Dirac-Born-Infeld action on the tachyon kink and vortex

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The tachyon effective field theory describing the dynamics of a non-Bogomol'nyi-Prasad-Sommerfield (BPS) D-brane in superstring theory has an infinitely thin but finite tension kink solution describing a codimension one BPS D-brane. We study the world-volume theory of massless modes on the kink, and show that the world volume action has precisely the Dirac-Born-Infeld (DBI) form without any higher derivative corrections. We generalize this to a vortex solution in the effective field theory on a brane-antibrane pair. As in the case of the kink, the vortex is infinitely thin, has finite energy density, and the world-volume action on the vortex is again given exactly by the DBI action on a BPS D-brane. We also discuss the coupling of fermions and restoration of supersymmetry and κ symmetry on the world volume of the kink. The absence of higher derivative corrections to the DBI action on the soliton implies that all such corrections are related to higher derivative corrections to the original effective action on the world volume of a non-BPS D-brane or brane-antibrane pair.

I. INTRODUCTION

The study of various aspects of tachyon dynamics on a non-Bogomol'nyi-Prasad-Sommerfield (BPS) D-brane of type IIA or IIB superstring field theory has led to some understanding of the tachyon dynamics near the tachyon vacuum. The proposed tachyon effective action, describing the dynamics of the tachyon field on a non-BPS Dp-brane of type IIA or IIB superstring theory, is given by [1–6]:[1]

$$S = \int d^{p+1}x \mathcal{L},$$

$$\mathcal{L} = -V(T)\sqrt{-\det \mathbf{A}}, \tag{1.1}$$

where

$$\mathbf{A}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \partial_\mu Y^I \partial_\nu Y^I + F_{\mu\nu}, \tag{1.2}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1.3}$$

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[1]Although we shall carry out our analysis for this action, our results are valid for a more general class of actions discussed in [6–9], where the Lagrangian density in the absence of gauge and massless scalar fields takes the form $-V(T)F(\eta^{\mu\nu}\partial_\mu T\partial_\nu T)$, and $F(u) \sim u^{1/2}$ for large u. This follows from the fact that for the solutions we shall be considering, $u \equiv \eta^{\mu\nu}\partial_\mu T\partial_\nu T$ is large everywhere, and hence in this regime all these actions reduce to Eq. (1.1). Generalization of the action $-\int d^{p+1}x V(T)F(\eta^{\mu\nu}\partial_\mu T\partial_\nu T)$ to include the world-volume gauge and scalar fields can be carried out by replacing $\eta^{\mu\nu}$ by the open string metric $G^{\mu\nu}$ [10,11], and by multiplying the action by an overall factor of $\sqrt{-\det(g_{\mu\nu}+F_{\mu\nu})}$, $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu Y^I \partial_\nu Y^I$ being the induced closed string metric on the D-brane world volume. This class of actions includes the action proposed in [7–9,12,13], motivated by boundary string field theory [14,15].

A_μ and Y^I for $0 \leq \mu, \nu \leq p$, $(p+1) \leq I \leq 9$ are the gauge and the transverse scalar fields on the world volume of the non-BPS brane, and T is the tachyon field. $V(T)$ is the tachyon potential which is symmetric under $T \to -T$, has a maximum at $T=0$, and has its minimum at $T = \pm \infty$ where it vanishes. We are using the convention where $\eta = \mathrm{diag}(-1,1,\ldots,1)$ and the fundamental string tension has been set equal to $(2\pi)^{-1}$ (i.e. $\alpha'=1$).

The effective field theory described by the action (1.1) is expected to be a good description of the system under the condition that (1) T is large, and (2) the second and higher derivatives of T are small. A kink solution in the full tachyon effective field theory, which is supposed to describe a BPS D-$(p-1)$-brane [16–18], interpolates between the vacua at $T = \pm \infty$, and hence T must pass through 0. Thus *a priori* we would expect that higher derivative corrections to the action (1.1) will be needed to provide a good description of the D-$(p-1)$-brane as a kink solution. Nevertheless it is known that the energy density on the kink in the theory described by the action (1.1) is localized strictly on a codimension one surface [7–9,19] as in the case of a BPS D-$(p-1)$-brane. We show that the world-volume theory on this kink solution is also given precisely by the Dirac-Born-Infeld (DBI) action on a BPS D-$(p-1)$-brane. This agreement continues to hold even after including the world-volume fermion fields in the action, and we recover the expected supersymmetry and κ symmetry on the BPS D-$(p-1)$-brane world volume [20–24]. Thus contrary to expectation, the kink solution of the effective field theory does provide a good description of the D-$(p-1)$-brane even without taking into account higher derivative corrections.

There have been several previous attempts to analyze the dynamics of fluctuations on a tachyon kink solution. Reference [7] analyzed the world-volume theory of the fluctuations on the kink. However, in this study they restricted to the analysis of small fluctuations. We shall not put such restrictions, since in order to see the full DBI action we need to keep terms involving arbitrary power of the world-volume

fields. Reference [25] analyzed the world-volume action on the kink keeping the non-linear terms, but including only the fluctuations in the transverse scalar field. Reference [26] addresses the problem of getting the DBI action on the soliton from the conformal field theory viewpoint, whereas Ref. [27] discusses construction of various special classical solutions of the tachyon effective field theory around the kink solution, without doing a general analysis of the equations of motion around this background. A general approach to getting the DBI action on the kink and vortex solutions has been described in [28,29]. These papers, however, worked with a very general form of the tachyon effective action, and arrived at the DBI action after ignoring the higher derivative terms. In contrast, we work with a specific form of the action given in Eq. (1.1), but given this form, we make no further approximation in our analysis. In particular, we keep all powers of fields and all derivative terms, and nevertheless arrive at the DBI action without any higher derivative terms. We should, of course, keep in mind that the action (1.1) itself is at best an approximate action for the tachyon in string theory, and corrections to this action will certainly modify the world-volume action on the kink. The significance of our result is that all such corrections involving higher derivative terms on the world-volume action of the BPS D-$(p-1)$-brane must come from an explicit addition of such corrections to the world-volume action of the non-BPS D-p-brane. This suggests a sense in which the action (1.1) is a "low energy effective action,"—namely that it reproduces the low energy effective action on the world volume of the soliton without any correction terms. In fact we also argue that in the world-volume theory on the kink, the would be massive modes, obtained by analyzing the linearized equations of motion of various fields around the kink solution [7], disappear when we take into account the effect of the non-linear terms. Thus the only perturbative excitations on the kink world volume are the massless degrees of freedom.

We also generalize our analysis to the construction of a vortex solution on a Dp-brane–anti-Dp-brane pair. For this we begin with a generalization of the tachyon effective action on the brane-antibrane pair—this is done in a way that satisfies various known consistency requirements for such an action. We then construct the vortex solution, and find that it has finite energy density per unit $(p-2)$-volume; however, the energy density is strictly localized on a codimension 2 subspace. Furthermore, the world-volume theory on the vortex is given by the DBI action expected for a BPS D-$(p-2)$-brane.

The rest of the paper is organized as follows. In Sec. II we review construction of the kink solution on a non-BPS D-brane. In Sec. III we analyze the world-volume theory of the bosonic fields on the kink. In Sec. IV we discuss the coupling of fermions on the non-BPS and BPS D-brane world volume, and show how the supersymmetry and κ symmetry, expected to be present on the world-volume action of a BPS D-$(p-1)$-brane, appear in the world-volume action on the tachyon kink in a non-BPS D-p-brane. Section V is devoted to construction of the vortex solution on the brane-

antibrane pair, and in Sec. VI we construct the world-volume action on the vortex. We conclude with a few general comments in Sec. VII.

II. THE KINK SOLUTION

The construction of the kink solution follows [4,7–9]. The energy momentum tensor[2] associated with the action (1.1) is given by [30,5,31]

$$T^{\mu\nu} = -V(T)(\mathbf{A}^{-1})_S^{\mu\nu}\sqrt{-\det \mathbf{A}}, \qquad (2.1)$$

where the subscript S denotes the symmetric part of a matrix. In order to construct a kink solution, we look for a solution for which the tachyon depends on one spatial direction $x \equiv x^p$ and is time independent, and furthermore, the gauge fields and the transverse scalar fields are set to zero. For such a background the energy momentum tensor is given by

$$T_{xx} = -V(T)/\sqrt{1+(\partial_x T)^2}, \qquad T_{\alpha x} = 0,$$

$$T_{\alpha\beta} = -V(T)\sqrt{1+(\partial_x T)^2}\,\eta_{\alpha\beta} \quad \text{for} \quad 0 \leq \alpha,\beta \leq (p-1). \qquad (2.2)$$

The energy-momentum conservation gives

$$\partial_x T_{xx} = 0. \qquad (2.3)$$

Thus T_{xx} is independent of x. Since for a kink solution $T \to \pm\infty$ as $x \to \pm\infty$, and $V(T) \to 0$ in this limit, T_{xx} vanishes as $x \to \infty$. Thus T_{xx} must vanish for all x. This, in turn, shows that we must have

$$T = \pm\infty, \quad \text{or} \quad \partial_x T = \infty \quad \text{(or both)} \quad \text{for all } x. \quad (2.4)$$

Clearly the solution looks singular. We shall now see that despite this singularity, the solution has finite energy density which is independent of the way we regularize the singularity. Also the energy density is localized on a codimension 1 subspace, just as is expected of a D$(p-1)$-brane [7,9]. For this let us consider the field configuration

$$T(x) = f(ax), \qquad (2.5)$$

where $f(u)$ satisfies

$$f(-u) = -f(u), \quad f'(u) > 0 \quad \forall \quad u, \quad f(\pm\infty) = \pm\infty, \qquad (2.6)$$

but is otherwise an arbitrary function of its argument u. a is a constant that we shall take to ∞ at the end. In this limit we have $T = \infty$ for $x > 0$ and $T = -\infty$ for $x < 0$. Thus the kink is singular as expected. Equation (2.2) gives the non-zero components of $T_{\mu\nu}$ for this background to be

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[2]In writing down the expression for the energy momentum tensor, it will be understood that these are localized on the plane of the original D-p-brane by a position space delta function in the transverse coordinates. Also only the components of the energy-momentum tensor along the world volume of the original D-p-brane are non-zero.

$$T_{xx} = -V(f(ax))/\sqrt{1+a^2(f'(ax))^2},$$

$$T_{\alpha\beta} = -V(f(ax))\sqrt{1+a^2(f'(ax))^2}\,\eta_{\alpha\beta}. \qquad (2.7)$$

Clearly in the $a \to \infty$ limit, T_{xx} vanishes everywhere since the numerator vanishes (except at $x=0$) and the denominator becomes infinite at $x=0$. Hence the conservation law (2.3) is automatically satisfied.

Let us now check that this configuration satisfies the full set of equations of motion. The non-trivial components of the equations of motion are

$$\partial_x\left(\frac{V(T)\partial_x T}{\sqrt{1+(\partial_x T)^2}}\right) - V'(T)\sqrt{1+(\partial_x T)^2}=0. \qquad (2.8)$$

Taking $T=f(ax)$ we get the left hand side to be

$$\partial_x\left(\frac{V(f(ax))af'(ax)}{\sqrt{1+a^2(f'(ax))^2}}\right) - V'(f(ax))\sqrt{1+a^2(f'(ax))^2}$$

$$= V'(f(ax))\frac{a^2(f'(ax))^2}{\sqrt{1+a^2(f'(ax))^2}} + \frac{V(f(ax))a^2f''(ax)}{[1+a^2(f'(ax))^2]^{3/2}}$$

$$- V'(f(ax))\sqrt{1+a^2(f'(ax))^2} = \mathcal{O}\!\left(\frac{1}{a}\right), \qquad (2.9)$$

assuming that $V(f(y))f''(y)/(f'(y))^3$ is finite everywhere. Thus in the $a \to \infty$ limit the configuration satisfies the equations of motion.

We shall now compute the energy density associated with this solution. From Eq. (2.7) we see that in the $a \to \infty$ limit T_{xx} vanishes, and we can write $T_{\alpha\beta}$ as

$$T_{\alpha\beta} = -a\,\eta_{\alpha\beta}V(f(ax))f'(ax). \qquad (2.10)$$

Thus the integrated $T_{\alpha\beta}$, associated with the codimension 1 soliton, is given by

$$T_{\alpha\beta}^{kink} = -a\,\eta_{\alpha\beta}\int_{-\infty}^{\infty}dx\,V(f(ax))f'(ax) = -\eta_{\alpha\beta}\int_{-\infty}^{\infty}dy\,V(y), \qquad (2.11)$$

where $y=f(ax)$. Thus $T_{\alpha\beta}^{kink}$ depends only on the form of $V(y)$ and not on the shape of the function $f(u)$ used to describe the soliton [9,27]. It is also clear from the exponential falloff in $V(y)$ for large y that most of the contribution to $T_{\alpha\beta}^{kink}$ is contained within a finite range of y. From the relation $y=f(ax)$ we see that this means that the contribution comes from a region of x integral of width $1/a$ around $x=0$. In the $a \to \infty$ limit such a distribution approaches a δ function. Thus the $(p+1)$-dimensional energy-momentum tensor associated with this solution is given by

$$T_{xx}=0, \quad T_{\alpha\beta}=-\eta_{\alpha\beta}\delta(x)\int_{-\infty}^{\infty}dy\,V(y). \qquad (2.12)$$

This is precisely what is expected of a D-$(p-1)$-brane, provided the integral $\int_{-\infty}^{\infty}dy\,V(y)$ equals the tension of the

D-$(p-1)$-brane. For comparison, we also recall that $V(0)$ denotes the tension of a Dp-brane. These relations can be written as[3]

$$\mathcal{T}_p = V(0), \quad \mathcal{T}_{p-1}=\int_{-\infty}^{\infty}V(y)dy. \qquad (2.13)$$

If we also require that the tachyon around $T=0$ has mass^2 $=-\frac{1}{2}$, we get [7] $V''(0)/V(0)=-1/2$. However, higher derivative contribution to the action could modify this result.

Incidentally, we might note that one possible choice of the function $f(u)$ is $f(u)=u$. For this choice, the second and higher derivatives of the tachyon field vanish everywhere.[4] Thus the tachyon satisfies at least one of the two conditions under which the effective action (1.1) is expected to be valid. The agreement between the properties of the soliton and those of a D-$(p-1)$-brane suggests that corrections to the action (1.1) organize themselves in a way so as not to affect the desired features of the kink solution of (1.1).

We conclude this section by giving an intuitive argument for the infinite spatial gradient of T. From Eq. (2.2) we see that the total energy associated with a static configuration depending on only one spatial direction x, and interpolating between $T=\pm\infty$ at $x=\pm\infty$, is given by

$$E=\int_{-\infty}^{\infty}dx\,V(T(x))\sqrt{1+(\partial_x T)^2}$$

$$\geq \int_{-\infty}^{\infty}dx\,V(T(x))|\partial_x T|$$

$$\geq \int_{-\infty}^{\infty}dx\,V(T(x))\partial_x T=\int_{-\infty}^{\infty}dy\,V(y). \qquad (2.14)$$

The right hand side of Eq. (2.14) is independent of the choice of $T(x)$. Since a static solution of the equations of motion must minimize (extremize) the total energy, we conclude that in order to get a solution of the equations of motion the bound given in Eq. (2.14) must be saturated. This requires $|\partial_x T|\to\infty$ and $\partial_x T>0$ everywhere. This is precisely the result we obtained by explicitly analyzing the equations of motion.

III. STUDY OF FLUCTUATIONS AROUND THE KINK

In this section we shall study fluctuations of various bosonic fields around the kink background and compare the

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[3]For more general actions of the kind discussed in footnote I, if F is normalized such that $F(0)=1$, and if for large u, $F(u)/u^{1/2} \simeq C$, then we have $\mathcal{T}_p=V(0)$, $\mathcal{T}_{p-1}=C\int_{-\infty}^{\infty}V(y)dy$. $V(T)$ and $F(u)$, motivated by boundary string field theory, automatically gives the correct ratio of the Dp-brane and D-$(p-1)$-brane tensions [7].

[4]We note the similarity between such solutions and those in boundary superstring field theory [14]. This of course is consistent with the proposal that the effective action from boundary string field theory has the general form given in footnote I [7,8,12,13].

effective action describing the dynamics of these fluctuations to the expected DBI action on the D-$(p-1)$-brane world volume. First as a warm-up exercise we shall consider the dynamics of the translation zero mode along the x direction, keeping the gauge fields A_μ and the transverse scalar fields Y^I to zero. Such fluctuations correspond to fluctuation of T of the form:

$$T(x,\xi)=f\{a[x-t(\xi)]\}, \tag{3.1}$$

where we have denoted by $\{\xi^\alpha\}$ for $0\leq\alpha\leq(p-1)$ the coordinates tangential to the kink world volume. Here $t(\xi)$ is the $(p-1,1)$-dimensional field associated with the translational zero mode of the kink.[5] For this configuration,

$$-\det(\mathbf{A})=(1+\eta^{\mu\nu}\partial_\mu T\partial_\nu T)=1+a^2(f')^2(1+\eta^{\alpha\beta}\partial_\alpha t\partial_\beta t), \tag{3.2}$$

where for brevity we have denoted $f'\{a[x-t(\xi)]\}$ by f', and $f\{a[x-t(\xi)]\}$ by f. Substituting this into the action (1.1) we get, for $a\to\infty$:

$$S=-\int d^p\xi\int dx V(f)af'\sqrt{1+\eta^{\alpha\beta}\partial_\alpha t\partial_\beta t}. \tag{3.3}$$

We now make a change of variables from x to y:

$$y=f\{a[x-t(\xi)]\}. \tag{3.4}$$

Equation (3.3) may then be rewritten as

$$S=-\int d^p\xi\int_{-\infty}^{\infty}dy V(y)\sqrt{1+\eta^{\alpha\beta}\partial_\alpha t\partial_\beta t}. \tag{3.5}$$

Performing the y integral, and using Eq. (2.13) we get

$$S=-\mathcal{T}_{p-1}\int d^p\xi\sqrt{1+\eta^{\alpha\beta}\partial_\alpha t\partial_\beta t}. \tag{3.6}$$

This is precisely the action involving the scalar field t associated with the coordinate x transverse to a D-$(p-1)$-brane, lying in the $\xi^1,\ldots,\xi^{p-1}$ plane. For the boundary string field theory action, this analysis was carried out previously in [25].

Note, however, that this does not yet establish that the dynamics of the kink is described by the action (3.6). In order to do so, we need to establish that given any solution of the equations of motion derived from Eq. (3.6), it will produce a solution of the original equations of motion derived from the action (1.1) under the identification (3.1). Put another way, since S given in (1.1) reduces to that given in (3.6) when (3.1) holds, we already know that given a solution of the equations of motion of (3.6), δS vanishes for any variation of T that is induced due to a variation of $t(\xi)$ through (3.1). What needs to be shown is that δS also vanishes for a δT with more general x dependence that is not

necessarily induced due to a variation $\delta t(\xi)$ of t. For this we need to look at the general equation of motion of T following from Eq. (1.1). It is

$$\eta^{\alpha\beta}\partial_\alpha\left(\frac{V(T)\partial_\beta T}{\sqrt{1+\eta^{\mu\nu}\partial_\mu T\partial_\nu T}}\right)$$
$$+\partial_x\left(\frac{V(T)\partial_x T}{\sqrt{1+\eta^{\mu\nu}\partial_\mu T\partial_\nu T}}\right)-V'(T)\sqrt{1+\eta^{\mu\nu}\partial_\mu T\partial_\nu T}=0. \tag{3.7}$$

Substituting (3.1) into (3.7), and using the equations of motion of $t(\xi)$ derived from Eq. (3.6) we can easily verify that the left hand side of (3.7) vanishes in the $a\to\infty$ limit. This, in turn, shows that the dynamics of the field $t(\xi)$ is described precisely by the action (3.6).

Let us now turn to the inclusion of the gauge fields A_i and the scalar fields Y^I. We expect that appropriate fluctuations in these fields will be responsible for the transverse scalar field excitations y^I and gauge field excitations a_α on the D-$(p-1)$-brane. Thus the first step is to make a suitable ansatz for the fluctuations in the $(p+1)$-dimensional fields A_μ and Y^I in terms of the $(p-1+1)$-dimensional fields $a_\alpha(\xi)$ and $y^I(\xi)$. We make the following ansatz:

$$A_x(x,\xi)=0, \quad A_\alpha(x,\xi)=a_\alpha(\xi), \quad Y^I(x,\xi)=y^I(\xi), \tag{3.8}$$

together with Eq. (3.1). In other words we take the fields A_μ and Y^I to be independent of x. This seems surprising at first sight, since the fluctuations on a kink are expected to be localized around $x=0$ where the kink is sitting. We note, however, that the dynamics of the gauge fields A_μ and the scalar fields Y^I away from the location of the kink is essentially trivial [1,32,30,33,34], and hence although we allow fluctuations in A_μ and Y^I far away from the location of the kink, the energy momentum tensor associated with such fluctuations is localized in the plane of the brane due to the explicit factor of $V(T)$ in Eq. (2.1) which vanishes away from the plane of the kink.[6] We shall discuss this issue further at the end of this section.

The next step will be to show that with the ansatz (3.1), (3.8) the action (1.1) reduces to the DBI action on a BPS D-$(p-1)$ brane. Computation of $\mathbf{A}_{\mu\nu}$ defined in Eq. (1.2) with this ansatz yields

$$\mathbf{A}_{xx}=1+a^2(f')^2, \quad \mathbf{A}_{x\alpha}=\mathbf{A}_{\alpha x}=-a^2(f')^2\partial_\alpha t,$$

$$\mathbf{A}_{\alpha\beta}=(a^2(f')^2-1)\partial_\alpha t\partial_\beta t+\mathbf{a}_{\alpha\beta}, \tag{3.9}$$

where $f\equiv f\{a[x-t(\xi)]\}$, $f'\equiv f'\{a[x-t(\xi)]\}$, and

$$\mathbf{a}_{\alpha\beta}=\eta_{\alpha\beta}+f_{\alpha\beta}+\partial_\alpha y^I\partial_\beta y^I+\partial_\alpha t\partial_\beta t, \quad f_{\alpha\beta}=\partial_\alpha a_\beta-\partial_\beta a_\alpha. \tag{3.10}$$

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[5]Since the soliton solution is infinitely thin, we do not need to rescale the argument of f by $\sqrt{1+\partial^\alpha t\partial_\alpha t}$ as in [28].

[6]Only exceptions to this arises when the field strengths are at their critical values [35,30,33].

We can simplify the evaluation of det $\mathbf{A}$ by adding appropriate multiples of the first row and first column to other rows and columns. More specifically, we define

$$\hat{\mathbf{A}}_{\mu\beta} = \mathbf{A}_{\mu\beta} + \mathbf{A}_{\mu x}\partial_\beta t, \quad \hat{\mathbf{A}}_{\mu x} = \mathbf{A}_{\mu x},$$

$$\tilde{\mathbf{A}}_{\alpha\nu} = \hat{\mathbf{A}}_{\alpha\nu} + \hat{\mathbf{A}}_{x\nu}\partial_\alpha t, \quad \tilde{\mathbf{A}}_{x\nu} = \hat{\mathbf{A}}_{x\nu}.$$
$$(3.11)$$

Clearly this operation does not change the determinants, so we have

$$\det(\mathbf{A}) = \det(\hat{\mathbf{A}}) = \det(\tilde{\mathbf{A}}). \quad (3.12)$$

On the other hand, we have, from Eqs. (3.9), (3.11),

$$\tilde{\mathbf{A}}_{xx} = 1 + a^2(f')^2, \quad \tilde{\mathbf{A}}_{x\alpha} = \tilde{\mathbf{A}}_{\alpha x} = \partial_\alpha t,$$

$$\tilde{\mathbf{A}}_{\alpha\beta} = \mathbf{a}_{\alpha\beta}. \quad (3.13)$$

Using Eqs. (3.12), (3.13), we get

$$\det(\mathbf{A}) = a^2(f')^2 \left[\det\mathbf{a} + \mathcal{O}\left(\frac{1}{a^2}\right) \right]. \quad (3.14)$$

Substituting this into Eq. (1.1), we get, in the $a\rightarrow\infty$ limit,

$$S = -\int d^p\xi \int dx V(f) a f' \sqrt{-\det\mathbf{a}}. \quad (3.15)$$

Making the change of variables (3.4) and using (2.13) we can write this as

$$S = -\mathcal{T}_{p-1}\int d^p\xi\sqrt{-\det\mathbf{a}}, \quad (3.16)$$

with $\mathbf{a}_{\alpha\beta}$ given by Eq. (3.10). This is precisely the world-volume action on a BPS D-$(p-1)$-brane if we identify the field t as the coordinate y^p associated with the pth direction.

In order to establish that the dynamics of the kink is described by the action (3.16), we now need to show that any solution of the equations of motion derived from the action (3.16) also provides a solution of the full $(p+1)$-dimensional equations of motion. The p-dimensional equations, derived from Eq. (3.16) are

$$\partial_\alpha[(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\beta t\sqrt{-\det\mathbf{a}}] = 0,$$

$$\partial_\alpha[(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\beta y^I\sqrt{-\det\mathbf{a}}] = 0,$$

$$\partial_\alpha[(\mathbf{a}^{-1})_A^{\alpha\beta}\sqrt{-\det\mathbf{a}}] = 0, \quad (3.17)$$

where the subscripts S and A denote the symmetric and antisymmetric components of a matrix, respectively. On the other hand, the $(p+1)$-dimensional equations, which need to be verified, are

$$\partial_\mu[V(T)(\mathbf{A}^{-1})_S^{\mu\nu}\partial_\nu T\sqrt{-\det\mathbf{A}}] - V'(T)\sqrt{-\det\mathbf{A}} = 0,$$

$$\partial_\mu[V(T)(\mathbf{A}^{-1})_S^{\mu\nu}\partial_\nu Y^I\sqrt{-\det\mathbf{A}}] = 0,$$

$$\partial_\mu[V(T)(\mathbf{A}^{-1})_A^{\mu\nu}\sqrt{-\det\mathbf{A}}] = 0. \quad (3.18)$$

Equations (3.1) and (3.8) expresses the $(p+1)$-dimensional fields in terms of p-dimensional fields. We also need expressions for $\mathbf{A}^{-1}$ and det$(\mathbf{A})$ in terms of $\mathbf{a}_{\alpha\beta}$. These are summarized in the relations:

$$(\mathbf{A}^{-1})^{xx} \simeq (\mathbf{a}^{-1})^{\alpha\beta}\partial_\alpha t\partial_\beta t, \quad (\mathbf{A}^{-1})^{x\alpha} \simeq \partial_\beta t(\mathbf{a}^{-1})^{\beta\alpha},$$

$$(\mathbf{A}^{-1})^{\alpha x} \simeq (\mathbf{a}^{-1})^{\alpha\beta}\partial_\beta t, \quad (\mathbf{A}^{-1})^{\alpha\beta} \simeq (\mathbf{a}^{-1})^{\alpha\beta},$$
$$(3.19)$$

together with Eq. (3.14). All the relations given in Eq. (3.19) hold up to corrections of order $1/a^2$.

We shall now verify that Eqs. (3.17), together with Eqs. (3.1), (3.8), implies Eqs. (3.18). Besides the relations (3.14), (3.19), an identity that is particularly useful in carrying out this analysis is

$$\partial_\alpha F[x - t(\xi)] = -\partial_\alpha t\partial_x F[x - t(\xi)], \quad (3.20)$$

for any function F. We begin our discussion with the verification of the second equation of (3.18). Using Eqs. (3.1), (3.8), (3.14) and (3.19) we can express the left hand side of this equation as

$$\partial_x\{V(T)(\mathbf{A}^{-1})_S^{x\beta}\partial_\beta Y^I\sqrt{-\det\mathbf{A}}\}$$
$$+ \partial_\alpha\{V(T)(\mathbf{A}^{-1})_S^{\alpha\beta}\partial_\beta Y^I\sqrt{-\det\mathbf{A}}\}$$
$$\simeq \partial_x\{V(f)\partial_\alpha t\partial_\beta y^I(\mathbf{a}^{-1})_S^{\alpha\beta}af'\sqrt{-\det\mathbf{a}}\}$$
$$+ \partial_\alpha\{V(f)(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\beta y^Iaf'\sqrt{-\det\mathbf{a}}\}$$
$$= (\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\beta y^I\sqrt{-\det\mathbf{a}}\{\partial_\alpha t\partial_x(V(f)af')$$
$$+ \partial_\alpha(V(f)af')\} = 0, \quad (3.21)$$

where in going from the second to the third line we have used the second equation in (3.17), and in the last step we have used Eq. (3.20). Note, however, that only terms of order a^2 and a cancel, leaving behind a contribution of order 1. These finite contributions come, for example, from product of $\mathcal{O}(a^{-2})$ corrections to the right hand side of Eqs. (3.14), (3.19) with the $\mathcal{O}(a^2)$ contribution from $\partial_x(V(f)af')$. However, since $V(T)$ is non-zero only within a range of order $1/a$ in the x space, the contribution to a variation δS in the action due to the finite terms in the equations of motion will be of order $1/a$ for any finite δY^I. This goes to zero in the $a\rightarrow\infty$ limit, and hence we conclude that the y^I equations of motion given in (3.17) implies $\delta S = 0$ for arbitrary finite δY^I.

Verification of the third equation of (3.18) proceeds in the same way. For $\nu = \beta$ the left hand side of this equation is given by

$$\partial_x\{V(T)(\mathbf{A}^{-1})_A^{x\beta}\sqrt{-\det\mathbf{A}}\}+\partial_\alpha\{V(T)(\mathbf{A}^{-1})_A^{\alpha\beta}\sqrt{-\det\mathbf{A}}\}$$

$$\simeq\partial_x\{V(f)\partial_\alpha t(\mathbf{a}^{-1})_A^{\alpha\beta}af'\sqrt{-\det\mathbf{a}}\}$$

$$+\partial_\alpha\{V(f)(\mathbf{a}^{-1})_A^{\alpha\beta}af'\sqrt{-\det\mathbf{a}}\}$$

$$=(\mathbf{a}^{-1})_A^{\alpha\beta}\sqrt{-\det\mathbf{a}}\{\partial_\alpha t\partial_x(V(f)af')+\partial_\alpha(V(f)af')\}=0.$$

$$(3.22)$$

In going from the second to the third line in Eq. (3.22) we have used the last equation in Eqs. (3.17). Again (3.22) has a finite left-over contribution, but this is sufficient to establish that the variation of δS vanishes for arbitrary finite δA_α when the equations (3.17) are satisfied.

For $\nu=x$, the left hand side of the third equation in (3.18) has the form

$$\partial_\alpha\{V(T)(\mathbf{A}^{-1})_A^{\alpha x}\sqrt{-\det\mathbf{A}}\}$$

$$\simeq\partial_\alpha\{V(f)(\mathbf{a}^{-1})_A^{\alpha\beta}\partial_\beta taf'\sqrt{-\det\mathbf{a}}\}$$

$$=(\mathbf{a}^{-1})_A^{\alpha\beta}\partial_\beta t\sqrt{-\det\mathbf{a}}\partial_\alpha(V(f)af')$$

$$=-(\mathbf{a}^{-1})_A^{\alpha\beta}\sqrt{-\det\mathbf{a}}\partial_\beta t\partial_\alpha t\partial_x(V(f)af')=0, \qquad (3.23)$$

where in going from the second to the third line of Eq. (3.23) we have used the third equation in (3.17) and the antisymmetry of $(\mathbf{a}^{-1})_A^{\alpha\beta}$, and in the last step we have used the antisymmetry of $(\mathbf{a}^{-1})_A^{\alpha\beta}$.

Verification of the first equation of (3.18) is a little more involved due to the following reasons. First of all, here the leading contribution from individual terms is of order a^3, with one factor of a coming from $\sqrt{-\det\mathbf{A}}$ and two more factors of a coming from the two derivatives of $f\{a[x-t(\xi)]\}$. Thus we cannot, from the beginning, use (3.14) and (3.19), since the corrections of order a^{-2} in these equations could combine with the a^3 terms to give a contribution of order a. Furthermore, since finite δt induces a $\delta T=-af'\delta t\sim a$, the equations of motion of T must hold, including finite terms, since such terms will give a contribution of order 1 in δS. We proceed with our analysis as follows. Using the equations $(\mathbf{A}^{-1})^{\mu\nu}\mathbf{A}_{\nu x}=\delta_x^\mu$, $\mathbf{A}_{x\nu}(\mathbf{A}^{-1})^{\nu\mu}=\delta_x^\mu$, we get the following exact relations:

$$(\mathbf{A}^{-1})_S^{\mu x}-(\mathbf{A}^{-1})_S^{\mu\beta}\partial_\beta t=\frac{1}{a^2(f')^2}[\delta_x^\mu-(\mathbf{A}^{-1})_S^{\mu x}].$$

$$(3.24)$$

Using Eq. (3.24) and that $\partial_\beta T=-\partial_x T\partial_\beta t=-af'\partial_\beta t$, we can now express the left hand side (lhs) of the first equation of (3.18) as

$$\text{lhs}=\partial_\mu\left(V(f)af'\frac{1}{a^2(f')^2}[\delta_x^\mu-(\mathbf{A}^{-1})_S^{\mu x}]\sqrt{-\det\mathbf{A}}\right)$$

$$-V'(f)\sqrt{-\det\mathbf{A}}. \qquad (3.25)$$

Due to the explicit factor of $a^2(f')^2$ in the denominator of the first term, the leading contribution from individual terms in this expression is now of order a, and hence we can now

use Eqs. (3.14), (3.19) to analyze Eq. (3.25) if we are willing to ignore contributions of order $1/a$. Using these equations Eq. (3.25) can be simplified as follows:

$$\text{lhs}\simeq\partial_x\{V(f)\sqrt{-\det\mathbf{a}}[1-(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\alpha t\partial_\beta t]\}$$

$$-\partial_\alpha\{V(f)\sqrt{-\det\mathbf{a}}(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\beta t\}-V'(f)af'\sqrt{-\det\mathbf{a}}$$

$$=V'(f)af'\sqrt{-\det\mathbf{a}}[1-(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\alpha t\partial_\beta t]$$

$$+V'(f)af'\sqrt{-\det\mathbf{a}}(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\alpha t\partial_\beta t$$

$$-V(f)\partial_\alpha[\sqrt{-\det\mathbf{a}}(\mathbf{a}^{-1})_S^{\alpha\beta}\partial_\beta t]$$

$$-V'(f)af'\sqrt{-\det\mathbf{a}}=0, \qquad (3.26)$$

using the first equation of (3.17). This establishes that any solution of Eqs. (3.17) automatically gives a solution of Eqs. (3.18).

Before concluding this section, let us note that if we consider a general expansion of the fields Y^I and A_μ of the form:

$$Y^I(x,\xi)=y^I(\xi)+\sum_{n=1}^\infty f_n[x-t(\xi)]y_{(n)}^I(\xi),$$

$$A_x(x,\xi)=\phi_{(0)}(\xi)+\sum_{n=1}^\infty f_n[x-t(\xi)]\phi_{(n)}(\xi)$$

$$\equiv\phi(x,\xi),$$

$$A_\alpha(x,\xi)=a_\alpha(\xi)+\sum_{n=1}^\infty f_n[x-t(\xi)]a_\alpha^{(n)}(\xi)$$

$$-\phi(x,\xi)\partial_\alpha t, \qquad (3.27)$$

where $\{f_n(u)\}$ for $n\geq1$ is a basis of smooth functions which vanish at $u=0$, and which are bounded including at $u=\pm\infty$,[7] then the action will be independent of $y_{(n)}^I(\xi)$, $a_\alpha^{(n)}(\xi)$ for $n\geq1$ and $\phi_{(n)}(\xi)$ for $n\geq0$. This can be seen by carrying out the same manipulations on the matrix $\mathbf{A}_{\mu\nu}$ as given in Eqs. (3.9)–(3.16). This has the following implication. As was argued in [32,38], at the tachyon vacuum a finite deformation of the A_μ and the Y^I fields do not change the action, and hence it is natural to identify all such field configurations as a single point in the configuration space, just like in the polar coordinate system different values of the polar angle θ give rise to the same physical point at $r=0$. This can be made into a general principle by postulating that whenever we encounter a local transformation that does not change the action, we should identify the different points in the configuration space related by this local transformation. In this spirit, the deformations associated with $\phi(x,\xi)$, $y_{(n)}^I(\xi)$ and $a_\alpha^{(n)}(\xi)$ should be regarded as pure gauge deformations. This general principle means, however, that the dimension of the gauge group may change from one point to another in the

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[7]This condition is imposed so that $-\det(\mathbf{A})$ remains positive for all x for arbitrary finite amplitude fluctuations of $y_{(n)}^I$, $a_\alpha^{(n)}$ and $\phi_{(n)}$.

configuration space, e.g. while around the tachyon background all deformations in A_μ and Y^I are pure gauge, around the non-BPS D-p-brane solution most of these deformations are physical, while around a kink solution some of these deformations are physical. This should not come as a surprise, as it simply indicates that the coordinate system that we have chosen—the fields T, A_μ and Y^I—are not good coordinates everywhere in the configuration space just like the polar coordinate system is not a good system near the origin.

To summarize, what we see from this analysis is that not only is the effective field theory of low energy modes on the world volume of the kink described by DBI action, but all the other smooth excitations on the kink world volume associated with gauge and transverse scalar fields are pure gauge deformations. The action depends only on the pull-back of the fields Y^I and the gauge field strength $F_{\mu\nu}$ along the surface $x = t(\xi)$ along which the kink world-volume lies. In particular, invariance of the action under the deformations generated by $y^I_{(n)}$, $a^{(n)}_\alpha$ and $\phi_{(n)}$ for $n \geq 1$ reflects that the action does not depend on the fields away from the location of the kink, whereas $\phi_{(0)}$ independence of the action reflects that the action depends only on the components of the gauge field strength along the world volume of the kink.

In this context we would like to note that Ref. [7] analyzed the non-zero mode excitations involving the A_μ (and the tachyon) fields and found a non-trivial spectrum for these modes by working to quadratic order in these fields in the action. For potential $V(T)$ motivated by the boundary string field theory analysis, these eigenmodes turned out to be Hermite polynomials with their arguments scaled by a. Since these are not smooth functions in the $a \to 0$ limit, and grow for large x except for the constant mode, there is no conflict with our result. However, we should note that in general, for actions of the kind considered here where the overall multiplicative factor vanishes away from the core of the soliton, the results based on the linearized analysis of the equations of motion may be somewhat misleading, since the non-linear terms could dominate even for small amplitude oscillations. In particular, if we consider the fluctuation of a mode of A_μ associated with a Hermite polynomial that grows for large x, then for any small but finite amplitude oscillation the $F_{\mu\nu}$ in $\mathbf{A}_{\mu\nu}$ will become comparable with $\eta_{\mu\nu}$ for sufficiently large x, and could drive $-\det(\mathbf{A})$ to be negative, thereby invalidating the analysis. We can see this explicitly by taking the linear tachyon profile $T \propto ax$ as in [7] and considering a fluctuation in the gauge field $A_1(x, \xi)$ of the form $H_n(ax) a_1(\xi^0)$, where H_n denotes the nth Hermite polynomial. Let us further consider a specific instant of time when $a_1(\xi^0)$ vanishes but $\partial_0 a_1(\xi^0)$ is non-zero. As this instant $\sqrt{-\det(\mathbf{A})} \propto a\sqrt{1 - [H_n(ax)]^2 (\partial_0 a_1)^2}$. Since $H_n(ax)$ grows for large ax, we see that for any finite $\partial_0 a_1$, however small, the expression under the square root will become negative for sufficiently large ax. The only mode that does not suffer from this problem is the constant mode. A similar argument holds for fluctuations in Y^I and T. This leads us to suspect that the only surviving modes on the kink world volume are

the massless modes associated with t, y^I and a_α.[8] A similar argument works for potentials $V(T)$ with different asymptotic behavior, e.g. $V(T) \sim e^{-\beta T}$ for large T where β is some constant. The only difference is that instead of the Hermite polynomials $H_n(ax)$, we have some other functions which grow for large ax.

A simpler version of this problem can be seen even for studying gauge (and scalar) field fluctuations around the tachyon vacuum. If we expand the action $-C\int d^{p+1}x \sqrt{-\det(\eta + F)}$ to quadratic order in F, then we can absorb a factor of $\sqrt{C}$ in A_μ and get the standard kinetic term for the gauge fields. This would lead to a conclusion that the spectrum contains a massless photon for all C. However, in the $C \to 0$ limit (relevant for the tachyon vacuum) this procedure is clearly incorrect since this will give an action $-C\int d^{p+1}x \sqrt{-\det(\eta + C^{-1/2}F)}$, and even a small fluctuation in F could drive the term under the square root negative, invalidating the analysis. In this case a Hamiltonian analysis of the system gives a much better understanding of the possible fluctuations around the tachyon vacuum [30] (see also [37]). A similar analysis in the kink background may provide useful insight into what type of fluctuations are present around this background.

IV. WORLD-VOLUME FERMIONS, SUPERSYMMETRY AND κ SYMMETRY

So far in our discussion we have ignored the world-volume fermions. We shall now discuss inclusion of these fields in our analysis.

For definiteness we shall restrict our analysis to D-branes in type IIA string theory, but generalization to type IIB theory is straightforward following the analysis of Ref. [1]. On a non-BPS Dp-brane world volume in type IIA string theory, we have a 32 component anti-commuting field Θ which transforms as a Majorana spinor of the ten dimensional Lorentz group [1]. We shall denote by Γ_M the ten dimensional γ matrices, and take the indices M, N to run from 0 to 9. In order to construct the world-volume action involving the fields A_μ, Y^I, Θ and T ($0 \leq \mu \leq p$, $(p+1) \leq I \leq 9$) in static gauge, we first define

$$\Pi^\nu_\mu = \delta^\nu_\mu - \overline{\Theta}\Gamma^\nu \partial_\mu \Theta, \quad \Pi^I_\mu = \partial_\mu Y^I - \overline{\Theta}\Gamma^I \partial_\mu \Theta, \quad (4.1)$$

$$\mathbf{G}_{\mu\nu} = \eta_{MN}\Pi^M_\mu \Pi^N_\nu + \partial_\mu T \partial_\nu T, \quad (4.2)$$

and

$$\mathbf{F}_{\mu\nu} = F_{\mu\nu} - [\{\overline{\Theta}\Gamma_{11}\Gamma_\nu \partial_\mu \Theta + \overline{\Theta}\Gamma_{11}\Gamma_I \partial_\mu \Theta \partial_\nu Y^I$$
$$- \tfrac{1}{2}\overline{\Theta}\Gamma_{11}\Gamma_M \partial_\mu \Theta \overline{\Theta}\Gamma^M \partial_\nu \Theta\} - \{\mu \leftrightarrow \nu\}], \quad (4.3)$$

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[8]This argument of course does not affect the analysis for other types of action discussed in [36,7] where the action takes the form of a kinetic plus a singular potential term.

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{4.4}$$

In terms of these variables, the DBI part of the world-volume action is given by [1–3]

$$S_{DBI} = -\int d^{p+1}x\, V(T)\sqrt{-\det(\mathbf{G}+\mathbf{F})}. \tag{4.5}$$

The action is invariant under the supersymmetry transformation parametrized by a ten dimensional Majorana spinor ϵ. In the static gauge in which we are working, the infinitesimal supersymmetry transformation laws are given by [1]

$$\delta_p \Theta = \epsilon - (\bar{\epsilon}\Gamma^\mu\Theta)\partial_\mu\Theta,$$

$$\delta_p Y^I = \bar{\epsilon}\Gamma^I\Theta - (\bar{\epsilon}\Gamma^\mu\Theta)\partial_\mu Y^I, \quad \delta_p T = -(\bar{\epsilon}\Gamma^\mu\Theta)\partial_\mu T,$$

$$\delta_p A_\nu = \bar{\epsilon}\Gamma_{11}\Gamma_\nu\Theta + \bar{\epsilon}\Gamma_{11}\Gamma_I\Theta\,\partial_\nu Y^I$$

$$-\tfrac{1}{6}(\bar{\epsilon}\Gamma_{11}\Gamma_M\Theta\bar{\Theta}\Gamma^M\partial_\nu\Theta + \bar{\epsilon}\Gamma_M\Theta\bar{\Theta}\Gamma_{11}\Gamma^M\partial_\nu\Theta)$$

$$-(\bar{\epsilon}\Gamma^\mu\Theta)\partial_\mu A_\nu - \partial_\nu(\bar{\epsilon}\Gamma^\mu\Theta)A_\mu. \tag{4.6}$$

The subscript p in δ_p denotes that these are the supersymmetry transformation laws on the D-p-brane world volume. The supersymmetry transformation parameter ϵ is a Majorana spinor of the ten dimensional Lorentz group.

Besides the DBI term, the world-volume action also contains a Wess-Zumino term. In the bosonic sector this term is important only for non-vanishing RR background field, but once we take into account the world-volume fermions, this term survives even for zero RR background. The structure of this term is [39,18,45,3]

$$S_{WZ} = \int W(T) dT \wedge \mathbf{C} \wedge e^{\mathbf{F}}, \tag{4.7}$$

where $\mathbf{F} = \mathbf{F}_{\mu\nu} dx^\mu \wedge dx^\nu$, $W(T)$ is an even function of T which vanishes as $T \to \pm\infty$, and $\mathbf{C}$ is a specific combination of background RR fields and the world-volume fields Y^I, Θ on the D-brane [3]. In particular, the bosonic part of $\mathbf{C}$ is given by $\Sigma_{q\geq0}C^{(p-2q)}$ where $C^{(p-2q)}$ denotes the pullback of the RR $(p-2q)$-form field on the D-p-brane world volume. This vanishes for vanishing RR background, but there is a part of $\mathbf{C}$ involving the world-volume fermion fields that survives even in the absence of any RR background [20–24,3]. Since we shall not need the explicit form of $\mathbf{C}$ for our analysis, we shall not give it here. (See, for example, [3] for the component form of this term for trivial supergravity background.) The Wess-Zumino term is also invariant under the supersymmetry transformations (4.6). Later we shall see that consistency requires

$$\int_{-\infty}^{\infty} W(T) dT = \int_{-\infty}^{\infty} V(T) dT = \mathcal{T}_{p-1}, \tag{4.8}$$

where in the last step we have used Eq. (2.13).

Since we want to compare the world-volume action on a kink solution with that on the BPS D-$(p-1)$-brane, we need to first know the form of the world-volume action on a BPS D-$(p-1)$-brane. The world-volume fields in this case consist of a vector field $a_\alpha(\xi)$ $(0\leq\alpha\leq(p-1))$, a set of $(9-p+1)$ scalar fields which we shall denote by $y^I(\xi)$ $((p+1)\leq I\leq9)$ and $y^p(\xi)\equiv t(\xi)$, respectively, in the convention of Sec. III, and a Majorana spinor $\theta(\xi)$ of the ten dimensional Lorentz group. Here $\{\xi^\alpha\}$ denotes the world-volume coordinate on the D-$(p-1)$-brane as in Sec. III. The DBI part of the action is given by [20–24]

$$S_{dbi} = -\mathcal{T}_{p-1}\int d^p\xi\sqrt{-\det(\mathbf{g}+\mathbf{f})}, \tag{4.9}$$

where

$$\mathbf{g}_{\alpha\beta} = \eta_{MN}\pi_\alpha^M\pi_\beta^N, \tag{4.10}$$

$$\pi_\alpha^\beta = \delta_\alpha^\beta - \bar{\theta}\Gamma^\beta\partial_\alpha\theta, \quad \pi_\alpha^I = \partial_\alpha y^I - \bar{\theta}\Gamma^I\partial_\alpha\theta,$$

$$\pi_\alpha^p = \partial_\alpha t - \bar{\theta}\Gamma^p\partial_\alpha\theta, \tag{4.11}$$

$$\mathbf{f}_{\alpha\beta} = f_{\alpha\beta} - [\{\bar{\theta}\Gamma_{11}\Gamma_\beta\partial_\alpha\theta + \bar{\theta}\Gamma_{11}\Gamma_I\partial_\alpha\theta\partial_\beta y^I + \bar{\theta}\Gamma_{11}\Gamma_p\partial_\alpha\theta\partial_\beta t$$

$$-\tfrac{1}{2}\bar{\theta}\Gamma_{11}\Gamma_M\partial_\alpha\theta\bar{\theta}\Gamma^M\partial_\beta\theta\} - \{\alpha\leftrightarrow\beta\}], \tag{4.12}$$

$$f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha. \tag{4.13}$$

The Wess-Zumino term, on the other hand, has the form

$$S_{wz} = \mathcal{T}_{p-1}\int \mathbf{c}\wedge e^{\mathbf{f}}, \tag{4.14}$$

where $\mathbf{f} = \mathbf{f}_{\alpha\beta} d\xi^\alpha \wedge d\xi^\beta$, and $\mathbf{c}$ is an expression containing the RR background and the world-volume fields y^I, t, θ [20–24]. The bosonic part of $\mathbf{c}$ is given by $\Sigma_{q\geq0}C^{(p-2q)}$ where $C^{(p-2q)}$ denotes the pullback of the RR $(p-2q)$-form field on the D-$(p-1)$-brane world volume. Like $\mathbf{C}$, $\mathbf{c}$ also contains a term involving y^I and θ which survive even for trivial RR background. If we think of the world volume of the D-$(p-1)$-brane as sitting inside that of a D-p-brane along the surface $x^p = t(\xi)$, then $\mathbf{c}$ is in fact the pullback of $\mathbf{C}$ appearing in (4.7) provided we identify θ and y^I as the restriction of Θ and Y^I along the surface $x^p = t(\xi)$.

Both S_{dbi} and S_{wz} are separately invariant under the infinitesimal supersymmetry transformation:

$$\delta_{p-1}\theta = \epsilon - (\bar{\epsilon}\Gamma^\alpha\theta)\partial_\alpha\theta, \quad \delta_{p-1}y^I = \bar{\epsilon}\Gamma^I\theta$$

$$-(\bar{\epsilon}\Gamma^\alpha\theta)\partial_\alpha y^I, \quad \delta_{p-1}t = \bar{\epsilon}\Gamma^p\theta - (\bar{\epsilon}\Gamma^\alpha\theta)\partial_\alpha t,$$

$$\delta_{p-1}a_\beta = \bar{\epsilon}\Gamma_{11}\Gamma_\beta\theta + \bar{\epsilon}\Gamma_{11}\Gamma_I\theta\partial_\beta y^I + \bar{\epsilon}\Gamma_{11}\Gamma_p\theta\partial_\beta t$$

$$-\tfrac{1}{6}(\bar{\epsilon}\Gamma_{11}\Gamma_M\theta\bar{\theta}\Gamma^M\partial_\beta\theta + \bar{\epsilon}\Gamma_M\theta\bar{\theta}\Gamma_{11}\Gamma^M\partial_\beta\theta)$$

$$-(\bar{\epsilon}\Gamma^\alpha\theta)\partial_\alpha a_\beta - \partial_\beta(\bar{\epsilon}\Gamma^\alpha\theta)a_\alpha. \tag{4.15}$$

The subscript $(p-1)$ on δ_{p-1} indicates that these represent supersymmetry transformation laws on the world volume of a BPS D-$(p-1)$-brane.

In order to show that the world-volume action $S_{dbi}+S_{wz}$ on the BPS D-$(p-1)$-brane arises from the world-volume action on the tachyon kink solution of Sec. II, we need to first propose an ansatz relating the fields $T(x,\xi)$, $A_\mu(x,\xi)$, $Y^I(x,\xi)$ and $\Theta(x,\xi)$ to the fields $a_\alpha(\xi)$, $y^I(\xi)$, $t(\xi)$ and $\theta(\xi)$ on the BPS D-brane. For this we propose the following ansatz:

$$T(x,\xi)=f\{a[x-t(\xi)]\}, \quad Y^I(x,\xi)=y^I(\xi),$$

$$\Theta(x,\xi)=\theta(\xi),$$

$$A_x(x,\xi)=0 \quad A_\alpha(x,\xi)=a_\alpha(\xi). \tag{4.16}$$

We can now compute $\mathbf{G}_{\mu\nu}$ and $\mathbf{F}_{\mu\nu}$ in terms of the variables a_α, y^I, t and θ using Eqs. (4.1)–(4.4) and (4.16). The result is

$$\mathbf{G}_{xx}=1+a^2(f')^2, \quad \mathbf{G}_{\alpha x}=\mathbf{G}_{x\alpha}=-a^2(f')^2\partial_\alpha t-\bar\theta\Gamma^p\partial_\alpha\theta,$$

$$\mathbf{G}_{\alpha\beta}=\mathbf{g}_{\alpha\beta}+\partial_\alpha t\bar\theta\Gamma^p\partial_\beta\theta+\partial_\beta t\bar\theta\Gamma^p\partial_\alpha\theta$$
$$+[a^2(f')^2-1]\partial_\alpha t\partial_\beta t,$$

$$\mathbf{F}_{\alpha x}=-\mathbf{F}_{x\alpha}=-\bar\theta\Gamma_{11}\Gamma^p\partial_\alpha\theta,$$

$$\mathbf{F}_{\alpha\beta}=\mathbf{f}_{\alpha\beta}-\partial_\alpha t\bar\theta\Gamma_{11}\Gamma^p\partial_\beta\theta+\partial_\beta t\bar\theta\Gamma_{11}\Gamma^p\partial_\alpha\theta, \tag{4.17}$$

with $\mathbf{g}_{\alpha\beta}$ and $\mathbf{f}_{\alpha\beta}$ defined as in Eqs. (4.10)–(4.13). Using manipulations similar to those in Eqs. (3.11)–(3.16) we can now show that

$$\det(\mathbf{G}+\mathbf{F})=a^2(f')^2\{\det(\mathbf{g}+\mathbf{f})+\mathcal{O}(a^{-2})\}, \tag{4.18}$$

and

$$S_{DBI}=-\int d^{p+1}x V(T)\sqrt{-\det(\mathbf{G}+\mathbf{F})}$$
$$=-\mathcal{T}_{p-1}\int d^p\xi\sqrt{-\det(\mathbf{g}+\mathbf{f})}=S_{dbi}. \tag{4.19}$$

The analysis for S_{WZ} is even simpler; indeed, this term was designed to reproduce the Wess-Zumino term on the world volume of a kink solution [18,3]. For this let us define

$$u=x-t(\xi). \tag{4.20}$$

Then from Eqs. (4.17) we get

$$\mathbf{F}\equiv\mathbf{F}_{\mu\nu}dx^\mu\wedge dx^\nu$$
$$=2\mathbf{F}_{x\beta}dx\wedge d\xi^\beta+\mathbf{F}_{\alpha\beta}d\xi^\alpha\wedge d\xi^\beta$$
$$=2\bar\theta\Gamma_{11}\Gamma_p\partial_\alpha\theta du\wedge d\xi^\alpha+\mathbf{f}_{\alpha\beta}d\xi^\alpha\wedge d\xi^\beta. \tag{4.21}$$

Since we have

$$dT=af'(au)du, \tag{4.22}$$

only the second term on the right hand side of Eq. (4.21) will contribute to S_{WZ} given in Eq. (4.7). Thus we can replace $\mathbf{F}$ by $\mathbf{f}$ in Eq. (4.7). On the other hand, we can analyze $\mathbf{C}$ by writing it as

$$\mathbf{C}=\sum_q \mathbf{C}^{(q)}_{\mu_1\cdots\mu_q}dx^{\mu_1}\wedge\cdots dx^{\mu_q}$$

$$=\sum_q (q\mathbf{C}^{(q)}_{x\alpha_2\cdots\alpha_q}dx\wedge d\xi^{\alpha_2}\wedge\cdots d\xi^{\alpha_q}$$
$$+\mathbf{C}^{(q)}_{\alpha_1\cdots\alpha_q}d\xi^{\alpha_1}\wedge\cdots d\xi^{\alpha_q})$$

$$=\sum_q [q\mathbf{C}^{(q)}_{x\alpha_2\cdots\alpha_q}du\wedge d\xi^{\alpha_2}\wedge\cdots d\xi^{\alpha_q}+(q\mathbf{C}^{(q)}_{x\alpha_2\cdots\alpha_q}\partial_{\alpha_1}t$$
$$+\mathbf{C}^{(q)}_{\alpha_1\cdots\alpha_q})d\xi^{\alpha_1}\wedge\cdots d\xi^{\alpha_q}], \tag{4.23}$$

where in the last step we have used $dx=du+\partial_\alpha t d\xi^\alpha$. The term proportional to du does not contribute to Eq. (4.7) due to Eq. (4.22), whereas the term proportional to $d\xi^{\alpha_1}\wedge\cdots d\xi^{\alpha_q}$, after being summer over q, is precisely the pullback of $\mathbf{C}$ on the kink world volume along $x=t(\xi)$ and hence can be identified with $\mathbf{c}$. Thus we get

$$S_{WZ}=\int W(f(au))af'(au)du\wedge\mathbf{c}\wedge e^{\mathbf{f}}=\mathcal{T}_{p-1}\int \mathbf{c}\wedge e^{\mathbf{f}}=S_{wz}, \tag{4.24}$$

using Eq. (4.8).

This shows that $S_{DBI}+S_{WZ}$ reduces to $S_{dbi}+S_{wz}$ under the identification (4.16). In principle we also need to check that any solution of the equations of motion derived from $S_{dbi}+S_{wz}$ is automatically a solution of the equations of motion derived from $S_{DBI}+S_{WZ}$. Presumably this can be done following the analysis of Sec. III, but we have not worked out all the details.

Finally, we need to check that the supersymmetry transformations (4.15) are compatible with the supersymmetry transformations (4.6). For this we need to calculate $\delta_{p-1}A_\mu$, $\delta_{p-1}Y^I$ and $\delta_{p-1}T$ using Eqs. (4.15), (4.16) and compare them with Eqs. (4.6). The calculation is straightforward, and we get

$$\delta_p A_x=\delta_{p-1}A_x+\bar\epsilon\Gamma_{11}\Gamma_p\theta,$$

$$\delta_p A_\alpha=\delta_{p-1}A_\alpha-\bar\epsilon\Gamma_{11}\Gamma_p\theta\partial_\alpha t, \tag{4.25}$$

$$\delta_p Y^I=\delta_{p-1}Y^I, \quad \delta_p T=\delta_{p-1}T.$$

Thus we see that δ_p and δ_{p-1} differ for the transformation laws of A_x and A_α. This difference, however, is precisely of the form induced by the function $\phi(x,\xi)$ in Eq. (3.27) with $\phi(x,\xi)=\bar\epsilon\Gamma_{11}\Gamma_p\theta(\xi)$. As was argued below (3.27), this is a gauge transformation. Thus we see that the action of δ_p and δ_{p-1} differ by a gauge transformation in the world-volume theory on the D-p-brane.

This establishes that the world-volume action on the kink reduces to that on a D-$(p-1)$-brane. The latter has a local

κ-symmetry which can be used to gauge away half of the world-volume fermion fields [20–24]. This leads to a puzzle. Whereas on a BPS D-brane the local κ symmetry is postulated to be a gauge symmetry, i.e. different configurations related by κ transformation are identified, on a kink solution the appearance of the κ symmetry seems accidental and *a priori* there is no reason to identify field configurations which are related by κ symmetry. We believe the resolution of this puzzle lies in the general principle advocated below (3.27) that any local transformation of the fields which does not change the action must be a gauge symmetry. This will automatically imply that the κ transformation is a gauge transformation and we should identify the configurations related by κ transformation. This κ symmetry can now be used to gauge away half of the fermion fields on the world volume of the kink.

V. VORTEX SOLUTION ON THE BRANE-ANTIBRANE PAIR

In this section we shall generalize the construction of Sec. II to a vortex solution on a brane-antibrane pair. For this we need to begin with a tachyon effective action on a brane-antibrane pair. In this case we have a complex tachyon field T, besides the massless gauge fields $A_\mu^{(1)}$, $A_\mu^{(2)}$ and scalar fields $Y_{(1)}^I$, $Y_{(2)}^I$ corresponding to the transverse coordinates of individual branes. We shall work with the following effective action that generalizes (1.1):[9],[10]

$$S = -\int d^{p+1}x \, V(T, Y_{(1)}^I - Y_{(2)}^I)(\sqrt{-\det \mathbf{A}_{(1)}} + \sqrt{-\det \mathbf{A}_{(2)}}),$$
$$(5.1)$$

where

$$\mathbf{A}_{(i)\mu\nu} = \eta_{\mu\nu} + F_{\mu\nu}^{(i)} + \partial_\mu Y_{(i)}^I \partial_\nu Y_{(i)}^I + \tfrac{1}{2}(D_\mu T)^*(D_\nu T)$$
$$+ \tfrac{1}{2}(D_\nu T)^*(D_\mu T), \qquad (5.2)$$

$$F_{\mu\nu}^{(i)} = \partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}, \quad D_\mu T = (\partial_\mu - iA_\mu^{(1)} + iA_\mu^{(2)})T,$$
$$(5.3)$$

_____

[9]As in Sec. II, we expect our analysis to be valid for a more general action of the form:

$$-\int d^{p+1}x V(T, Y_{(1)}^I - Y_{(2)}^I)[\sqrt{-\det(g_{\mu\nu}^{(1)} + F_{\mu\nu}^{(1)})} F(G_{(1)}^{\mu\nu} D_\mu T^* D_\nu T)$$
$$+ \sqrt{-\det(g_{\mu\nu}^{(2)} + F_{\mu\nu}^{(2)})} F(G_{(2)}^{\mu\nu} D_\mu T^* D_\nu T)]$$

where $g_{\mu\nu}^{(i)} = \eta_{\mu\nu} + \partial_\mu Y_{(i)}^I \partial_\nu Y_{(i)}^I$ is the induced closed string metric on the ith brane, $G_{(i)}^{\mu\nu}$ is the open string metric on the ith brane and the function $F(u)$ grows as $u^{1/2}$ for large u.
[10]There have been various other proposals for the tachyon effective action and/or vortex solutions on brane-antibrane pair, see, e.g. [40–44].

and the potential $V(T)$ depends on $|T|$ and $\Sigma_I (Y_{(1)}^I - Y_{(2)}^I)^2$ only. For small T, V behaves as

$$V(T, Y_{(1)}^I - Y_{(2)}^I) = \mathcal{T}_p \left[1 + \frac{1}{2} \left\{ \sum_I \left(\frac{Y_{(1)}^I - Y_{(2)}^I}{2\pi} \right)^2 - \frac{1}{2} \right\} |T|^2 \right.$$
$$\left. + \mathcal{O}(|T|^4) \right]. \qquad (5.4)$$

$\mathcal{T}_p$ denotes the tension of the individual D-p-branes. Although this action has not been derived from first principles, we note that this obeys the following consistency conditions:

(1) The action has the required invariance under the gauge transformation:

$$T \to e^{2i\alpha(x)}T, \quad A_\mu^{(1)} \to A_\mu^{(1)} + \partial_\mu \alpha(x),$$

$$A_\mu^{(2)} \to A_\mu^{(2)} - \partial_\mu \alpha(x). \qquad (5.5)$$

(2) For $T=0$ the action reduces to the sum of the usual DBI action on the individual branes.

(3) If we require the fields to be invariant under the symmetry $(-1)^{F_L}$ that exchanges the brane and the antibrane, we get the restriction

$$T = \text{real}, \quad A_\mu^{(1)} = A_\mu^{(2)} \equiv A_\mu, \quad Y_{(1)}^I = Y_{(2)}^I \equiv Y^I. \quad (5.6)$$

Under this restriction the action becomes proportional to that on a non-BPS D-p-brane, as given in Eq. (1.1). This is a necessary consistency check, as modding out a brane-antibrane configuration by $(-1)^{F_L}$ is supposed to produce a non-BPS D-p-brane [45].

We should keep in mind however that these constraints do not fix the form of the action uniquely. Nevertheless we shall make the specific choice given in (5.1) and proceed to study the vortex solution in this theory.

The energy momentum tensor $T^{\mu\nu}$ associated with this action is given by

$$T^{\mu\nu} = -V(T, Y_{(1)}^I - Y_{(2)}^I)[\sqrt{-\det(\mathbf{A}_{(1)})}(\mathbf{A}_{(1)}^{-1})_S^{\mu\nu}$$
$$+ \sqrt{-\det(\mathbf{A}_{(2)})}(\mathbf{A}_{(2)}^{-1})_S^{\mu\nu}]. \qquad (5.7)$$

In order to construct a vortex solution we begin with the ansatz

$$T(r, \theta) = \bar{f}(r)e^{i\theta}, \quad A_\theta^{(1)} = -A_\theta^{(2)} = \tfrac{1}{2}\,\bar{g}(r), \qquad (5.8)$$

where r and θ denote the polar coordinates in the (x^{p-1}, x^p) plane, and $\bar{f}(r)$ and $\bar{g}(r)$ are real functions of r satisfying the boundary conditions

$$\bar{f}(0) = 0, \quad \bar{f}(\infty) = \infty, \quad \bar{g}(0) = 0, \quad \bar{g}'(0) = 0. \quad (5.9)$$

All other fields vanish. For such a background

$$D_r T = \bar{f}'(r) e^{i\theta}, \quad D_\theta T = i \bar{f}(r)(1 - \bar{g}(r)) e^{i\theta},$$

$$F_{r\theta}^{(1)} = -F_{r\theta}^{(2)} = \tfrac{1}{2} \bar{g}'(r). \tag{5.10}$$

Also in the polar coordinate that we have been using:

$$\eta = \begin{pmatrix} \eta_{\alpha\beta} & & \\ & 1 & \\ & & r^2 \end{pmatrix}, \quad 0 \leq \alpha, \beta \leq (p-2). \tag{5.11}$$

This gives

$$\mathbf{A}_{(1)} = \begin{pmatrix} \eta_{\alpha\beta} & & \\ & 1+(\bar{f}')^2 & \tfrac{1}{2}\bar{g}' \\ & -\tfrac{1}{2}\bar{g}' & r^2+\bar{f}^2(1-\bar{g})^2 \end{pmatrix},$$

$$\mathbf{A}_{(2)} = \begin{pmatrix} \eta_{\alpha\beta} & & \\ & 1+(\bar{f}')^2 & -\tfrac{1}{2}\bar{g}' \\ & \tfrac{1}{2}\bar{g}' & r^2+\bar{f}^2(1-\bar{g})^2 \end{pmatrix}. \tag{5.12}$$

$$-\det(\mathbf{A}_{(1)}) = -\det(\mathbf{A}_{(2)})$$

$$= [\{1+(\bar{f}')^2\}\{r^2+\bar{f}^2(1-\bar{g})^2\} + \tfrac{1}{4}(\bar{g}')^2], \tag{5.13}$$

$$T_{\alpha\beta} = -2\eta_{\alpha\beta}V(T)$$
$$\times \sqrt{\{1+(\bar{f}')^2\}\{r^2+\bar{f}^2(1-\bar{g})^2\} + \tfrac{1}{4}(\bar{g}')^2},$$

$$T_{rr} = -2V(T)\{r^2+\bar{f}^2(1-\bar{g})^2\}/$$
$$\sqrt{\{1+(\bar{f}')^2\}\{r^2+\bar{f}^2(1-\bar{g})^2\} + \tfrac{1}{4}(\bar{g}')^2},$$

$$T_{\theta\theta} = -2V(T)\{1+(\bar{f}')^2\}/$$
$$\sqrt{\{1+(\bar{f}')^2\}\{r^2+\bar{f}^2(1-\bar{g})^2\} + \tfrac{1}{4}(\bar{g}')^2}, \tag{5.14}$$

where we have used the shorthand notation $V(T)$ to denote $V(T,0)$. All other components of $T_{\mu\nu}$ vanish. The energy momentum conservation

$$0 = \partial^\mu T_{\mu r} = \partial_r T_{rr} \tag{5.15}$$

now shows that T_{rr} must be a constant. Since $V(T) = V(\bar{f}e^{i\theta})$ falls off exponentially for large $|T|$, we see from Eq. (5.14) that T_{rr} vanishes at ∞, unless $\bar{g}(r)$ increases sufficiently fast. Shortly, we shall see that $\bar{g}$ varies monotonically between 0 and 1, and hence is bounded. This leads us to the conclusion that T_{rr} does vanish at infinity, and hence must be zero everywhere due to the conservation law (5.15).

To see that $\bar{g}(r)$ varies monotonically between 0 and 1, we proceed as follows. As a consequence of the equations of motion of the gauge fields, the $(p-2)$-dimensional energy density $\int r\,dr\,d\theta T_{00}$, with T_{00} given in Eq. (5.14), must be minimized with respect to the function $\bar{g}(r)$ subject to the boundary condition (5.9). Now if $\bar{g}(r)$ exceeds 1 for some range of r, then we can lower T_{00} in that range by replacing the original $\bar{g}(r)$ by another continuous function which is equal to the original function when the latter is less than 1, and which is equal to 1 when the latter exceeds 1. Thus the original $\bar{g}(r)$ does not minimize energy and hence is not a solution of the equations of motion. This shows that a solution of the equations of motion must have $\bar{g}(r) \leq 1$ everywhere. An exactly similar argument can be used to show that $\bar{g}(r) \geq 0$ everywhere. Furthermore, if $\bar{g}(r)$ is not a monotone increasing function, then it will have a local maximum at some point a. We can now define a range (a,b) on the r axis such that $\bar{g}(r) < \bar{g}(a)$ for $a < r < b$. (b could be infinity.) In this case we can lower the energy of the configuration by replacing the original function by another continuous function that agrees with the original function outside the range (a,b) and is equal to $\bar{g}(a)$ in the range (a,b). Since this should not be possible if the original $\bar{g}(r)$ is a solution of the equations of motion, we see that a solution of the equations of motion must have a monotone increasing $\bar{g}(r)$.

The vanishing of T_{rr} requires that for every value of r, either the numerator in the expression for T_{rr} vanishes, which requires $V(T)$ to vanish, or the denominator becomes infinite, which requires $\bar{f}'$ and/or $\bar{g}'$ to be infinite. $V(T)$ is finite at $r=0$ where T vanishes, thus it is not zero everywhere. Thus at least for $r=0$, $\bar{f}'$ and/or $\bar{g}'$ must be infinite. In analogy with the kink solution, we look for $\bar{f}$ and $\bar{g}$ of the form:

$$\bar{f}(r) = f(ar), \quad \bar{g}(r) = g(ar), \tag{5.16}$$

and at the end take the $a \to \infty$ limit, keeping the functions f and g fixed. The boundary conditions (5.9) now translate to

$$f(0) = 0, \quad f(\infty) = \infty, \quad g(0) = 0, \quad g'(0) = 0. \tag{5.17}$$

We shall also impose the condition

$$f'(u) > 0 \quad \text{for} \quad 0 \leq u < \infty. \tag{5.18}$$

This guarantees that $\bar{f}'(r) = af'(ar)$ is infinite everywhere in the $a \to \infty$ limit. Once we have chosen $\bar{f}$ this way, we do not need to take $\bar{g}$ in the form given in Eq. (5.16). But this form allows for more general possibilities since without this the term involving $\bar{g}'$ will simply drop out in the scaling limit $a \to \infty$. On the other hand, by allowing $\bar{g}$ to scale as in Eq. (5.16) we do not preclude the case where $\bar{g}$ approaches a finite function in the $a \to \infty$ limit, since this will just correspond to choosing $g(r) \equiv \bar{g}(r/a)$ to be a nearly constant function except for very large r.

Substituting Eq. (5.16) into Eqs. (5.13), (5.14) we get, for large a,

$$-\det(\mathbf{A}_{(1)}) = -\det(\mathbf{A}_{(2)}) \simeq a^2 (f'(ar))^2 \{r^2 + f(ar)^2$$
$$\times [1 - g(ar)]^2 + \tfrac{1}{4}[g'(ar)/f'(ar)]^2\}, \tag{5.19}$$

and

$$T_{\alpha\beta} \simeq -2\eta_{\alpha\beta} V(f(ar)) af'(ar)$$
$$\times \sqrt{r^2 + f(ar)^2 [1 - g(ar)]^2 + \tfrac{1}{4}[g'(ar)/f'(ar)]^2}, \tag{5.20}$$

$$T_{rr} \simeq -2V(f(ar)) \frac{r^2 + f(ar)^2 [1 - g(ar)]^2}{af'(ar)\sqrt{r^2 + f(ar)^2 [1 - g(ar)]^2 + \tfrac{1}{4}[g'(ar)/f'(ar)]^2}}. \tag{5.21}$$

Thus T_{rr} vanishes everywhere in the $a \to \infty$ limit as required. On the other hand, integrating (5.20) over the (r, θ) coordinates gives the $(p-2+1)$-dimensional energy momentum tensor $T_{\alpha\beta}^{vortex}$ on the vortex:

$$T_{\alpha\beta}^{vortex} = -4\pi\eta_{\alpha\beta} \int_0^\infty dr V(f(ar)) af'(ar)$$
$$\times \sqrt{r^2 + f(ar)^2 [1 - g(ar)]^2 + \tfrac{1}{4}[g'(ar)/f'(ar)]^2}. \tag{5.22}$$

Defining

$$y = f(ar), \quad \hat{r}(y) = a^{-1} f^{-1}(y), \quad \hat{g}(y) = g(ar) = g(a\hat{r}(y)), \tag{5.23}$$

where f^{-1} denotes the inverse function of f, we can rewrite Eq. (5.22) as

$$T_{\alpha\beta}^{vortex} = -4\pi\eta_{\alpha\beta} \int_0^\infty dy V(y)$$
$$\times \sqrt{\hat{r}(y)^2 + y^2 \{1 - \hat{g}(y)\}^2 + \tfrac{1}{4}\hat{g}'(y)^2}. \tag{5.24}$$

From Eq. (5.23) it follows that in the $a \to \infty$ limit, $\hat{r}(y)$ vanishes for any finite y. Thus Eq. (5.24) further simplifies to

$$T_{\alpha\beta}^{vortex} = -4\pi\eta_{\alpha\beta} \int_0^\infty dy V(y) \sqrt{y^2 \{1 - \hat{g}(y)\}^2 + \tfrac{1}{4}\hat{g}'(y)^2}. \tag{5.25}$$

We now see that as in the case of the kink solution, Eq. (5.25) is completely insensitive to the choice of the function

f, although it does depend on the choice of $\hat{g}(y)$. $\hat{g}(y)$ in turn is determined by the equations of motion of the gauge fields, or equivalently, by minimizing the expression for the energy T_{00}^{vortex}, subject to the boundary conditions:

$$\hat{g}(0) = 0, \quad \hat{g}'(0) = 0. \tag{5.26}$$

This leads to the following differential equation for $\hat{g}(y)$:

$$\tfrac{1}{4} \partial_y \left[V(y)\hat{g}'(y) / \sqrt{y^2 \{1 - \hat{g}(y)\}^2 + \tfrac{1}{4}\hat{g}'(y)^2} \right]$$
$$+ V(y) y^2 [1 - \hat{g}(y)] / \sqrt{y^2 \{1 - \hat{g}(y)\}^2 + \tfrac{1}{4}\hat{g}'(y)^2} = 0. \tag{5.27}$$

Thus $\hat{g}(y)$ and the final expression for $T_{\alpha\beta}^{vortex}$ are determined completely in terms of the potential $V(T)$, independently of the choice of the function f.[11] Furthermore, as in the case of the kink solution, most of the contribution to $T_{\alpha\beta}^{vortex}$ comes

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[11] The choice $f(ar) = ar$ gives $T = a(x^{p-1} + ix^p)$ in the Cartesian coordinate system. This resembles the vortex solution in boundary string field theory [40,41]. However, unlike in [40,41], here we have background gauge fields present. This is not necessarily a contradiction, since the fields used here could be related to those in [40,41] by a non-trivial field redefinition. In fact, we would like to note that generically, when both the real and the imaginary parts of the tachyon are non-zero and are not proportional to each other, we have a source for the gauge field $A_\mu^{(1)} - A_\mu^{(2)}$, and hence it is not possible to find a solution of the equations of motion keeping the gauge fields to zero. Boundary string field theory seems to use a very special definition of fields where this is possible in the $a \to \infty$ limit.

from a finite range of values of y, which corresponds to a region in r space of width $1/a$ around the origin. Thus in the $a \rightarrow 0$ limit, $T_{\alpha\beta}$ has the form of a δ function centered around the origin of the (x^{p-1}, x^p) plane:

$$T_{\alpha\beta} = -4\pi\eta_{\alpha\beta}\delta(x^{p-1})\delta(x^p)\int_0^\infty dy\, V(y)$$

$$\times \sqrt{y^2\{1-\hat{g}(y)\}^2 + \tfrac{1}{4}\hat{g}'(y)^2}. \tag{5.28}$$

This agrees with the identification of the vortex solution as a D-$(p-2)$-brane; as for the latter, the energy-momentum tensor is localized on a $(p-2)$-dimensional surface. [This can be seen by examining the boundary state describing a D-$(p-2)$-brane.] The tension of the D-$(p-2)$-brane is identified as

$$\mathcal{T}_{p-2} = 4\pi\int_0^\infty dy\, V(y)\sqrt{y^2\{1-\hat{g}(y)\}^2 + \tfrac{1}{4}\hat{g}'(y)^2}. \tag{5.29}$$

Before concluding this section, we shall determine the asymptotic behavior of $\hat{g}(y)$ satisfying Eqs. (5.26) and (5.27). Our previous arguments for the function $\bar{g}(r)$, when translated to $\hat{g}(y)$, shows that $\hat{g}(y)$ must be a monotone increasing function of y, and must lie between 0 and 1. The boundary condition forces $\hat{g}(y)$ to vanish at $y=0$. We shall now show that given a mild constraint on the potential $V(T)$, $\hat{g}(y)$ must approach 1 as $y \rightarrow \infty$. We shall begin by assuming that $\hat{g}(y)$ approaches some constant value $(1-C)$ as $y \rightarrow \infty$, and then show that C must vanish. If $C \neq 0$, then the dominant term inside the square root for large y is the first term which takes the value $y^2 C^2$, since $\hat{g}'(y)$ vanishes for large y. Thus for large y, Eq. (5.27) takes the form

$$\tfrac{1}{4}\partial_y[V(y)\hat{g}'(y)/yC] + yV(y) = 0. \tag{5.30}$$

Since $\partial_y(\hat{g}'(y)/yC)$ approaches 0 as $y \rightarrow \infty$, clearly the only part of the first term in Eq. (5.30) that can possibly cancel the second term is $V'(y)\hat{g}'(y)/(4yC)$. If this has to cancel the second term, we require

$$V'(y)/V(y) \simeq -4y^2 C/\hat{g}'(y) \quad \text{for large } y. \tag{5.31}$$

Since $\hat{g}(y)$ approaches a constant as $y \rightarrow \infty$, $\hat{g}'(y)$ must fall off faster than $1/y$ for large y. Thus the magnitude of the right hand side of Eq. (5.31) increases faster than y^3 for large y. This, in turn, shows that $-V'(y)/V(y)$ must also increase faster than y^3 for large y. Neither a potential of the form $e^{-\beta y}$ obtained from the analysis of time dependent solutions [5] nor a potential of the form $e^{-\beta y^2}$ given by boundary string field theory [40,41] satisfies this condition. Thus our original assumption must be wrong and C must vanish for either of these choices of $V(T)$.

This leads us to the conclusion that if $-V'(y)/V(y)$ does not increase faster than y^3 for large y, we must have

$$\lim_{y\rightarrow\infty}\hat{g}(y) = 1. \tag{5.32}$$

This, in turn, has the following consequence. From Eqs. (5.10), (5.23), (5.32) we have

$$\int dr\, d\theta(F_{r\theta}^{(1)} - F_{r\theta}^{(2)}) = 2\pi[\bar{g}(\infty) - \bar{g}(0)]$$

$$= 2\pi[\hat{g}(\infty) - \hat{g}(0)] = 2\pi. \tag{5.33}$$

This answer is universal, independent of the choice of the potential $V(T)$, provided $V(T)$ satisfies the mild asymptotic condition given above Eq. (5.32). This is also the same answer that we would have gotten if we had a usual Abelien Higgs model with an action given by the sum of a kinetic and a potential term. Finally, for this gauge field background, if we compute the Ramond-Ramond (RR) charge of the vortex using the usual coupling between the world-volume gauge fields and the RR fields at zero tachyon background, we get the correct expression for the RR charge of the vortex. Thus the net additional contribution to the RR charge from the tachyon dependent coupling of the RR fields [46] must vanish. This is in contrast with the boundary string field theory result [40,41] where the complete contribution to the RR charge comes from the tachyon fields. This again reflects that the fields used here are related to those in boundary string field theory by non-trivial field redefinition.

VI. WORLD-VOLUME ACTION ON THE VORTEX

We shall now study the world-volume action on the vortex. We begin by introducing some notation. We shall denote by x^i for $(p-1) \leq i \leq p$ the coordinates transverse to the world volume of the vortex but tangential to the original brane and by ξ^α for $0 \leq \alpha \leq (p-2)$ the coordinates tangential to the vortex. We shall express the classical vortex solution of Eq. (5.8) in Cartesian coordinates as

$$A_i^{(1)} = -A_i^{(2)} = \bar{h}_i(\vec{x}), \quad T(\vec{x}) = \bar{f}(\vec{x}), \tag{6.1}$$

where

$$\bar{h}_{p-1}(\vec{x}) = -\frac{x^p}{2r^2}\bar{g}(r), \quad \bar{h}_p(\vec{x}) = \frac{x^{p-1}}{2r^2}\bar{g}(r), \quad \bar{f}(\vec{x}) = \bar{f}(r),$$

$$r \equiv |\vec{x}|, \quad \vec{x} = (x^{p-1}, x^p). \tag{6.2}$$

We now make the following ansatz for the fluctuating fields on the world volume of the vortex:

$$A_i^{(1)}(\vec{x}, \xi) = \bar{h}_i[\vec{x} - \vec{t}(\xi)], \quad A_i^{(2)}(\vec{x}, \xi) = -\bar{h}_i[\vec{x} - \vec{t}(\xi)],$$

$$A_\alpha^{(1)}(\vec{x}, \xi) = -\bar{h}_i[\vec{x} - \vec{t}(\xi)]\partial_\alpha t^i + a_\alpha(\xi),$$

$$A_\alpha^{(2)}(\vec{x}, \xi) = \bar{h}_i[\vec{x} - \vec{t}(\xi)]\partial_\alpha t^i + a_\alpha(\xi),$$

$$Y_{(1)}^I(\vec{x}, \xi) = Y_{(2)}^I(\vec{x}, \xi) = y^I(\xi), \quad T(\vec{x}, \xi) = \bar{f}[\vec{x} - \vec{t}(\xi)]. \tag{6.3}$$

Thus the world-volume fields on the vortex are $y^I(\xi)$, $t^i(\xi)$ and $a_\alpha(\xi)$.

We shall now substitute this ansatz into the action (5.1) and evaluate the action. Using the ansatz (6.3) and the definitions (5.3) we get

$$D_i T = \partial_i \bar{f} - 2i\bar{h}_i \bar{f} \equiv D_i \bar{f}, \quad D_\alpha T = -D_i \bar{f} \partial_\alpha t^i,$$

$$F_{ij}^{(1)} = (\partial_i \bar{h}_j - \partial_j \bar{h}_i), \quad F_{ij}^{(2)} = -(\partial_i \bar{h}_j - \partial_j \bar{h}_i),$$

$$F_{i\beta}^{(1)} = -F_{i\beta}^{(2)} = -(\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\beta t^j,$$

$$F_{\alpha j}^{(1)} = -F_{\alpha j}^{(2)} = -(\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\alpha t^i,$$

$$F_{\alpha\beta}^{(1)} = f_{\alpha\beta} + (\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\alpha t^i \partial_\beta t^j,$$

$$F_{\alpha\beta}^{(2)} = f_{\alpha\beta} - (\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\alpha t^i \partial_\beta t^j, \quad (6.4)$$

where

$$f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha. \quad (6.5)$$

In each expression the arguments of $\bar{h}_i$ and $\bar{f}$ are $[\vec{x} - \vec{t}(\xi)]$ which we have suppressed. From Eq. (5.2) we now get

$$\mathbf{A}_{(1)ij} = \delta_{ij} + (\partial_i \bar{h}_j - \partial_j \bar{h}_i) + \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]$$

$$\mathbf{A}_{(1)i\beta} = -(\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\beta t^j$$
$$- \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]\partial_\beta t^j$$

$$\mathbf{A}_{(1)\alpha j} = -(\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\alpha t^i$$
$$- \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]\partial_\alpha t^i$$

$$\mathbf{A}_{(1)\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha y^I \partial_\beta y^I + (\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\alpha t^i \partial_\beta t^j$$
$$+ \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]\partial_\alpha t^i \partial_\beta t^j$$

$$\mathbf{A}_{(2)ij} = \delta_{ij} - (\partial_i \bar{h}_j - \partial_j \bar{h}_i) + \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]$$

$$\mathbf{A}_{(2)i\beta} = (\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\beta t^j$$
$$- \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]\partial_\beta t^j$$

$$\mathbf{A}_{(2)\alpha j} = (\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\alpha t^i$$
$$- \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]\partial_\alpha t^i$$

$$\mathbf{A}_{(2)\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha y^I \partial_\beta y^I - (\partial_i \bar{h}_j - \partial_j \bar{h}_i)\partial_\alpha t^i \partial_\beta t^j$$
$$+ \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]\partial_\alpha t^i \partial_\beta t^j. \quad (6.6)$$

We now simplify the computation of the determinants by subtracting appropriate multiples of the first two rows/columns from the rest of the rows/columns. This does not change the determinant of the matrix. More precisely, we define

$$\hat{\mathbf{A}}_{(s)\alpha\nu} = \mathbf{A}_{(s)\alpha\nu} + \mathbf{A}_{(s)i\nu}\partial_\alpha t^i, \quad \hat{\mathbf{A}}_{(s)i\nu} = \mathbf{A}_{(s)i\nu},$$

$$\tilde{\mathbf{A}}_{(s)\mu\beta} = \hat{\mathbf{A}}_{(s)\mu\beta} + \hat{\mathbf{A}}_{(s)\mu j}\partial_\beta t^j,$$

$$\tilde{\mathbf{A}}_{(s)\mu j} = \hat{\mathbf{A}}_{(s)\mu j} \quad \text{for} \quad 0 \leq \mu, \nu \leq p. \quad (6.7)$$

Under this transformation we have

$$\det(\mathbf{A}_{(s)}) = \det(\hat{\mathbf{A}}_{(s)}) = \det(\tilde{\mathbf{A}}_{(s)}), \quad s = 1,2. \quad (6.8)$$

On the other hand, we have, from Eqs. (6.6), (6.7)

$$\tilde{\mathbf{A}}_{(1)ij} = \delta_{ij} + (\partial_i \bar{h}_j - \partial_j \bar{h}_i) + \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]$$

$$\tilde{\mathbf{A}}_{(1)i\beta} = \partial_\beta t^i, \quad \tilde{\mathbf{A}}_{(1)\alpha j} = \partial_\alpha t^j,$$

$$\tilde{\mathbf{A}}_{(1)\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha y^I \partial_\beta y^I + \partial_\alpha t^i \partial_\beta t^i,$$

$$\tilde{\mathbf{A}}_{(2)ij} = \delta_{ij} - (\partial_i \bar{h}_j - \partial_j \bar{h}_i) + \tfrac{1}{2}[(D_i \bar{f})^* D_j \bar{f} + (D_j \bar{f})^* D_i \bar{f}]$$

$$\tilde{\mathbf{A}}_{(2)i\beta} = \partial_\beta t^i, \quad \tilde{\mathbf{A}}_{(2)\alpha j} = \partial_\alpha t^j,$$

$$\tilde{\mathbf{A}}_{(2)\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha y^I \partial_\beta y^I + \partial_\alpha t^i \partial_\beta t^i. \quad (6.9)$$

Examining the form of the ij component of the matrices $\tilde{\mathbf{A}}_{(1)}$ and $\tilde{\mathbf{A}}_{(2)}$ we see that they are precisely of the same form as one would get for the classical vortex solution without fluctuation, except for the replacement of $\vec{x}$ by $[\vec{x} - \vec{t}(\xi)]$ in the argument of $\bar{h}_i$ and $\bar{f}$. Since this determinant given in Eq. (5.19) has an explicit factor of a^2 which becomes large in the $a \to \infty$ limit, and since $\tilde{\mathbf{A}}_{(s)i\beta}$, $\tilde{\mathbf{A}}_{(s)\alpha j}$ and $\tilde{\mathbf{A}}_{(s)\alpha\beta}$ are all of order one, in this limit we can ignore the contribution from the off-diagonal elements $\tilde{\mathbf{A}}_{(s)i\beta}$ and $\tilde{\mathbf{A}}_{(s)\alpha j}$ in evaluating $\det(\tilde{\mathbf{A}}_{(s)})$. Thus the resulting action is given by

$$-2 \int d^{p-1} \xi \int dr \, d\theta \, V(f(ar)) a f'(ar)$$

$$\times \sqrt{r^2 + f(ar)^2 [1 - g(ar)]^2 + \tfrac{1}{4} (g'(ar)/f'(ar))^2}$$

$$\times \sqrt{-\det \mathbf{a}}, \tag{6.10}$$

where

$$\mathbf{a}_{\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha y^I \partial_\beta y^I + \partial_\alpha t^i \partial_\beta t^i. \tag{6.11}$$

In Eq. (6.10) we have redefined r to be $|\vec{x} - \vec{t}(\xi)|$, and θ to be $\tan^{-1}\{[x^{p-1} - t^{p-1}(\xi)]/[x^p - t^p(\xi)]\}$. We can now explicitly perform the r and θ integrals as in Sec. V and use Eq. (5.29) to rewrite the action (6.10) as

$$-\mathcal{T}_{p-2} \int d^{p-1} \xi \sqrt{-\det \mathbf{a}}. \tag{6.12}$$

This is precisely the world-volume action on a BPS D-$(p-2)$-brane with t^i and y^I interpreted as the coordinates transverse to the brane for $(p-1) \le i \le p$ and $(p+1) \le I \le 9$ and a_α interpreted as the gauge field on the D-brane world volume.

As in Sec. III, in order to establish completely that the dynamics of the world-volume theory on the vortex is governed by the action (6.12) we need to show that given any solution of the equations of motion derived from this action, (6.3) provides us with a solution of the full $(p+1)$-dimensional equations of motion. We have not checked this, but believe that this can be done following techniques similar to that discussed in Sec. III.

VII. DISCUSSION

In this paper we have analyzed kink and vortex solutions in tachyon effective field theory by postulating a suitable form of the tachyon effective action on the non-BPS D-brane and brane-antibrane system, respectively. In both cases the topological soliton has all the right properties for describing a BPS D-brane. These properties include localization of the energy-momentum tensor on subspaces of codimensions 1 and 2, respectively, as is expected of a D-brane and also the DBI form of the effective action describing the world-volume theory on the soliton. For the kink solution we have also done the analysis including the world-volume fermions, and shown the appearance of κ symmetry in the world-volume theory on the kink.

One feature of both the solutions is an infinite spatial gradient of the tachyon field away from the core of the soliton. If we want to construct a solution describing tachyon matter [47,48,5,49–53] in the presence of such a soliton, then the spatial gradient of the tachyon field represents local velocity of the tachyon matter [5,6]. More precisely, the local $(p+1)$-velocity of the dust is given by $u_\mu = -\partial_\mu T$. Thus large positive gradient of the tachyon implies large local velocity *towards* the core of the soliton. This shows that tachyon matter in the presence of such a solution will fall towards the core of the soliton. If this feature survives in the

full string theory, then it will imply that any tachyon matter in contact with the soliton will be sucked in immediately. This is consistent with the analysis of [54,55] where a similar effect was found by analyzing the boundary state associated with the time dependent solutions.[12] This might provide a very effective means of absorbing tachyon matter from the surrounding by a defect brane, and drastically modify the results of Refs. [57,58] for the formation of topological defects during the rolling of the tachyon. The appearance of an infinite slope during the dynamical process of defect formation has already been observed in [19]. We should note, however, that a different type of solution where a codimension 1 soliton and tachyon matter coexist has been constructed in [56].

Another surprising feature of both the kink and the vortex solutions is that the world-volume theory on the soliton has exactly the DBI form without any higher derivative corrections. This means that all such corrections must come from higher derivative corrections to the original actions (1.1) and (5.1). This may seem accidental, but may be significant for the following reasons. This result suggests that there is a close relation between the systematic derivative (of field strength) expansion of the world-volume action of the non-BPS D-p-brane (D-p-brane–$\bar{\text{D}}$-p-brane pair) and that of the BPS soliton solution representing the D-$(p-1)$ brane [D-$(p-2)$-brane]. It will be interesting to explore this line of thought to see if one can establish a precise connection between the two. Since the derivative expansion on the world volume of BPS D-branes is well understood, finding a connection of the type mentioned above will provide a better understanding of the derivative expansion of the world-volume action of a non-BPS D-brane/brane-antibrane system.

One question that we have not addressed in this paper is the analysis of the world-volume theories on (multiple) kink-antikink pairs and multivortex solutions. The construction of these solutions should be quite straightforward following, e.g. the analysis of [27,9,42]. In a finite region around the location of each soliton the solution will have the form discussed in Secs. II and V, and we need to ensure that before taking the $a \to \infty$ limit, the various fields match smoothly, keeping $|T|$ of order a or larger in the intervening space. Analysis of the world-volume theory around such a background will clearly yield the sum of the world-volume actions on the individual solitons, since essentially the field configurations around individual solitons do not talk to each other in the $a \to \infty$ limit. The interesting question is whether we can see the excitations associated with the fundamental string stretched between the solitons. We believe these excitations must come from classical solutions ("solitons") describing fundamental string along the line of Refs. [35,30,33,59]. We can, for example, take the solutions in the DBI theory given in [60–63] and lift them to solutions of the

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[12]We should keep in mind, however, that this result is exact only for bosonic string theory. For superstring theory the corresponding boundary conformal field theory is not solvable, and hence no exact result can be obtained.

equations of motion derived from Eqs. (1.1) or (5.1) using Eqs. (3.1), (3.8) or (6.3). The (spontaneously broken) gauge symmetry that mixes the states of the open string living on individual D-branes with states of the open string stretched between different D-branes, exchanges perturbative states with 'solitonic' states, and hence is analogous to the electric magnetic duality symmetry in gauge theories [64–70].

The general lesson that one could learn from the results of this paper is that for many purposes, it is useful to complement the supergravity action, describing low energy effective action of closed string theory, by coupling it to the tachyon effective action of the type described in this paper. In such a theory, BPS D-branes arise naturally as topological solitons rather than having to be added by hand, and we get the correct low energy effective action on these D-branes. Furthermore, we have seen earlier that this effective action is capable of describing certain time dependent solutions of open string theory [47,48,5], and solutions describing the fundamental string [35,30,33]. Coupling the tachyon field to supergravity does not give rise to any new perturbative physical states, and hence does not violate any known result in string theory. Finally, as was argued in [6], coupling of the tachyon effective action to gravity may resolve some of the conceptual problems involving "time" in quantum gravity.

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