# Nearly scale-invariant spectrum of adiabatic fluctuations may be from a very slowly expanding phase of the Universe

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(Received 2 April 2003; revised manuscript received 4 August 2003; published 31 October 2003)

In this paper we construct an expanding phase with phantom matter, in which the scale factor expands very slowly but the Hubble parameter increases gradually, and assume that this expanding phase could be matched to our late observational cosmology by the proper mechanism. We obtain the nearly scale-invariant spectrum of adiabatic fluctuations in this scenario; different from the simplest inflation and usual ekpyrotic or cyclic scenario, the tilt of the nearly scale-invariant spectrum in this scenario is blue. Although there exists an uncertainty surrounding the way in which the perturbations propagate through the transition in our scenario, which is dependent on the details of possible ''bounce'' physics, compared with inflation and the ekpyrotic or cyclic scenario, our work may provide another feasible cosmological scenario generating the nearly scale-invariant perturbation spectrum.

DOI: 10.1103/PhysRevD.68.083515 PACS number(s): 98.80.Cq, 98.70.Vc

The inflation scenario [1] plays an important role in modern cosmology, which solves many problems of standard cosmology. The physics of the usual inflation models is dependent on the inflaton potential; in general, such a potential that yields a large enough *e*-folding number and the correct magnitude of density perturbation is fine-tuned. A lot of observations, specifically the recent Wilkinson Microwave Anisotropy Probe (WMAP) results [2] such as the flatness of space, the near scale invariance, adiabaticity and Gaussian of the density perturbations imply that inflation is very consistent in the early cosmological scenario. But inflation may not be a uniquely consistent scenario with the WMAP data. There remain some alternatives [3]. An example is the ekpyrotic or cyclic scenario [4,5], which is motivated by the string or M theory, in which the visible universe is a boundary brane in a five-dimensional bulk space-time and the collision between two boundary branes leads to a reheating in the visible universe corresponding to the big bang of standard cosmology. The relevant dynamics can be described by a 4D effective theory in which the separation of the branes in the extra dimensions is modeled as a scalar field. In this scenario, the perturbation leaving the horizon during the contracting phase reenters the horizon after the bounce to an expanding phase corresponding to our observational cosmology. If the proper matching conditions during the bounce are considered [5–7] (see also [8,9]) the nearly scale-invariant spectrum could be obtained.

Both the inflation and the ekpyrotic or cyclic models rely on the parameter $\omega$ of state equation having a specific qualitative behavior throughout the period when the perturbations are generated. For inflation, the condition on $\omega$ is $\omega \simeq -1$ and for the ekpyrotic or cyclic model, it is $\omega \gg 1$. Correspondingly, the Hubble parameter is nearly constant during inflation, and the 4D scale factor contracts very slowly and is nearly constant during the ekpyrosis or cyclic. In some sense, both scenarios can give some results satisfying the WMAP observation. The spectrum is nearly scale invariant and the tilt of the spectrum is red in its simplest realization [10]. In this paper, an early universe model with phantom matter[1] is proposed, in which the 4D scale factor expands very slowly and is nearly constant which corresponds to $\omega \ll -1$. We find that when this expanding phase is ''linked'' to another expanding phase with the usual radiation and matter by the proper mechanism, a nearly scale-invariant spectrum may be generated and the tilt of spectrum is blue.

We start with such a 4D effective action of the phantom field as follows:

$$\mathcal{L}_{\mathrm{mat}} = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi), \tag{1}$$

where the metric signature $(- + + +)$ is used. If taking the field $\varphi$ spatially homogeneous but time dependent, the energy density $\rho$ and pressure $p$ can be written as

$$\rho = -\frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p = -\frac{1}{2}\dot{\varphi}^2 - V(\varphi). \tag{2}$$

[1]The state equation of matter is $p \equiv \omega\rho$. The matter with the state parameter $\omega < -1$, dubbed ''phantom matter'' in general, has received increased attention recently [11,12], especially applied to the late universe as an explanation for dark energy, because it has some strange properties. The phantom matter violates the dominant energy condition, prohibiting time machines and wormholes, and its energy density increases with time, and may be up to infinite in a finite time and lead to a ''big rip.'' But in our work, we focus on the early universe, especially the generation of a nearly scale-invariant adiabatic spectrum.

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From Eqs. (2), for $\rho>0$ the state parameter $\omega\equiv p/\rho<-1$ can be seen. We minimally couple the action (1) to the 4D gravitational action

$$\mathcal{L}_{\rm gra}=\frac{1}{16\pi G}R, \tag{3}$$

where $G$ is the Newton gravitational constant and $R$ is the 4D curvature scalar. The Friedmann universe, described by the scale factor $a(t)$, satisfies the equations

$$h^2=\frac{8\pi G}{3}\left(-\frac{1}{2}\dot{\varphi}^2+V(\varphi)\right) \tag{4}$$

and the dynamical equation of phantom field is

$$\ddot{\varphi}+3h\dot{\varphi}-V'(\varphi)=0, \tag{5}$$

where $h=\dot{a}/a$ is the Hubble parameter. Combining Eqs. (4) and (5), the relation

$$\epsilon\equiv\frac{3}{2}(1+\omega)=-\frac{\dot{h}}{h^2} \tag{6}$$

can be obtained. Since $\omega(t)<-1$, considering Eq. (6), $\dot{h}>0$ is required. Therefore, a reasonable selection for the scale factor $a(t)$ is

$$a(t)\sim\frac{1}{(-t)^{n(t)}}. \tag{7}$$

For $t$ initially from $-\infty$ to $0_-$, it corresponds to an expanding phase. We assume $(-t/n)dn/dt\ll 1$—i.e., the variable rate of $n(t)$ is very small and near constant—and have

$$h\simeq\frac{n}{(-t)},\quad \dot{h}\simeq\frac{n}{(-t)^2}, \tag{8}$$

and thus

$$\epsilon\simeq-\frac{1}{n}. \tag{9}$$

Consequently, the background values of all relevant quantities can be determined. From Eqs. (4), (5), and (8),

$$\dot{\varphi}^2\simeq\frac{n}{4\pi G}\frac{1}{(-t)^2} \tag{10}$$

can be given, and thus

$$V(\varphi)\simeq\frac{n(3n+1)}{8\pi G}\frac{1}{(-t)^2}. \tag{11}$$

We see that for arbitrary value $n$, $V(\varphi)$ is positive, which is different from the usual scalar field, in which $n<\frac{1}{3}$, the potential being negative. From Eqs. (10) and (11), we can also check and obtain Eq. (9). Integrating Eq. (10) and instituting this result into Eq. (11), we derive the effective potential

$$V(\varphi)\simeq\frac{n(3n+1)}{8\pi G}\exp\left(-\sqrt{\frac{16\pi G}{n}}\varphi\right). \tag{12}$$

Instituting Eqs. (10) and (11) into Eq. (2), we see that the energy density of the phantom field,

$$\rho\simeq\frac{3n^2}{8\pi G}\frac{1}{(-t)^2}\sim\frac{3n^2}{8\pi G}a^{2/n}, \tag{13}$$

increases with expansion, while the energy density of usual matter and radiation decreases, respectively, at

$$\rho_{\rm matt}\sim\frac{1}{a^3},\quad \rho_{\rm rad}\sim\frac{1}{a^4}. \tag{14}$$

Thus the evolving solution dominated by the phantom field will be an attractor, which is generic and likes the case that the phantom matter is regarded as dark energy.

When $t\to 0_-$, the Hubble parameter $h$ increases gradually, and a singularity will appear. For the transition to the observational cosmology, we may expect that at some time the phantom field could decay into the usual radiation by some coupling[2] or through the singular ‘‘big rip’’ by some other mechanism from high-energy and -dimension theory, which may be regarded as a ‘‘bounce’’ to an observational cosmology, like pre-big-bang [13] (see [14] for recent reviews) and the ekpyrotic or cyclic scenario.

We discuss the metric perturbations of this scenario in the following. In longitudinal gauge and in the absence of anisotropic stresses, the scalar metric perturbation can be written as

$$ds^2=a^2(\eta)[-(1+2\Phi)d\eta^2+(1-2\Phi)\delta_{ij}dx^i dx^j], \tag{15}$$

where $\Phi$ is the Bardeen potential and $\eta$ is conformal time $d\eta=dt/a$; thus,

$$\eta\sim-(-t)^{n+1}, \tag{16}$$

$$a(\eta)\sim(-\eta)^{-n/(n+1)}\simeq(-\eta)^{1/(\epsilon-1)}, \tag{17}$$

$$\tilde{h}=\frac{a'}{a^2}\simeq\frac{1}{(\epsilon-1)a\eta}. \tag{18}$$

In general the curvature perturbation $\zeta$ on a uniform comoving hypersurface,

$$\zeta=-\frac{h}{\dot{h}}(\dot{\Phi}+h\Phi)+\Phi, \tag{19}$$

and the Bardeen potential $\Phi$ change dramatically across the bounce; thus, the usual matching conditions used in the inflation model may be not proper. More recently, as is pointed

[2]The phantom matter may be unstable and will decay into ordinary particles and other phantom particles, whose lifetimes depends on the cutoff scale [12], which, in some sense, is similar to the reheating mechanism after the usual inflation.

to in Ref. [9], the resulting spectral index in the late radiation-dominated universe depends on which of $\zeta$ and $\Phi$ passes regularly through the bounce, which is dependent on detailed features of the scenarios at the "bounce." At the moment we can hardly describe the detail of "bounce" physics, and paying attention to the generation of the nearly scale-invariant spectrum of adiabatic fluctuations, we assume that the Bardeen potential $\Phi$ is regular through the "bounce" from the preexpanding phase with phantom field to the late expanding phase with the usual radiation, and straightly focus on the evolution of the Bardeen potential $\Phi$.[3]

We define $u \equiv a\Phi/\varphi'$ and obtain the differential equation that the Fourier mode $u_k$ of $u$ obeys

$$u_k'' + \left(k^2 - \frac{\beta(\eta)}{\eta^2}\right)u_k = 0, \tag{20}$$

with

$$\beta(\eta) \equiv \eta^2 \tilde{h}^2 a^2 \left\{ \epsilon - \frac{(1-\epsilon^2)}{2}\left(\frac{d\ln|\epsilon|}{d\mathcal{N}}\right) + \frac{(1-\epsilon^2)}{4}\left(\frac{d\ln|\epsilon|}{d\mathcal{N}}\right)^2 - \frac{(1-\epsilon)^2}{2}\left(\frac{d^2\ln|\epsilon|}{d\mathcal{N}^2}\right)\right\}, \tag{21}$$

in which Eqs. (7) and (16) are considered, and the prime denotes differentiation with respect to the conformal time $\eta$ and the variable $\mathcal{N}$, which measures the $e$-folding number which exits the horizon before the end of the preexpanding phase,

$$\mathcal{N} \equiv \ln\left(\frac{a_e \tilde{h}_e}{a\tilde{h}}\right), \tag{22}$$

where the subscript $e$ denotes the quantity evaluated at the end of the expanding phase. Assuming that $\beta$ is near constant for all interesting modes $k$, we can solve Eq. (20) analytically and obtain

$$u_k = \sqrt{-k\eta}[C_1(k)J_v(-k\eta) + C_2(k)J_{-v}(-k\eta)], \tag{23}$$

where $v \equiv \sqrt{\beta + \frac{1}{4}}$,[4] and $J_v$ is the first kind of the Bessel function with order $v$ and the function $C_i(k)$ can be determined by specifying the initial conditions.

During the preexpanding, the Hubble parameter $h$ increases gradually; thus, the initial perturbation in the horizon will exit the horizon and reenter the horizon after the "bounce" to an expanding phase corresponding to our observational cosmology in which the Hubble parameter $h$ decreases gradually. In the regime $k^2\eta^2 \gg |\beta|$, in which the mode $u_k$ is very deep in the horizon, the mode equation (20) reduced to the equation for a simple harmonic oscillator and $u_k$ is stable. In this limit, we may make the usual assumption that at $\eta \to -\infty$ the normalized initial conditions is

$$u_k \sim \frac{e^{-ik(\eta-\eta_i)}}{(2k)^{3/2}}, \tag{24}$$

where $\eta_i$ is an arbitrary conformal time having no influence on the subsequent evolution. In the regime $k^2\eta^2 \ll |\beta|$, in which the mode $u_k$ is far out the horizon, the mode is unstable and grows. In the long-wave limit, $\Phi_k$ can be given and expanded to the leading term of $k$:

$$k^{3/2}\Phi_k \sim k^{-v+1/2} \simeq k^{-\beta}. \tag{25}$$

Thus the corresponding spectrum index can be regarded as

$$n_s - 1 \simeq -2\beta \simeq -\frac{2}{(1-\epsilon)^2}\left\{\epsilon - \frac{(1-\epsilon^2)}{2}\left(\frac{d\ln|\epsilon|}{d\mathcal{N}}\right)\right\}, \tag{26}$$

where in the second equation, the higher-order terms $(d\ln|\epsilon|/d\mathcal{N})^2$ and $d^2\ln|\epsilon|/d\mathcal{N}^2$ have been neglected. We see, from Eq. (26), that since $\epsilon < 0$, the nearly scale-invariant spectrum requires $\epsilon \ll 0$,[5] which corresponds to $\omega \ll -1$: i.e.,

$$\omega + 1 = \frac{\rho + p}{\rho} = \frac{-\dot{\varphi}^2}{-\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} \ll 0. \tag{27}$$

Considering Eq. (9), $n \sim 0$ is required; i.e., the expansion of the scale factor is very slow. Thus, from (12), we see that the effective potential of the phantom field is very steep and the phantom field rolls up quickly during a period of very slow expansion. In this case, we have

$$n_s - 1 \simeq \frac{2}{|\epsilon|} - \frac{d\ln|\epsilon|}{d\mathcal{N}}. \tag{28}$$

Since $a(\eta)$ is near constant, the end of this very slowly expanding phase may occur when $a(\eta)$ begins to change significantly. Similar to the analysis of Ref. [10], from Eqs. (17) and (18), we obtain

$$a \sim \left(\frac{1}{(1-\epsilon)a\tilde{h}}\right)^{1/(\epsilon-1)}; \tag{29}$$

thus, for $\epsilon \simeq \text{const}$, we have

$$\frac{a_e}{a} = \left(\frac{a\tilde{h}}{a_e\tilde{h}_e}\right)^{1/(\epsilon-1)} \simeq e^{-\mathcal{N}/\epsilon}. \tag{30}$$

[3]In fact, due to the lack of "bounce" physics, there may exist an uncertainty about the perturbation spectrum, which is from the possible effects of the "bounce" on $\zeta$ and $\Phi$ propagating through the transition. We will go back to this issue in the future.

[4]For $a \sim (-\eta)^q$, $q$ is constant, $\beta = q(q+1)$; thus, $\beta + \frac{1}{4} = (q + \frac{1}{2})^2 > 0$ is always satisfied, independent the value of $q$. In our model, $q \simeq -1/(1-\epsilon)$.

[5]In fact, when $\epsilon \simeq 0$, the nearly scale-invariant spectrum can be also obtained, which corresponds to $\omega \simeq -1$, and may be regarded as the phantom inflation.

Since in such a very slowly expanding phase the significant changing of $a$ can be written as $a_e/a \simeq e$, one obtains

$$\epsilon \simeq -\mathcal{N}. \tag{31}$$

Thus combining Eqs. (9) and (31), we have

$$n_s - 1 \simeq \frac{2}{\mathcal{N}} - \frac{1}{\mathcal{N}} = \frac{1}{\mathcal{N}} \simeq n. \tag{32}$$

Different from the usual ekpyrotic and cyclic models, since $n$ is very small, a nearly scale-invariant blue spectrum is obtain.

In summary, we construct an expanding phase with phantom matter, in which the scale factor expands very slowly. We expect that at some time the phantom matter may decay into the usual radiation by some coupling or through the singular ''big rip'' into the radiation-dominated phase by some other mechanism, which may be from string or M theory or other high-energy and -dimension theory. If regarding such a transition as a ''bounce'' from this preexpanding phase to the late expanding phase denoting our observational cosmology, after the ''bounce,'' the nearly scale-invariant spectrum of adiabatic fluctuations may be obtain. Different from the ekpyrotic and cyclic scenarios in which the bounce is from the contracting phase to the expanding phase, the ''bouncing'' in our scenario is from the expanding phase to the expanding phase; however, instead of the red spectrum in its simplest realization, the spectrum is blue in our scenario. We also gave a phantom field action describing such a very slowly expanding phase. Since in our scenario the phantom field will roll up quickly along its steep effective potential, after it decays or passes through the singularity, it may be placed again in the bottom of the effective potential and then rolled up again, which may lead to a variable ''cyclic'' scenario. Compared with usual inflation and ekpyrotic or cyclic scenario, our work may provide another feasible cosmological scenario to generate the nearly scale-invariant perturbation spectrum, which is worth studying further.

Y.S.P. would like to thank all his collaborators regarding his Ph.D. thesis, especially his advisor Y.Z. Zhang and X.M. Zhang, and also all colleagues at the Institute of Theoretical Physics, Chinese Academy of Science.

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