Two-body charmless B decays involving η and η'

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We discuss the implications of recent experimental data for B decays into two pseudoscalar mesons, with an emphasis on those with η and η' in the final states. Applying a U-spin argument, we show that tree and penguin amplitudes, both in $B^+\to\pi^+\eta$ and in $B^+\to\pi^+\eta'$, are of comparable magnitudes. Nontrivial relative weak and strong phases between the tree-level amplitudes and penguin-loop amplitudes in the $B^\pm\to\pi^\pm\eta$ modes are extracted. We predict the possible values for the averaged branching ratio and CP asymmetry of the $B^\pm\to\pi^\pm\eta'$ modes. We test the assumption of a singlet-penguin amplitude with the same weak and strong phases as the QCD penguin amplitude in explaining the large branching ratios of $\eta' K$ modes, and show that it is consistent with current branching ratio and CP asymmetry data of the $B^+\to(\pi^0,\eta,\eta')K^+$ modes. We also show that the strong phases of the singlet-penguin and tree-level amplitudes can be extracted with further input of electroweak penguin contributions and a sufficiently well-known branching ratio of the ηK^+ mode. Using $SU(3)$ flavor symmetry, we also estimate required data samples to detect modes that have not yet been seen.

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I. INTRODUCTION

The KEK-B and SLAC PEP-II e^+e^- colliders and Belle and BaBar detectors have permitted the study of B decays with unprecedented sensitivity. CP-violating asymmetries in $B^0\to J/\psi K_S$ and related modes have been observed [1,2] and agree with predictions based on the Kobayashi-Maskawa theory [3]. These asymmetries are associated with the interference between B^0-$\overline{B}^0$ mixing and a single decay amplitude. The observation of *direct CP* asymmetries in B decays, associated with two amplitudes differing in both weak and strong phases, has remained elusive. In this paper we demonstrate that the data on $B\to PP$ branching ratios, where P denotes a pseudoscalar meson, now indicate that substantial direct CP asymmetries in the decays $B^+\to\pi^+\eta$ and $B^+\to\pi^+\eta'$, anticipated previously [4–6], are likely. Indeed, a recent BaBar result [7] favors a large $\pi^+\eta$ asymmetry.

We shall discuss $B^0\to PP$ and $B^+\to PP$ decays within the framework of $SU(3)$ flavor symmetry [8–13], introducing corrections for $SU(3)$ breaking or assigning appropriate uncertainties. Our treatment will be an update of previous discussions [14,15], to which we refer for further details. We shall be concerned here mainly with the decays of charged and neutral B mesons to $K\eta$, $K\eta'$, $\pi\eta$, and $\pi\eta'$. We shall compare our results with a recent treatment also based on flavor $SU(3)$ symmetry [16].

In Sec. II we review notation and amplitude decompositions using flavor symmetry. We compare these with experimental rates, obtaining magnitudes of amplitudes, in Sec. III. We can then extract amplitudes corresponding to specific flavor topologies in Sec. IV. Section V is devoted to $B\to\eta K$ and $B\to\eta' K$, while discussions of $B^+\to\pi^+\eta$ and $B^+\to\pi^+\eta'$ occupy Sec. VI. Some progress on testing amplitude relations proposed in Refs. [5,12] is noted in Sec. VII. Relations among all charged B decays, obtained by applying only the U-spin subgroup [17,18] of flavor $SU(3)$, are studied in Sec. VIII. We remark on as yet unseen processes such as $B^+\to K^+\overline{K}^0$ and $B^0\to(K^0\overline{K}^0,\ \pi^0\pi^0,\ \pi^0\eta,\ \pi^0\eta')$ in Sec. IX, and conclude in Sec. X. An Appendix compares our methods with those used in Ref. [19] to estimate non-penguin contributions to $B^0\to\eta' K^0$.

II. NOTATION

Our quark content and phase conventions [11,12] are:

Bottom mesons: $B^0=d\bar{b}$, $\overline{B}^0=b\bar{d}$, $B^+=u\bar{b}$, $B^-=-b\bar{u}$, $B_s=s\bar{b}$, $\overline{B}_s=b\bar{s}$.

Charmed mesons: $D^0=-c\bar{u}$, $\overline{D}^0=u\bar{c}$, $D^+=c\bar{d}$, $D^-=d\bar{c}$, $D_s^+=c\bar{s}$, $D_s^-=s\bar{c}$.

Pseudoscalar mesons: $\pi^+=u\bar{d}$, $\pi^0=(d\bar{d}-u\bar{u})/\sqrt{2}$, $\pi^-=-d\bar{u}$, $K^+=u\bar{s}$, $K^0=d\bar{s}$, $\overline{K}^0=s\bar{d}$, $K^-=-s\bar{u}$, $\eta=(s\bar{s}-u\bar{u}-d\bar{d})/\sqrt{3}$, $\eta'=(u\bar{u}+d\bar{d}+2s\bar{s})/\sqrt{6}$.

The η and η' correspond to octet-singlet mixtures

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TABLE I. Experimental branching ratios of selected $\Delta S=0$ decays of B mesons. Branching ratios are quoted in units of 10^{-6}. Numbers in parentheses are upper bounds at 90% C.L. References are given in square brackets. Additional lines, if any, give the *CP* asymmetry $\mathcal{A}_{CP}$ (second line) or $(\mathcal{S},\mathcal{A})$ (second and third lines) for charged or neutral modes, respectively.

	Mode	CLEO	BaBar	Belle	Average
$B^+\to$	$\pi^+\pi^0$	$4.6^{+1.8+0.6}_{-1.6-0.7}$ [24]	$5.5^{+1.0}_{-0.9}\pm0.6$ [26]	$5.3\pm1.3\pm0.5$ [33]	5.27 ± 0.79
		-	$-0.03^{+0.18}_{-0.17}\pm0.02$ [26]	$-0.14\pm0.24^{+0.05}_{-0.04}$ [33]	-0.07 ± 0.14
	$K^+\overline{K}^0$	<3.3 [24]	$-0.6^{+0.6}_{-0.7}\pm0.3$ (<1.3) [27]	$1.7\pm1.2\pm0.1$ (<3.4) [33]	<1.3
	$\pi^+\eta$	$1.2^{+2.8}_{-1.2}$ (<5.7) [25]	$4.2^{+1.0}_{-0.9}\pm0.3$ [7]	$5.2^{+2.0}_{-1.7}\pm0.6$ [33]	4.12 ± 0.85
		-	$-0.51^{+0.20}_{-0.18}\pm0.01$ [7]	-	-0.51 ± 0.19
	$\pi^+\eta'$	$1.0^{+5.8}_{-1.0}$ (<12) [25]	$5.4^{+3.5}_{-2.6}\pm0.8$ (<12)[a] [28]	<7 [34]	<7[a]
$B^0\to$	$\pi^+\pi^-$	$4.5^{+1.4+0.5}_{-1.2-0.4}$ [24]	$4.7\pm0.6\pm0.2$ [29]	$4.4\pm0.6\pm0.3$ [33]	4.55 ± 0.44
		-	$(0.02\pm0.34\pm0.05$ [29]	$(-1.23\pm0.41^{+0.08}_{-0.07}$, [35]	$(-0.49\pm0.27,$
		-	$0.30\pm0.25\pm0.04)$ [29]	$0.77\pm0.27\pm0.08)$ [35]	$0.51\pm0.19)$
	$\pi^0\pi^0$	<4.4 [24]	$1.6^{+0.7+0.6}_{-0.6-0.3}$ (<3.6)[a] [26]	$1.8^{+1.4+0.5}_{-1.3-0.7}$ (<4.4)[a] [33]	<3.6[a]
	K^+K^-	<0.8 [24]	<0.6 [29]	<0.7 [33]	<0.6
	$K^0\overline{K}^0$	<3.3 [24]	<2.4 [30]	$0.8\pm0.8\pm0.1$ (<3.2) [33]	<2.4
	$\pi^0\eta$	$0.0^{+0.8}_{-0.0}$ (<2.9) [25]	-	-	<2.9
	$\pi^0\eta'$	$0.0^{+1.8}_{-0.0}$ (<5.7) [25]	-	-	<5.7

[a]This mode has now been detected; see text in Sec. IX.

$$\eta=\eta_8\cos\theta_0-\eta_1\sin\theta_0,\quad \eta'=\eta_8\sin\theta_0+\eta_1\cos\theta_0, \tag{1}$$

with $\theta_0=\sin^{-1}(1/3)=19.5°$.

In the present approximation there are seven types of independent amplitudes: a "tree" contribution t; a "color-suppressed" contribution c; a "penguin" contribution p; a "singlet penguin" contribution s, in which a color-singlet $q\bar{q}$ pair produced by two or more gluons or by a Z or γ forms an $SU(3)$ singlet state; an "exchange" contribution e, an "annihilation" contribution a, and a "penguin annihilation" contribution pa. These amplitudes contain both the leading-order and electroweak penguin contributions:

$$t\equiv T+P^C_{\rm EW},\quad c\equiv C+P_{\rm EW},$$

$$p\equiv P-\frac{1}{3}P^C_{\rm EW},\quad s\equiv S-\frac{1}{3}P_{\rm EW}, \tag{2}$$

$$a\equiv A,\quad e+pa\equiv E+PA,$$

where the capital letters denote the leading-order contributions [5,11,12,20] while $P_{\rm EW}$ and $P^C_{\rm EW}$ are, respectively, color-favored and color-suppressed electroweak penguin amplitudes [20]. We shall neglect smaller terms [21,22] $PE_{\rm EW}$ and $PA_{\rm EW}$ [the (γ,Z) exchange and (γ,Z) direct channel electroweak penguin amplitudes]. We shall denote $\Delta S=0$ transitions by unprimed quantities and $|\Delta S|=1$ transitions by primed quantities. The hierarchy of these amplitudes can be found in Ref. [15].

The partial decay width of two-body B decays is

$$\Gamma(B\to M_1M_2)=\frac{p_c}{8\pi m_B^2}|\mathcal{A}(B\to M_1M_2)|^2, \tag{3}$$

where p_c is the momentum of the final state meson in the rest frame of B, m_B is the B meson mass, and M_1 and M_2 can be either pseudoscalar or vector mesons. Using Eq. (3), one can extract the invariant amplitude of each decay mode from its experimentally measured branching ratio. To relate partial widths to branching ratios, we use the world-average lifetimes $\tau^+=(1.656\pm0.014)$ ps and $\tau^0=(1.539\pm0.014)$ ps computed by the LEPBOSC group [23]. Unless otherwise indicated, for each branching ratio quoted we imply the average of a process and its *CP* conjugate.

III. AMPLITUDE DECOMPOSITIONS AND EXPERIMENTAL RATES

The experimental branching ratios and *CP* asymmetries on which our analysis is based are listed in Tables I and II. Contributions from the CLEO [24,25], BaBar [7,26–32], and Belle [33–38] Collaborations are included. In addition we shall make use of the 90% C.L. upper bounds [39] $\overline{\mathcal{B}}(B^0\to\eta\eta,\ \eta\eta',\ \eta'\eta')<(18,\ 27,\ 47)\times10^{-6}$. (*Note added.* Several of these branching ratios have been updated. See, e.g., Refs. [40,41] for summaries and references.)

We list theoretical predictions and averaged experimental data for charmless $B\to PP$ decays involving $\Delta S=0$ transitions in Table III and those involving $|\Delta S|=1$ transitions in Table IV. Numbers in italics are assumed inputs. All other numbers are inferred using additional assumptions and $SU(3)_F$-breaking and CKM factors. Terms of order λ^2 and smaller relative to dominant amplitudes are omitted. These results update ones quoted most recently in Ref. [15]. The magnitudes of individual amplitudes are based on predicted values (see Table V below) and include the appropriate Clebsch-Gordan coefficients for each mode.

IV. EXTRACTING AMPLITUDES

We begin with those amplitudes or combinations for which information is provided by a single decay or by an

TABLE II. Same as Table I for $|\Delta S|=1$ decays of B mesons.

Mode	CLEO	BaBar	Belle	Average
$B^+\to\pi^+K^0$	$18.8^{+3.7+2.1}_{-3.3-1.8}$ [24]	$17.5^{+1.8}_{-1.7}\pm1.3$ [27]	$22.0\pm1.9\pm1.1$ [33]	19.61 ± 1.44
	-	$-0.17\pm0.10\pm0.02$ [27]	$0.07^{+0.09+0.01}_{-0.08-0.03}$ [36]	-0.032 ± 0.066
π^0K^+	$12.9^{+2.4+1.2}_{-2.2-1.1}$ [24]	$12.8^{+1.2}_{-1.1}\pm1.0$ [26]	$12.8\pm1.4^{+1.4}_{-1.0}$ [33]	12.82 ± 1.07
	-	$-0.09\pm0.09\pm0.01$ [26]	$0.23\pm0.11^{+0.01}_{-0.04}$ [33]	0.035 ± 0.071
ηK^+	$2.2^{+2.8}_{-2.2}$ (<6.9) [25]	$2.8^{+0.8}_{-0.7}\pm0.2$ [7]	$5.3^{+1.8}_{-1.5}\pm0.6$ [33]	3.15 ± 0.69
	-	$-0.32^{+0.22}_{-0.18}\pm0.01$ [7]	-	-0.32 ± 0.20
$\eta'K^+$	$80^{+10}_{-9}\pm7$ [25]	$76.9\pm3.5\pm4.4$ [31]	$78\pm6\pm9$ [33]	77.57 ± 4.59
	-	$0.037\pm0.045\pm0.011$ [31]	$-0.015\pm0.070\pm0.009$ [37]	-0.002 ± 0.040
$B^0\to\pi^-K^+$	$18.0^{+2.3+1.2}_{-2.1-0.9}$ [24]	$17.9\pm0.9\pm0.7$ [29]	$18.5\pm1.0\pm0.7$ [33]	18.16 ± 0.79
	-	$-0.102\pm0.050\pm0.016$ [29]	$-0.07\pm0.06\pm0.01$ [33]	-0.088 ± 0.040
π^0K^0	$12.8^{+4.0+1.7}_{-3.3-1.4}$ [24]	$10.4\pm1.5\pm0.8$ [32]	$12.6\pm2.4\pm1.4$ [33]	11.21 ± 1.36
	-	$0.03\pm0.36\pm0.09$ [32]	-	0.03 ± 0.37
ηK^0	$0.0^{+3.2}_{-0.0}$ (<9.3) [25]	$2.6^{+0.9}_{-0.8}\pm0.2$ (<4.6) [7]	<12 [33]	<4.6
$\eta'K^0$	$89^{+18}_{-16}\pm9$ [25]	$55.4\pm5.2\pm4.0$ [31]	$68\pm10^{+9}_{-8}$ [33]	60.57 ± 5.61
	-	$(0.02\pm0.34\pm0.03$, [31]	$(0.71\pm0.37^{+0.05}_{-0.06}$ [38]	$(0.33\pm0.25$,
	-	$-0.10\pm0.22\pm0.03)$ [31]	$-0.26\pm0.22\pm0.03)$ [38]	$-0.18\pm0.16)$

independent analysis. We then indicate how the remaining amplitudes may be determined or bounded. We expect p', $t+c$, and s' to dominate most decays in which they occur, while p, $t'+c'$, and s should be of relative order λ with respect to them.

The decay $B^+\to\pi^+K^0$ is expected to be dominated by the amplitude $|p'|$ aside from a very small annihilation contribution, as shown in Table IV. We thus extract $|p'|=(45.7\pm1.7)\times10^{-9}$ GeV from the $B^+\to\pi^+K^0$ branching ratio.

[*Note added*. The BaBar Collaboration [42] has reported a new branching ratio $\mathcal{B}(B^+\to K^0\pi^+)=(22.3\pm1.7\pm1.1)\times10^{-6}$, which modifies the average in Table II to $(21.8\pm1.4)\times10^{-6}$. The invariant amplitude for this process in Table IV then becomes $|A_{\rm exp}|=(48.1\pm1.5)\times10^{-9}$ eV. A new CP asymmetry $\mathcal{A}_{CP}(B^+\to K^0\pi^+)=-0.053\pm0.079\pm0.013$ has also been reported [42] and changes the world average value to 0.003 ± 0.059.]

In principle, $|p|$ for $\Delta S=0$ transitions could be directly obtained from the $B^+\to K^+\overline{K^0}$ and $B^0\to K^0\overline{K^0}$ modes. However, current experiments only give upper bounds on their branching ratios. Instead, we use the relation $|p/p'|=|V_{td}/V_{ts}|=\lambda|1-\overline{\rho}-i\overline{\eta}|$, assuming both p and p' to be dominated by the top quark loop. The central values $(\overline{\rho},\overline{\eta})=(0.21,0.34)$ quoted in one analysis [43], together with their 68% C.L. limits, imply $|p/p'|=0.197\pm0.012$ for $\lambda=0.2240$ (see [44]), and hence $|p|=(9.00\pm0.64)\times10^{-9}$ GeV. Although this is the nominal 1σ error, the range $\overline{\rho}\in[0.08,0.34]$, $\overline{\eta}\in[0.25,0.43]$ quoted in Ref. [43] implies an error for $|p|$ more like 20% when theoretical uncertainties affecting $\overline{\rho}$ and $\overline{\eta}$ are taken into account. We shall see that the prospects are good for reducing this error by direct measurement of the $K\overline{K}$ branching ratios mentioned above.

In the majority of our discussion we will be using a convention in which penguin amplitudes are governed by CKM factors $V^*_{tb}V_{ts}$ and $V^*_{tb}V_{td}$, corresponding to strangeness changing and strangeness conserving decays, respectively. In an alternative convention [45] one integrates out the top quark in the $\overline{b}\to\overline{s}(\overline{d})$ loops and uses the unitarity relations $V^*_{tb}V_{ts(d)}=-V^*_{cb}V_{cs(d)}-V^*_{ub}V_{us(d)}$. In this convention penguin amplitudes are governed by $V^*_{cb}V_{cs}$ and $V^*_{cb}V_{cd}$. The ratio of these CKM factors is better known than that occurring in the other conventions. However, $SU(3)$ breaking corrections, possibly of the form f_K/f_π would introduce an uncertainty of about 20% in $|p/p'|$, similar to the above. We will return to this convention when discussing the consequences of U-spin symmetry in Sec. VIII.

Another combination which can be extracted directly from data is $t+c$. The electroweak penguin contribution to this amplitude is expected to be small and we shall neglect it. The average branching ratio $\overline{\mathcal{B}}(B^+\to\pi^+\pi^0)=(5.27\pm0.79)\times10^{-6}$ quoted in Table I gives $|t+c|=(33.3\pm2.5)\times10^{-9}$ GeV. Two subsequent routes permit the separate determination of t and c.

Factorization calculations [46] in principle can yield the ratio $|C/T|$ of leading color-suppressed to color-favored amplitudes [exclusive of the electroweak penguin amplitudes in Eq. (2)]. However, at present $|C/T|$ is only bracketed between 0.08 and 0.37 [47]. For comparison, the corresponding $|C/T|$ ratio in $B^+\to\overline{D}^0\pi^+$ is about 0.4 [48]. With the corresponding estimate $|C+T|/|T|=1.23\pm0.15$, adding errors in quadrature, we find $|t|\simeq|T|=(27.1\pm3.9)\times10^{-9}$ GeV. The error associated with this estimate is superior to that obtained by applying factorization to $B\to\pi l\nu$ [49], which yields $|t|=(28.8\pm6.4)\times10^{-9}$ GeV. We shall use the former estimate for now. (An improved estimate based on new $B\to\pi l\nu$ data [50] yields $|t|=(24.4^{+3.9}_{-1.2})\times10^{-9}$ GeV [51].)

A delicate point arises when passing from T and C to the $|\Delta S|=1$ amplitudes T' and C'. In the combination $t'+c'$

TABLE III. Summary of predicted contributions to $\Delta S=0$ decays of B mesons to two pseudoscalars. Amplitude magnitudes $|A_{\rm exp}|$ extracted from experiments are quoted in units of 10^{-9} GeV. Numbers in italics are assumed inputs. Others are inferred using additional assumptions and $SU(3)_F$-breaking and CKM factors.

Mode	Amplitudes	$\|t+c\|$	$\|p\|$	$\|s\|$ [a]	p_c (GeV)	$\|A_{\rm exp}\|$	$\mathcal{A}_{CP}$
$B^+\to\pi^+\pi^0$	$-\frac{1}{\sqrt{2}}(t+c)$	*23.59*	0	0	2.636	23.59±1.76	−0.07±0.14
$K^+\overline{K}^0$	p	0	9.00	0	2.593	<11.82	
$\pi^+\eta$	$-\frac{1}{\sqrt{3}}(t+c+2p+s)$	19.26	10.39	2.15	2.609	20.95±2.15	−0.51±0.19
$\pi^+\eta'$	$\frac{1}{\sqrt{6}}(t+c+2p+4s)$	13.62	7.35	6.09	2.551	<27.64	
$B^0\to\pi^+\pi^-$	$-(t+p)$	*27.12*[b]	9.01	0	2.636	22.73±1.09	$(\mathcal{S},\mathcal{A})$ [c]
$\pi^0\pi^0$	$-\frac{1}{\sqrt{2}}(c-p)$	-	6.36	0	2.636	<20.21	
K^+K^-	$-(e+pa)$	0	0	0	2.593	<8.32	
$K^0\overline{K}^0$	p	0	9.00	0	2.592	<16.64	
$\pi^0\eta$	$-\frac{1}{\sqrt{6}}(2p+s)$	-	7.35	1.52	2.610	<18.25	
$\pi^0\eta'$	$\frac{1}{\sqrt{3}}(p+2s)$	-	5.20	4.30	2.551	<25.87	
$\eta\eta$	$\frac{\sqrt{2}}{3}(c+p+s)$	-	4.24	1.76	2.582	<45.70	
$\eta\eta'$	$-\frac{\sqrt{2}}{3}(c+p+\frac{5}{2}s)$	-	4.24	4.40	2.523	<56.63	
$\eta'\eta'$	$\frac{1}{3\sqrt{2}}(c+p+4s)$	-	2.12	3.52	2.460	<75.66	

[a] Assuming constructive interference between s' and p' in $B\to\eta' K$ (Table IV).
[b] $T\simeq t$ contribution alone.
[c] $(\mathcal{S},\mathcal{A})=(-0.49\pm0.27,0.51\pm0.19)$.

$=(T'+P'^{C}_{EW})+(C'+P'_{EW})$, the electroweak penguin terms contribute in magnitude about 2/3 of the $|T'+C'|$ terms [52]. Aside from an overall strong phase, one expects

$$t'+c'=|T'+C'|[e^{i\gamma}-\delta_{EW}],\quad \delta_{EW}=0.65\pm0.15, \quad (4)$$

where the second term is the estimate of the electroweak penguin term. $|T'+C'|=(9.35\pm0.70)\times10^{-9}$ GeV is obtained by multiplying $|t+c|\simeq|T+C|$ by the factor $|(V_{us}f_K)/(V_{ud}f_\pi)|\simeq0.280$, with $\lambda=0.2240$ [44]. The corresponding electroweak penguin term contribution is $|T'+C'|\delta_{EW}=(6.1\pm1.5)\times10^{-9}$ GeV. It is expected to have the same weak phase as the strangeness-changing penguin contribution p' [52].

We next extract the "singlet penguin" amplitude $|s'|$ by comparing the $B\to\eta' K$ branching ratios with those expected on the basis of p' alone. The p' contribution to $B\to\eta' K$ is much larger than that to $B\to\eta K$ [53], vanishing altogether for the latter for our choice $\eta=(s\bar{s}-u\bar{u}-d\bar{d})/\sqrt{3}$ as a result of cancellation of the nonstrange and strange quark contributions. In $\eta'=(2s\bar{s}+u\bar{u}+d\bar{d})/\sqrt{6}$ the nonstrange and strange quarks contribute constructively in the p' term, but not enough to account for the total amplitude. A flavor-singlet penguin term s' added constructively to p' with no relative strong phase and with $|s'/p'|\simeq0.41$ can account for the $B\to\eta' K$ decay rates. We shall consider mainly a minimal s' term interfering constructively with p', discussing in Sec. VII the possibility that $|s'|$ could be larger than its minimal value. (The weak phases of p' and s' are expected to be the same [5,12], but their strong phases need not be.)

The amplitude for $B^+\to\eta' K^+$ is better known than that for $B^0\to\eta' K^0$ (see Table IV). In the limit of p',s' dominance they should be equal, while that for the charged mode is slightly larger. This could be a consequence of a statistical fluctuation or a contribution from t'. The combination $t'+c'$ which appears in $A(B^+\to\eta' K^+)=(92.4\pm2.7)\times10^{-9}$ GeV includes an electroweak penguin term $|T'+C'|\delta_{EW}/\sqrt{6}=(2.5\pm0.6)\times10^{-9}$ GeV which we subtract from the total amplitude (the weak p' and s' phases are expected to be π as well) to obtain the estimate $|(3p'$

TABLE IV. Same as Table III for $|\Delta S|=1$ decays of B mesons.

Mode		Amplitudes	$\lvert T'+C'\rvert$	$\lvert p'\rvert$	$\lvert s'\rvert$ [a]	p_c (GeV)	$\lvert A_{\rm exp}\rvert$	$\mathcal{A}_{CP}$
$B^+\to$	π^+K^0	p'	0	*45.70*	0	2.614	45.70 ± 1.68	-0.032 ± 0.066
	π^0K^+	$-\frac{1}{\sqrt{2}}(p'+t'+c')$	6.61	32.32	0	2.615	36.94 ± 1.54	0.035 ± 0.071
	ηK^+	$-\frac{1}{\sqrt{3}}(s'+t'+c')$	5.40	0	10.92	2.588	18.40 ± 2.01	-0.32 ± 0.20
	$\eta' K^+$	$\frac{1}{\sqrt{6}}(3p'+4s'+t'+c')$	3.82	55.97	30.88	2.530	92.42 ± 2.74	-0.002 ± 0.040
$B^0\to$	π^-K^+	$-(p'+t')$	7.59 [b]	45.70	0	2.615	45.57 ± 0.99	-0.088 ± 0.040
	π^0K^0	$\frac{1}{\sqrt{2}}(p'-c')$	-	32.32	0	2.614	35.81 ± 2.17	
	ηK^0	$-\frac{1}{\sqrt{3}}(s'+c')$	-	0	10.92	2.587	<23.06	
	$\eta' K^0$	$\frac{1}{\sqrt{6}}(3p'+4s'+c')$	-	55.97	30.88	2.528	84.73 ± 3.93	$(\mathcal{S},\mathcal{A})$ [c]

[a]Assuming constructive interference between s' and p' in $B\to\eta' K$.
[b]T' contribution alone.
[c]$(\mathcal{S},\mathcal{A})=(0.02\pm0.34,-0.10\pm0.22)$.

$+4s')/\sqrt{6}|=(89.9\pm2.8)\times10^{-9}$ GeV. In addition the term $|T'+C'|/\sqrt{6}=(3.8\pm0.3)\times10^{-9}$ GeV contributes with unknown phase. We thus combine it in quadrature as an additional error to obtain $|(3p'+4s')/\sqrt{6}|=(89.9\pm4.7)\times10^{-9}$ GeV from $B^+\to\eta' K^+$. We average this value with $A(B^0\to\eta' K^0)=(84.7\pm3.9)\times10^{-9}$ GeV, neglecting in the latter all c' contributions, including a possible electroweak penguin term. We then obtain $|(3p'+4s')/\sqrt{6}|=(86.9\pm3.0)\times10^{-9}$ GeV. Assuming that p' and s' contribute constructively as mentioned above, we subtract the p' contribution to find $|s'|=(18.9\pm2.2)\times10^{-9}$ GeV.

The value of $|s/s'|$ is assumed to be governed by the same ratio of CKM factors $|V_{td}/V_{ts}|=0.197\pm0.012$ as $|p/p'|$, bearing in mind that the full range of uncertainty, including theoretical errors, could be as much as 20%. We summarize the extracted magnitudes of amplitudes along with their associated errors in Table V.

TABLE V. Values and errors of the topological amplitudes extracted according to the method outlined in the text.

Amplitude	Magnitude ($\times10^{-9}$ GeV)
$\lvert t+c\rvert\simeq\lvert T+C\rvert$	33.3 ± 2.5
$\lvert t\rvert\simeq\lvert T\rvert$	27.1 ± 3.9
$\lvert c\rvert\simeq\lvert C\rvert$	6.2 ± 3.3
$\lvert p\rvert$	9.00 ± 0.64
$\lvert s\rvert$	3.73 ± 0.50
$\lvert T'+C'\rvert$	9.35 ± 0.70
$\lvert T'\rvert$	7.6 ± 1.1
$\lvert C'\rvert$	1.74 ± 0.93
$\lvert (T'+C')\delta_{EW}\rvert$	6.1 ± 1.5
$\lvert p'\rvert$	45.7 ± 1.7
$\lvert s'\rvert$	18.9 ± 2.2

Much theoretical effort has been expended on attempts to understand the magnitude of the singlet penguin amplitude s' [54]. An alternative treatment [55] finds an enhanced standard-penguin contribution to $B\to\eta' K$ without the need for a large singlet penguin contribution. A key feature of this work is the description of η-η' mixing along the lines of Ref. [56], involving a slightly different octet-singlet mixing angle [$\theta_0=(15.4\pm1.0)°$ instead of our value of 19.5°]. The effect of the $s\bar{s}$ component of the wave function for both η and η' is enhanced with respect to the symmetry limit. We shall comment upon one distinction between this scheme and ours at the end of the next section. Predictions are given also for $|\Delta S|=1$ decays involving one pseudoscalar and one vector meson (see also Ref. [15]), but not for $\Delta S=0$ decays. (*Note added*. A complete QCD factorization analysis of charmless $B\to PP$ and $B\to VP$ decays has now been performed in Ref. [57].)

V. $B\to\eta K$ AND $B\to\eta' K$ DECAYS

The singlet penguin contribution to the $B\to\eta K$ amplitude is expected to be $1/(2\sqrt{2})$ of that for $B\to\eta' K$, amounting to $(10.9\pm1.3)\times10^{-9}$ GeV. As seen from Table IV, this is an appreciable fraction of the observed amplitude $|A(B^+\to\eta K^+)|=(18.4\pm2.0)\times10^{-9}$ GeV. An additional electroweak penguin contribution of $|(T'+C')\delta_{EW}|/\sqrt{3}=(3.50\pm0.85)\times10^{-9}$ GeV leaves only $(4.0\pm2.5)\times10^{-9}$ GeV to be accounted for via interference with $|T'+C'|/\sqrt{3}=(5.4\pm0.4)\times10^{-9}$ GeV. This favors, but does not prove, constructive interference between $t' + c'$ and s'.

Taking into account the s' contribution alone (neglecting c' including its electroweak penguin part), one predicts $\overline{\mathcal{B}}(B^0 \to \eta K^0) = (1.03 \pm 0.24) \times 10^{-6}$. We shall compare this with the current upper bound in Sec. IX.

We mentioned in Sec. IV that the value of $|A(B^+ \to \eta' K^+)|$, after subtracting an electroweak penguin contribution, was $(89.9 \pm 2.8) \times 10^{-9}$ GeV, which is composed of the combination $(3p' + 4s')/\sqrt{6}$ [whose magnitude, averaging between charged and neutral modes, we found to be $(86.9 \pm 3.0) \times 10^{-9}$ GeV], and a $T' + C'$ contribution with magnitude $(3.8 \pm 0.3) \times 10^{-9}$ GeV. Again, as in $B^+ \to \eta K^+$, this favors but does not prove constructive interference between the $t' + c'$ and penguin contributions.

Having now specified the necessary amplitudes, we can predict decay amplitudes and *CP* asymmetries for $B^+ \to \eta K^+$ and $B^+ \to \eta' K^+$, as well as for the related process $B^+ \to \pi^0 K^+$, as functions of the CKM angle γ and a relative strong phase. For the purpose of this discussion we may write the decay amplitude for $B^+ \to M K^+$ ($M = \pi^0,\ \eta,\ \eta'$) as

$$A(B^+ \to M K^+) = a(e^{i\gamma} - \delta_{EW}) e^{i\delta_T} - b, \qquad (5)$$

where the sign before b takes account of the weak phase π in the $|\Delta S| = 1$ penguin term. For $M = \pi^0,\ \eta,\ \eta'$ the values of a are (6.61, 5.40, 3.82)$\times 10^{-9}$ GeV, while those of b are (32.32, 10.92, 86.85)$\times 10^{-9}$ GeV, as one may see from the entries in Table IV. The *CP* rate asymmetries are

$$\mathcal{A}_{CP}(f) \equiv \frac{|A(B^- \to \overline{f})|^2 - |A(B^+ \to f)|^2}{|A(B^- \to \overline{f})|^2 + |A(B^+ \to f)|^2}, \qquad (6)$$

while the *CP*-averaged amplitudes, to be compared with the experimental amplitudes quoted in Tables III and IV, are

$$|A(f)| \equiv \left\{ \frac{1}{2} [|A(B^+ \to f)|^2 + |A(B^- \to \overline{f})|^2] \right\}^{1/2}. \qquad (7)$$

Here we have assumed the penguin and singlet penguin amplitudes s' and p' to have the same strong phase, which we take to be zero.

The *CP* asymmetries are most sensitive to δ_T, varying less significantly as a function of γ over the 95% C.L. allowed range [43] $38° < \gamma < 80°$. For illustration we present the asymmetries calculated for $\gamma = 60°$ in Fig. 1.

The constraints on δ_T from $\mathcal{A}_{CP}(\pi^0 K^+)$ are fairly stringent: $-34° \leq \delta_T \leq 19°$ and a region of comparable size around $\delta_T = \pi$. The allowed range of $\mathcal{A}_{CP}(\eta K^+)$ restricts these regions further, leading to net allowed regions $-7° \leq \delta_T \leq 19°$ or $163° \leq \delta_T \leq 185°$. These allowed regions do not change much if we vary γ over its range between 38° and 80°.

The predicted magnitudes $|A(f)|$ are very insensitive to δ_T within the above ranges. In Fig. 2 we exhibit them for the two cases $\delta_T = 0$ and $\delta_T = \pi$. Values of δ_T near zero are favored over those near π, and there is some preference for the higher values of γ within its standard model range. The experimental value of $|A(\eta K^+)|$ tends to exceed the prediction for all but the highest allowed values of γ.

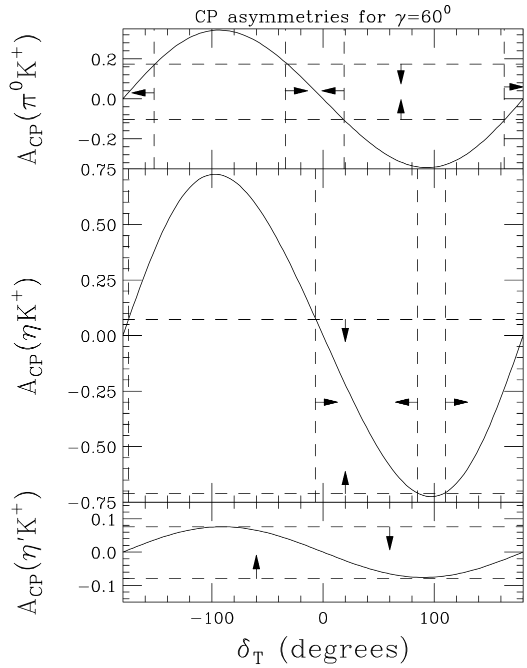

FIG. 1. Predicted *CP* rate asymmetries when $\gamma = 60°$ for $B^+ \to \pi^0 K^+$ (top), $B^+ \to \eta K^+$ (middle), and $B^+ \to \eta' K^+$ (bottom). Horizontal dashed lines denote 95% C.L. ($\pm 1.96\sigma$) upper and lower experimental bounds, leading to corresponding bounds on δ_T denoted by vertical dashed lines. Arrows point toward allowed regions.

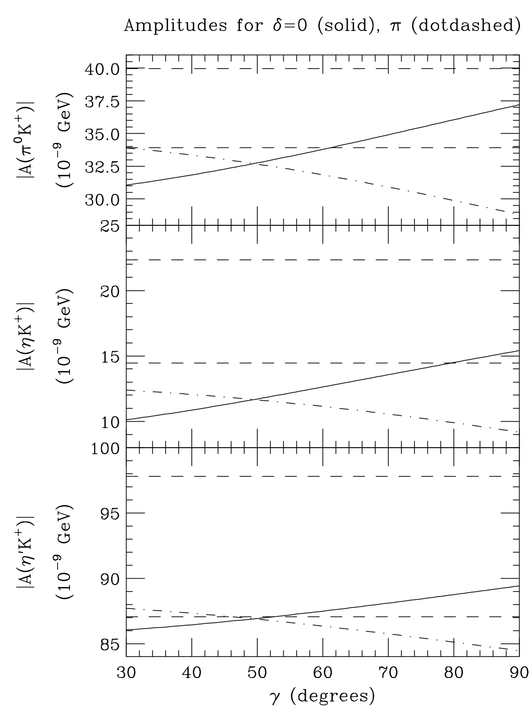

FIG. 2. Predicted magnitudes $|A|$ of amplitudes (based on *CP*-averaged rates) for $B^+ \to \pi^0 K^+$ (top), $B^+ \to \eta K^+$ (middle), and $B^+ \to \eta' K^+$ (bottom). Solid and dot-dashed curves correspond to $\delta_T = 0$ and π, respectively. Horizontal dashed lines denote 95% C.L. ($\pm 1.96\sigma$) upper and lower experimental bounds.

In contrast to our description of $B\to\eta K$ and $B\to\eta' K$, the calculation of Ref. [55] has very small singlet penguin contributions to both decays. In the case of $B\to\eta' K$ a very small CP asymmetry is predicted as a result of the overwhelming dominance of the penguin amplitude. The $B\to\eta K$ penguin amplitude does not vanish (in contrast to our approach), but is predicted to be the dominant (small) contribution to the decay, with a sign *opposite* to that in $B\to\eta' K$. (See Table 3 of Ref. [55].) Thus, for a given final-state phase, the CP asymmetry predicted in Ref. [55] for $B^+\to\eta K^+$ will have the *opposite sign* to that which we predict. This has interesting consequences for a comparison with the CP asymmetries in $B^+\to\pi\eta$ and $B^+\to\pi\eta'$, which we will discuss in the next section.

A clearcut difference between our formalism and that of Ref. [55] is in the CP asymmetries of $B\to\eta K$ and $B\to\eta' K$. Assuming that the singlet penguin amplitude has the same strong phase as the QCD penguin, we predict both asymmetries to have the same sign for a fixed final-state phase of penguin amplitudes relative to tree-level amplitudes. However, the central values of the predictions given in Ref. [55] favor the asymmetries to have opposite signs. Better measurements of $\mathcal{A}_{CP}(\eta K^+)$ and $\mathcal{A}_{CP}(\eta' K^+)$ (although the latter could be quite difficult) will be very useful to justify which approach is more favored.

VI. CHARGED $\pi\eta^{(\prime)}$ MODES

As seen in Table III, the magnitudes of $t+c$ and p contributions to the $\pi^\pm\eta^{(\prime)}$ modes are comparable to each other. The CKM factors associated with these amplitudes are $V_{ub}^* V_{ud}\propto e^{i\gamma}$ and dominantly $V_{tb}^* V_{td}\propto e^{-i\beta}$, respectively. One therefore expects to observe sizable direct CP asymmetries in these decay modes if there is a nontrivial relative strong phase in the amplitudes. Indeed, a rate asymmetry of -0.51 ± 0.19 for $B^\pm\to\pi^\pm\eta$ has been observed at BaBar [7]. On the other hand, the fact that the invariant amplitude prediction for $B^\pm\to\pi^\pm\eta$ with both maximal constructive and destructive interference schemes will be in conflict with the one extracted from experiments also indicates a nontrivial phase between $t+c$ and p.

As outlined in Ref. [15], by combining the branching ratio and CP rate asymmetry information of the $\pi^\pm\eta$ modes, one should be able to extract the values of the relative strong phase δ and the weak phase α, assuming maximal constructive interference between p and s (no relative strong phase). The solution thus obtained can be used to predict the branching ratio and CP asymmetry of the $\pi^\pm\eta'$ modes.

Let us write the decay amplitudes for the $\pi^+\eta$ and $\pi^+\eta'$ modes as

$$A(\pi^+\eta)=-\frac{1}{\sqrt{3}}[|t+c|e^{i\gamma}+|2p+s|e^{i(-\beta+\delta)}], \quad (8)$$

$$A(\pi^+\eta')=\frac{1}{\sqrt{6}}[|t+c|e^{i\gamma}+|2p+4s|e^{i(-\beta+\delta)}]. \quad (9)$$

Then the CP rate asymmetries $\mathcal{A}_{CP}(f)$ and the CP-averaged branching ratios

$$\overline{\mathcal{B}}(f)\equiv\frac{\mathcal{B}(B^-\to\overline{f})+\mathcal{B}(B^+\to f)}{2} \quad (10)$$

are found to be

$$\mathcal{A}_{CP}(\pi^+\eta)\simeq-\frac{0.91\sin\delta\sin\alpha}{1-0.91\cos\delta\cos\alpha}, \quad (11)$$

$$\mathcal{A}_{CP}(\pi^+\eta')\simeq-\frac{\sin\delta\sin\alpha}{1-\cos\delta\cos\alpha}, \quad (12)$$

$$\overline{\mathcal{B}}(\pi^+\eta)\simeq4.95\times10^{-6}(1-0.91\cos\delta\cos\alpha), \quad (13)$$

$$\overline{\mathcal{B}}(\pi^+\eta')\simeq3.35\times10^{-6}(1-\cos\delta\cos\alpha), \quad (14)$$

where the relation $\alpha=\pi-\beta-\gamma$ has been used and the amplitudes have been substituted by the preferred values given in Table III.

Note that Eqs. (11)–(14) are invariant under the exchange $\alpha\leftrightarrow\delta$ and the transformation $\alpha\to\pi-\alpha$ and $\delta\to\pi-\delta$. (Although $\{\alpha\to-\alpha,\delta\to-\delta\}$ is also an invariant transformation, negative α is disfavored by current unitarity triangle constraints.) In comparison, we have world averages of $\overline{\mathcal{B}}(\pi^+\eta)=(4.12\pm0.85)\times10^{-6}$ and $\mathcal{A}_{CP}(\pi^+\eta)=-0.51\pm0.19$. We use the central values to solve for the phases α and δ and obtain four possibilities:

$$(\alpha,\delta)\simeq(78°,28°), \quad (15)$$

and those related by the $\alpha\leftrightarrow\delta$ and $(\alpha,\delta)\to(\pi-\alpha,\pi-\delta)$ symmetries. This information leads us to the prediction of the branching ratio and CP asymmetry for the $\pi^+\eta'$ mode:

$$\overline{\mathcal{B}}(\pi^+\eta')\simeq2.7\times10^{-6}, \quad (16)$$

$$\mathcal{A}_{CP}(\pi^+\eta')\simeq-0.57. \quad (17)$$

In general, we also allow the amplitude parameters ($|t+c|$, $|p|$, and $|s|$), the branching ratio and direct CP asymmetry of the $\pi^\pm\eta$ mode to vary. We use a normal distribution to sample 500 sets of input parameters within the 1σ ranges as extracted in Sec. IV and of the experimental data. For each set of input parameters, we go through similar processes as outlined above to solve from $\mathcal{A}_{CP}(\pi^+\eta)$ and $\overline{\mathcal{B}}(\pi^+\eta)$ for the weak and strong phases. They are found to fall within the cross-marked area in Fig. 3. At the 1σ level, the weak phase α ranges from $\sim60°$ to $\sim100°$ and the strong phase from $\sim15°$ to $\sim55°$. As mentioned before, there are three other possibilities related to Fig. 3 by the $\alpha\leftrightarrow\delta$ and $(\alpha,\delta)\to(\pi-\alpha,\pi-\delta)$ symmetries. In either of these cases, the predicted values of the branching ratio and the direct CP asymmetry for the $\pi^\pm\eta'$ mode are the same. As shown in Fig. 4, the averaged branching ratio of the $\pi^\pm\eta'$ modes is predicted to fall in the range $2.0\times10^{-6}\lesssim\overline{\mathcal{B}}(\pi^+\eta')\lesssim3.5\times10^{-6}$, which is well below the best upper bound given in Table I. (See, however, the note at the end of Ref. [28].) A sizable

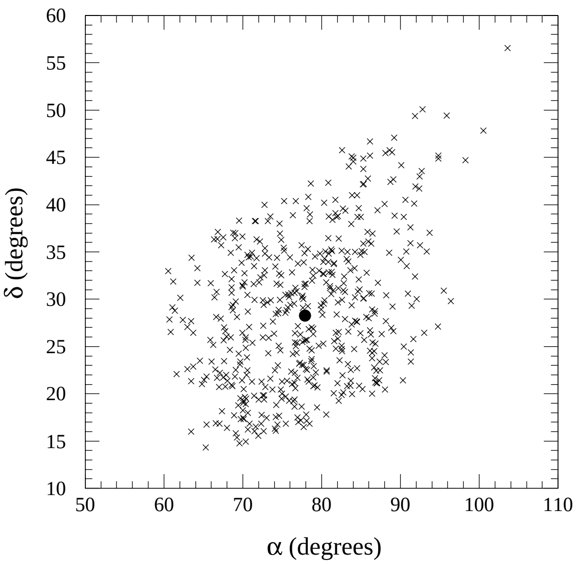

FIG. 3. Phases α and δ governing the decays $B^{\pm}\rightarrow\pi^{\pm}\eta$, obtained by solving constraints provided by the branching ratio and direct *CP* asymmetry along with amplitude inputs varying over allowed values, are depicted by scattered crosses. The solution corresponding to the preferred central values is marked with a thick dot.

direct *CP* asymmetry between ~ -0.34 and ~ -0.80 is expected from current data.

The amplitude relation (8) and the corresponding charge-conjugate amplitude may be written in the form

$$A(\pi^{+}\eta)=-\frac{1}{\sqrt{3}}|t+c|e^{i\gamma}[1-r_{\eta}e^{i(\alpha+\delta)}], \tag{18}$$

$$A(\pi^{-}\eta)=-\frac{1}{\sqrt{3}}|t+c|e^{-i\gamma}[1-r_{\eta}e^{i(-\alpha+\delta)}], \tag{19}$$

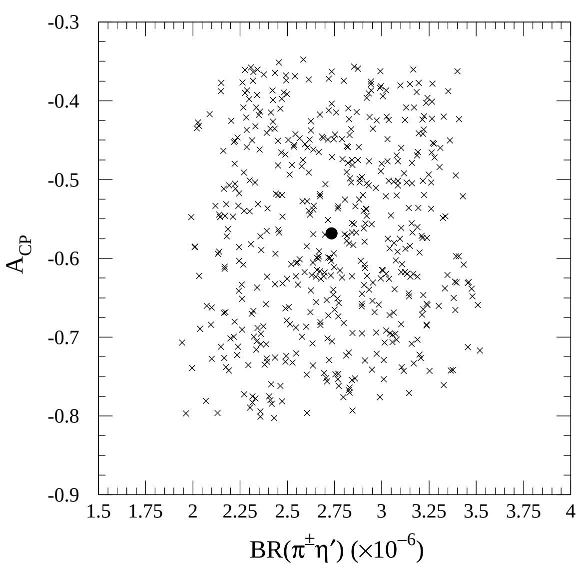

FIG. 4. Predicted values of the averaged branching ratio and direct *CP* asymmetry for the decays $B^{\pm}\rightarrow\pi^{\pm}\eta'$ corresponding to the points in Fig. 3.

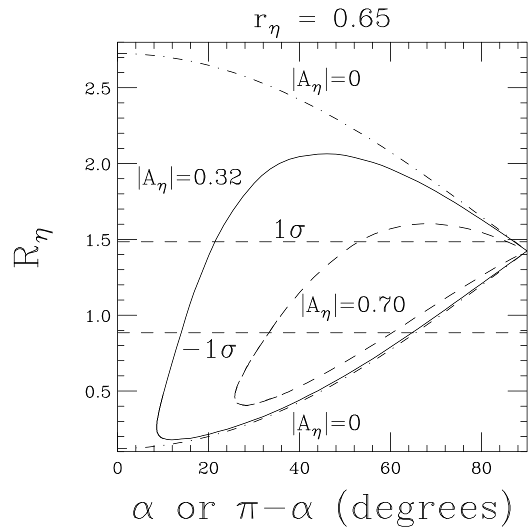

FIG. 5. Predicted value of R_{η} (ratio of observed *CP*-averaged $B^{\pm}\rightarrow\pi^{\pm}\eta$ decay rate to that predicted for tree amplitude alone) as a function of α for various values of *CP* asymmetry $|A_{\eta}|$. (The values 0.70 and 0.32 correspond to $\pm 1\sigma$ errors on this asymmetry.)

where $r_{\eta}\equiv|2p+s|/|t+c|=0.65\pm0.06$ is the ratio of penguin to tree contributions to the $B^{\pm}\rightarrow\pi^{\pm}\eta$ decay amplitudes. In analogy with our previous treatments of $B^{0}\rightarrow\pi^{+}\pi^{-}$ [58] and $B^{0}\rightarrow\phi K_{s}$ [59], we may define a quantity R_{η} which is the ratio of the observed *CP*-averaged $B^{\pm}\rightarrow\pi^{\pm}\eta$ decay rate to that which would be expected in the limit of no penguin contributions. We find

$$R_{\eta}=1+r_{\eta}^{2}-2r_{\eta}\cos\alpha\cos\delta=1.18\pm0.30. \tag{20}$$

One can then use the information on the observed *CP* asymmetry in this mode to eliminate δ and constrain α. (For a related treatment with a different convention for penguin amplitudes see Ref. [60].) The asymmetry is

$$A_{\eta}=-2r_{\eta}\sin\alpha\sin\delta/R_{\eta}=-0.51\pm0.19, \tag{21}$$

so one can either use the simple result

$$R_{\eta}=1+r_{\eta}^{2}\pm\sqrt{4r_{\eta}^{2}\cos^{2}\alpha-(A_{\eta}R_{\eta})^{2}\cot^{2}\alpha} \tag{22}$$

with experimental ranges of R_{η} and A_{η} or solve Eq. (22) for R_{η} in terms of α and A_{η}. The result of this latter method is illustrated in Fig. 5.

The range of α allowed at 95% C.L. in standard-model fits to CKM parameters is $78°\leqslant\alpha\leqslant122°$ [43]. For comparison, Fig. 5 permits values of α in the three ranges

$$14°\leqslant\alpha\leqslant53°, \tag{23}$$

$$60°\leqslant\alpha\leqslant120°, \tag{24}$$

$$127°\leqslant\alpha\leqslant166° \tag{25}$$

if R_{η} and $|A_{\eta}|$ are constrained to lie within their 1σ limits. These limits coincide with those extracted from Fig. 3 when one considers all the possible solutions related by symmetries. Only the middle range overlaps the standard-model parameters, restricting them very slightly. Better constraints on α in this region mainly would require reduction of errors on R_η.

There are important questions of the consistency of the range of δ as exhibited in Fig. 3 with other determinations of the relative strong phases between penguin and tree amplitudes. They involve comparisons with two classes of processes: (a) the $\Delta S=0$ decays $B^0\to\pi^+\pi^-$, and (b) the $|\Delta S|=1$ decays $B^+\to(\pi^0,\eta,\eta')K^+$.

For $B^0\to\pi^+\pi^-$ the amplitude $t+p$ is not exactly the same as the amplitude $A(B^+\to\eta\pi^+)=-(t+c+2p+s)/\sqrt{3}$, which has small c and s contributions and a larger penguin-to-tree ratio. Nonetheless one should expect the same sign of the CP asymmetries $\mathcal{A}_{CP}(\eta\pi^+)$ and $\mathcal{A}_{\pi\pi}$, whereas the first is -0.51 ± 0.19 while the second is 0.51 ± 0.19. It would be interesting to see whether explicit calculations (e.g., using the methods of Refs. [46] and [55]) could cope with this opposite sign.

In comparing the $|\Delta S|=1$ decays $B^+\to(\pi^0,\eta,\eta')K^+$ discussed in Sec. V with $B^+\to(\eta,\eta')\pi^+$ discussed in the present section, one expects in the flavor-$SU(3)$ limit that $\delta=-\delta_T$. (We have associated the strong phase in each case with the less-dominant amplitude: tree for $|\Delta S|=1$ in Sec. V and penguin for $\Delta S=0$ in the present section.) With the preference for $\delta>0$ exhibited in Fig. 3, we would then expect to prefer $\delta_T<0$ in Fig. 1, which is disfavored by the negative central value of the CP asymmetry for $B^+\to\eta K^+$.

To say it more succinctly, there is not a consistent pattern of direct CP asymmetries within the present framework when one considers $\mathcal{A}_{CP}(\eta\pi^+)<0$ (favoring $\delta>0$) on the one hand, and both $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{CP}(\eta K^+)$ (favoring $\delta<0$) on the other hand. The measurement of a significant CP asymmetry for $B^+\to\eta'\pi^+$ would provide valuable additional information in this respect.

As we mentioned at the end of the preceding section, for a given final-state phase we expect the calculation of Ref. [55] to give an opposite sign to ours for $\mathcal{A}_{CP}(\eta K^+)$. The current experimental central value of $\mathcal{A}_{CP}(\eta K^+)$ (consistent with the range predicted in Ref. [55]) favors $\delta>0$ in accord with $\mathcal{A}_{CP}(\eta\pi^+)$ which also favors $\delta>0$ (Fig. 3). It is then $\mathcal{A}_{\pi\pi}$ which is "odd man out," favoring $\delta<0$.

VII. $|\Delta S|=1$ CHARGED B DECAYS AND THE RATIO s'/p'

Several relations among amplitudes were proposed in Refs. [12] and [5] (see also [61]). Notable among these was the quadrangle relation for B^+ decay amplitudes

$$A(\eta'K^+)=\sqrt{6}A(\pi^+K^0)+\sqrt{3}A(\pi^0K^+)-2\sqrt{2}A(\eta K^+). \tag{26}$$

We will show in the next section that this relation and a similar quadrangle relation among $\Delta S=0$ amplitudes follows from U-spin symmetry alone. A quadrangle construction was suggested for $|\Delta S|=1$ processes and their charge conjugates which permits the determination of the weak phase γ as long as the two quadrangles are not degenerate. In order for this to be the case, at least two of the three processes $B^+\to\eta'K^+$, $B^+\to\pi^0K^+$, and $B^+\to\eta K^+$ must have non-vanishing CP asymmetries. The CP asymmetry for $B^+\to\pi^+K^0$ must be very small if our assumption that this decay is dominated by the penguin amplitude is correct.

We shall discuss a relation between CP-violating rate differences which follows from the amplitude decompositions in Table IV:

$$A(\pi^+K^0)=p', \tag{27}$$

$$\sqrt{2}A(\pi^0K^+)=-(p'+t'+c'), \tag{28}$$

$$\sqrt{3}A(\eta K^+)=-(s'+t'+c'), \tag{29}$$

$$\sqrt{6}A(\eta'K^+)=3p'+4s'+t'+c'. \tag{30}$$

We shall assume that the amplitudes p' and s' have the same weak phase but not necessarily the same strong phase (in contrast to the simplified case assumed in previous sections). The amplitude $t'+c'$ has a weak phase γ associated with its $T'+C'$ piece, and an electroweak penguin piece with the same weak phase as p' and s'. Now let us define CP-violating rate asymmetries $\Delta(f)\equiv\Gamma(\overline{f})-\Gamma(f)$. These may be calculated by taking the difference between the absolute squares of the amplitudes defined above and those for their charge-conjugate processes. Under the above assumptions about weak phases, we predict $\Delta(\pi^+K^0)=0$ [which is satisfied since $\mathcal{A}_{CP}(\pi^+K^0)=-0.032\pm0.066$] and

$$\Delta(\pi^0K^+)+2\Delta(\eta K^+)=\Delta(\eta'K^+). \tag{31}$$

This may be written in terms of observable quantities as

$$\mathcal{A}_{CP}(\pi^0K^+)\overline{\mathcal{B}}(\pi^0K^+)+2\mathcal{A}_{CP}(\eta K^+)\overline{\mathcal{B}}(\eta K^+)$$
$$=\mathcal{A}_{CP}(\eta'K^+)\overline{\mathcal{B}}(\eta'K^+). \tag{32}$$

The individual terms in this equation (in units of 10^{-6}) read

$$(0.4\pm0.9)+(-2.0\pm1.3)=-0.2\pm3.1; \tag{33}$$

the sum on the left-hand side is -1.6 ± 1.6. The sum rule is satisfied, but at least two terms in it must be individually non-vanishing to permit the quadrangle construction of Ref. [12]. It does not make sense to attempt such a construction with the present central values of the CP asymmetries since they do not satisfy the sum rule exactly.

A related sum rule can be written for the rate asymmetries in $B\to\pi K$ decays. Using similar methods, we find

$$\Delta(\pi^0K^0)=\frac{1}{2}\Delta(\pi^-K^+)-\Delta(\pi^0K^+). \tag{34}$$

This may be written as a prediction

$$\mathcal{A}_{CP}(\pi^0K^0)=[\overline{\mathcal{B}}(\pi^0K^0)]^{-1}\left[\frac{1}{2}\mathcal{A}_{CP}(\pi^-K^+)\overline{\mathcal{B}}(\pi^-K^+)\right.$$
$$\left.-\frac{\tau_0}{\tau_+}\mathcal{A}_{CP}(\pi^0K^+)\overline{\mathcal{B}}(\pi^0K^+)\right]$$

$$= -0.11 \pm 0.08. \quad (35)$$

The most general check of our assumption that p' and s' have the same strong phases (made in extracting the minimal value of $|s'|$ which would reproduce the large $B \to \eta' K$ branching ratios) would rely on the quadrangle construction of Ref. [12], which utilizes the rates for the processes in Eqs. (27)–(30) and their charge conjugates. As noted, in order to be able to perform this construction, one must have quadrangles for processes and their charge conjugates which are of different shapes, and thus (by virtue of the sum rule for rate differences) at least two of the decays $B^+ \to \pi^0 K^+$, $B^+ \to \eta K^+$, and $B^+ \to \eta' K^+$ must have non-zero *CP* asymmetries. Independently of whether such asymmetries exist, one can still check the consistency of taking $s' = \mu p'$ (where μ is a real constant) by noting that under this assumption one has

$$|A(\pi^+ K^0)|^2(1+\mu)(1+2\mu)(1-\mu) + |A(\pi^0 K^+)|^2(1+\mu) - |A(\eta K^+)|^2(1+2\mu) - |A(\eta' K^+)|^2(1-\mu) = 0. \quad (36)$$

Using the amplitudes quoted in Table IV, one obtains the three roots $\mu = (-2.21,\ 0.47,\ 1.24)$ to this cubic equation. The value of $|t'+c'|^2$ is a function of μ and squares of amplitudes. Other ways of writing $|t'+c'|^2$ give equivalent results,

$$|t'+c'|^2 = |A(\pi^+ K^0)|^2(1+4\mu+2\mu^2) + |A(\pi^0 K^+)|^2 + 2|A(\eta K^+)|^2 - |A(\eta' K^+)|^2. \quad (37)$$

The negative μ root gives negative $|t'+c'|^2$, while $\mu = 1.24$ gives much too large a value in comparison with the $|T'+C'|$ amplitude in Table V. The root $\mu = 0.47 \pm 0.05$ is not far from the ratio $|s'/p'| = 0.41 \pm 0.05$ implied by the values in Table V. It implies a value of $|t'+c'| = (21.6 \pm 10.1) \times 10^{-9}$ GeV, still somewhat large in comparison with the value $|T'+C'| = (9.53 \pm 0.70) \times 10^{-9}$ GeV but consistent with it given the large error and the uncertain relative phase between $e^{i\gamma}$ and δ_{EW}.

The error in the determination of $|t'+c'|$ using the above method is dominated by that (22%) in $\bar{\mathcal{B}}(\eta K^0)$ (to be compared with 6–8 % in the other three branching ratios). To see the effect of a change in $\bar{\mathcal{B}}(\eta K^+)$, let us imagine that it is instead slightly below its present 1σ limit, or $(2.39 \pm 0.52) \times 10^{-6}$. We then find $|t'+c'| = (9.4 \pm 19.2) \times 10^{-9}$ GeV. Alternatively, if $\bar{\mathcal{B}}(\eta K^+)$ retains its present central value but its error is decreased by a factor of 3 while the errors in the other branching ratios remain the same, we find $|t'+c'| = (21.6 \pm 6.5) \times 10^{-9}$ GeV.

If the error in $|t'+c'|$ as determined by the above method decreases to the point that an inconsistency with Table V develops, we would be led to question at least one of the assumptions that (a) η and η' are the specific octet-singlet mixtures assumed here, and (b) the strong phases of s' and p' are equal.

With improved knowledge of branching ratios and amplitudes one could extract a relative strong phase between s' and p' from data. In this approach, instead of extracting $|t'+c'|$ from $|\Delta S| = 1$ B^+ decays as in the above example, one would determine $|T'+C'|$ from $\Delta S = 0$ transitions. One also needs the relative size of the electroweak penguin δ_{EW}, the magnitude of p' based on the $B^+ \to K^0 \pi^+$ decay rate, and the measured *CP*-averaged branching ratios for $B^+ \to (\pi^0, \eta, \eta') K^+$. With these, one can solve for the magnitude and relative strong phase of s'/p' and the strong phase δ_T between the $T'+C'$ and p' amplitudes.

Let $s' = \mu p' e^{i\delta_S}$ with $\mu > 0$ here and note that the weak phase of the p' amplitude is π (as in Sec. V). Then Eqs. (27)–(30) may be rewritten as

$$A(\pi^+ K^0) = -|p'|, \quad (38)$$

$$\sqrt{2} A(\pi^0 K^+) = -[|T'+C'|(e^{i\gamma} - \delta_{EW}) e^{i\delta_T} - |p'|], \quad (39)$$

$$\sqrt{3} A(\eta K^+) = -[|T'+C'|(e^{i\gamma} - \delta_{EW}) e^{i\delta_T} - \mu |p'| e^{i\delta_S}], \quad (40)$$

$$\sqrt{6} A(\eta' K^+) = |T'+C'|(e^{i\gamma} - \delta_{EW}) e^{i\delta_T} - 3|p'| - 4\mu |p'| e^{i\delta_S}. \quad (41)$$

The first equation determines $|p'|$. The remaining three then determine μ, δ_S, and δ_T as functions of γ.

Taking the central values of the input parameters noted in the previous paragraph, including $|p'| = 45.7 \times 10^{-9}$ GeV, $|T'+C'| = 9.7 \times 10^{-9}$ GeV and $\delta_{EW} = 0.65$, we find that γ has to be greater than 88° in order to have solutions for μ, δ_S and δ_T. This feature arises from the need to reproduce the branching ratio for $B^+ \to \pi^0 K^+$ which is slightly higher than expected on the basis of penguin dominance. One then needs maximal constructive interference between the $|T'+C'|(e^{i\gamma} - \delta_{EW})$ and $-|p'|$ terms in the $B^+ \to \pi^0 K^+$ amplitude, which forces γ toward larger values. This is the basis of bounds originally presented in Ref. [52].

To exhibit a less restrictive set of solutions, we take $\delta_{EW} = 0.80$ and the 95% C.L. lower bound on the $B^+ \to \pi^0 K^+$ branching ratio, $\bar{\mathcal{B}} \geq 10.7 \times 10^{-6}$. The minimum value of γ permitting a solution is 51.9°. This is to be compared with the result [62] based on consideration of all possible errors on the ratio $2\bar{\mathcal{B}}(B^+ \to \pi^0 K^+)/\bar{\mathcal{B}}(B^+ \to \pi^+ K^0) = 1.24 \pm 0.13$: $\gamma \gtrsim 52°$ at the 1σ level, and no lower bound at 95% C.L.

In Fig. 6 we show solutions for μ, δ_S and δ_T in the range $50° \leq \gamma \leq 90°$. As γ increases from its minimum, one obtains two branches of solutions for δ_T differing by a sign. Furthermore, there are two sets of possible δ_S values for each sign of δ_T, one set with larger absolute values forming a branch that corresponds to larger values of μ while the other with smaller absolute values forming the other branch that corresponds to smaller values of μ. For a given μ, $\delta_T \to -\delta_T$ corresponds to $\delta_S \to -\delta_S$.

With $\delta_{EW} = 0.80$, $\bar{\mathcal{B}} \geq 10.7 \times 10^{-6}$, and the central values of other input parameters, we find that the *CP* asymmetry of the $\pi^0 K^+$ mode is predicted to be zero at the minimal value of $\gamma \simeq 51.9°$, since the relative strong phase δ_T vanishes at that point. The *CP* asymmetries of the ηK^+ and $\eta' K^+$

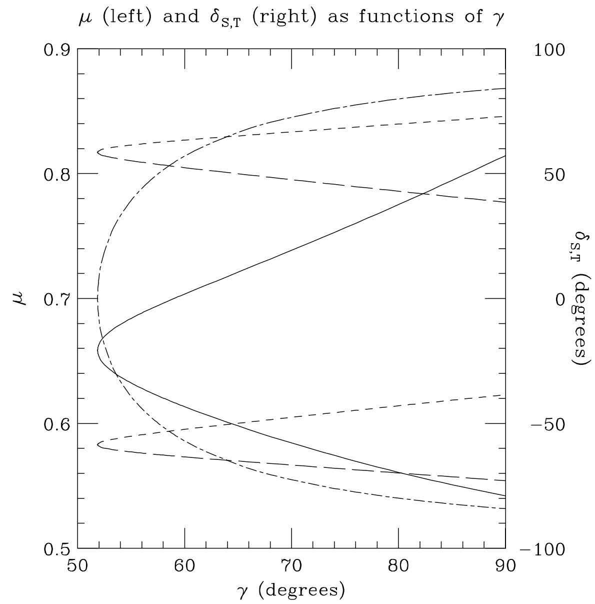

FIG. 6. Extracted values of $\mu \equiv |s'/p'|$ (solid) and strong phases δ_S (dashed) and δ_T (dash-dotted) as functions of the weak phase γ. Positive (negative) values of δ_T are plotted with long (short) dash-dotted curves, with the corresponding δ_S values using long (short) dashed curves. For either sign of δ_T, the branch of δ_S with larger (smaller) absolute values corresponds to the upper (lower) branch of μ. Branches are joined at the point with minimum γ. Here $\overline{\mathcal{B}}(B^+ \to \pi^0 K^+) = 10.7 \times 10^{-6}$ and $\delta_{EW} = 0.80$.

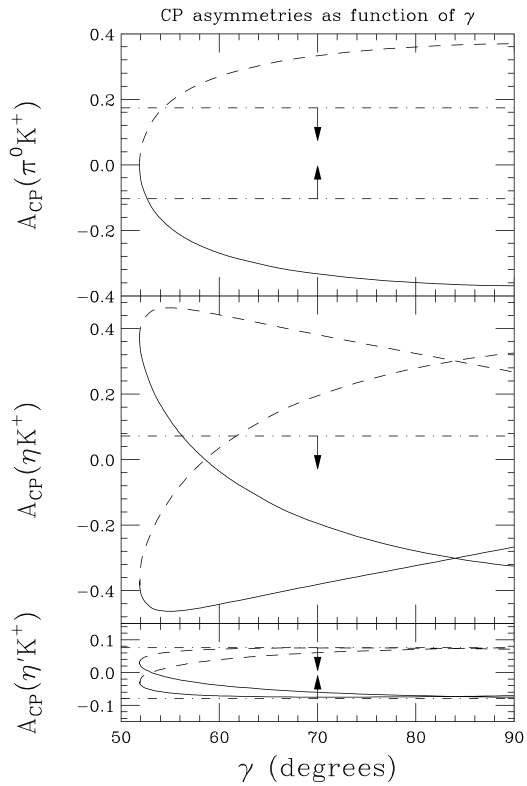

FIG. 7. Predicted *CP* asymmetries for the $\pi^0 K^+$, ηK^+ and $\eta' K^+$ modes. For all three plots, solid (dashed) curves correspond to the long dash-dotted positive (short dash-dotted negative) δ_T branch in Fig. 6. The outer curves at low γ's correspond to the branches of larger $|\delta_S|$ and larger μ. The corresponding 95% C.L. bounds are also drawn in dash-dotted lines. [The lower bound for $\mathcal{A}_{CP}(\eta K^+)$ is outside the plotting range.] Here $\overline{\mathcal{B}}(B^+ \to \pi^0 K^+) = 10.7 \times 10^{-6}$ and $\delta_{EW} = 0.80$.

modes at the same value of γ, however, are predicted to be ± 0.37 and ± 0.03, respectively. The set of negative *CP* asymmetries (corresponding to positive δ_T) is consistent with the current data as given in Table IV. We plot the *CP* asymmetries as functions of γ in Fig. 7. While the measured *CP* asymmetry of the $\pi^0 K^+$ mode [26,33] gives the strongest bound, $\gamma < 55°$, given the above-mentioned input conditions, this conclusion depends strongly on the assumed branching ratios, particularly of the $\pi^0 K^+$ and ηK^+ modes. In any case it is clear that a solution is possible in principle for both the relative magnitude and the relative phase of the singlet penguin and ordinary penguin amplitude, given sufficiently reliable data.

VIII. U-SPIN RELATIONS AMONG ALL CHARGED *B* DECAYS

While in previous sections we have employed the complete flavor $SU(3)$ symmetry group, neglecting small annihilation-type amplitudes, we will rely in the present section only on U-spin [17,18], an important subgroup of $SU(3)$. We will show that the eight charged *B* decay amplitudes in Tables III and IV, for both $|\Delta S| = 1$ and $\Delta S = 0$ transitions, are given in terms of two triplets of U-spin amplitudes describing penguin and tree contributions. This implies several relations among these amplitudes, including Eq. (26), and a similar quadrangle relation among $\Delta S = 0$ amplitudes. Relations will also be derived among penguin amplitudes in strangeness changing and strangeness conserving decays, and among tree amplitudes in these decays. Such relations may constrain tree amplitudes in $|\Delta S| = 1$ decays and penguin amplitudes in $\Delta S = 0$ decays. Values calculated for these contributions in previous sections, where stronger assumptions than U-spin were made, must obey these constraints.

The U-spin subgroup of $SU(3)$ is the same as the I-spin (isospin) except that the doublets with $U = 1/2, U_3 = \pm 1/2$ are for quarks

$$\begin{bmatrix} \left|\frac{1}{2} \quad \frac{1}{2}\right\rangle \\ \left|\frac{1}{2} - \frac{1}{2}\right\rangle \end{bmatrix} = \begin{bmatrix} |d\rangle \\ |s\rangle \end{bmatrix}, \tag{42}$$

and for antiquarks

$$\begin{bmatrix} \left|\frac{1}{2} \quad \frac{1}{2}\right\rangle \\ \left|\frac{1}{2} - \frac{1}{2}\right\rangle \end{bmatrix} = \begin{bmatrix} |\overline{s}\rangle \\ -|\overline{d}\rangle \end{bmatrix}. \tag{43}$$

The charged *B* is a U-spin singlet, while the charged kaon and pion belong to a U-spin doublet,

$$|0\quad 0\rangle = |B^+\rangle = |u\bar{b}\rangle, \tag{44}$$

$$\begin{bmatrix} \left|\frac{1}{2}\quad \frac{1}{2}\right\rangle \\ \left|\frac{1}{2} - \frac{1}{2}\right\rangle \end{bmatrix} = \begin{bmatrix} |u\bar{s}\rangle = |K^+\rangle \\ -|u\bar{d}\rangle = -|\pi^+\rangle \end{bmatrix}. \tag{45}$$

Nonstrange neutral mesons belong either to a U-spin triplet or a U-spin singlet. The U-spin triplet residing in the pseudoscalar meson octet is

$$\begin{bmatrix} |1\quad 1\rangle \\ |1\quad 0\rangle \\ |1\quad -1\rangle \end{bmatrix} = \begin{bmatrix} |K^0\rangle = |d\bar{s}\rangle \\ \frac{\sqrt{3}}{2}|\eta_8\rangle - \frac{1}{2}|\pi^0\rangle = \frac{1}{\sqrt{2}}|s\bar{s} - d\bar{d}\rangle \\ -|\overline{K^0}\rangle = -|s\bar{d}\rangle \end{bmatrix}, \tag{46}$$

and the corresponding singlet is

$$|0\quad 0\rangle = \frac{1}{2}|\eta_8\rangle + \frac{\sqrt{3}}{2}|\pi^0\rangle = \frac{1}{\sqrt{6}}|s\bar{s} + d\bar{d} - 2u\bar{u}\rangle. \tag{47}$$

In addition, the η_1 is, of course, a U-spin singlet. We take $\eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$. We shall also use $\eta_1 \equiv (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, and recall our definition $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$.

The $\Delta C = 0$, $\Delta S = 1$ effective Hamiltonian transforms like a $\bar{s}$ component ($\Delta U_3 = \frac{1}{2}$) of a U-spin doublet, while the $\Delta C = 0, \Delta S = 0$ Hamiltonian transforms like a $-\bar{d}$ component ($\Delta U_3 = -\frac{1}{2}$) of another U-spin doublet. Furthermore, one may decompose the two Hamiltonians into members of the same two U-spin doublets multiplying given CKM factors. For practical purposes, it is convenient to use a convention in which the CKM factors involve the u and c quarks, rather than the u and t quarks [18],

$$\mathcal{H}_{\rm eff}^{\bar{b} \to \bar{s}} = V_{ub}^* V_{us} O_s^u + V_{cb}^* V_{cs} O_s^c, \tag{48}$$

$$\mathcal{H}_{\rm eff}^{\bar{b} \to \bar{d}} = V_{ub}^* V_{ud} O_d^u + V_{cb}^* V_{cd} O_d^c. \tag{49}$$

Here $O_{d,s}^u$ and $O_{d,s}^c$ are two U-spin doublet operators, which for simplicity of nomenclature will be called tree and penguin operators.

Since the initial B^+ meson is a U-spin singlet, the final states are U-spin doublets, which can be formed in three different ways from the two U-spin singlets and the U-spin triplet, each multiplying the U-spin doublet meson states. Consequently, the eight decay processes can be expressed in terms of three U-spin reduced matrix elements of the tree operator and three U-spin penguin amplitudes. Amplitudes corresponding to the U-spin singlet and triplet in the octet and the $SU(3)$ singlet, will be denoted by A_0^u, A_1^u and B_0^u, respectively, for tree amplitudes and A_0^c, A_1^c and B_0^c for penguin amplitudes. Complete $\Delta S = 1$ and $\Delta S = 0$ amplitudes for U-spin final states made of two octets are given by

$$A_{0,1}^s = V_{ub}^* V_{us} A_{0,1}^u + V_{cb}^* V_{cs} A_{0,1}^c, \tag{50}$$

$$A_{0,1}^d = V_{ub}^* V_{ud} A_{0,1}^u + V_{cb}^* V_{cd} A_{0,1}^c. \tag{51}$$

Similar expressions for B_0^s and B_0^d describe decays to final states involving η_1.

Absorbing a factor 1/2 in the definition of $A_{0,1}^{u,c}$ one finds

$$A(\eta_1 K^+) = B_0^s, \quad A(K^0\pi^+) = -\frac{4}{\sqrt{6}}A_1^s, \tag{52}$$

$$A(\eta_8 K^+) = A_0^s - A_1^s, \tag{53}$$

$$A(\pi^0 K^+) = \sqrt{3}A_0^s + \frac{1}{\sqrt{3}}A_1^s, \tag{54}$$

$$A(\eta_1 \pi^+) = B_0^d, \quad A(\overline{K^0}K^+) = -\frac{4}{\sqrt{6}}A_1^d, \tag{55}$$

$$A(\eta_8 \pi^+) = A_0^d + A_1^d, \tag{56}$$

$$A(\pi^0 \pi^+) = \sqrt{3}A_0^d - \frac{1}{\sqrt{3}}A_1^d. \tag{57}$$

The physical η and η' states are mixtures of the octet and singlet. In our convention if we write

$$\eta = c_\theta \eta_8 - s_\theta \eta_1, \quad \eta' = c_\theta \eta_1 + s_\theta \eta_8, \tag{58}$$

the states defined in Sec. II correspond to $c_\theta \equiv \cos\theta = 2\sqrt{2}/3$, $s_\theta \equiv \sin\theta = 1/3$, $\theta = 19.5°$. Since the four physical $|\Delta S| = 1$ amplitudes are expressed in terms of three U-spin amplitudes, B_0^s, A_0^s and A_1^s, they obey one linear relation given by Eq. (26). Thus, this relation follows purely from U-spin and does not require further approximations. A similar U-spin quadrangle relation holds for $\Delta S = 0$ amplitudes,

$$A(\eta'\pi^+) = -\sqrt{6}A(K^+\overline{K^0}) + \sqrt{3}A(\pi^0\pi^+) - 2\sqrt{2}A(\eta\pi^+). \tag{59}$$

Combining Eqs. (50)–(51) and Eqs. (52)–(58), one may relate penguin amplitudes A^c (or tree amplitudes A^u) in $\Delta S = 0$ and $|\Delta S| = 1$ decays. One finds

$$A^c(\eta'\pi^+) = A^c(\eta' K^+) - \frac{1}{\sqrt{6}}A^c(K^0\pi^+), \tag{60}$$

$$A^c(\eta\pi^+) = A^c(\eta K^+) - \frac{2}{\sqrt{3}}A^c(K^0\pi^+), \tag{61}$$

$$A^u(\eta' K^+) = A^u(\eta'\pi^+) + \frac{1}{\sqrt{6}}A^u(\overline{K^0}K^+), \tag{62}$$

$$A^u(\eta K^+) = A^u(\eta\pi^+) + \frac{2}{\sqrt{3}} A^u(\overline{K}^0 K^+). \quad (63)$$

We note that, because of the different conventions used here and in Tables III and IV, the amplitudes A^c and A^u do not correspond *exactly* to penguin and tree amplitudes in the tables. It is straightforward to translate amplitudes in one convention to the other convention [45].

Let us focus our attention first on Eq. (60). The two penguin contributions on the right-hand side dominate the corresponding measured amplitudes. Therefore, the complex triangle relation implies a lower bound, at 90% confidence level, on the penguin contribution to $B^+ \to \eta' \pi^+$ in terms of measured amplitudes,

$$|V_{cb}^* V_{cd} A^c(\eta'\pi^+)| \geq \frac{|V_{cd}|}{|V_{cs}|}\left[|A(\eta' K^+)| - \frac{1}{\sqrt{6}}|A(K^0\pi^+)|\right] > 16.1\times 10^{-9}\ \text{GeV}. \quad (64)$$

This should be compared with the tree contribution to this process,

$$|V_{ub}^* V_{ud} A^u(\eta'\pi^+)| \approx \frac{1}{\sqrt{3}}|A(\pi^+\pi^0)| = 13.6\times 10^{-9}\ \text{GeV}. \quad (65)$$

We conclude that *U-spin symmetry alone implies that the penguin contribution in $B^+ \to \eta'\pi^+$ is at least comparable in magnitude to the tree amplitude of this process*. This confirms our more detailed estimate in Sec. VI. (The small differences between the lower bound and this estimate follow from the different convention used and from the small non-penguin contribution in $B^+ \to \eta' K^+$.)

Equation (61) implies a somewhat weaker lower bound on the penguin contribution in $B^+ \to \eta\pi^+$,

$$|V_{cb}^* V_{cd} A^c(\eta\pi^+)| \geq \frac{|V_{cd}|}{|V_{cs}|}\left[-|A(\eta K^+)| + \frac{2}{\sqrt{3}}|A(K^0\pi^+)|\right] > 7.1\times 10^{-9}\ \text{GeV}. \quad (66)$$

Namely, the penguin amplitude in $B^+ \to \eta\pi^+$ is at least 37% of the tree contribution to this process. This bound is weakened somewhat by using the complete amplitude of $B^+ \to \eta K^+$, which contains a sizable tree amplitude.

Equations (62) and (63) may be used in order to obtain upper bounds on tree contributions with weak phase γ in $B^+ \to \eta' K^+$ and $B^+ \to \eta' K^+$. Assuming that the physical amplitudes of $B^+ \to \eta\pi^+$ and $B^+ \to \overline{K}^0 K^+$ are not smaller than the corresponding tree amplitudes, one finds

TABLE VI. Predicted branching ratios for some as-yet-unseen modes and present 90% C.L. upper limits in units of 10^{-6}.

	Decay mode	Predicted: This work	Predicted: Ref. [16]	Upper limit
B^+	$\to \pi^+\eta'$	2.7 ± 0.7 [a]	$16.8^{+16.0}_{-9.7}$	7.0[b] [34]
	$\to K^+\overline{K}^0$	0.75 ± 0.11	$0.8^{+0.4}_{-0.2}$	1.3 [27]
B^0	$\to \pi^0\pi^0$	0.4 to 1.6 [c]	$1.9^{+0.8}_{-0.7}$	3.6[b] [26]
	$\to \pi^0\eta$	0.69 ± 0.10	$1.2^{+0.6}_{-0.4}$	2.9 [25]
	$\to \pi^0\eta'$	0.77 ± 0.11	$7.8^{+3.8}_{-4.3}$	5.7 [25]
	$\to K^0\overline{K}^0$	0.70 ± 0.10	$0.7^{+0.4}_{-0.2}$	2.4 [30]
	$\to \eta\eta$	0.3 to 1.1 [c]	$3.1^{+1.3}_{-1.1}$	18 [39]
	$\to \eta\eta'$	0.6 to 1.7 [c]	$7.6^{+5.3}_{-3.4}$	27 [39]
	$\to \eta'\eta'$	0.3 to 0.6 [c]	$5.4^{+4.5}_{-3.1}$	47 [39]
	$\to \eta K^0$	1.03 ± 0.24	$2.4^{+0.5}_{-0.6}$	4.6 [7]

[a]Predicted $\mathcal{A}_{CP} = -0.57\pm 0.23$.
[b]This mode has now been detected; see text.
[c]Lower value from the central value of penguin amplitudes alone; upper value with constructive c–penguin interference and maximal $|c|$, $|p|$, and $|s|$ (1σ).

$$|V_{ub}^* V_{us} A^u(\eta' K^+)| \leq \frac{f_K}{f_\pi}\frac{|V_{us}|}{|V_{ud}|}\left[|A(\eta'\pi^+)| + \frac{1}{\sqrt{6}}|A(\overline{K}^0 K^+)|\right] < 9.1\times 10^{-9}\ \text{GeV}, \quad (67)$$

$$|V_{ub}^* V_{us} A^u(\eta K^+)| \leq \frac{f_K}{f_\pi}\frac{|V_{us}|}{|V_{ud}|}\left[|A(\eta\pi^+)| + \frac{2}{\sqrt{3}}|A(\overline{K}^0 K^+)|\right] < 10.5\times 10^{-9}\ \text{GeV}. \quad (68)$$

These upper bounds imply that the tree contributions to $B^+ \to \eta' K^+$ and $B^+ \to \eta K^+$ are less than 10% and 66% of the total amplitudes of these processes, respectively. The bounds become 7% and 42% if one neglects the small annihilation amplitude in $B^+ \to \overline{K}^0 K^+$.

IX. MODES TO BE SEEN

We summarize predicted branching ratios for some as-yet-unseen decay modes in Table VI. We have already discussed the $B^+ \to \pi^+\eta'$ mode in Sec. VI. The spread in the predicted charge-averaged branching ratio, $\overline{\mathcal{B}}(\pi^+\eta') = (2.7\pm 0.7)\times 10^{-6}$, reflects that shown in Fig. 2. (See note at end of Ref. [28].) The predicted *CP* asymmetry is large: $\mathcal{A}_{CP}(\pi^+\eta') = -0.57\pm 0.23$. By contrast, Ref. [16] finds $\overline{\mathcal{B}}(\pi^+\eta') = (16.8^{+16.0}_{-9.7})\times 10^{-6}$ and $\mathcal{A}_{CP}(\pi^+\eta') = -0.18^{+0.15}_{-0.09}$.

The prediction $\overline{\mathcal{B}}(B^0 \to \eta K^0) = (1.03\pm 0.24)\times 10^{-6}$ (discussed in Sec. IV) is given for the s' contribution alone. It is a factor of about 2 below the value of $(2.4^{+0.5}_{-0.6})\times 10^{-6}$ found in Ref. [16].

Using approximate $SU(3)_F$ symmetry, the amplitudes of both $B^+\to K^+\bar{K}^0$ and $B^0\to K^0\bar{K}^0$ are the same (p) and related to the one extracted from the π^+K^0 mode. Thus, their branching ratios are expected to be $\sim 7.5\times 10^{-7}$ and $\sim 7.0\times 10^{-7}$, respectively. These are rather close to central values (quoted with rather large errors) in Ref. [16]. To observe these decay modes, the data sample should be enlarged by a factor of ~ 1.7 and ~ 3.4. These estimates do not include additional possible theoretical errors on p associated with the methods of Sec. IV.

The decay $B^0\to\pi^0\pi^0$ receives contributions from the p and c amplitudes: $A(B^0\to\pi^0\pi^0)=(p-c)/\sqrt{2}$. This amplitude is to be compared with $A(B^0\to\pi^+\pi^-)=-(t+p)$, in which Table III indicates that the tree and penguin amplitudes may be interfering destructively. Since one expects c/t to be mainly real and positive [46,47], one then expects either no interference or constructive interference between p and c in $B^0\to\pi^0\pi^0$. The p contribution alone gives a branching ratio of about 0.4×10^{-6}, while Table V indicates that the c contribution could be as large as p. If c and p then add constructively, one could have a branching ratio as large as 1.6×10^{-6}. This still lies below the present upper limit, by a little more than a factor of 2. A lower bound at the 90% C.L. of $\bar{\mathcal{B}}(B^0\to\pi^0\pi^0)\gtrsim 0.2\times 10^{-6}$ may be obtained using the observed $B^+\to\pi^+\pi^0$ and $B^0\to\pi^+\pi^-$ branching ratios and isospin alone [60]. For comparison, Ref. [16] predicts $\bar{\mathcal{B}}(B^0\to\pi^0\pi^0)=(1.9^{+0.8}_{-0.7})\times 10^{-6}$. [*Note added*. New measurements of this branching ratio are $(2.1\pm 0.6\pm 0.3)\times 10^{-6}$ [63] and $(1.7\pm 0.6\pm 0.3)\times 10^{-6}$ [64].]

Since the $\pi^0\eta^{(\prime)}$ modes involve linear combinations of p and s that are believed to have the same weak phase and no sizable relative strong phase, we predict their branching ratios to be $(0.69\pm 0.10)\times 10^{-6}$ and $(0.77\pm 0.11)\times 10^{-6}$. We therefore need about 4 and 7 times more data than the sample on which the upper limits in Table VI are based in order to see these decays. This should not be difficult since those limits were based on CLEO data alone [25]. The corresponding predictions of Ref. [16] are $\bar{\mathcal{B}}(B^0\to\pi^0\eta)=(1.2^{+0.6}_{-0.4})\times 10^{-6}$ (slightly above ours) and $\bar{\mathcal{B}}(B^0\to\pi^0\eta')=(7.8^{+3.8}_{-4.3})\times 10^{-6}$ (far above ours, with the upper values excluded by experiment).

X. SUMMARY

We have discussed implications of recent experimental data for B decays into two pseudoscalar mesons, with emphasis on those with η and η' in the final states. We present a preferred set of amplitude magnitudes in Tables III and IV, where quantities are either extracted directly from data or related to one another by appropriate CKM and $SU(3)_F$ breaking factors. In particular, we make the assumption that the singlet penguin amplitude and the QCD penguin in $|\Delta S|=1$ transitions have the same strong phase in the tables. We show that this assumption is consistent with current measurements of the branching ratios and *CP* asymmetries of the charged B meson decays. We also study the consequences of relaxing this assumption but assuming that electroweak penguin contributions and branching ratios are sufficiently well known.

We have extracted relative weak and strong phases between the tree-level amplitudes and penguin-loop amplitudes in the $B^\pm\to\eta\pi^\pm$ modes, and shown how improved data will lead to stronger constraints. Remarkably, branching ratio data can be at least as useful as *CP* asymmetries in this regard. We use U-spin alone to argue for a large penguin contribution in $B^+\to\eta'\pi^+$, and we predict a range of values for the branching ratio and *CP* asymmetry of this decay. In particular, we predict $\bar{\mathcal{B}}(B^+\to\eta'\pi^+)=(2.7\pm 0.7)\times 10^{-6}$ for the charge-averaged branching ratio and, as a consequence of the apparent large *CP* asymmetry in $B^+\to\eta\pi^+$, an even larger *CP* asymmetry of $\mathcal{A}_{CP}(B^+\to\eta'\pi^+)=-0.57\pm 0.23$. We show that the present sign of the direct *CP* asymmetry in $B^+\to\eta\pi^+$ conflicts with that in $B^0\to\pi^+\pi^-$ and, assuming flavor $SU(3)$, with that in $B^+\to\eta K^+$. (The *CP* asymmetry in $B^+\to\eta K^+$ predicted by Ref. [55] would be opposite in sign for the same final-state phase, agreeing with that in $B^+\to\eta\pi^+$ and disagreeing with the direct asymmetry in $B^0\to\pi^+\pi^-$.) Since none of these asymmetries has yet been established at the 3σ level, there is not cause for immediate concern, but it would be interesting to see whether any other explicit calculations (e.g., those of Ref. [46,55]) for $B^+\to\eta\pi^+$ can reproduce such a pattern.

Using $SU(3)$ flavor symmetry, we also have estimated the required data samples to detect modes that have not yet been seen. The one closest to being observed is $B^+\to K^+\bar{K}^0$, which should be visible with about twice the present number of observed B decays.

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APPENDIX: NONPENGUIN CONTRIBUTIONS IN $B^0\to\eta' K_S$

The angle β can be measured through several different B decay modes in addition to the "golden" $B^0\to J/\psi K_S$ channel and others involving the $\bar{b}\to\bar{c}c\bar{s}$ subprocess. The large branching ratio for $B^0\to\eta' K_S$ makes this mode particularly appealing; it is dominated in our approach by the $\bar{b}\to\bar{s}$ penguin amplitude p' and the flavor-singlet penguin amplitude s'. Within the standard model there are several other possible contributions to this decay, including c' in our treatment and smaller amplitudes (e',pa') which we neglect.

An estimate was performed [19] with terms which could alter the effective value of β extracted from the *CP*-violating asymmetry parameter $\mathcal{S}_{\eta' K_S}$. While the full machinery of flavor $SU(3)$ was used, we shall demonstrate that the U-spin subgroup employed in Sec. VIII suffices. We derive a linear relation among decay amplitudes differing from that in Ref. [19], who neglected subtleties of symmetrization in dealing with identical particles in an S-wave final state. When these are taken into account, the amplitudes listed in Tables III and IV satisfy the corrected linear relation. Finally, we estimate the corrections due to non-(p',s') terms within our framework, finding them to be much less important than in the more general treatment of Ref. [19]. In this respect we are much closer to the earlier approach of London and Soni [65], who concluded that such correction terms were insignificant.

Since we are considering S-wave decays of B to two spinless final particle, one must symmetrize the two-particle U-spin states. The amplitudes in Tables III and IV are defined in such a way that their squares times appropriate kinematic factors always give partial widths. For identical particles, amplitudes satisfying Clebsch-Gordan relations are defined with factors of $1/\sqrt{2}$ with respect to those in Tables III and IV. We then reproduce results of Ref. [19] using U-spin. The first relation, written for our notation and phase convention, is

$$A^{u,c}(\eta_1 K^0) = \frac{1}{\sqrt{2}} A^{u,c}(\eta_1 \pi_0) - \sqrt{\frac{3}{2}} A^{u,c}(\eta_1 \eta_8), \quad \text{(A1)}$$

which refers to a single $U=1$ amplitude. In the combination of π^0 and η_8 on the right-hand side, the $U=0$ pieces cancel. The η_1 is of course a U-spin singlet.

The final state in $B^0 \to \eta_8 K^0$ involves $U=U_3=1$, since both $|B^0\rangle$ and the weak $|\Delta S|=1$ Hamiltonian transform as $|\frac{1}{2}\,\frac{1}{2}\rangle$. The final states in $\Delta S=0$ B^0 decays, on the other hand, involve several possible U-spin combinations. The total U-spin can be either 0 or 1. There are two ways of getting $U=0$: Each of the final mesons can have either $U=0$ or $U=1$. There is only one way of getting $U=1$: One final meson must have $U=0$ and the other must have $U=1$. This follows from the symmetry of the final state; two $|1\ 0\rangle$ states cannot make a $|1\ 0\rangle$ state. Thus there are three invariant amplitudes describing four decays. The appropriate relation between them is

$$A^{u,c}(\eta_8 K^0) = \frac{1}{2}\sqrt{\frac{3}{2}}[A^{u,c}(\pi^0\pi^0) - A^{u,c}(\eta_8\eta_8)] - \frac{1}{2\sqrt{2}} A^{u,c}(\eta_8 \pi^0). \quad \text{(A2)}$$

Aside from some signs due to different conventions, this agrees with the result of Ref. [19].

The physical η and η' states are given in Eq. (58) in terms of the η-η' mixing angle θ. Using this general parametrization, and respecting the above-mentioned symmetrization rule, we find for $B^0 \to \eta' K^0$:

$$\begin{aligned} A^u(\eta' K^0) = {} & \frac{2c_\theta^2 - s_\theta^2}{2\sqrt{2}} A^u(\eta' \pi^0) - \frac{3 s_\theta c_\theta}{2\sqrt{2}} A^u(\eta \pi^0) \\ & + \frac{s_\theta}{2}\sqrt{\frac{3}{2}} A^u(\pi^0\pi^0) - \sqrt{\frac{3}{2}}\left(2 s_\theta c_\theta^2 \right. \\ & \left. + \frac{1}{2} s_\theta^3\right) A^u(\eta' \eta') + \frac{3}{2}\sqrt{\frac{3}{2}} s_\theta c_\theta^2 A^u(\eta\eta) \\ & + \sqrt{\frac{3}{2}}\left[c_\theta \left(\frac{1}{2} s_\theta^2 - c_\theta^2\right)\right] A^u(\eta\eta'). \quad \text{(A3)} \end{aligned}$$

Applying this relation in order to obtain an upper bound on the tree contribution in $B^0 \to \eta' K^0$ requires assuming that the amplitudes on the right-hand side dominate the corresponding processes. Using present upper bounds on the magnitudes of these amplitudes would have led to a rather weak bound, of about 40% of the measured amplitude of $B^0 \to \eta' K^0$. However, Table III shows that this assumption cannot be justified. On the other hand, estimating the C' contribution to the amplitude using the Clebsch-Gordan coefficients of Table IV and the range quoted in Table V, we find it to be less than 1%. It is clear that dynamical assumptions such as those made in Ref. [65] have considerable effects in limiting non-penguin contributions to the decay $B^0 \to \eta' K^0$.

[1] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **89**, 201802 (2002).

[2] Belle Collaboration, K. Abe *et al.*, Phys. Rev. D **66**, 071102(R) (2002).

[3] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[4] S. Barshay, D. Rein, and L.M. Sehgal, Phys. Lett. B **259**, 475 (1991).

[5] A.S. Dighe, M. Gronau, and J.L. Rosner, Phys. Lett. B **367**, 357 (1996); **377**, 325(E) (1996); Phys. Rev. Lett. **79**, 4333 (1997).

[6] M.R. Ahmady and E. Kou, Phys. Rev. D **59**, 054014 (1999).

[7] BaBar Collaboration, B. Aubert *et al.*, presented at XXXVIII Rencontres de Moriond on QCD and High Energy Hadronic Interactions, Les Arcs, France, 2003, SLAC Report No. SLAC-PUB-9962, hep-ex/0303039.

[8] D. Zeppenfeld, Z. Phys. **8**, 77 (1981).

[9] M. Savage and M. Wise, Phys. Rev. D **39**, 3346 (1989); **40**, 3127(E) (1989).

[10] L.L. Chau *et al.*, Phys. Rev. D **43**, 2176 (1991).

[11] M. Gronau, O.F. Hernandez, D. London, and J.L. Rosner, Phys. Rev. D **50**, 4529 (1994).

[12] M. Gronau and J.L. Rosner, Phys. Rev. D **53**, 2516 (1996).

[13] B. Grinstein and R.F. Lebed, Phys. Rev. D **53**, 6344 (1996).

[14] M. Gronau and J.L. Rosner, Phys. Rev. D **61**, 073008 (2000).

[15] C.W. Chiang and J.L. Rosner, Phys. Rev. D **65**, 074035 (2002).

[16] H.K. Fu, X.G. He, and Y.K. Hsiao, hep-ph/0304242.

[17] S. Meshkov, C.A. Levinson, and H.J. Lipkin, Phys. Rev. Lett. **10**, 361 (1963).

[18] M. Gronau, Phys. Lett. B **492**, 297 (2000). For recent applications of U-spin to multibody B decays see M. Gronau and J.L. Rosner, *ibid.* **564**, 90 (2003), and Ref. [19].

[19] Y. Grossman, Z. Ligeti, Y. Nir, and H. Quinn, Phys. Rev. D **68**, 015004 (2003).

[20] M. Gronau, O.F. Hernandez, D. London, and J.L. Rosner, Phys. Rev. D **52**, 6374 (1995).
[21] M. Gronau, Phys. Rev. D **62**, 014031 (2000).
[22] M. Gronau and J.L. Rosner, Phys. Rev. D **65**, 013004 (2002).
[23] LEP *B* Oscillation Working Group, presented at XXXVIII Rencontres de Moriond, ElectroWeak Interactions and Unified Theories, Les Arcs, France, 2003.
[24] CLEO Collaboration, A. Bornheim *et al.*, Phys. Rev. D **68**, 052002 (2003).
[25] CLEO Collaboration, S.J. Richichi *et al.*, Phys. Rev. Lett. **85**, 520 (2000).
[26] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **91**, 021801 (2003).
[27] BABAR Collaboration, B. Aubert *et al.*, presented at Flavor Physics and *CP* Violation (FPCP), Philadelphia, 2002, SLAC Report No. SLAC-PUB-9270, hep-ex/0206053.
[28] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **87**, 221802 (2001). An updated result is $\mathcal{B}(B^+\to\eta'\pi^+) = (2.8^{+1.3}_{-1.0}\pm 0.3)\times 10^{-6}$ (3.4σ), or $<4.5\times10^{-6}$ (90% C.L.); B. Aubert *et al.*, SLAC Report No. SLAC-PUB-10099, hep-ex/0308015.
[29] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **89**, 281802 (2002).
[30] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **87**, 151802 (2001).
[31] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. (to be published), SLAC Report No. SLAC-PUB-9698, hep-ex/0303046.
[32] BABAR Collaboration, B. Aubert *et al.*, SLAC Report No. SLAC-PUB-9304, hep-ex/0207065.
[33] T. Tomura, presented at XXXVIII Rencontres de Moriond on Electroweak Interactions and Unified Theories, Les Arcs, France, 2003, hep-ex/0305036.
[34] Belle Collaboration, K. Abe *et al.*, Phys. Lett. B **517**, 309 (2001).
[35] Belle Collaboration, K. Abe *et al.*, Phys. Rev. D **68**, 012001 (2003).
[36] Belle Collaboration, Y. Unno *et al.*, Phys. Rev. D **68**, 011103(R) (2003).
[37] Belle Collaboration, K.F. Chen *et al.*, Phys. Lett. B **546**, 196 (2002).
[38] Belle Collaboration, K. Abe *et al.*, Phys. Rev. D **67**, 031102(R) (2003).
[39] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[40] J. Fry, presented at XXI International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, 2003.
[41] Updated results and references are tabulated periodically by the Heavy Flavor Averaging Group: http://www.slac.stanford.edu/xorg/hfag/rare.
[42] BaBar Collaboration, presented by M. Bona at FPCP Conference, Paris, 2003; presented by J. Ocariz at EPS 2003 (International Europhysics Conference on High Energy Physics), Aachen, 2003.
[43] See http://ckmfitter.in2p3.fr for updated sets of parameters. We have used those from the 2002 International Conference on High Energy Physics.
[44] M. Battaglia *et al.*, hep-ph/0304132.
[45] Relations between amplitudes in the two conventions are given in M. Gronau and J.L. Rosner, Phys. Rev. D **66**, 053003 (2002); **66**, 119901(E) (2002).
[46] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
[47] M. Neubert (private communication).
[48] M. Neubert and A.A. Petrov, Phys. Lett. B **519**, 50 (2001); C.W. Chiang and J.L. Rosner, Phys. Rev. D **67**, 074013 (2003).
[49] Z. Luo and J.L. Rosner, Phys. Rev. D **65**, 054027 (2002).
[50] CLEO Collaboration, S.B. Athar *et al.*, Phys. Rev. D **68**, 072003 (2003).
[51] Z. Luo and J.L. Rosner, Phys. Rev. D **68**, 074010 (2003). .
[52] M. Neubert and J.L. Rosner, Phys. Lett. B **441**, 403 (1998); Phys. Rev. Lett. **81**, 5076 (1998); M. Neubert, J. High Energy Phys. **02**, 014 (1999); M. Gronau, D. Pirjol, and T.M. Yan, Phys. Rev. D **60**, 034021 (1999).
[53] H.J. Lipkin, Phys. Rev. Lett. **46**, 1307 (1981); Phys. Lett. B **254**, 247 (1991); **415**, 186 (1997); **433**, 117 (1998).
[54] D. Atwood and A. Soni, Phys. Lett. B **405**, 150 (1997); Phys. Rev. Lett. **79**, 2506 (1997); I. Halperin and A. Zhitnitsky, Phys. Rev. D **56**, 7247 (1997); Phys. Rev. Lett. **80**, 438 (1998); F. Yuan and K.T. Chao, Phys. Rev. D **56**, R2495 (1997); H. Fritzsch, Phys. Lett. B **415**, 83 (1997); H.Y. Cheng and B. Tseng, *ibid.* **415**, 263 (1997); A. Ali and C. Greub, Phys. Rev. D **57**, 2996 (1998); A. Ali, J. Chay, C. Greub, and P. Ko, Phys. Lett. B **424**, 161 (1998); W.S. Hou and B. Tseng, Phys. Rev. Lett. **80**, 434 (1998); A. Datta, X.G. He, and S. Pakvasa, Phys. Lett. B **419**, 369 (1998); D. Du, Y. Yang, and G. Zhu, Phys. Rev. D **59**, 014007 (1999); E. Kou and A.I. Sanda, Phys. Lett. B **525**, 240 (2002).
[55] M. Beneke and M. Neubert, Nucl. Phys. **B651**, 225 (2003).
[56] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998); T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000).
[57] M. Beneke and M. Neubert, Cornell University Report No. CLNS-03/1835, hep-ph/0308039.
[58] M. Gronau and J.L. Rosner, Phys. Rev. D **65**, 093012 (2002); **66**, 053003 (2002); **66**, 119901(E) (2002).
[59] C.W. Chiang and J.L. Rosner, Phys. Rev. D **68**, 014007 (2003).
[60] M. Gronau, presented at Flavor Physics and *CP* Violation Conference, Paris, France, 2003.
[61] N.G. Deshpande and X.G. He, Phys. Rev. Lett. **75**, 3064 (1995).
[62] M. Gronau and J. L. Rosner, Enrico Fermi Institute Report No. EFI 03-34, Phys. Lett. B (to be published), hep-ph/0307095.
[63] BaBar Collaboration, B. Aubert *et al.*, Stanford Linear Accelerator Center Report No. SLAC-PUB-10092, hep-ex/0308012.
[64] Belle Collaboration, quoted in Ref. [40].
[65] D. London and A. Soni, Phys. Lett. B **407**, 61 (1997).