Limit on the fermion masses in technicolor models

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Recently it has been pointed out that no limits can be put on the scale of fermion mass generation (M) in technicolor models, because the relation between the fermion masses (m_f) and M depends on the dimensionality of the interaction responsible for generating the fermion mass. Depending on this dimensionality it may happen that m_f does not depend on M at all. We show that exactly in this case m_f may reach its largest value, which is almost saturated by the top quark mass. We make a few comments on the question of how large a dynamically generated fermion mass can be.

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The mechanism that breaks the electroweak gauge symmetry $SU(2)_L\times U(1)_Y$ down to the gauge symmetry of electromagnetism $U(1)_{em}$ is still the only obscure part of the standard model. It is known that up to the scale of 1 TeV some sign of this mechanism has to become manifest in future experiments. In the same way that an upper bound on the scale of electroweak symmetry breaking has been put forward (the 1 TeV scale), it was thought that the scale of fermion mass generation also had an upper bound, and that bound would be within the reach of the next generation of accelerators [1]. Recently it was shown that no upper bound can be put on the scale of fermion mass generation beyond that on the scale of electroweak symmetry breaking [2]. This result was obtained by considering the scattering of the same helicity fermions into a large number of longitudinal weak vector bosons in the final state, and it was also obtained in a more involved way in Ref. [3]. This result is important and at the same time disappointing, because an upper bound on the scale of fermion mass generation would provide a target for future accelerators in order to understand the origin of fermion masses.

The scale related to the origin of fermion masses cannot be bounded but the fermion mass itself is bounded. The bound on the fermion masses comes out from the upper limit on the Yukawa coupling ($\lambda_y \leq \sqrt{8\pi}$) [4]. In the standard scenario this is not very interesting because it also leads to a bound on the fermion masses of the order of 1 TeV. Therefore there is still space for a heavy new family (respecting the constraints provided by the high precision experiments). This problem becomes much more interesting in theories with dynamical symmetry breaking such as technicolor theories, where, in principle, some of the free parameters of the standard model are calculable as long as we know the symmetries of the underlying theory that is responsible for the mass generation. Let us recall some of the arguments about the nonexistence of a bound on the scale for fermion mass generation in technicolor (tc) models [2]. In these models the fermion mass is given by

$$m_f \approx c\,\frac{\langle \overline{\psi}_{tc}\psi_{tc}\rangle}{M_{etc}^2}, \qquad (1)$$

where c is a constant and $\langle \overline{\psi}_{tc}\psi_{tc}\rangle$ is the technifermion condensate. M_{etc} is the mass of the extended technicolor (etc) boson and is the mass scale that reproduces an effective Yukawa coupling. According to Ref. [1] M_{etc} should be bounded in the following way. If technicolor is a QCD-like model we can assume $\langle \overline{\psi}_{tc}\psi_{tc}\rangle \approx v^3$ [5], where $v \approx 246$ GeV, and assuming $c \simeq g_{etc}^2$ we obtain

$$M_{etc} \approx \left(g_{etc}^2 \frac{v^3}{m_f}\right)^{1/2}, \qquad (2)$$

which gives the bound on the mass scale responsible for fermion mass generation. Nowadays it is known that composite operators like $\langle \overline{\psi}_{tc}\psi_{tc}\rangle$ (and the technifermion self-energy) may have a large anomalous dimension ($\gamma_m \gtrsim 1$) in such a way that the fermion mass is given by

$$m_f \approx c\,\frac{\langle \overline{\psi}_{tc}\psi_{tc}\rangle}{M_{etc}^2}\left(\frac{M_{etc}^2}{v^2}\right)^{\gamma_m}, \qquad (3)$$

from which we notice that for $\gamma_m = 1$ there is no relation at all between m_f and M_{etc}, indicating that no bound exists on this last mass scale. The anomalous dimension $\gamma_m = 1$ can be obtained in the extreme limit of a walking technicolor dynamics [6], corresponding to a near critical extended technicolor interaction with increased importance of four-fermion operators, and, if the M_{etc} scale is raised, $\gamma_m = 1$ possibly only happens with a fine-tuning of the theory. Let us still continue to discuss the case $\gamma_m = 1$. Exactly in this case we cannot establish a bound on M_{etc}, but note that it also implies that the maximum dynamical fermion mass is limited by

$$m_f \leq cv. \qquad (4)$$

In this Brief Report we propose to discuss what is the maximum value admitted by Eq. (4) or by the dynamical fermion mass in general. We will compute the dynamical fermion mass described in Fig. 1, where the ordinary fermions (f) are connected to technifermions (T_f) through an extended technicolor gauge boson associated with some gauge group $[SU(N_{etc})$ with coupling $\alpha_{etc} = g_{etc}^2/4\pi]$.

We perform the calculation of Fig. 1 using the following general expression for the techniquark self-energy [7]:

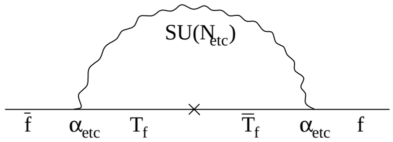

FIG. 1. Diagram for ordinary fermion dynamical masses in technicolor models.

$$\Sigma(p)_g=\mu\left(\frac{\mu^2}{p^2}\right)^{\theta}[1+bg_{tc}^2(\mu^2)\ln(p^2/\mu^2)]^{-\gamma\cos(\theta\pi)}, \tag{5}$$

where in the last equation we identified $\gamma=\gamma_{tc}$. The scale μ (or v) is related to the technicolor condensate $\langle\overline{\psi_{tc}\psi_{tc}}\rangle\equiv\mu^3$ and is ultimately fixed by the experimental value of the weak gauge boson masses. The advantage of using this expression is that it interpolates between the extreme possibilities for the technifermion self-energy. That is, when $\theta=1$ we have the soft self-energy given by

$$\Sigma_s(p)=\frac{\mu^3}{p^2}[1+bg_{tc}^2(\mu^2)\ln(p^2/\mu^2)]^{\gamma}, \tag{6}$$

which is the one obtained when the composite operator $\langle\overline{\psi_{tc}}\psi_{tc}\rangle$ has the canonical dimension. When $\theta=0$ operators of higher dimension may lead to the hard self-energy expression

$$\Sigma_h(p)=\mu[1+bg_{tc}^2(\mu^2)\ln(p^2/\mu^2)]^{-\gamma}, \tag{7}$$

where γ must be larger than 1/2 and the self-energy behaves like a bare mass [8]. Therefore no matter what the dimensionality of the operators responsible for the mass generation in technicolor theories, the self-energy can always be described by Eq. (5). In the above equations g_{tc} is the technicolor coupling constant and $\gamma=3C_{tc}/16\pi^2 b$, where $C_{tc}=\frac{1}{2}[C_2(R_1)+C_2(R_2)-C_2(R_{\bar{\psi}\psi})]$, with the quadratic Casimir operators $C_2(R_1)$ and $C_2(R_2)$ associated with the right- and left-handed fermionic representations of the technicolor group, and $C_2(R_{\bar{\psi}\psi})$ related to the condensate representation. $b_{tc}=(1/16\pi^2)[11N-\frac{2}{3}n_f]$ is the g_{tc}^3 coefficient of the technicolor group β function. The complete equation for the dynamical fermion mass is

$$m_f=\frac{3C_{etc}\mu}{16\pi^4}\int dq^4\left(\frac{\mu^2}{q^2}\right)^{\theta}\frac{g_{etc}^2(q)[1+b_{tc}g_{tc}^2\ln(q^2/\mu^2)]^{-\delta}}{(q^2+M_{etc}^2)(q^2+\mu^2)}, \tag{8}$$

where C_{etc} is the Casimir operator related to the etc fermionic representations, a factor μ remains in the fermion propagator as a natural infrared regulator, $\delta=\gamma\cos\theta\pi$, and $g_{etc}^2(q)$ is assumed to be given by

$$g_{etc}^2(q^2)\simeq\frac{g_{etc}^2(M_{etc}^2)}{1+b_{etc}g_{etc}^2(M_{etc}^2)\ln(q^2/M_{etc}^2)}. \tag{9}$$

Note that in Eq. (8) we have two terms of the form $[1+b_i g_i^2\ln q^2]$, where the index i can be related to tc or etc. To obtain an analytical formula for the fermion mass we will consider the substitution $q^2\rightarrow xM_{etc}^2/\mu^2$, and we will assume that $b_{etc}g_{etc}^2(M_{etc})\approx b_{tc}g_{tc}^2(M_{etc})$, which will considerably simplify the calculation. Knowing that the etc group is usually larger than the tc one, we computed the error in this approximation numerically for a few examples found in the literature. The resulting expression for m_f will be overestimated by a factor 1.1–1.3 and is given by

$$m_f\simeq\frac{3C_{etc}g_{etc}^2(M_{etc})\mu}{16\pi^2}\left(\frac{\mu^2}{M_{etc}^2}\right)^{\theta}\left[1+b_{tc}g_{tc}^2\ln\frac{M_{etc}^2}{\mu^2}\right]^{-\delta}I, \tag{10}$$

where

$$I=\frac{1}{\Gamma(\sigma)}\int_0^{\infty}d\sigma\sigma^{\epsilon-1}e^{-\sigma}\frac{1}{\theta+\alpha\sigma},$$

and $\epsilon=\delta+1=\gamma\cos\theta\pi+1$, $\alpha=b_{tc}g_{tc}^2(M_{etc})$. To obtain Eq. (10) we made use of the following Mellin transform:

$$\left[1+\kappa\ln\frac{x}{\mu^2}\right]^{-\epsilon}=\frac{1}{\Gamma(\epsilon)}\int_0^{\infty}d\sigma e^{-\sigma}\left(\frac{x}{\mu^2}\right)^{-\sigma\kappa}\sigma^{\epsilon-1}. \tag{11}$$

Finally, we obtain

$$m_f\simeq\frac{3C_{etc}g_{etc}^2(M_{etc})\mu}{16\pi^2}\left(\frac{\mu^2}{M_{etc}^2}\right)^{\theta}F(\cos\theta\pi,\gamma,\alpha), \tag{12}$$

where

$$F(\cos\theta\pi,\gamma,\alpha)=\left[1+b_{tc}g_{tc}^2\ln\frac{M_{etc}^2}{\mu^2}\right]^{-\gamma\cos(\theta\pi)}\times\Gamma(-\gamma\cos(\theta\pi),\theta/\alpha)\times\exp\left(\frac{\theta}{\alpha}\right)\alpha^{-1-\gamma\cos(\theta\pi)}\theta^{\gamma\cos(\theta\pi)}.$$

Simple inspection of the above equation shows that (as long as $M_{etc}>\mu$) the largest value for the fermion mass happens for $\theta=0$, and expanding Eq. (12) near this point we have

$$m_f\simeq\frac{3C_{etc}g_{etc}^2(M_{etc})\mu}{16\pi^2}\left[1+b_{tc}g_{tc}^2\ln\frac{M_{etc}^2}{\mu^2}\right]^{-\gamma}\times\frac{1}{\gamma b_{tc}g_{tc}^2(M_{etc})}[1+O(\theta)+\cdots]. \tag{13}$$

For $\theta=0$ we obtain

$$m_f \simeq \frac{C_{etc} g_{etc}^2(M_{etc})\mu}{C_{tc} g_{tc}^2(\mu)} \left[1 + b_{tc} g_{tc}^2 \ln \frac{M_{etc}^2}{\mu^2}\right]^{-\gamma+1}, \quad (14)$$

which gives the largest dynamical fermion mass that we can generate. Although this result is simple and quite intuitive we have not been able to find it stated anywhere.

Note that using the expression for the running coupling $g_{etc}^2(M_{etc})$ Eq. (14) can be written in the following form:

$$m_f \sim \frac{C_{etc}}{C_{tc}} \left(\frac{\alpha_{etc}}{\alpha_{tc}}\right)^{\gamma_{tc}} \mu \sim c v, \quad (15)$$

where γ of the previous expressions indicates γ_{tc}, and the factor c is now given by $c=(C_{etc}/C_{tc})(\alpha_{etc}/\alpha_{tc})^{\gamma_{tc}}$. The possible values of c will determine the maximum value of the fermion mass. To find some limits on the dynamical fermion mass, let us consider some possible ways to introduce the extended technicolor theory. We may, for example, consider that the etc theory may be a kind of grand unified theory (GUT) based on the group $SU(k)$ containing technicolor and the standard Georgi-Glashow group [9]; we then have

$$SU(k) \supset SU(k-5)_{tc} \otimes SU(5)_{gg},$$

where $SU(k-5)_{tc}$ is the tc group, and $SU(5)_{gg}$ is the GUT of Ref. [9]. As tc is a strongly interacting theory it is natural to have $k \geqslant 7$. Therefore, associating the $SU(k)$ group with etc, we obtain the following ratio of Casimir operators:

$$\frac{C_{etc}}{C_{tc}} = \frac{(k^2-1)(k-5)}{k((k-5)^2-1)}.$$

On the other hand, we must also preserve asymptotic freedom, which implies $k \leqslant 11$ [5,12], and the ratio $r_c = C_{etc}/C_{tc}$ will take values in the range $r_c = 1.7$–4.5. We still have to look at the ratio of coupling constants. As tc is a QCD-like theory we can assume as usual that $\alpha_{tc} \sim 1$. The etc theory can be associated with a GUT in this case. Actually, there is no reason at all (specially when the self-energy is the expression with $\theta=0$) to expect a low value for M_{etc}, and a natural one could be $M_{etc} = \Lambda_{gut} \sim 10^{16}$ GeV with $\alpha_{etc} \approx \alpha_{GUT} \sim (40)^{-1}$. The coefficient $\gamma_{tc} = 3C_{tc}/16\pi^2 b_{tc}$ must be larger than 0.5, and in fact if the *tc* group is $SU(2)_{tc}$ we have $\gamma_{tc} \sim 0.5$; for other (and larger) models this coefficient will be larger than 1/2. Therefore we roughly have

$$\left(\frac{\alpha_{etc}}{\alpha_{tc}}\right)^{\gamma_{tc}} \sim \left(\frac{1}{40}\right)^{1/2} \sim \frac{1}{6}.$$

Finally, considering all the estimates, we obtain

$$m_f^{max} \sim O(0.3\text{–}0.8) v \sim O(75\text{–}200) \text{ GeV}. \quad (16)$$

Note that this is a rough estimate and possibly the best that we can do considering the present knowledge of strongly interacting theories. Our calculation is possibly overestimated and it should be divided by a factor of 1.1–1.3 as we indicated in the paragraph after Eq. (9). We also assumed an extreme case for the self-energy maximizing the fermion mass, and it is not clear if a realistic model can exactly reproduce this behavior. Therefore, considering only the smaller factor (1.1) discussed above, it seems that the maximum value of the dynamical mass is already saturated by the top quark mass. There is a possible way to circumvent this limit, i.e., we could build a model with a fermion more massive than the limit given by Eq. (16) where the mass comes from the contribution of several diagrams. In this case the fermion mass could be given by $m_f = n m_f^{max}$, where n is the number of diagrams contributing to the mass of one specific fermion. Models of this kind are similar to the ones of Ref. [11] [a $SU(9)_{gut} \otimes SU(3)_H$ theory, with a technicolor GUT and a horizontal symmetry group], which is based on the model of Ref. [10] [a $SU(7)$ technicolor GUT]. In Table I we show the maximum fermion mass that we can obtain in such models. In the $SU(9)$ model we have two diagrams feeding up the heaviest fermion; even so it is difficult to obtain a mass larger than the limit of Eq. (16). Note also that this result is quite dependent on the model, and the introduction of a horizontal symmetry is necessary for building a realistic model and to give several contributions to the fermion masses.

TABLE I. Maximum fermion mass of various models.

$SU(k)$	$r_c = C_{etc=GUT}/C_{tc}$	γ_{tc}	n	m_f^{max}
$SU(7)$	4.5	0.50	1	$O(177)$ GeV
$SU(9)$	2.4	0.65	2	$O(110)$ GeV

We can also consider a different class of models where the etc group (G_{etc}) and the standard model (G_{SM}) obey [13]

$$G_{etc} \otimes G_{SM} = SU(N_{etc}) \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_y, \quad (17)$$

where $SU(N_{etc})$ must be large enough to accommodate technicolor. No realistic model has yet been found along this line, but let us consider a model based on the $SU(5)_{etc}$ group [14,15], which contains $SU(2)_{tc}$ and one technifermion generation. To obtain the hierarchy among the three generations the model has the following symmetry breaking structure:

$$\begin{gathered} SU(5)_{etc} \otimes G_{SM} \\ \downarrow \Lambda_1 \sim 1000 \text{ TeV} \\ SU(4)_{etc} \otimes G_{SM} \\ \downarrow \Lambda_2 \sim 100 \text{ TeV} \\ SU(3)_{etc} \otimes G_{SM} \\ \downarrow \Lambda_3 \sim 10 \text{ TeV} \\ SU(2)_{tc} \otimes G_{SM}. \end{gathered} \quad (18)$$

We will not discuss the details of this model but just assume that the tc dynamics has the behavior of Eq. (14) and compute the mass of the heaviest family, which is given by

$$m_3 \sim \frac{C_3}{C_{tc}} \left(\frac{\alpha_3(\Lambda_3)}{\alpha_{tc}}\right)^{\gamma_{tc}} v \sim c_3 v. \quad (19)$$

With the values discussed in Refs. [14,15], we see that we do not obtain a very large mass and the limit of Eq. (16) seems to be common to all models.

It is also interesting to discuss another kind of constraint that can be put on the dynamical fermion masses. Fermion masses have been limited in the standard model by making an analysis of the partial wave amplitudes ($J=0$) of the processes $\overline{f}f \rightarrow \overline{f}f$ at high energies. These amplitudes will be proportional to $a_0 \propto m_f^2 G_F$, and the unitarity condition of the S matrix implies that $|a_0| \leqslant 1$, which gives the bound [16]

$$m_f^2 \lesssim \frac{2\pi\sqrt{2}}{3G_F}. \tag{20}$$

The question that we address now is if this limit can be directly applied to tc theories. If we follow Ref. [17] it seems that this result could be the case. In Ref. [17] it was shown that in one technicolor theory where the dynamical symmetry breaking is generated due to the effect of higher order operators the resulting effective theory exactly reproduces the standard model (their self-energy solution is identical to the one we are discussing here). The gauge interactions of the ordinary fermions with the standard gauge boson is obviously the same, but more importantly the Higgs boson coupling is also reduced to the standard model one, which is fundamental to obtain the result of Eq. (20). Only a light degree of freedom (a scalar composite boson) appears below the TeV scale. Therefore the limit of Eq. (20) could be valid for technicolor when the tc dynamics is the harder one that is discussed here. Of course, this is not true for a softer self-energy solution, when the fermion mass is smaller due to the dependence on the scale M_{etc}. If we impose the limit of Eq. (20) over Eq. (15) we obtain

$$\frac{\alpha_{etc}(M_{etc})}{\alpha_{tc}(\mu)} \lesssim \left(\frac{2\pi\sqrt{2}}{3G_F} \frac{1}{v^2 r_c^2} \right)^{1/2\gamma_{tc}}. \tag{21}$$

Considering the numerical values $G_F \sim 1.166 \times 10^{-5}$ GeV^{-2}, $v \sim 246$ GeV and the ratio $r_c = 1.7$–4.5, we obtain $\alpha_{etc}(M_{etc})/\alpha_{tc}(\mu) < 1$. This limit does not imply a very strong constraint on the dynamical fermion mass. However, there is a problem in the above argument. If the dynamical symmetry breaking is generated by the effect of higher order operators, the effective low energy theory may reproduce exactly the standard model as claimed in Ref. [17], but the effective Yukawa coupling will also be proportional to a form factor which, just on dimensional grounds, should be of the form $F(q^2 \rightarrow 0) \propto (1 - q^2/\mu^2 + \cdots)$, because any other mass scale (like M_{etc}) is erased from the self-energy (or appears only in a logarithm). Therefore, at low energies the Yukawa coupling is equal to the one of the standard model, but for momenta q^2 near the tc scale (μ) this coupling should be quite suppressed, leading to a dynamical fermion mass not higher than μ.

In conclusion, it seems very difficult to generate dynamical fermion masses in technicolor models larger than the technicolor scale. The largest mass that can be obtained appears when we consider the hardest (concerning the momentum dependence) expression for the technicolor dynamics, which is also consistent with the nonexistence of a bound on the scale of fermion mass generation. Maybe models with some extra symmetry (possibly a horizontal symmetry), implying that the heaviest fermion receives mass contribution from several diagrams, could be one possibility to have fermions heavier than the top quark within the technicolor scheme, although we do not know any realistic model along these lines. Otherwise, if technicolor is responsible for the standard model symmetry breaking, it seems that no other ordinary heavier fermion family will be found in the next generation of accelerators.

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