Puzzles and resolutions of information duplication in de Sitter space

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In this paper we consider a scenario consisting of a de Sitter phase followed by a phase described by a scale factor $a(t)\sim t^q$, where $1/3<q<1$, which can be viewed as an inflationary toy model. It is argued that this scenario naively could lead to an information paradox. We propose that the phenomenon of Poincaré recurrences plays a crucial role in the resolution of the paradox. This is suggested by the fact that the time it takes for an observer to actually experience information duplication is of the order of the recurrence time for the de Sitter phase in question.

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I. INTRODUCTION

Recently a lot of attention has been devoted to physics in a space-time with a positive cosmological constant Λ, i.e., de Sitter space. There are observational as well as theoretical reasons for this attention. From the observational point of view, there is strong evidence in favor of an accelerating universe, which might be due to a positive cosmological constant [1,2]. Understanding de Sitter space is also important in inflationary scenarios, where the possibility of trans-Planckian imprints in the cosmic microwave background radiation (CMBR) spectra has led to renewed interest in the field [3,4].

On the theoretical side the recent interest is partly due to the progress made in the understanding of quantum gravity for AdS spaces with a negative Λ [5–7]. In this case holography has played a key role and the hope has been that similar ideas should be important for de Sitter space as well. The cosmological horizons present in de Sitter space, and the possible parallels with black hole physics, make the problem even more interesting and challenging.

In this paper we will focus on the problem of complementarity in de Sitter space. Our purpose is to investigate the possibilities of an information paradox and compare with the corresponding situation in the case of black holes. In black hole physics the general view that has emerged is that a kind of complementarity principle is at work, implying that two observers, one traveling into a black hole and the other remaining on the outside, have very different views of what is going on. According to the observer staying behind, the black hole explorer will experience temperatures approaching the Planck scale close to the horizon and, as a consequence, the black hole explorer will be completely evaporated and all information transferred into Hawking radiation. According to the explorer herself, however, nothing peculiar happens as she crosses the horizon.

As explained in [8–11] the apparent paradox is resolved when one realizes that the two observers can never meet again to compare notes. Any attempts of the observers to communicate again, after the outside observer has extracted the information from the Hawking radiation, will necessarily make use of Planckian energies and presumably fail. An interesting question to pose, and this is the subject of the present paper, is whether a similar mechanism could be at work also in de Sitter space. It is important to emphasize that the potential information paradox that we will be investigating is not in the sense of losing information, but rather in the sense of duplicating information (“quantum Xeroxing” [8]).

In order to probe the possibility of a paradox in the case of de Sitter space, we will consider a scenario where at some point in time the de Sitter phase is turned off and replaced by a $\Lambda=0$ phase with scale factor $a(t)\sim t^q$, for $1/3<q<1$. We will refer to this latter phase as the post–de Sitter phase. As we will see, the above model, which is nothing but an inflationary toy model, will be of great use for the general understanding of holography and complementarity. While our main purpose is to investigate the general physical principles behind a possible information paradox in de Sitter space, it is interesting that the most suitable framework to do this is in the context of inflation. Our discussion is therefore of great relevance to the ongoing discussions of whether or not holography and other effects of quantum gravity are of importance for inflation.

As already stated, considering our inflationary toy model one might naively be led to a possible information paradox. The paradox is a result of assuming that an object receding towards the de Sitter horizon of an inertial de Sitter observer will return its information content to the observer in the form of de Sitter radiation. If the cosmological constant turns off, the object itself will eventually return to the observer’s causal patch, and one has the threat of an information paradox.

As we will explain, the time scale for Poincaré recurrences will play an important role in the resolution of the paradox. This is similar to earlier work [12], where recurrence plays a crucial role in another setting, namely, the tunneling from de Sitter space to flat space. In that case, as well as in ours, the characteristic time scale for the process is of the order the recurrence time, implying that the process is unphysical. In fact, the time one has to wait for the information to return classically is of the same order as for an arbitrary miracle to occur, that is, a breakdown of the second law by chance.

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The outline of this paper is as follows. In Sec. II we start by reviewing some basic classical and semiclassical properties of de Sitter space. We also briefly comment on quantum gravity in de Sitter space and discuss similarities and differences to the black hole case. In Sec. III we describe the paradox in more detail, comment on various possible loopholes and make attempts to resolve it. We end, in Sec. IV, with some conclusions.

II. SOME ASPECTS OF DE SITTER SPACE AND THE RELATION TO BLACK HOLE PHYSICS

A. Classical de Sitter space

de Sitter space is the maximally symmetric vacuum solution to the Einstein equations with a positive cosmological constant Λ. One way to realize de Sitter space is to view it as a hyperboloid embedded in ordinary flat Minkowski space. Four-dimensional de Sitter space, which is of interest in this paper, is then described as the hypersurface

$$-X_0^2+X_1^2+X_2^2+X_3^2+X_4^2=\frac{3}{\Lambda}=R^2. \tag{1}$$

There are numerous coordinate systems that can be used in discussing the various aspects of de Sitter space (see [13] for a review). In the so-called static coordinates, which are useful when we want to focus on observations made by a particular observer, the metric takes the form

$$ds^2=-\left(1-\frac{r^2}{R^2}\right)dt^2+\left(1-\frac{r^2}{R^2}\right)^{-1}dr^2+r^2d\Omega^2. \tag{2}$$

At $r=R$ we notice the presence of a horizon, the de Sitter horizon. From the point of view of an observer at $r=0$, the horizon acts as a one-way membrane preventing anything that leaves through it to ever come back again, as long as the space-time continues to be de Sitter. We can easily generalize to a situation with a black hole in de Sitter space described by

$$ds^2=-\left(1-\frac{2E}{r}-\frac{r^2}{R^2}\right)dt^2+\left(1-\frac{2E}{r}-\frac{r^2}{R^2}\right)^{-1}dr^2 +r^2d\Omega^2. \tag{3}$$

In general this space has two horizons, the black hole and the de Sitter horizon, respectively. As E is increased it can be seen that the two horizons approach each other, and for $E=3^{-3/2}R$ the two horizons coincide at $r=3^{-1/2}R$. As a consequence there is a maximal sized black hole which can fit inside the de Sitter space.

In discussing cosmology, a more convenient choice of coordinates is the Firedman Robertson Walker (FRW) coordinates

$$ds^2=-dt^2+a(t)^2(dr^2+r^2d\Omega^2), \tag{4}$$

where $a(t)=Re^{t/R}$, and the de Sitter radius R is related to the Hubble constant H through $R=1/H$. With these coordinates, covering half of de Sitter space, one can address global questions such as those relevant for a cosmology with a period of inflation.

B. Semiclassical de Sitter space

Gibbons and Hawking [14] noted that the de Sitter horizon possesses surface gravity and obeys laws analogous to those that govern the physics of black holes. This analogy suggests that the de Sitter horizon can be associated with a temperature and an entropy, similar to what is the case for the black hole event horizon. A black hole steadily emits Hawking radiation, which can be interpreted as the spontaneous creation of particles at a point just outside the black hole horizon, and one would expect that the same kind of radiation would be emitted from the de Sitter horizon. Gibbons and Hawking demonstrated that an observer in de Sitter space indeed detects thermal radiation at a temperature

$$T_{dS}=\frac{1}{2\pi R}, \tag{5}$$

in Planckian units, and that the dS horizon can be endowed with an entropy

$$S_{dS}=\pi R^2=\frac{3\pi}{\Lambda}, \tag{6}$$

supporting the analogy between the de Sitter horizon and the event horizon of a black hole.

Pointing out these similarities, there are some important differences between the two cases worth mentioning. The black hole horizon is an observer independent construction, while the de Sitter horizon is an observer dependent. Any observer in de Sitter space is surrounded by a de Sitter horizon, and if the observer moves, the horizon does so as well. So every observer in de Sitter space lives in the center of a "bubble," bounded by an event horizon with the radius R. As a consequence, the relation between different observers in de Sitter space is, unlike the black hole case, symmetric.

Another crucial difference is that de Sitter space usually is assumed to be in thermal equilibrium.[1] The horizon not only emits radiation, it also absorbs radiation, previously emitted by itself, at the same rate, keeping the radius fixed (under the assumption that Λ is kept fixed). In the case of black holes, on the other hand, one has the option of either studying black holes in thermal equilibrium with a heat bath, or black holes that are truly evaporating.

C. Quantum gravity in de Sitter space?

Lacking a true quantum gravity description of de Sitter space, one can only speculate on what kind of features it

[1] An exception is the work [15,16] where the cosmological constant is claimed to relax due to the radiation.

would possess. Many believe that the describing theory should be holographic. That is, the fundamental degrees of freedom should be the boundary degrees of freedom, with no more than one degree of freedom per Planck area [17–19], see also [20]. This gives particularly strong constraints for de Sitter spaces, simply because the operationally meaningful part of de Sitter space is bounded by the (finite sized) de Sitter horizon. This implies that a microscopic description of de Sitter space should only have a finite number of degrees of freedom, i.e. the entropy $S_{dS}=\pi R^2$ should better be thought of as the total number of degrees of freedom describing the universe. Microscopically deriving this number is a great challenge.[2]

Much of this parallels the situation with black holes, where, indeed, the entropy has been microscopically derived in some special cases; for early references see [21–23]. A microscopic description of black holes implies that they are not as featureless as the classical and semiclassical description suggests, and, in particular, the radiation should not be expected to be purely thermal, but rather be able to carry information. It is reasonable to assume that the same is true for the de Sitter radiation, even though it should be noted that radiation carrying information does not necessarily imply that an observer will be able to extract information from it. We will return to this important point later on.

III. THE PARADOX AND POSSIBLE RESOLUTIONS

A. The scenario

As mentioned in the Introduction, our scenario (see Fig. 1) can globally be viewed as an inflationary toy model. The universe starts out in a de Sitter phase with a scale factor $a(t)\sim e^{t/R}$. At a certain point in time, t_0, the de Sitter phase is turned off and replaced by a post–de Sitter phase where the universe is filled with matter with an equation of state given by $p=\sigma\rho$, and a scale factor $a(t)\sim t^q$ with $q=2/3(1+\sigma)$. We will consider values of q in the range $1/3<q<1$, where the lower bound comes from the requirement that $|\sigma|\leqslant 1$, and the upper bound is the condition for not having an accelerating universe, which would prevent our paradox from being realized.

At the time t_0 of transition we would like to match the scale factor $a(t)$ and the Hubble constant $H=\dot{a}/a$ smoothly. We imagine that the cosmological constant rapidly decays and the energy is transferred into matter through reheating. In the de Sitter phase we have

$$a(t)=Re^{t/R},\quad H=\frac{1}{R},\tag{7}$$

and in the post–de Sitter phase we make the following ansatz for the scale factor:

[2]There are, however, suggestions that this number is a defining property of the theory rather than some derivable consequence of it [17], i.e. in order to end up with a theory with a finite number of degrees of freedom one should postulate it from the beginning rather than deriving it in the end.

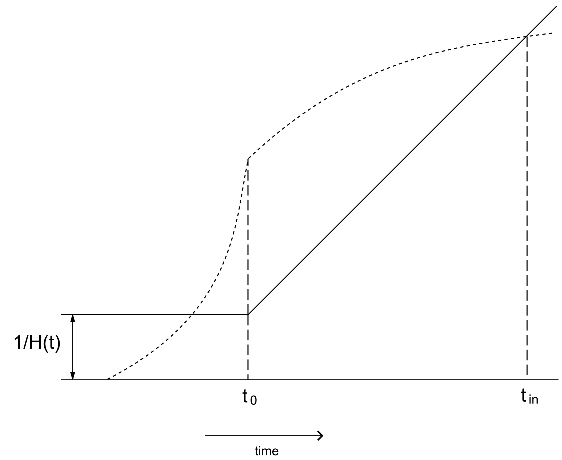

FIG. 1. The de Sitter phase turns into a post–de Sitter phase at $t=t_0$. The Hubble radius is given by the distance between the inertial observer (thin line) and the horizon (solid line). The object (dotted line) leaving the inertial observer crosses the horizon at the initial time $t=0$, and returns its information to the observer in the form of radiation at a time t before t_0. The object itself returns to the observer's causal region in the post–de Sitter phase at $t=t_{in}$, thereby (classically) returning the information a second time.

$$a(t)=A(t-B)^q.\tag{8}$$

The continuity of $a(t)$ and H at $t=t_0$ requires that

$$A=\frac{Re^{t_0/R}}{(qR)^q}\tag{9}$$

$$B=t_0-qR,\tag{10}$$

which gives us the scale factor during the post–de Sitter phase according to

$$a(t)=\frac{Re^{t_0/R}}{(qR)^q}(t-t_0+qR)^q.\tag{11}$$

After these introductory remarks we will move to the possibility of obtaining a paradox.

B. An information paradox?

Let us consider an inertial observer during the de Sitter phase, who drops an object that recedes towards the horizon. Taking the perspective of global coordinates, the object will eventually leave the causal patch of the observer and information is apparently lost. However, as mentioned in Sec. II C, the radiation coming from the horizon is expected, as in the case of black holes, to carry information. By itself, this gives rise to some highly non-trivial questions and issues, discussed in Sec. III C.

In our scenario this is, however, not the end of the story. By turning off the de Sitter phase, a new possibility for the information to get back emerges, namely, the object itself can return to within the causal patch of our observer (see Fig.

1), confusing her by apparently duplicating the information.

One natural question at this stage would be "how long does it take for the object to get back inside the causal patch of the observer?" A short calculation gives us the answer. The condition for the return of the object to the observer's causal region is that the distance, in global coordinates, between the observer and the object should be equal to the causal size of the observer's part of the universe. Since the object moves away from the observer at the rate of the expansion of the universe, i.e. as $\sim t^q$ with $q<1$, and the observer's light cone grows like $\sim t$, one concludes that at some point in time the two distances will coincide. For concreteness we will consider an object at constant comoving coordinate x with $x \lesssim 1$. At time $t=0$ the object is consequently at a physical distance $xR \lesssim R$ from the observer. The physical distance between the object and the observer at the time of return, t_{in}, is given by

$$xRe^{t_0/R}\left(\frac{a(t_{in})}{a(t_0)}\right)=xRe^{t_0/R}\left(\frac{t_{in}-t_0+qR}{qR}\right)^q. \tag{12}$$

The causal size of the universe for the observer at $t=t_{in}$ is

$$(t_{in}-t_0+qR)^q\int_{t_0}^{t_{in}}\frac{dt}{(t-t_0+qR)^q} = \frac{1}{1-q}(t_{in}-t_0+qR)^q[(t_{in}-t_0+qR)^{1-q}-(qR)^{1-q}]. \tag{13}$$

Identifying Eqs. (12) and (13) and solving for t_{in} gives

$$t_{in}=\left(\frac{(1-q)xRe^{t_0/R}}{(qR)^q}+(qR)^{1-q}\right)^{1/(1-q)}+t_0-qR. \tag{14}$$

As a consequence it seems like the information has returned twice to our observer, under the assumption that the observer can get complete information from the de Sitter radiation. In the following two sections we discuss possible loopholes in the arguments leading to this apparent puzzle.

Before doing this, however, it is illuminating to consider what this would look like locally, i.e. what our observer actually would see happen. Let us assume that the object is continuously emitting signals towards our observer. As the object approaches the de Sitter horizon, the signals become increasingly redshifted and effectively disappear from the observer's sight. As long as we are in the de Sitter phase, the redshift increases exponentially. In the post–de Sitter phase, however, we have a redshift that steadily decreases, meaning that the object eventually will become visible to the observer at a certain time in the distant future. Let us calculate this time and compare it with the time t_{in} derived above.

For simplicity we focus on the case where not only the time of detection of the signal, t_{obs}, but also the time of emission, t_{em}, are in the post–de Sitter phase. We then find that

$$\begin{aligned} x &= \int_{t_{em}}^{t_{obs}}\frac{dt}{a(t)} \\ &= \frac{1}{A}\left[\frac{1}{1-q}(t-B)^{1-q}\right]_{t_{em}}^{t_{obs}} \\ &= \frac{1}{(1-q)A}[(t_{obs}-B)^{1-q}-(t_{em}-B)^{1-q}], \end{aligned} \tag{15}$$

where A and B was calculated in Sec. III A. Using this, the redshift z becomes

$$\begin{aligned} 1+z &= \frac{a(t_{obs})}{a(t_{em})} \\ &= \frac{(t_{obs}-B)^q}{(t_{em}-B)^q} \\ &= \frac{(t_{obs}-B)^q}{[(t_{obs}-B)^{1-q}-Ax(1-q)]^{q/(1-q)}}, \end{aligned} \tag{16}$$

where it can be noted that for $t_{obs}\to\infty$ the expression goes to unity as expected. The condition for the object to become visible again is

$$(t_{obs}-B)^{1-q} \gtrsim Ax(1-q), \tag{17}$$

rendering Eq. (16) of order one. Solving for t_{obs}, we get

$$\begin{aligned} t_{obs} &\gtrsim [Ax(1-q)]^{1/(1-q)}+B \\ &= \left(\frac{(1-q)xR}{(qR)^q}\right)^{1/(1-q)} e^{t_0/(1-q)R}+t_0-qR, \end{aligned} \tag{18}$$

which indeed is of the same order as t_{in}, given by Eq. (14).

C. Measuring information in the de Sitter radiation

In the preceding section, a crucial point for the occurrence of the paradox was that one can actually extract information from the de Sitter radiation. Is this really possible? If so, how much radiation is needed and how long will it take to receive the appropriate amount?

In order to answer these questions, one would need a full quantum analysis of the process. Following a less ambitious route, where one considers some very general entropy relations, one can at least obtain a reasonable estimate of the quantities involved. This was done by Page in [24], where the main idea was to consider a total system, in a random pure state, being built up by two subsystems. The question then was how much information could be expected to be contained in the different parts, i.e. the smaller subsystem, the larger subsystem and the correlations between the two. In our situation this would translate into asking how much information we have in the horizon, in the radiation and in the correlations between them. Page's calculations revealed that the information in the smaller subsystem is always less than one-half bit of information, thus basically containing no information at all.

Let us now consider a measuring process where a detector collects radiation and, perhaps, information. As we saw from the discussion of the Schwarzschild–de Sitter metric in Sec. II B our detector must be limited by the size of the largest black hole that can fit in de Sitter space. This means that it can never access more than one-third of the degrees of freedom. The argument of Page would, furthermore, suggest that almost no information will be located in the detector but rather it resides in the correlations (as was also discussed in [25]).[3]

This discussion crucially hinges on the applicability of Page's argument to the discussion of complementarity. In his calculation it is assumed that one is considering a *random* pure state of the total system. In our case it is not clear that this is a fair description. After all, a detector (and observer) prepared to take part in a measuring process is not just any state. But, as we will show, even if the information *can* be retrieved in the de Sitter phase, this will not necessarily lead to a paradox anyway.

Now, what is the minimum time needed to measure the information in the radiation? A reasonable estimate would be the time it takes for the R^2 degrees of freedom of the horizon to, at least in principle, become available to our observer. One would expect this to be the time it takes for an amount of entropy (or information) of the order of R^2 to circulate once through the system. That is, being emitted by the horizon and reabsorbed again. The total flow of entropy (per unit time) from the horizon is given by T^3R^2, and the time to transfer R^2 is then $\tau \sim R^2/(T^3R^2) \sim T^{-3} \sim R^3$, where we have used that $T \sim 1/R$.

One can argue for the same result in the following illuminating way. A single particle in de Sitter space would need roughly the time R to pass through the causal patch of the observer. We then need to know how many particles there are in the de Sitter radiation at a given time. A rough calculation uses the fact that the number density n of blackbody radiation at the temperature T goes like $n \sim T^3$. This means that the total number of particles N in a horizon volume $V \sim R^3$ becomes

$$N = nV \sim \frac{1}{R^3} R^3 = 1. \tag{19}$$

This result[4] might seem somewhat surprising and is not really compatible with what seems to be the general picture of an observer bathing in a sea of de Sitter radiation. Now it is straightforward to estimate the time it takes for R^2 particles to, at least in principle, become available. Since one particle needs the time $t \sim R$ and we have roughly one particle per horizon volume, we recover $\tau \sim R^3$ in the case of R^2 particles. This estimate will be used in the following section.

D. Exceeding the recurrence time?

In Sec. III B we calculated the time it takes for the object to be available again for the observer. The result, Eq. (14), has a dependence on t_0 which is the time passed from the moment the object is released to the time the de Sitter phase is turned off. Then it is natural to let this time be at least the time τ estimated in the last section, i.e. the time needed for the observer to, at least in principle, be able to collect enough radiation during the de Sitter phase.

Now, since τ is a large time, being the cube of R, we only consider the dominant term in Eq. (14). Hence the time of return is

$$t_{in} \sim e^{1/(1-q)\,\frac{\tau}{R}} \sim e^{R^2} \sim e^{S_{dS}}, \tag{20}$$

where it is used that the entropy in de Sitter space is $S_{dS} = \pi R^2$, up to factors of order one in the exponential. But this is nothing but the Poincaré recurrence time for our de Sitter space. The recurrence time is the time it takes for a trajectory in phase space (for an isolated finite system) to return arbitrarily close to its initial value. In particular, this means that discussing experiments lasting longer than their recurrence time is meaningless, since the memory of the system then has been erased. Since the detector obviously has a smaller entropy than the entire de Sitter space, this suggests that we are considering an experiment lasting for too long to make any sense. Thus if the detector can be considered as an isolated system by itself in the post–de Sitter phase (until the expected signal is coming back), the paradox is eliminated. On the other hand, if the detector is allowed to interact with the environment, with more degrees of freedom coming into play, the relevance of Poincaré recurrences is not clear. At the same time, however, the issue of unitarity and a possible information paradox comes in a different light since we are considering an open system.

A related discussion regarding non-unitary processes can be found in [12], where tunneling from de Sitter to flat space was considered. In that case the entire causal diamond disappears, corresponding to a severe violation of unitarity. However, it was argued that this process should be considered unphysical, since the time needed for the process to occur was of the order of the recurrence time for the de Sitter space. This is in the same spirit as the suggested resolution of our paradox, as discussed above.

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[3]Even if this is correct, this does not necessarily imply that the situation is clearcut. We still have to consider the possibility of extracting quantum information in the post–de Sitter phase, where the state of the detector must be correlated with something replacing the de Sitter horizon. One possibility is that the state of the detector is correlated with the matter created through reheating. Then it may, at least in principle, be possible to eventually extract the quantum information.

[4]The fact that there is just one particle per horizon volume was also pointed out in [26].

To summarize, if we consider a detector in de Sitter space, there does not seem to exist a physical process such that information beyond the horizon can be made accessible to the detector before the recurrence time of the detector has passed. This includes tunneling processes as discussed in [12], as well as an abrupt end of inflation as discussed in this paper. In other words, the observer needs to wait as long as the time needed for a thermodynamical miracle to happen.[5]

IV. CONCLUSIONS

In this paper we have discussed the possibility of an information paradox, arising in a specific cosmological scenario. The scenario resembles an idealized inflationary scenario, where we go from a pure de Sitter phase to a phase described by a scale factor $a(t) \sim t^q$, where $1/3 < q < 1$. This could possibly lead to a duplication of information, by allowing an observer to receive information about an object, previously disappeared through the horizon, in two different ways. First, she could extract the information from the de Sitter radiation, assuming this could be done. Second, the object itself could return to the observer, well after the time of transition between the two different phases involved.

Assuming that it is possible to retrieve information during the de Sitter phase, we estimated the time needed to collect the appropriate amount of radiation, which turned out to be $\tau \sim R^3$. We then turned to a calculation of the time it would take for the object to return to the observer, thereby apparently duplicating the information. This time turned out to be dominated by a factor $\sim e^{R^2}$, using τ as input data. The appearance of this factor suggested that a recurrence argument could be invoked, related to issues discussed in [12]. As a consequence, the observer will have to wait as long for the classical return of the information as she must wait for a miracle.

Note added. While this work was being completed we received [27] that discusses the measurement problem in de Sitter space.

[5]At this point it is interesting to compare our results with the time scales involved in a realistic inflationary cosmology, just to give a taste of the relevant order of magnitudes. With 70 e-foldings and $H \sim 10^{-4} m_{Pl}$ one finds $t_{infl} \sim 70/H \sim 7 \times 10^5 t_{Pl}$. The fluctuations that eventually will be visible in the CMBR emerges out of Planckian scales well within this time frame, counted backwards from the end of inflation. Clearly this is much shorter than

$$\tau \sim R^3 = 1/H^3 \sim 10^{12} t_{Pl}, \tag{21}$$

and as a consequence the issue of complementarity would not be expected to be relevant. Another way to say the same thing is to estimate the time t_{in} needed for an object that left at time τ before the end of inflation, to reenter the causal patch of the observer. This is given by

$$t_{in} \sim e^{R^2} = e^{1/H^2} \sim e^{10^8} t_{Pl}, \tag{22}$$

which exceeds the present age of the universe by many orders of magnitude.

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