Leptogenesis and neutrino oscillations within a predictive G(224)/SO(10) framework

Jogesh C. Pati
*Department of Physics, University of Maryland, College Park, Maryland 20742, USA**
and Stanford Linear Accelerator Center, Stanford University, Menlo Park, California 94025, USA
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A framework based on an effective symmetry that is either $G(224)=SU(2)_L\times SU(2)_R\times SU(4)^c$ or SO(10) has been proposed (a few years ago) that successfully describes the masses and mixings of all fermions including neutrinos, with seven predictions, in good accord with the data. Baryogenesis via leptogenesis is considered within this framework by allowing for natural phases ($\sim$1/20–1/2) in the entries of the Dirac and Majorana mass matrices. It is shown that the framework leads quite naturally, for both thermal as well as nonthermal leptogenesis, to the desired magnitude for the baryon asymmetry. This result is obtained in full accord with the observed features of the atmospheric and solar neutrino oscillations, as well as with those of the quark and charged lepton masses and mixings, and the gravitino constraint. Hereby one obtains a *unified description* of fermion masses, neutrino oscillations and baryogenesis (via leptogenesis) within a single predictive framework.

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I. INTRODUCTION

The observed matter-antimatter asymmetry of the Universe [1,2] is an important clue to physics at truly short distances. A natural understanding of its magnitude (not to mention its sign) is thus a worthy challenge. Since the discovery of the electroweak sphaleron effect [3], baryogenesis via leptogenesis [4,5] appears to be the most attractive and promising mechanism to generate such an asymmetry [6]. In the context of a unified theory of quarks and leptons, leptogenesis involving decays of heavy right-handed (RH) neutrinos, is naturally linked to the masses of quarks and leptons, neutrino oscillations and, of course, *CP* violation.

In this regard, the route to higher unification based on an effective four-dimensional gauge symmetry of either $G(224)=SU(2)_L\times SU(2)_R\times SU(4)^C$ [7], or SO(10) [8] [that may emerge from a string theory near the string scale and breaks spontaneously to the standard model symmetry near the grand unified theory (GUT) scale [9]] offers some distinct advantages, which are directly relevant to understanding neutrino masses and implementing leptogenesis. These in particular include (a) the existence of the RH neutrinos as a compelling feature, (b) $B-L$ as a local symmetry, and (c) quark-lepton unification through SU(4) color. These three features, first introduced in Ref. [7] in the context of the symmetry G(224), are of course available within any symmetry that contains G(224) as a subgroup; thus, they are available within SO(10) and E_6 [10], though not in SU(5) [11]. Effective symmetries such as flipped $SU(5)\times U(1)$ [12] or $[SU(3)]^3$ [13], or $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}\times SU(3)^C$ [14] possess the first two features (a) and (b), but not (c). Now, the *combination* of the four ingredients—that is (i) the existence of the RH neutrino as an integral member of each family, (ii) the supersymmetric unification scale $M_X\sim 2\times 10^{16}$ GeV [15] (which provides the Majorana mass of the RH neutrinos), (iii) the symmetry SU(4) color (which provides the Dirac mass of the tau neutrino in terms of the top quark mass), *and* (iv) the seesaw mechanism [16]—yields even quantitatively [17] just about the right value of $\Delta m^2(\nu_2-\nu_3)$, as observed at SuperKamiokande [18].

Furthermore, these three features (a)–(c) noted above also provide just the needed ingredients—that is superheavy ν_R's and spontaneous violation of $B-L$ at high temperatures—for implementing baryogenesis via leptogenesis.

Now, in a theory with RH neutrinos having heavy Majorana masses, the magnitude of the lepton-asymmetry is known to depend crucially on both the Dirac as well as Majorana mass matrices of the neutrinos [19]. In this regard, a predictive G(224)/SO(10) framework, describing the masses and mixings of all fermions, including neutrinos, has been proposed [20] that appears to be remarkably successful. In particular it makes seven predictions including $m_b(m_b)\approx 4.7$–4.9 GeV, $m(\nu_3)\sim(1/20)$ eV(1/2–2), $V_{cb}\approx 0.044$, $\sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau}\approx 0.9$–0.99, $V_{us}\approx 0.20$, $V_{ub}\approx 0.003$ and $m_d\approx 8$ MeV, all in good accord with observations, to within 10% (see Sec. II). It has been noted recently [21] that the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution, which is preferred by experiments [22], can arise quite plausibly within the same framework through SO(10)-invariant higher dimensional operators which can contribute directly to the Majorana masses of the left-handed neutrinos (especially to the $\nu_L^e\nu_L^\mu$ mixing mass) without involving the familiar seesaw.

As an additional point, it has been noted by Babu and myself [23] that the framework proposed in Ref. [20] can naturally accommodate *CP* violation by introducing complex phases in the entries of the fermion mass matrices, which preserve the pattern of the mass matrices suggested in Ref. [20] as well as its successes.

*Present address.

The purpose of the present paper is to estimate the lepton and thereby the baryon excess that would typically be expected within this realistic G(224)/SO(10)-framework for fermion masses and mixings [20,23], by allowing for natural *CP* violating phases ($\sim$1/20 to 1/2, say) in the entries of the mass matrices as in Ref. [23]. The goal would thus be to obtain a *unified description* of (a) fermion masses, (b) neutrino oscillations, and (c) leptogenesis within a single predictive framework [24].

It should be noted that there have in fact been several attempts in the literature [25] at estimating the lepton and baryon asymmetries, many of which have actually been carried out in the context of SO(10) [26], though (to my knowledge) without an accompanying realistic framework for the masses and mixing of quarks, charged leptons as well as neutrinos [27]. Also the results in these attempts as regards leptogenesis have not been uniformly encouraging [28].

The purpose of this paper is to note that the G(224)/SO(10) framework, proposed in Refs. [20] and [23], leads quite naturally, for both thermal as well as nonthermal leptogenesis, to the desired magnitude for baryon asymmetry. This result is obtained in full accord with the observed features of atmospheric and solar neutrino oscillations, as well as with those of quark and charged lepton masses and mixings, and the gravitino-constraint. To present the analysis it would be useful to recall the salient features of these prior works [20,23] on fermion masses and mixings. This is what is done in the next section.

II. FERMION MASSES AND NEUTRINO OSCILLATIONS IN G(224)/SO(10): A BRIEF REVIEW OF PRIOR WORK

The 3×3 Dirac mass matrices for the four sectors (u,d,l,ν) proposed in Ref. [20] were motivated in part by the notion that flavor symmetries [29] are responsible for the hierarchy among the elements of these matrices (i.e., for "33"$\gg$"23"$\gg$"22"$\gg$"12"$\gg$"11", etc.), and in part by the group theory of SO(10)/G(224), relevant to a minimal Higgs system (see below). Up to minor variants [30], they are as follows:

$$M_u=\begin{bmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \zeta_{22}^u & \sigma+\epsilon \\ 0 & \sigma-\epsilon & 1 \end{bmatrix}\mathcal{M}_u^0; \quad M_d=\begin{bmatrix} 0 & \eta'+\epsilon' & 0 \\ \eta'-\epsilon' & \zeta_{22}^d & \eta+\epsilon \\ 0 & \eta-\epsilon & 1 \end{bmatrix}\mathcal{M}_d^0 \tag{1}$$

$$M_\nu^D=\begin{bmatrix} 0 & -3\epsilon' & 0 \\ -3\epsilon' & \zeta_{22}^u & \sigma-3\epsilon \\ 0 & \sigma+3\epsilon & 1 \end{bmatrix}\mathcal{M}_u^0; \quad M_l=\begin{bmatrix} 0 & \eta'-3\epsilon' & 0 \\ \eta'+3\epsilon' & \zeta_{22}^d & \eta-3\epsilon \\ 0 & \eta+3\epsilon & 1 \end{bmatrix}\mathcal{M}_d^0.$$

These matrices are defined in the gauge basis and are multiplied by $\overline{\Psi}_L$ on left and Ψ_R on right. For instance, the row and column indices of M_u are given by $(\overline{u}_L,\overline{c}_L,\overline{t}_L)$ and (u_R,c_R,t_R) respectively. Note the group-theoretic up-down and quark-lepton correlations: the same σ occurs in M_u and M_ν^D, and the same η occurs in M_d and M_l. It will become clear that the ϵ and ϵ' entries are proportional to $B-L$ and are antisymmetric in the family space (as shown above). Thus, the same ϵ and ϵ' occur in both (M_u and M_d) and also in (M_ν^D and M_l), but $\epsilon\to-3\epsilon$ and $\epsilon'\to-3\epsilon'$ as $q\to l$. Such correlations result in enormous reduction of parameters and thus in increased predictivity. Such a pattern for the mass matrices can be obtained, using a minimal Higgs system $\mathbf{45}_H$, $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ and $\mathbf{10}_H$ and a singlet S of SO(10), through effective couplings as follows [31]:

$$\begin{aligned}\mathcal{L}_{\rm Yuk}=&h_{33}\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H+[h_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(S/M)+a_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H(\mathbf{45}_H/M')(S/M)^p+g_{23}\mathbf{16}_2\mathbf{16}_3\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^q]\\ &+[h_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{10}_H(S/M)^2+g_{22}\mathbf{16}_2\mathbf{16}_2\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^{q+1}]+[g_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{16}_H^d(\mathbf{16}_H/M'')(S/M)^{q+2}\\ &+a_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{10}_H(\mathbf{45}_H/M')(S/M)^{p+2}].\end{aligned} \tag{2}$$

Typically we expect M', M'' and M to be of order $M_{\rm string}$ [32]. The VEV's of $\langle \mathbf{45}_H \rangle$ (along $B-L$), $\langle \mathbf{16}_H \rangle = \langle \overline{\mathbf{16}}_H \rangle$ (along standard model singlet sneutrino-like component) and of the SO(10)-singlet $\langle S \rangle$ are of the GUT scale, while those of $\mathbf{10}_H$ and of the down type $\mathrm{SU}(2)_L$-doublet component in $\mathbf{16}_H$ (denoted by $\mathbf{16}_H^d$) are of the electroweak scale [20,33]. Depending upon whether $M'(M'') \sim M_{\rm GUT}$ or $M_{\rm string}$ (see [32]), the exponent $p(q)$ is either one or zero [34].

The entries 1 and σ arise respectively from h_{33} and h_{23} couplings, while $\hat{\eta} \equiv \eta - \sigma$ and η' arise respectively from g_{23} and g_{12}-couplings. The $(B-L)$-dependent antisymmetric entries ϵ and ϵ' arise respectively from the a_{23} and a_{12} couplings. [Effectively, with $\langle \mathbf{45}_H \rangle \propto B-L$, the product $\mathbf{10}_H \times \mathbf{45}_H$ contributes as a $\mathbf{120}$, whose coupling is family-antisymmetric.] The small entry ζ_{22}^u arises from the h_{22}-coupling, while ζ_{22}^d arises from the joint contributions of h_{22} and g_{22}-couplings. As discussed in [20], using some of the observed masses as inputs, one obtains $|\hat{\eta}| \sim |\sigma| \sim |\epsilon| \sim \mathcal{O}(1/10)$, $|\eta'| \approx 4 \times 10^{-3}$ and $|\epsilon'| \sim 2 \times 10^{-4}$. The success of the framework presented in Ref. [20] (which set $\zeta_{22}^u = \zeta_{22}^d = 0$) in describing fermion masses and mixings remains essentially unaltered if $|(\zeta_{22}^u, \zeta_{22}^d)| \leqslant (1/3)(10^{-2})$ (say).

Such a hierarchical form of the mass matrices, with h_{33}-term being dominant, is attributed in part to flavor gauge symmetry(ies) that distinguishes between the three families, and in part to higher dimensional operators involving for example $\langle \mathbf{45}_H \rangle / M'$ or $\langle \mathbf{16}_H \rangle / M''$, which are suppressed by $M_{\rm GUT}/M_{\rm string} \sim 1/10$, if M' and/or $M'' \sim M_{\rm string}$. The basic presumption here is that effective dimensionless couplings allowed by $SO(10)/G(224)$ and flavor symmetries are of order unity [i.e., $(h_{ij}, g_{ij}, a_{ij}) \approx 1/3$–$3$ (say)]. The need for appropriate powers of (S/M) with $\langle S \rangle / M \sim M_{\rm GUT}/M_{\rm string} \sim (1/10 - 1/20)$ in the different couplings leads to a hierarchical structure. As an example, introduce just one U(1)-flavor symmetry, together with a discrete symmetry D, with one singlet S. The hierarchical form of the Yukawa couplings exhibited in Eqs. (1) and (2) would follow, for the case of $p=1$, $q=0$, if, for example, the $U(1)$ flavor charges are assigned as follows:

$$\begin{array}{cccccccc} \mathbf{16}_3 & \mathbf{16}_2 & \mathbf{16}_1 & \mathbf{10}_H & \mathbf{16}_H & \overline{\mathbf{16}}_H & \mathbf{45}_H & \mathbf{S} \\ a & a+1 & a+2 & -2a & -a-1/2 & -a & 0 & -1. \end{array} \tag{3}$$

All the fields are assumed to be even under the discrete symmetry D, except for $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ which are odd. It is assumed that other fields are present that would make the U(1) symmetry anomaly-free. With this assignment of charges, one would expect $|\zeta_{22}^{u,d}| \sim (\langle S \rangle / M)^2$; one may thus take $|\zeta_{22}^{u,d}| \sim (1/3) \times 10^{-2}$ without upsetting the success of Ref. [20]. In the same spirit, one would expect $|\zeta_{13}, \zeta_{31}| \sim (\langle S \rangle / M)^2 \sim 10^{-2}$ and $|\zeta_{11}| \sim (\langle S \rangle / M)^4 \sim 10^{-4}$ (say), where ζ_{11}, ζ_{13}, and ζ_{31} denote the "11," "13," and "31," elements respectively. The value of "a" would get fixed by the presence of other operators (see later).

To discuss the neutrino sector one must specify the Majorana mass-matrix of the RH neutrinos as well. These arise from the effective couplings of the form [35]

$$\mathcal{L}_{\rm Maj} = f_{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H / M \tag{4}$$

where the f_{ij}'s include appropriate powers of $\langle S \rangle / M$, in accord with flavor charge assignments of $\mathbf{16}_i$ [see Eq. (3)]. For the f_{33}-term to be leading, we must assign the charge $-a$ to $\overline{\mathbf{16}}_H$. This leads to a hierarchical form for the Majorana mass matrix [20]:

$$M_R^\nu = \begin{bmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{bmatrix} M_R. \tag{5}$$

Following the flavor-charge assignments given in Eq. (3), we expect $|y| \sim \langle S/M \rangle \sim 1/10$, $|z| \sim (\langle S/M \rangle)^2 \sim (1/200)(1$ to $1/2$, say), $|x| \sim (\langle S/M \rangle)^4 \sim (10^{-4}$–$10^{-5})$ (say). The "22" element (not shown) is $\sim (\langle S/M \rangle)^2$ and its magnitude is taken to be $< |y^2/3|$, while the "12" element (not shown) is $\sim (\langle S/M \rangle)^3$. We expect

$$M_R = f_{33} \langle \overline{\mathbf{16}}_H \rangle^2 / M_{\rm string} \approx (10^{15}\ \mathrm{GeV})(1/2 - 2) \tag{6}$$

for $\langle \overline{\mathbf{16}}_H \rangle \approx 2 \times 10^{16}$ GeV, $M_{\rm string} \approx 4 \times 10^{17}$ GeV [36] and $f_{33} \approx 1$. Allowing for 2-3 mixing, this value of M_R [together with the SU(4)-color relation $m(\nu_3^{\rm Dirac}) = m_t(M_{\rm GUT}) \approx 120$ GeV] leads to $m(\nu_3) \approx (1/24\ \mathrm{eV})(1/2$–$2)$ [17,20,37], in good accord with the SuperK data.

Ignoring possible phases in the parameters and thus the source of *CP* violation for a moment, as was done in Ref. [20], the parameters (σ, η, ϵ, ϵ', η', $\mathcal{M}_u^0$, $\mathcal{M}_D^0$, and y) can be determined by using, for example, $m_t^{\rm phys} = 174$ GeV, $m_c(m_c) = 1.37$ GeV, $m_S(1\ \mathrm{GeV}) = 110$–$116$ MeV, $m_u(1\ \mathrm{GeV}) = 6$ MeV, the observed masses of e, μ, and τ and $m(\nu_2)/m(\nu_3) \approx 1/(7 \pm 1)$ (as suggested by a combination of atmospheric and solar neutrino data, the latter corresponding to the LMA MSW solution, see below) as inputs. One is thus led, *for this CP conserving case*, to the following fit for the parameters, and the associated predictions [20]. [In this fit, we drop $|\zeta_{22}^{u,d}| \leqslant (1/3)(10^{-2})$ and leave the small quantities x and z in M_R^ν undetermined and proceed by assuming that they have the magnitudes suggested by flavor symmetries (i.e., $x \sim (10^{-4}$–$10^{-5})$ and $z \sim (1/200)(1$ to $1/2)$ [see remarks below Eq. (5)])],

$$\sigma \approx 0.110, \quad \eta \approx 0.151, \quad \epsilon \approx -0.095, \quad |\eta'| \approx 4.4 \times 10^{-3},$$

$$\epsilon' \approx 2 \times 10^{-4}, \quad \mathcal{M}_u^0 \approx m_t(M_X) \approx 120\ \ \mathrm{GeV}, \tag{7}$$

$$\mathcal{M}_D^0 \approx m_b(M_X) \approx 1.5\ \ \mathrm{GeV}, \quad y \approx -1/17.$$

These output parameters remain stable to within 10% corresponding to small variations ($\lesssim 10\%$) in the input parameters of m_t, m_c, m_s, and m_u. These in turn lead to the following predictions for the quarks and light neutrinos [20,37]:

$$
\begin{aligned}
&m_b(m_b)\approx(4.7\text{–}4.9)\ \ \text{GeV},\\
&\sqrt{\Delta m_{23}^2}\approx m(\nu_3)\approx(1/24\ \ \text{eV})(1/2\text{–}2),\\
&V_{cb}\approx\left|\sqrt{\frac{m_s}{m_b}}\left|\frac{\eta+\epsilon}{\eta-\epsilon}\right|^{1/2}-\sqrt{\frac{m_c}{m_t}}\left|\frac{\sigma+\epsilon}{\sigma-\epsilon}\right|^{1/2}\right|\approx 0.044,\\
&\left\{\begin{array}{l}\theta^{\rm osc}_{\nu_\mu\nu_\tau}\approx\left|\sqrt{\frac{m_\mu}{m_\tau}}\left|\frac{\eta-3\epsilon}{\eta+3\epsilon}\right|^{1/2}+\sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}\right|\approx|0.437+(0.378\pm0.03)|,\\ \text{Thus,}\ \ \sin^2 2\theta^{\rm osc}_{\nu_\mu\nu_\tau}\approx 0.99\ \ [\text{for}\ \ m(\nu_2)/m(\nu_3)\approx 1/7],\end{array}\right. \qquad (8)\\
&V_{us}\approx\left|\sqrt{\frac{m_d}{m_s}}-\sqrt{\frac{m_u}{m_c}}\right|\approx 0.20,\\
&\left|\frac{V_{ub}}{V_{cb}}\right|\approx\sqrt{\frac{m_u}{m_c}}\approx 0.07,\\
&m_d(1\ \ \text{GeV})\approx 8\ \ \text{MeV},\\
&\theta^{\rm osc}_{\nu_e\nu_\mu}\approx 0.06\ \ (\text{ignoring nonseesaw contributions});\ \text{see remarks below.}
\end{aligned}
$$

The Majorana masses of the RH neutrinos ($N_{iR}\equiv N_i$) are given by [37]

$$
\begin{aligned}
&M_3\approx M_R\approx 10^{15}\ \ \text{GeV}\ \ (1/2\text{–}1),\\
&M_2\approx|y^2|M_3\approx(2.5\times10^{12}\ \ \text{GeV})\ (1/2\text{–}1), \qquad (9)\\
&M_1\approx|x-z^2|M_3\sim(1/2\text{–}2)10^{-5}M_3\sim10^{10}\ \ \text{GeV}\ (1/4\text{–}2).
\end{aligned}
$$

Note that we necessarily have a hierarchical pattern for the light as well as the heavy neutrinos (see discussions below on m_{ν_1}). Leaving out the ν_e-ν_μ oscillation angle for a moment, it seems remarkable that the first seven predictions in Eq. (8) agree with observations, to within 10% [38]. Particularly intriguing is the $(B-L)$-dependent *group-theoretic correlation* between the contribution from the first term in V_{cb} and that in $\theta^{\rm osc}_{\nu_\mu\nu_\tau}$, which explains simultaneously why one is small (V_{cb}) and the other is large ($\theta^{\rm osc}_{\nu_\mu\nu_\tau}$) [39]. That in turn provides some degree of confidence in the gross structure of the mass matrices.

As regards ν_e-ν_μ and ν_e-ν_τ oscillations, the standard seesaw mechanism would typically lead to rather small angles as in Eq. (8), within the framework presented above [20]. It has, however, been noted recently [21] that small intrinsic (nonseesaw) masses $\sim10^{-3}$ eV of the LH neutrinos can arise quite plausibly through higher dimensional operators of the form [40]: $W_{12}\supset\kappa_{12}\mathbf{16}_1\mathbf{16}_2\mathbf{16}_H\mathbf{16}_H\mathbf{10}_H\mathbf{10}_H/M_{\rm eff}^3$, without involving the standard seesaw mechanism [16]. One can verify that such a term would lead to an intrinsic Majorana mixing mass term of the form $m_{12}^0\nu_L^e\nu_L^\mu$, with a strength given by $m_{12}^0\approx\kappa_{12}\langle\mathbf{16}_H\rangle^2(175\ \text{GeV})^2/M_{\rm eff}^3\sim(1.5\text{–}6)\times10^{-3}$ eV, for $\langle\mathbf{16}_H\rangle\approx(1\text{-}2)M_{\rm GUT}$ and $\kappa_{12}\sim1$, if $M_{\rm eff}\sim M_{\rm GUT}\approx2\times10^{16}$ GeV [41]. Such an intrinsic Majorana $\nu_e\nu_\mu$ mixing mass $\sim$few$\times10^{-3}$ eV, though small compared to $m(\nu_3)$, is still much larger than what one would generically get for the corresponding term from the standard seesaw mechanism (as in Ref. [20]). Now, the diagonal ($\nu_\mu\nu_\mu$) mass-term, arising from standard seesaw can naturally be $\sim(3\text{–}8)\times10^{-3}$ eV for $|y|\approx1/20\text{–}1/15$, say [20]. Thus, taking the net values of $m_{22}^0\approx(6-7)\times10^{-3}$ eV, $m_{12}^0\sim3\times10^{-3}$ eV as above and $m_{11}^0\leqslant10^{-3}$ eV, which are all plausible, we obtain $m_{\nu_2}\approx(6\text{–}7)\times10^{-3}$ eV, $m_{\nu_1}\sim(1\ \text{to few})\times10^{-3}$ eV, so that $\Delta m_{12}^2\approx(3.6\text{–}5)\times10^{-5}$ eV^2 and $\sin^2 2\theta^{\rm osc}_{\nu_e\nu\mu}\approx0.6\text{–}0.7$. These go well with the LMA MSW solution of the solar neutrino puzzle.

In summary, *the intrinsic nonseesaw contribution* to the Majorana masses of the LH neutrinos can possibly have the right magnitude for ν_e-ν_μ mixing so as to lead to the LMA solution within the G(224)/SO(10)-framework, without upsetting the successes of the first seven predictions in Eq. (8). (In contrast to the near maximality of the ν_μ-ν_τ oscillation angle, however, which emerges as a compelling prediction of the framework [20], the LMA solution, as obtained above, should be regarded as a consistent possibility, rather than as a compelling prediction, within this framework.)

Before discussing leptogenesis, we need to discuss the origin of *CP* violation within the G(224)/SO(10)-framework presented above. The discussion so far has ignored, for the sake of simplicity, possible *CP* violating phases in the parameters (σ, η, ϵ, η', ϵ', $\zeta_{22}^{u,d}$, y, z, and x) of the Dirac and Majorana mass matrices [Eqs. (1) and (5)]. In general, however, these parameters can and generically will have phases [42]. Some combinations of these phases enter into the CKM matrix and define the Wolfenstein parameters ρ_W and η_W [43], which in turn induce *CP* violation by utilizing the standard model interactions. As observed in Ref. [23], an additional and potentially important source of *CP* and flavor violations (as in $K^0\leftrightarrow\overline{K^0}$, $B_{d,s}\leftrightarrow\overline{B_{d,s}}$, $b\rightarrow s\overline{s}s$, etc. transitions) arises in the model through supersymmetry [44], involving squark and gluino loops (box and penguin), simply because of the embedding of MSSM within a string-unified G(224) or SO(10)-theory near the GUT scale, and the assumption that primordial SUSY-breaking occurs near the string scale ($M_{\rm string}>M_{\rm GUT}$) [45]. It is shown that complexification of the parameters (σ, η, ϵ, η', ϵ', etc.), through introduction of phases $\sim$1/20–1/2 (say) in them, can still preserve the successes of the predictions as regards fermion masses and neutrino oscillations shown in Eq. (8), as long as one maintains nearly the magnitudes of the real parts of the parameters and especially their relative signs as obtained in Ref. [20] and shown in Eq. (7) [46]. Such a picture is also in accord with the observed features of *CP* and flavor violations in ϵ_K, Δm_{Bd}, and asymmetry parameter in $B_d \to J/\Psi + K_s$, while predicting observable new effects in processes such as $B_s \to \overline{B_s}$ and $B_d \to \Phi + K_s$ [23].

We therefore proceed to discuss leptogenesis concretely within the framework presented above by adopting the Dirac and Majorana fermion mass matrices as shown in Eqs. (1) and (5) and assuming that the parameters appearing in these matrices can have natural phases $\sim$1/20–1/2 (say) with either sign up to addition of $\pm\pi$, *while their real parts have the relative signs and nearly the magnitudes given in Eq.* (8).

III. LEPTOGENESIS

In the context of an inflationary scenario [47], with a plausible reheat temperature $T_{RH} \sim (1 \text{ to few}) \times 10^9$ GeV (say), one can avoid the well known gravitino problem if $m_{3/2} \sim (1\text{–}2)$ TeV [48] and yet produce the lightest heavy neutrino N_1 efficiently from the thermal bath if $M_1 \sim (3\text{–}5) \times 10^9$ GeV (say), in accord with Eq. (9) (N_2 and N_3 are of course too heavy to be produced at $T \sim T_{RH}$). Given lepton number (and $B-L$) violation occurring through the Majorana mass of N_1, and C and *CP* violating phases in the Dirac and/or Majorana fermion mass-matrices as mentioned above, the out-of-equilibrium decays of N_1 (produced from the thermal bath) into $l+H$ and $\bar{l}+\bar{H}$ and into the corresponding SUSY modes $\tilde{l}+\tilde{H}$ and $\bar{\tilde{l}}+\bar{\tilde{H}}$ would produce a $B-L$ violating lepton asymmetry; so also would the decays of $\tilde{N}_1$ and $\bar{\tilde{N}}_1$. Part of this asymmetry would of course be washed out due to inverse decays and lepton number violating $2 \leftrightarrow 2$ scatterings. We will assume this commonly adopted mechanism for the so-called thermal leptogenesis (at the end, we will, however, consider an interesting alternative that would involve nonthermal leptogenesis). This mechanism has been extended to incorporate supersymmetry by several authors (see e.g., [49–51]). The net lepton asymmetry of the Universe [$Y_L \equiv (n_L - n_{\bar{L}})/s$] arising from decays of N_1 into $l+H$ and $\bar{l}+\bar{H}$ and into the corresponding SUSY modes ($\tilde{l}+\tilde{H}$ and $\bar{\tilde{l}}+\bar{\tilde{H}}$) and likewise from ($\tilde{N}_1$, $\bar{\tilde{N}}_1$) decays [49–51] is given by

$$Y_L = \kappa \epsilon_1 \left(\frac{n_{N_1} + n_{\tilde{N}_1} + n_{\bar{\tilde{N}}_1}}{s} \right) \approx \kappa \epsilon_1 / g^* \tag{10}$$

where ϵ_1 is the lepton-asymmetry produced per N_1 [or ($\tilde{N}_1 + \bar{\tilde{N}}_1$)-pair] decay (see below), κ is an efficiency or damping factor that represents the washout effects mentioned above (thus κ incorporates the extent of departure from thermal equilibrium in N_1-decays; such a departure is needed to realize lepton asymmetry). The entity $g^* \approx 228$ is the number of light degrees of freedom in MSSM.

The lepton asymmetry Y_L is converted to baryon asymmetry, by the sphaleron effects, which is given by

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} = C Y_L , \tag{11}$$

where, for MSSM, $C \approx -1/3$. Taking into account the interference between the tree and loop-diagrams for the decays of $N_1 \to lH$ and $\overline{lH}$ (and likewise for $N_1 \to \tilde{l}\tilde{H}$ and $\bar{\tilde{l}}\bar{\tilde{H}}$ modes and also for $\tilde{N}_1$ and $\bar{\tilde{N}}_1$-decays), the *CP* violating lepton asymmetry parameter in each of the four channels (see e.g., [50] and [51]) is given by

$$\epsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j=2,3} \mathrm{Im}[(M_D^\dagger M_D)_{j1}]^2 f(M_j^2/M_1^2) \tag{12}$$

where M_D is the Dirac neutrino mass matrix evaluated in a basis in which the Majorana mass matrix of the RH neutrinos M_R^ν [see Eq. (5)] is diagonal, $v = (174 \text{ GeV}) \sin\beta$ and the function $f \approx -3(M_1/M_j)$ for the case of SUSY with $M_j \gg M_1$.

The efficiency factor mentioned above, is often expressed in terms of the parameter $K \equiv [\Gamma(N_1)/2H]_{T=M_1}$ [47]. Assuming initial thermal abundance for N_1, κ is normalized so that it is 1 if N_1's decay fully out of equilibrium corresponding to $K \ll 1$ (in practice, this actually requires $K<0.1$). Including inverse decays as well as $\Delta L \neq 0$ scatterings in the Boltzmann equations, a recent analysis [52] shows that in the relevant parameter-range of interest to us (see below), the efficiency factor (for the SUSY case) is given by [53]

$$\kappa \approx (0.7 \times 10^{-4})(\mathrm{eV}/\tilde{m}_1) \tag{13}$$

where $\tilde{m}_1$ is an effective mass parameter (related to K [54]), and is given by [55]

$$\tilde{m}_1 \equiv (M_D^\dagger M_D)_{11}/M_1 . \tag{14}$$

Equation (14) should hold to better than 20% (say), when $\tilde{m}_1 \gg 5 \times 10^{-4}$ eV [52] (this applies well to our case, see below).

Given the Dirac and Majorana mass matrices of the neutrinos [Eqs. (1) and (5)], we are now ready to evaluate lepton asymmetry by using Eqs. (10)–(14).

The Majorana mass matrix [Eq. (5)] describing the mass-term $\nu_R^T C M_R^\nu \nu_R$ is diagonalized by the transformation $\nu_R = U_R^{(1)} U_R^{(2)} N_R$, where (to a good approximation)

$$U_R^{(1)} \approx \begin{bmatrix} 1 & 0 & z \\ 0 & 1 & y \\ -z & -y & 1 \end{bmatrix}, \tag{15}$$

and $U_R^{(2)} = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ is a diagonal phase matrix that ensures real positive eigenvalues. The phases ϕ_i can of course be derived from those of the parameters in M_R^ν [see Eq. (5)]. Applying this transformation to the neutrino Dirac mass-term $\overline{\nu_L} M_\nu^D \nu_R$ given by Eq. (1), we obtain $M_D = M_\nu^D U_R^{(1)} U_R^{(2)}$, which appears in Eqs. (12) and (14). In turn, this yields

$$\frac{(M_D^\dagger M_D)_{21}}{(\mathcal{M}_u^0)^2} = e^{i(\phi_1-\phi_2)}\{(-3\epsilon'^* - \zeta_{13}^* y^*)(\zeta_{11} - z\zeta_{13}) + [\zeta_{22}^{u*} - y^*(\sigma^* - 3\epsilon^*)][3\epsilon' - z(\sigma - 3\epsilon)] + (\zeta_{31} - z)[(\sigma^* + 3\epsilon^*) - y^*]\} \tag{16}$$

$$\frac{(M_D^\dagger M_D)_{11}}{(\mathcal{M}_u^0)^2} = |3\epsilon' - z(\sigma - 3\epsilon)|^2 + |\zeta_{31} - z|^2. \tag{17}$$

In writing Eqs. (16) and (17), we have allowed, for the sake of generality, the relatively small "11," "13," and "31" elements in the Dirac mass matrix M_ν^D, denoted by ζ_{11}, ζ_{13} and ζ_{31} respectively, which are not exhibited in Eq. (1). As mentioned before, guided by considerations of flavor symmetry [see Eq. (3)], we would expect $|\zeta_{11}| \sim (\langle S\rangle/M)^4 \sim 10^{-4}$–$10^{-5}$, and $|\zeta_{13}| \sim |\zeta_{31}| \sim (\langle S\rangle/M)^2 \sim 10^{-2}(1$ to $1/3)$ (say). These small elements (neglected in [20]) would not, of course, have any noticeable effects on the predictions of the fermion masses and mixings given in Eq. (8), except possibly on m_d.

We now proceed to make numerical estimates of lepton and baryon-asymmetries by taking the magnitudes and the relative signs of the real parts of the parameters (σ, η, ϵ, η', ϵ', and y) approximately the same as in Eq. (7), but allowing in general for natural phases in them. As mentioned before [see for example the fit given in Ref. [46] and Ref. [23] (to appear)] such a procedure introduces *CP* violation in accord with observation, while preserving the successes of the framework as regards its predictions for fermion masses and neutrino oscillations [20,23].

Given the magnitudes of the parameters [see Eqs. (7) and Ref. [46]], which are obtained from considerations of fermion masses and neutrino oscillations [20,23]—that is $|\sigma| \approx |\epsilon| \approx 0.1$, $|y| \approx 0.06$, $|\epsilon'| \approx 2\times 10^{-4}$, $|z| \sim (1/200)(1$ to $1/2)$, $|\zeta_{22}^u| \sim 10^{-3}(1$ to $3)$, $|\zeta_{13}| \sim |\zeta_{31}| \sim (1/200)(1$ to $1/2)$, with the real parts of (σ, ϵ and y) having the signs $(+,-,-)$ respectively—we would expect the typical magnitudes of the three terms of Eq. (16) to be as follows:

$$\begin{aligned} |\text{1st Term}| &= |(-3\epsilon'^* - \zeta_{13}^* y^*)(\zeta_{11} - z\zeta_{13})| \\ &\approx [(6 \text{ to } 8)\times 10^{-4}][(2.5\times 10^{-5})(1 \text{ to } 1/4)] \\ &\sim 10^{-8} \end{aligned}$$

$$\begin{aligned} |\text{2nd Term}| &= |\{\zeta_{22}^{u*} - y^*(\sigma^* - 3\epsilon^*)\}\{3\epsilon' - z(\sigma - 3\epsilon)\}| \\ &\approx (2\times 10^{-2})[2\times 10^{-3}(1 \text{ to } 1/2)] \\ &\approx (4\times 10^{-5})(1 \text{ to } 1/2) \end{aligned} \tag{18}$$

$$\begin{aligned} |\text{3rd Term}| &= |(\zeta_{31} - z)\{(\sigma^* + 3\epsilon^*) - y^*\}| \\ &\approx [(1/200)(1/2 \text{ to } 1/5)](0.13) \\ &\approx (0.7\times 10^{-3})(1/2 \text{ to } 1/5). \end{aligned}$$

Thus, assuming that the phases of the different terms are roughly comparable, the third term would clearly dominate. The RHS of Eq. (17) is similarly estimated to be

$$\begin{aligned} \frac{(M_D^\dagger M_D)_{11}}{(\mathcal{M}_u^0)^2} &= |3\epsilon' - z(\sigma - 3\epsilon)|^2 + |\zeta_{31} - z|^2 \\ &\approx |6\times 10^{-4} \mp 2\times 10^{-3}(1 \text{ to } 1/2)|^2 + |5\times 10^{-3}(1/2 \text{ to } 1/5)|^2 \\ &\approx 2.5\times 10^{-5}(1/4 \text{ to } 1/6). \end{aligned} \tag{19}$$

Since $|\zeta_{31}|$ and $|z|$ are each expected to be of order (1/200)(1 to 1/2), we have allowed in Eqs. (18) and (19) for a possible mild cancellation between their contributions to $|\zeta_{31} - z|$ by putting $|\zeta_{31} - z| \approx (1/200)(1/2$ to $1/5)$ (say). In going from the second to the third step of Eq. (19) we have assumed (for simplicity) that the second term of $(M_D^\dagger M_D)_{11}/(\mathcal{M}_u^0)^2$ given by $|\zeta_{31} - z|^2$ denominates over the first. This in fact holds for a large part of the expected parameter space, especially for values of $|z| \approx (1/200)(1/2) \lesssim |\zeta_{31}| \approx (1/200)(1$ to $3/4)$ (say). Note that the combination $|\zeta_{31} - z|$ also enters into the dominant term [i.e., the third term in Eq. (18)] of $(M_D^\dagger M_D)_{21}/(\mathcal{M}_u^0)^2$. As a result, to a good approximation (in the region of parameter space mentioned above), the lepton-asymmetry parameter ϵ_1 [given by Eq. (12)] becomes independent of the magnitude of $|\zeta_{31} - z|^2$, and is given by

$$\begin{aligned} \epsilon_1 &\approx \frac{1}{8\pi}\left(\frac{\mathcal{M}_u^0}{v}\right)^2 |(\sigma + 3\epsilon) - y|^2 \sin(2\phi_{21})(-3)\left(\frac{M_1}{M_2}\right) \\ &\approx -(2.0\times 10^{-6})\sin(2\phi_{21}), \end{aligned} \tag{20}$$

where, $\phi_{21} = \arg[(\zeta_{31} - z)(\sigma^* + 3\epsilon^* - y^*)] + (\phi_1 - \phi_2)$, and we have put $(\mathcal{M}_u^0/v)^2 \approx 1/2$, $|\sigma + 3\epsilon - y| \approx 0.13$ [see Eq. (7) and Ref. [46]], and for concreteness (for the present case of thermal leptogenesis) $M_1 \approx 4\times 10^9$ GeV and $M_2 \approx 2\times 10^{12}$ GeV [see Eq. (9)]. The parameter $\tilde{m}_1$, given by Eq. (14), is (approximately) proportional to $|\zeta_{31} - z|^2$ [see Eq. (19)]. It is given by

$$\begin{aligned} \tilde{m}_1 &\approx |\zeta_{31} - z|^2 (\mathcal{M}_u^0)^2 / M_1 \\ &\approx (1.9\times 10^{-2} \text{ eV})(1 \text{ to } 1/6)\left(\frac{4\times 10^9 \text{ GeV}}{M_1}\right) \end{aligned} \tag{21}$$

where, as before, we have put $|\zeta_{31} - z| \approx (1/200)(1/2$ to $1/5)$. The corresponding efficiency factor κ

TABLE I. Baryon asymmetry for the case of thermal leptogenesis.

	$\vert\zeta_{31}-z\vert$		
	(1/200)(1/3)	(1/200)(1/4)	(1/200)(1/5)
$\tilde{m}_1$ (eV)	0.83×10^{-2}	0.47×10^{-2}	0.30×10^{-2}
κ	1/73	1/39	1/24
$Y_L/\sin(2\phi_{21})$	-11.8×10^{-11}	-22.4×10^{-11}	-36×10^{-11}
$Y_B/\sin(2\phi_{21})$	4×10^{-11}	7.5×10^{-11}	12×10^{-11}
ϕ_{21}	$\sim\pi/4$	$\sim\pi/12-\pi/4$	$\sim\pi/18-\pi/4$

[given by Eq. (13)], lepton and baryon-asymmetries Y_L and Y_B [given by Eqs. (10) and (11)] and the requirement on the phase-parameter ϕ_{21} are listed in Table I.

The constraint on ϕ_{21} is obtained from considerations of big-bang nucleosynthesis, which requires $3.7\times10^{-11}\lesssim(Y_B)_{BBN}\lesssim9\times10^{-11}$ [1]; this is consistent with the CMB data [2], which suggest somewhat higher values of $(Y_B)_{CMB}\approx(7-10)\times10^{-11}$ (say). We see that the first case $|\zeta_{31}-z|\approx1/200(1/3)$ leads to a baryon asymmetry Y_B that is on the low side of the BBN-data, even for a maximal $\sin(2\phi_{21})\approx1$. The other cases with $|\zeta_{31}-z|\approx(1/200)(1/4\text{ to }1/5)$, which are of course perfectly plausible, lead to the desired magnitude of the baryon asymmetry for natural values of the phase parameter $\phi_{21}\sim(\pi/18\text{ to }\pi/4)$. We see that, for the thermal case, the CMB data, requiring higher values of Y_B, would suggest somewhat smaller values of $|\zeta_{31}-z|\sim10^{-3}$. This constraint would be eliminated for the case of nonthermal leptogenesis.

We next consider briefly the scenario of nonthermal leptogenesis [56,57]. In this case the inflaton is assumed to decay, following the inflationary epoch, directly into a pair of heavy RH neutrinos (or sneutrinos). These in turn decay into $l+H$ and $\bar{l}+\bar{H}$ as well as into the corresponding SUSY modes, and thereby produce lepton asymmetry, during the process of reheating. It turns out that this scenario goes well with the fermion mass pattern of Sec. II [in particular see Eq. (9)] and the observed baryon asymmetry, provided $2M_2>m_{\rm infl}>2M_1$, so that the inflation decays into $2N_1$ rather than into $2N_2$ (contrast this from the case proposed in Ref. [56]). In this case, the reheating temperature ($T_{\rm RH}$) is found to be much less than $M_1\sim10^{10}$ GeV (see below); thereby (a) the gravitino constraint is satisfied quite easily, even for a rather low gravitino-mass ~200 GeV (unlike in the thermal case); at the same time (b) while N_1's are produced nonthermally (and copiously) through inflaton decay, they remain out of equilibrium and the wash out process involving inverse decays and $\Delta L\neq0$ scatterings is ineffective, so that the efficiency factor κ is 1.

To see how the nonthermal case can arise naturally, we recall that the VEV's of the Higgs fields $\Phi=(1,2,4)_H$ and $\bar{\Phi}=(1,2,\bar{4})_H$ have been utilized to (i) break $SU(2)_R$ and $B-L$ so that G(224) breaks to the SM symmetry [7], and simultaneously (ii) to give Majorana masses to the RH neutrinos via the coupling in Eq. (4) (see e.g., Ref. [20]; for SO(10), $\bar{\Phi}$ and Φ would be in $\mathbf{16}_H$ and $\overline{\mathbf{16}_H}$ respectively), it is attractive to assume that the same Φ and $\bar{\Phi}$ (in fact their $\tilde{\nu}_{\rm RH}$ and $\bar{\tilde{\nu}}_{\rm RH}$-components), which acquire GUT-scale VEV's, also drive inflation [56]. In this case the inflaton would naturally couple to a pair of RH neutrinos by the coupling of Eq. (4). To implement hybrid inflation in this context, let us assume following Ref. [56], an effective superpotential $W_{\rm eff}^{\rm infl}=\lambda S(\bar{\Phi}\Phi-M^2)+(\text{nonrenormalizable terms})$, where S is a singlet field [58]. It has been shown in Ref. [56] that in this case a flat potential with a radiatively generated slope can arise so as to implement inflation, with G(224) broken during the inflationary epoch to the SM symmetry. The inflaton is made of two complex scalar fields [i.e., $\theta=(\delta\tilde{\nu}_H^C+\delta\bar{\tilde{\nu}}_H^C)/\sqrt{2}$ that represents the fluctuations of the Higgs fields around the SUSY minimum, and the singlet S]. Each of these have a mass $m_{\rm infl}=\sqrt{2}\lambda M$, where $M=\langle(1,2,4)_H\rangle\approx2\times10^{16}$ GeV and a width $\Gamma_{\rm infl}=\Gamma(\theta\to\Psi_{\nu_H}\Psi_{\nu_H})=\Gamma(S\to\tilde{\nu}_H\tilde{\nu}_H)\approx[1/(8\pi)](M_1/M)^2m_{\rm infl}$ so that

$$T_{\rm RH}\approx(1/7)(\Gamma_{\rm infl}M_{\rm Pl})^{1/2}\approx(1/7)(M_1/M)[m_{\rm infl}M_{\rm Pl}/(8\pi)]^{1/2}. \tag{22}$$

For concreteness, take [59] $M_2\approx2\times10^{12}$ GeV, $M_1\approx2\times10^{10}$ GeV(1 to 2) [in accord with Eq. (9)], and $\lambda\approx10^{-4}$, so that $m_{\rm infl}\approx3\times10^{12}$ GeV. We then get $T_{\rm RH}\approx(1.7\times10^8\text{ GeV})(1\text{ to }2)$, and thus (see e.g., Sec. 8 of Ref. [47])

$$\begin{aligned}(Y_B)_{nonthermal}&\approx-(Y_L/3)\\&\approx(-1/3)[(n_{N_1}+n_{\tilde{N}_1}+n_{\bar{\tilde{N}}_1})/s]\epsilon_1\\&\approx(-1/3)[(3/2)(T_{\rm RH}/m_{\rm infl})\epsilon_1]\\&\approx(30\times10^{-11})(\sin2\phi_{21})(1\ \text{ to }\ 2)^2.\end{aligned} \tag{23}$$

Here we have used Eq. (20) for ϵ_1 with appropriate (M_1/M_2), as above. Setting $M_1\approx2\times10^{10}$ for concreteness, we see that Y_B obtained above agrees with the (nearly central) observed value of $\langle Y_B\rangle_{\rm BBN(CMB)}^{\rm central}\approx[6(9)]\times10^{-11}$, again for a natural value of the phase parameter $\phi_{21}\approx\pi/30(\pi/20)$. As mentioned above, one possible advantage of the nonthermal over the thermal case is that the gravitino-constraint can be met rather easily, in the case of the former (because T_{RH} is rather low $\sim10^8$ GeV), whereas for the thermal case there is a significant constraint on the lowering of the T_{RH} (so as to satisfy the gravitino-constraint) vis à vis a raising of $M_1\sim T_{RH}$ so as to have sufficient baryon asymmetry [note that $\epsilon_1\propto M_1$, see Eq. (20)]. Furthermore, for the nonthermal case, the dependence of Y_B on the parameter $|\zeta_{31}-z|^2$ [which arises through κ and $\tilde{m}_1$ in the thermal case, see Eqs. (13), (14), and (19)] is largely eliminated. Thus the expected magnitude of Y_B [Eq. (23)] holds without a significant constraint on $|\zeta_{31}-z|$ (in contrast to the thermal case).

To conclude, we have considered two alternative scenarios (thermal as well as nonthermal) for inflation and leptogenesis. We see that the G(224)/SO(10) framework provides a simple and *unified description* of not only fermion masses and neutrino oscillations (consistent with maximal atmospheric and large solar oscillation angles) but also of baryogenesis via leptogenesis, treated within either scenario, in accord with the gravitino-constraint. Each of the features—(a) the existence of the right-handed neutrinos, (b) $B-L$ local symmetry, (c) quark-lepton unification through SU(4) color, (d) the magnitude of the supersymmetric unification-scale and (e) the seesaw mechanism—plays a crucial role in realizing this unified and successful description. These features in turn point to the relevance of either the G(224) or the SO(10) symmetry being effective between the string and the GUT scales, in four dimensions [9]. While the observed magnitude of the baryon asymmetry seems to emerge naturally from within the framework, understanding its observed sign (and thus the relevant *CP* violating phases) remains a challenging task [60].

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[24] By combining these results with the analysis of the forthcoming paper [23], one would incorporate *CP* violation into this unified picture as well.

[25] Many of these are based on phenomenological models just for neutrino masses, which are not linked to the masses and mixings of quarks and charged leptons. See, e.g., S.F. King, J. High Energy Phys. **09**, 011 (2002), for a recent analysis along these lines and references therein.

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[27] For example, many of the attempts in [26] assume that the Dirac mass matrix of the neutrinos is equal to that of the up-flavor quarks ($M_\nu^D = M_u$) at GUT scale. This simple equality would be true for SO(10) if only $\mathbf{10}_H$ contributes to the fermion masses. However, the minimal Higgs system permits a $(B-L)$-dependent antisymmetric "23" and "32" entry [20] (as discussed later), which plays a crucial role in explaining why $m_\mu \neq m_s$ and why V_{cb} is so small and yet $\theta^{\mathrm{osc}}_{\nu_2\nu_3}$ is rather maximal. Such entries do not respect $M_\nu^D = M_u$.

[28] For instance, in the first paper of Ref. [26], it is found that only the solar vacuum oscillation solution gives acceptable baryon asymmetry. In the second paper, it is noted that SUSY models with full quark-lepton symmetry give too small an asymmetry, while in the third paper it is found that the just-so and SMA solutions give viable leptogenesis, but the LMA solution is strongly disfavored [based on their assumption of $M_\nu^D = M_u$ (see comments in Ref. [27])]. In the fourth paper, it is observed

that the SMA and vacuum solutions produce reasonable asymmetry, but the LMA solution produces too large an asymmetry.

[29] These have been introduced in various forms in the literature. For a sample, see, e.g., C.D. Frogatt and H.B. Nielsen, Nucl. Phys. **B147**, 277 (1979); L. Hall and H. Murayama, Phys. Rev. Lett. **75**, 3985 (1995); P. Binetruy, S. Lavignac, and P. Ramond, Nucl. Phys. **B477**, 353 (1996). In the string theory context, see, e.g., A. Faraggi, Phys. Lett. B **278**, 131 (1992).

[30] The zeros in "11," "13" and "31" elements signify that they are relatively small quantities (specified below). While the "22" elements were set to zero in Ref. [20], because they are meant to be <"23""32"/"33"$\sim 10^{-2}$ (see below), and thus unimportant for purposes of Ref. [20], they are retained here, because such small ζ^u_{22} and ζ^d_{22} [$\sim(1/3)\times 10^{-2}$ (say)] can still be important for CP violation and thus leptogenesis.

[31] For G(224), one can choose the corresponding submultiplets—that is $(1,1,15)_H$, $(1,2,\overline{4})_H$, $(1,2,4)_H$, $(2,2,1)_H$—together with a singlet S, and write a superpotential analogous to Eq. (2).

[32] If the effective nonrenormalizable operator like $\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H/M'$ is induced through exchange of states with GUT-scale masses involving renormalizable couplings, rather than through quantum gravity, M' would, however, be of order GUT scale. In this case $\langle\mathbf{45}_H\rangle/M'\sim 1$, rather than 1/10.

[33] While $\mathbf{16}_H$ has a GUT-scale VEV along the SM singlet, it turns out it can also have a VEV of EW scale along the "$\tilde{\nu}_L$" direction due to its mixing with $\mathbf{10}^d_H$, so that the H_d of MSSM is a mixture of $\mathbf{10}^d_H$ and $\mathbf{16}^d_H$. This turns out to be the origin of nontrivial CKM mixings (see Ref. [20]).

[34] The flavor charge(s) of $\mathbf{45}_H(\mathbf{16}_H)$ would get determined depending upon whether $p(q)$ is one or zero (see below).

[35] These effective nonrenormalizable couplings can of course arise through exchange of (for example) **45** in the string tower, involving renormalizable $\mathbf{16}_i\overline{\mathbf{16}}_H\mathbf{45}$ couplings. In this case, one would expect $M\sim M_{\rm string}$.

[36] P. Ginsparg, Phys. Lett. B **197**, 139 (1987); V.S. Kaplunovsky, Nucl. Phys. **B307**, 145 (1988); **B382**, 436(E) (1992).

[37] J.C. Pati, to appear in the Proceedings of the Erice 2002 School, hep-ph/0305221.

[38] The range in M_3 and M_2 is constrained by the values of $m(\nu_3)$ and $m(\nu_2)$ suggested by the atmospheric and solar neutrino data.

[39] Note that the magnitudes of η, ϵ and σ are fixed by the input quark masses. Furthermore, one can argue that the two contributions for $\theta^{\rm osc}_{\nu_2\nu_3}$ [see Eq. (8)] necessarily add to each other as long as $|y|$ is hierarchical ($\sim 1/10$) [20]. As a result, once the sign of ϵ relative to η and σ is chosen to be negative, the actual magnitudes of $V_{cb}\approx(0.044)$ and $\sin^2 2\theta^{\rm osc}_{\nu_2\nu_3}\approx 0.99$ emerge as predictions of the model [20].

[40] Note that such an operator would be allowed by the flavor symmetry defined in Eq. (3) if one sets $a=1/2$. In this case, operators such as W_{23} and W_{33} that would contribute to $\nu^\mu_L\nu^\tau_L$ and $\nu^\tau_L\nu^\tau_L$ masses would be suppressed relative to W_{12} by flavor symmetry. As pointed out by other authors [see, e.g., S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979) and Proc. XXVI Int. Conf. on High Energy Physics, Dallas, Texas, 1992; E. Akhmedov, Z. Berezhiani, and G. Senjanovic, Phys. Rev. D **47**, 3245 (1993)], nonseesaw Majorana masses of the LH neutrinos can arise directly, even in the standard model, through operators of the form $L_iL_j\Phi_H\Phi_H/M$, by utilizing quantum gravity. [For SO(10), two $\mathbf{16}_H$'s are needed additionally to violate $B-L$ by two units.] In the case of the standard model, ordinarily, one would expect $M\sim M_{\rm Planck}$. Thus one would still need to find a reason (in the context of the standard model) why (a) $M\sim M_{\rm GUT}$ and also (b) why $L_1L_2\Phi_H\Phi_H/M$ is the leading operator in its class, rather than being suppressed (due to flavor symmetries) relative to $L_3L_3\Phi_H\Phi_H/M$ (for example). Both (a) and (b) are needed for this direct nonseesaw mass to be relevant to the LMA MSW solution.

[41] A term like W_{12} can be induced in the presence of, for example, a singlet $\hat{S}$ and a ten-plet $(\hat{\mathbf{10}})$, possessing effective renormalizable couplings of the form $a_i\mathbf{16}_i\mathbf{16}_H\hat{\mathbf{10}}$, $b\hat{\mathbf{10}}\mathbf{10}_H\hat{S}$ and mass terms $\hat{M}_S\hat{S}\hat{S}$ and $\hat{M}_{10}\hat{\mathbf{10}}\hat{\mathbf{10}}$. In this case $\kappa_{12}/M^3_{\rm eff}\approx a_1a_2b^2/(\hat{M}^2_{10}\hat{M}_S)$. Setting the charge $a=1/2$ [see Eq. (3) and [40]], and assigning charges $(-3/2,5/2)$ to $(\hat{\mathbf{10}},\hat{S})$, the couplings a_1, and b would be flavor-symmetry allowed, while a_2 would be suppressed but so also would be the mass of $\hat{\mathbf{10}}$ compared to the GUT scale. One can imagine that $\hat{S}$ on the other hand acquires a GUT-scale mass through for example the Dine-Seiberg-Witten mechanism, violating the U(1)-flavor symmetry. One can verify that in such a picture, one would obtain $\kappa_{12}/M^3_{\rm eff}\sim 1/M^3_{\rm GUT}$.

[42] For instance, consider the superpotential for $\mathbf{45}_H$ only: $W(\mathbf{45}_H)=M_{45}\mathbf{45}^2_H+\lambda\mathbf{45}^4_H/M$, which yields (setting $F_{\mathbf{45}_H}=0$), either $\langle\mathbf{45}_H\rangle=0$, or $\langle\mathbf{45}_H\rangle^2=-[2M_{45}M/\lambda]$. Assuming that "other physics" would favor $\langle\mathbf{45}_H\rangle\neq 0$, we see that $\langle\mathbf{45}_H\rangle$ would be pure imaginary, if the square bracket is positive, with all parameters being real. In a coupled system, it is conceivable that $\langle\mathbf{45}_H\rangle$ in turn would induce phases (other than "0" and π) in some of the other VEV's as well, and may itself become complex rather than pure imaginary.

[43] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[44] Within the framework developed in Ref. [23], the CP violating phases entering into the SUSY contributions (for example those entering into the squark-mixings) also arise entirely through phases in the fermion mass matrices.

[45] An intriguing feature is the prominence of the $\delta^{23}_{RR}(\tilde{b}_R\to\tilde{s}_R)$ parameter which gets enhanced in part because of the largeness of the ν_2-ν_3 oscillation angle. This leads to large departures from the predictions of the standard model, especially in transitions such as $B_s\to\overline{B_s}$ and $B_d\to\Phi K_s(b\to s\overline{s}s)$ [23]. This feature has independently been noted recently by D. Chang, A. Massiero, and H. Murayama, Phys. Rev. D **67**, 075013 (2003).

[46] As an example, one such fit with complex parameters assigns [23] $\sigma=0.10-0.012i$, $\eta=0.12-0.05i$, $\epsilon=-0.095$, $\eta'=4.0\times 10^{-3}$, $\epsilon'=1.54\times 10^{-4}e^{i\pi/4}$, $\zeta^u_{22}=1.25\times 10^{-3}e^{i\pi/9}$ and $\zeta^d_{22}=4\times 10^{-3}e^{i\pi/2}$, $\mathcal{M}^0_u\approx 110$ GeV, $\mathcal{M}^0_D\approx 1.5$ GeV, $y\approx -1/17$ [compare with Eq. (7) for which $\zeta^u_{22}=\zeta^d_{22}=0$]. One obtains as outputs $m_{b,s,d}\approx$(5 GeV, 132 MeV, 8 MeV), $m_{c,u}\approx$(1.2 GeV, 4.9 MeV), $m_{\mu,e}\approx$(102 MeV, 0.4 MeV) with $m_{t,\tau}\approx$(167 GeV, 1.777 GeV), $(V_{us},V_{cb},|V_{ub}|,|V_{td}|)\approx(0.217,0.044,0.0029,0.011)$, while preserving the predictions for neutrino masses and oscillations as in Eq. (8). The above serves to demonstrate that complexification of parameters of the sort presented above can preserve the successes of Eq. (8) [20]. This particular case leads to $\eta_W=0.29$ and $\rho_W=-0.187$ [23], to be compared with the corresponding stan-

dard model values (obtained from ϵ_K, V_{ub} and Δm_{Bd}) of $(\eta_W)_{\mathrm{SM}} \approx 0.33$ and $(\rho_W)_{\mathrm{SM}} \approx +0.2$. The consistency of such values for η_W and ρ_W (especially reversal of the sign of ρ_W compared to the SM value), in the light of having both standard model and SUSY-contributions to *CP* and flavor-violations, and their distinguishing tests, are discussed in Ref. [23].

[47] For reviews, see Chaps. 6 and 8 in E.W. Kolb and M.S. Turner, *The Early Universe* [6].

[48] J. Ellis, J.E. Kim, and D. Nanopoulos, Phys. Lett. **145B**, 181 (1984); M.Yu. Khlopov and A. Linde, *ibid.* **138B**, 265 (1984); E. Holtmann, M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D **60**, 023506 (1999).

[49] B.A. Campbell, S. Davidson, and K.A. Olive, Nucl. Phys. **B399**, 111 (1993).

[50] L. Covi, E. Roulet, and F. Vissani, Phys. Lett. B **384**, 169 (1996).

[51] M. Plumacher, Nucl. Phys. **B530**, 207 (1998).

[52] P. Di Bari, hep-ph/0211175; W. Buchmuller, P. Di Bari, and M. Plumacher, Nucl. Phys. **B643**, 367 (2002).

[53] The factor 0.7 in Eq. (13) [instead of 1 in Eq. (14) of Ref. [52]] is an estimate that incorporates the modification needed for SUSY corresponding to a doubling of N_1-decay width owing to the presence of both $l+H$ and $\tilde{l}+\tilde{H}$-modes and an increase of g^* from 106 for the standard model to 228 for SUSY.

[54] One can verify that $K \equiv [\Gamma(N_1)/2H]_{T=M_1} \approx (0.37)\{M_{\mathrm{Pl}}/[1.66\sqrt{g^*}(8\pi v^2)]\}\tilde{m}_1 \approx 234(\tilde{m}_1/\mathrm{eV})$, where 0.37 denotes the usual time-dilation factor, $g^*(\text{for SUSY}) \approx 228$ and $v \approx 174$ GeV. For comparison, we note that if one includes only inverse decays (thus neglecting $\Delta L \neq 0$ scatterings) in the Boltzmann equations, one would obtain $\kappa \approx 0.3/[K(\ln K)^{0.6}]$ for $K > 10$ [47], and $\kappa \approx 1/2K$ for $1 \lesssim K \lesssim 10$. As pointed out in Ref. [52], these expressions, frequently used in the literature, however, tend to overestimate κ by nearly a factor of 7. In what follows, we will therefore use Eq. (13) to evaluate κ.

[55] M. Plumacher, Z. Phys. C **74**, 549 (1997).

[56] For a specific scenario of inflation and leptogenesis in the context of SUSY G(224), see R. Jeannerot, S. Khalil, G. Lazarides, and Q. Shafi, J. High Energy Phys. **10**, 012 (2000), and references therein. As noted in this paper, with the VEV's of $(1,2,4)_H$ and $(1,2,\overline{4})_H$ breaking G(224) to the standard model, and also driving inflation, just the COBE measurement of $\delta T/T \approx 6.6 \times 10^{-6}$, interestingly enough, implies that the relevant VEV should be of order 10^{16} GeV. In this case, the inflation made of two complex scalar fields [i.e., $\theta = (\delta\tilde{\nu}_H^c + \delta\bar{\tilde{\nu}}_H^c)/\sqrt{2}$, given by the fluctuations of the Higgs fields, and a singlet S], each with a mass $\sim 10^{12}$–10^{13} GeV, would decay *directly* into a pair of heavy RH neutrinos—that is into N_2N_2 (or N_1N_1) if $m_{\mathrm{infl}} > 2M_2$ (or $2M_1$). The subsequent decays of N_2's (or N_1's), thus produced, into $l+\Phi_H$ and $\overline{l}+\overline{\Phi}_H$ would produce lepton-asymmetry *during the process of reheating*. I will comment later on the consistency of this possibility with the fermion mass-pattern exhibited in Sec. II. I would like to thank Qaisar Shafi for a discussion on these issues.

[57] K. Kumekawa, T. Moroi, and T. Yanagida, Prog. Theor. Phys. **92**, 437 (1994); G.F. Giudice, M. Peleso, A. Riotto, and T. Tkachev, J. High Energy Phys. **08**, 014 (1999); T. Asaka, K. Hamaguchi, M. Kawasaki, and T. Yanagida, Phys. Lett. B **464**, 12 (1999).

[58] Incorporating such an effective superpotential in accord with the assignment of flavor-changes suggested in Eq. (3) and Ref. [41] would involve two additional singlets with appropriate charges. The $(\mathrm{VEV})^2$ of one or both of these may represent M^2. Derivation of such a picture with appropriate flavor-charge assignments from an underlying (string/M) theory is of course beyond the state of the art at present.

[59] Note that for this nonthermal case, since the gravitino-constraint is relaxed, N_1 can be chosen heavier than for the case considered before (the thermal case), still in accord with Eq. (9). Since $Y_B \propto \epsilon_1 T_{\mathrm{RH}}/\mathcal{M}_{\mathrm{infl}}$, while $\epsilon_1 \propto (M_1/M_2)$, $T_{\mathrm{RH}} \propto M_1(\mathcal{M}_{\mathrm{infl}})^{1/2}$, and $\mathcal{M}_{\mathrm{infl}} \propto \lambda$, we see that $Y_B \propto (M_1^2/M_2)/\sqrt{\lambda}$, for a constant M, for the case of nonthermal leptogenesis.

[60] Note that the effective phase ϕ_{21}, relevant to leptogenesis, depends on the phases in both the Dirac (M_ν^D) and the Majorana (M_R^ν) mass matrices of the neutrinos. Thus, in general, it is quite distinct from the phase(s) entering into observed *CP* violations in the K and the B systems.