^2\equiv\sum\ (Y_i^{data}-Y_i^{calc})^2/\sigma_i^2, \tag{15}$$

where σ_i includes the velocity uncertainty added to the distance error.

The SNIa data imply a shallow minimum for $\Gamma^{-1}\approx 0.3$ Gyr and $\Omega_\Lambda=0.78$. The reduced χ^2 per degree of freedom at this minimum is $\chi_r^2=0.94$ compared with $\chi_r^2=1.07$ for a standard ΛCDM cosmology [25]. The 1σ confidence limit corresponds to $\Gamma^{-1}\leqslant 10$ Gyr, but the 2σ region is consistent with a broad range of Γ as long as $\Omega_\Lambda=0.75\pm 0.15$.

GALAXY CLUSTER *M/L* CONSTRAINT

Another interesting cosmological probe comes from galaxy cluster mass-to-light ratios as also shown on Fig. 3. This is the traditional technique to obtain the total universal matter content Ω_M. A most recent average value of $\Omega_M=0.17\pm 0.05$ has been determined in [28] based upon 21 galaxy clusters out to $z\approx 1$ corrected for their color and evolution with redshift. The very fact that the nearby cluster data seem to prefer a smaller value of Ω_M than the value of $\Omega_M=0.27\pm 0.02$ deduced [29] from the distant microwave back-

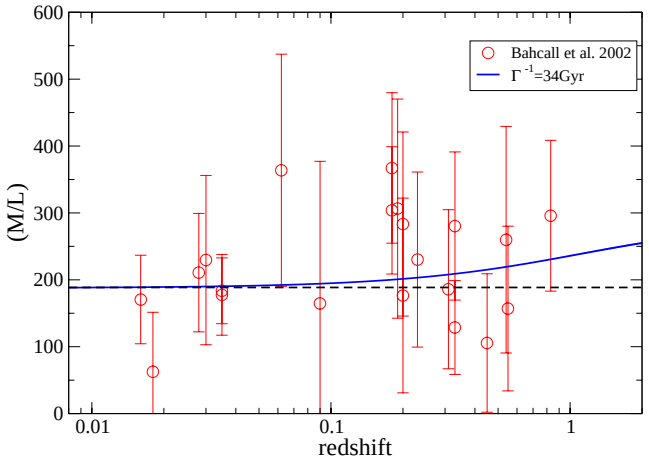

FIG. 4. Illustration of the evolution and temperature corrected galaxy cluster mass-to-light ratios (from [28]) as a function of redshift. The solid line shows the best fit cosmology with disappearing dark matter as described in the text. The dashed line shows the present value of Ω_M as deduced from the nearby cluster data.

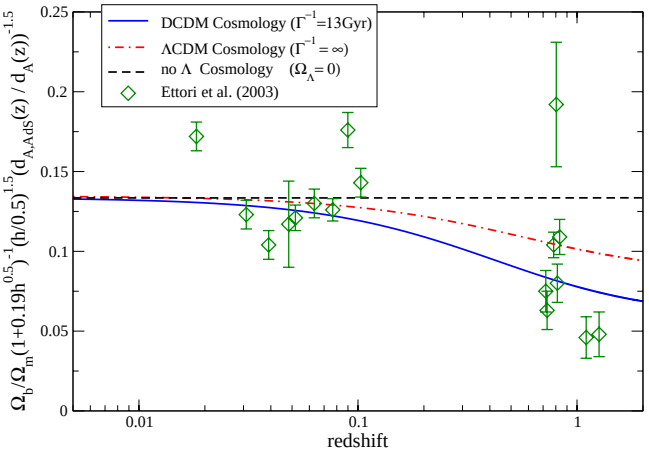

FIG. 5. Illustration of the fraction of x-ray emitting gas to total mass in rich clusters as a function of redshift (from [30]) as a function of redshift. The solid line shows the theoretical gas mass fraction from the disappearing dark matter cosmology as described in the text. The dashed line shows a $\Lambda=0$ cosmology and the dot-dashed line is for a standard ΛCDM cosmology. All theoretical models are normalized to have the same gas mass fraction at present $(z=0)$.

ground surface of photon last scattering is consistent with the notion of disappearing dark matter as discussed below.

In the present disappearing dark matter paradigm, the dark matter content diminishes with time, while the normal baryonic luminous matter remains mostly confined to the brane. Therefore, the M/L ratio should increase with look-back time. This is complicated, however, by two effects. One is that clusters at high redshift have had less time to evolve and dim. Hence, their M/L ratios are expected to decline with redshift. This effect is corrected in Table 1 of [28]. Another complication is an observational bias due to the fact that at high redshift a larger fraction of high-temperature clusters is observed. In essence, higher temperature clusters have deeper gravitational wells and are expected to have more dark matter and larger M/L ratios. Nevertheless, we have corrected for this temperature bias by using the power-law analysis described in [28] to adjust all clusters to a common temperature. Even after applying this correction we find a residual trend of increasing cluster M/L ratio with redshift which can be attributed to disappearing dark matter as depicted in Fig. 4.

Our standard χ^2 goodness of fit to the data of [28] (corrected for evolution and temperature bias) is labeled as Cluster M/L on Fig. 3. We find a minimum χ^2 per degree of freedom of $\chi^2_r=0.61$ for $\Gamma^{-1}=34$ Gyr as shown on Figs. 3 and 4. This is an improvement over the fit with a fixed M/L (shown as the straight dashed line on Fig. 4) for which $\chi^2_r=0.67$. The 2σ (95% confidence level) limits from the galaxy cluster data correspond to $\Gamma^{-1}\geqslant 7$ Gyr for our flat ΛDCDM model as shown in Fig. 3. This limit is concordant with the previously discussed type Ia supernova analysis.

Clearly, more work is needed to unambiguously identify evidence for enhanced dark matter in the past. In this regard we note that there is complementary data [30] to the cluster M/L ratios from *BeppoSax* and the *ROSAT* x-ray observations of rich clusters at high redshift. In this case, the x-ray emitting gas mass can be determined from the x-ray luminosity and the total mass deduced from the gravitational mass required to maintain the x-ray gas in hydrostatic equilibrium. There is, however, uncertainty in this method due to the model dependence of the inferred gas fractions [30]. Nevertheless, the observations clearly exhibit a trend of diminishing gas fraction for systems with $z>1$. Figure 5 shows a comparison of the deduced gas fractions for various cosmological models. These data are consistent with an increasing total mass content for these systems as predicted in this disappearing dark matter paradigm.

CMB CONSTRAINT

As noted above, the matter content $(\Omega_M=0.27\pm0.02)$ deduced from the recent high-resolution WMAP analysis [29] of the cosmic microwave background is larger than that deduced $(\Omega_M=0.17\pm0.05)$ from nearby galaxy cluster mass-to-light ratios [28]. This in itself is suggestive of the disappearing dark matter paradigm proposed here. However, this cosmology can also involve a shorter look back time and different expansion history between now and the epoch of photon last scattering. In particular there will be more dark matter at earlier times leading to earlier structure formation. There will also be a smaller integrated Sachs-Wolf effect (ISW) at early times, and a larger ISW effect at late times as photons propagate to the present epoch. Thus, the amplitudes and locations of the peaks in the power spectrum of microwave background fluctuations [31] can in principle be used to constrain this cosmology.

We caution, however, that there is a complication with using the CMB constraint. Inflation generated metric fluctuations which contribute to the CMB should also induce fluctuations in the dark radiation component. Unfortunately, however, calculations of the power spectrum from five-dimensional gravity are complicated and beyond the scope of

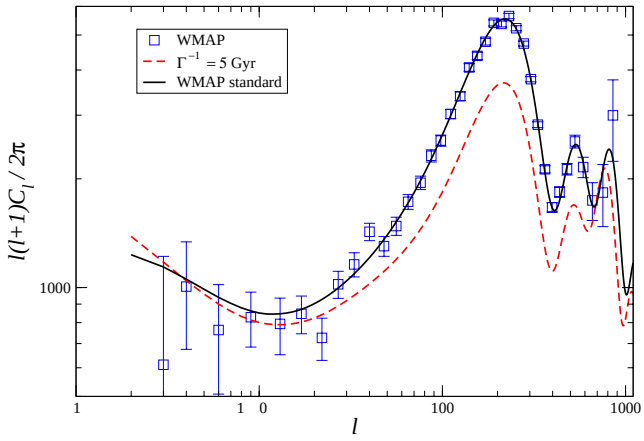

FIG. 6. Illustration of a disappearing dark matter cosmology (dashed line) which is ruled out by the WMAP power spectrum. The points are the WMAP data. The solid line is the standard best fit [29] for a normal ΛCDM cosmology.

the present work. A straightforward application of this disappearing dark matter paradigm without a proper treatment of the fluctuation power spectrum from the dark radiation should therefore probably be viewed with caution. Nevertheless, under the assumption that fluctuations in the dark radiation contribute insignificantly to the power spectrum at the surface of photon last scattering, a straightforward study of the CMB constraints on the disappearing dark matter cosmology is possible.

We have done calculations of the CMB power spectrum, $\Delta T^2 = l(l+1)C_l/2\pi$ based upon the CMBFAST code of Seljak and Zaldarriaga [32]. We have explicitly modified this code to account for the disappearing dark matter cosmology described in Eq. (8). Figure 6 shows an illustration of a disappearing dark matter model which can be ruled out by the CMB. In this example $\Gamma^{-1} = 5$ Gyr, and all other cosmological parameters set to their best fit WMAP values [29].

Nevertheless, it is quite possible to have a finite Γ and still fit the WMAP data. As an illustration of this, we have simultaneously varied Γ and Ω_Λ, and marginalized over the parameters of the matter power spectrum, while maintaining other cosmological parameters at the best fit WMAP values. The likelihood functions we computed from a combination of Gaussian and lognormal distributions as described in Verde *et al.* [33]. These were used to generate contours of 1, 2, and 3σ confidence limits for fits to the WMAP power spectrum [34,35] as shown on Fig. 3.

An important point is that we find that equivalent fits to that of the best-fit WMAP parameters [29] can be obtained for a broad range of values for Γ and Ω_Λ. This means that the CMB does not rule out this paradigm. On the contrary, the 2σ CMB contours nicely overlap the region allowed by the cluster M/L ratios. A 2σ concordance region of $15 \leqslant \Gamma^{-1} \leqslant 80$ Gyr survives this constraint. The essential requirements to fit the CMB in this model are that the matter content during photon decoupling be at the (higher) WMAP value, and that the dark radiation be an insignificant contributor to the background energy density during that epoch.

CONCLUSION

Obviously, there is great need for better type Ia supernova data in the crucial $z > 1$ regime as well as more galactic cluster mass-to-light ratios at high redshifts. Although the evidence for disappearing dark matter is of marginal statistical significance at the present time, the purpose of this paper is nevertheless to emphasize the potential importance of future studies aimed at unambiguously determining the decay width. If such a finite value of Γ were to be established, it would constitute the first observational indication for noncompact extra dimensions. It would also provide valuable insight into the physical parameters of the higher-dimensional space.

Rewriting the equation for the decay width, along with the relations [Eqs. (9)–(11)] between various quantities in the modified Friedmann equation, i.e. κ_5, G_N, M_4, M_5, Λ_4, and Λ_5, leads to the following relation between the five-dimensional Planck mass M_5 and quantities which can be measured in the four-dimensional spacetime, $M_5^6 = (M_4^4/64\pi^2)[\pi m_0^3/16\Gamma + \Lambda_4]$. Other fundamental parameters in five dimensions, e.g. Λ_5 and the brane tension τ, are derivable from M_5 via Eqs. (9) and (11). This implies that, should the dark-matter mass m_0 ever be known, all of the five-dimensional parameters could be determined. For example, a dark matter mass of $m_0 \approx 1$ TeV (as expected for the LSP), and a most optimistic decay lifetime of $\Gamma^{-1} = 15$ Gyr, would imply $(M_5/M_4) \approx 4(m_0/\mathrm{TeV})^{1/2}(\Gamma^{-1}/15\ \mathrm{Gyr})^{1/6}$.

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