Supersymmetric extension of the Lorentz- and CPT-violating Maxwell-Chern-Simons model

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Focusing on the gauge degrees of freedom specified by a (1+3)-dimensional model accommodating a Maxwell term plus a Lorentz and CPT noninvariant Chern-Simons-like contribution, we obtain a minimal extension of such a system to a supersymmetric framework. We comment on the resulting peculiar self-couplings for the gauge sector, as well as on the background contribution for gaugino masses. Furthermore, a nonpolynomial generalization is presented.

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I. INTRODUCTION

Lorentz and CPT invariances are cornerstones in modern quantum field theory, both symmetries being respected by the standard model for particle physics. Nevertheless, nowadays, one faces the possibility that this scenario is only an effective theoretical description of a low-energy regime, an assumption that leads to the idea that these fundamental symmetries could be violated when one deals with energies close to the Planck scale [1]. Taking this viewpoint, several approaches to analyze the violation of Lorentz symmetry have been proposed in the literature. Eventually, a common feature arises: the violation is implemented by keeping either a four-vector (in a CPT-odd term [1–8]) or a traceless symmetric tensor (CPT-preserving term [9]) unchanged by particle inertial frame transformations [10], which is generally called spontaneous violation. Furthermore, the issue of preserving supersymmetry (SUSY), while violating Lorentz symmetry, is addressed in Ref. [11]. This breaking of Lorentz symmetry is also phenomenologically motivated as a candidate to explain the patterns observed in the detection of ultrahigh energy cosmic rays, concerning the events with energy above the Greisen-Zatsepin-Kuz'min (GZK) ($E_{\mathrm{GZK}} \simeq 4 \times 10^{19}$ eV) cutoff [12]. Moreover, measurements of radio emission from distant galaxies and quasars verify that the polarization vectors of these radiations are not randomly oriented as naturally expected. This peculiar phenomenon suggests that the space-time intervening between the source and observer may be exhibiting some sort of optical activity, the origin of which is not known [13].

In a field-theoretic proposal where the breaking of Lorentz invariance is taken into account, an analysis of the unitarity, causality, and vortexlike solutions has been carried out in Ref. [14]. Another focus of interest points to planar gauge systems, which play a relevant role in condensed matter descriptions, as they happen to be related to issues such as high-T_c superconductivity and the fractional quantum Hall effect. Possible contributions from Lorentz-violating terms to the appearance of anisotropy in planar systems have been investigated in Refs. [15,16].

A first proposal to incorporate supersymmetry in connection with Lorentz violation was carried out in the work of Ref. [11]. The aim of that work was to investigate whether one could maintain the desired properties of supersymmetric systems, namely, the cancellation of divergences and the patterns of spontaneous breaking schemes, while violating Lorentz symmetry. A Lorentz-breaking tensor with constant entries has been adopted, following an original suggestion given by Colladay [10]. Working upon a modified Wess-Zumino model, the authors of Ref. [11] demonstrated that convenient corrections to the SUSY-algebra of fermionic charges and SUSY-covariant derivatives have to be taken into account to set up a SUSY-like invariance for the Lorentz-violating original theory. As a matter of fact, the modification of the algebra was achieved by adding a particular tensor-dependent central term, of the $k_{\mu\nu}\partial^{\nu}$-type, where $k_{\mu\nu}$ exhibits real symmetric traceless tensor properties.

As a net result, it was shown that a model for a modified-SUSY invariant, but Lorentz noninvariant, *matter* system can be built. Motivated by a different perspective, we now present an analysis on the Lorentz and SUSY breakings concerning degrees of freedom in the *gauge* field sector. We start off by establishing a supersymmetry-like minimal extension for the Chern-Simons-like term [1],

$$\Sigma_{\mathrm{CS}} = -\frac{1}{4}\int dx^4 \epsilon^{\mu\nu\alpha\beta} c_{\mu} A_{\nu} F_{\alpha\beta}, \tag{1}$$

preserving the usual $(1+3)$-dimensional SUSY algebra. The breaking of SUSY will follow the very same route to Lorentz breaking: the statement that c_{μ} is a constant (in the active sense) vector triggers both Lorentz and, as we shall comment on, SUSY breakings. It is convenient here to make more precise our statement on what we mean by supersymmetrizing the term in Eq. (1). The algebra of SUSY generators and covariant derivatives will not be changed; consequently, the component-field transformation laws under SUSY are not modified. However, it will become manifest later that the breaking of SUSY that accompanies the Lorentz violation is a sort of explicit SUSY breaking, realized at the Lagrangian level by means of terms that induce a mass splitting between the photon and its partner, the photino. This means that, instead of introducing the vector c_μ at the level of the SUSY generators, we adopt a different strategy: we rather decide to accommodate this background vector inside a suitable superfield, accompanied by a background fermion.

Choosing appropriate superfield extensions for the background prevents the model from displaying higher-spin excitations, and interesting self-couplings for the gauge sector as well as background contribution for the gaugino masses come up naturally, as a consequence of the (initially) supersymmetric structure.

In the next section, we present the SUSY minimal extension for the action (1). In Sec. III, a first generalization, with nonpolynomial couplings, shows up. Finally, we comment on conclusions and perspectives in Sec. IV.

II. THE SUPERSYMMETRIC EXTENSION OF THE MAXWELL-CHERN-SIMONS MODEL

Adopting covariant superspace-superfield formulation, we propose the following minimal extension for the action (1):

$$A=\int d^4x\, d^2\theta\; d^2\bar{\theta}\{W^a(D_aV)S+\bar{W}_{\dot{a}}(\bar{D}^{\dot{a}}V)\bar{S}\}, \tag{2}$$

where the superfields W_a, V, S and the SUSY-covariant derivatives D_a, $\bar{D}_{\dot{a}}$ hold the definitions

$$D_a=\frac{\partial}{\partial\theta^a}+i\sigma^\mu{}_{a\dot{a}}\bar{\theta}^{\dot{a}}\partial_\mu, \tag{3}$$

$$\bar{D}_{\dot{a}}=-\frac{\partial}{\partial\bar{\theta}^{\dot{a}}}-i\theta^a\sigma^\mu{}_{a\dot{a}}\partial_\mu; \tag{4}$$

from $\bar{D}_{\dot{b}}W_a(x,\theta,\bar{\theta})=0$ and $D^aW_a(x,\theta,\bar{\theta})=\bar{D}_{\dot{a}}\bar{W}^{\dot{a}}(x,\theta,\bar{\theta})$, it follows that

$$W_a(x,\theta,\bar{\theta})=-\frac{1}{4}\bar{D}^2D_aV. \tag{5}$$

Its θ expansion reads as below,

$$W_a(x,\theta,\bar{\theta})=\lambda_a(x)+i\theta^b\sigma^\mu{}_{b\dot{a}}\bar{\theta}^{\dot{a}}\partial_\mu\lambda_a(x)-\frac{1}{4}\bar{\theta}^2\theta^2\Box\lambda_a(x)+2\theta_aD(x)-i\theta^2\bar{\theta}^{\dot{a}}\sigma^\mu{}_{a\dot{a}}\partial_\mu D(x)+\sigma^{\mu\nu b}{}_a\theta_bF_{\mu\nu}(x)$$
$$-\frac{i}{2}\sigma^{\mu\nu b}{}_a\sigma^\alpha{}_{b\dot{a}}\theta^2\bar{\theta}^{\dot{a}}\partial_\alpha F_{\mu\nu}(x)-i\sigma^\mu{}_{a\dot{a}}\partial_\mu\bar{\lambda}^{\dot{a}}(x)\theta^2 \tag{6}$$

and $V=V^\dagger$. The Wess-Zumino gauge choice is taken as usually done:

$$V_{WZ}=\theta\sigma^\mu\bar{\theta}A_\mu(x)+\theta^2\bar{\theta}\bar{\lambda}(x)+\bar{\theta}^2\theta\lambda(x)+\theta^2\bar{\theta}^2D, \tag{7}$$

with no loss of generality, since the action (2) is gauge-invariant.

The background superfield is so chosen to be chiral: $\bar{D}_{\dot{a}}S(x)=0$. Such a constraint restricts the highest spin component of the background to be an $s=\frac{1}{2}$ component-field, showing up as a SUSY-partner for a spinless dimensionless scalar field. Also, one should notice that S turns out to be dimensionless. The superfield expansion for S then reads

$$S(x)=s(x)+i\theta\sigma^\mu\bar{\theta}\partial_\mu s(x)-\frac{1}{4}\bar{\theta}^2\theta^2\Box s(x)+\sqrt{2}\theta\psi(x)+\frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x)+\theta^2F(x). \tag{8}$$

The component-field version of the action (2) is as follows:

$$A_{comp.}=\int d^4x\;\Big\{-\frac{1}{2}(s+s^*)F_{\mu\nu}F^{\mu\nu}+\frac{i}{2}\partial_\mu(s-s^*)\varepsilon^{\mu\alpha\beta\nu}F_{\alpha\beta}A_\nu+4D^2(s+s^*)-2is\;\lambda\;\;\sigma^\mu\partial_\mu\bar{\lambda}-2is^*\;\bar{\lambda}\;\bar{\sigma}^\mu\partial_\mu\lambda$$
$$-\sqrt{2}\lambda(\sigma^{\mu\nu})F_{\mu\nu}\psi+\sqrt{2}\;\bar{\lambda}(\bar{\sigma}^{\mu\nu})F_{\mu\nu}\bar{\psi}+\lambda\;\;\lambda F+\bar{\lambda}\;\;\bar{\lambda}F^*-2\sqrt{2}\lambda\;\;\psi D-2\sqrt{2}\;\bar{\lambda}\;\;\bar{\psi}D\Big\}\;. \tag{9}$$

As one can easily recognize, the first line displays the 4D Chern-Simons-like term (1), where the vector c_μ is expressed as the gradient of a real background scalar:

$$c_\mu=\partial_\mu\sigma \qquad \text{for} \qquad s=\xi+i\sigma. \tag{10}$$

Such a reduction of the vector into a gradient of a scalar field stems directly from the simultaneous requirements of gauge symmetry[1] and minimal supersymmetry.

Another interesting feature of this model concerns the presence of self-couplings for the gauge sector: the fermionic background field, ψ, triggers the coupling of the gauge boson (through the field-strength) to the gaugino. Moreover, using the equation of motion for the gauge auxiliary field D yields a quartic fermionic field coupling—$\lambda\lambda\psi\psi$—and the background nature of ψ indicates a background contribution for the gaugino mass.[2]

Concerning the breaking of Lorentz symmetry, realized by assuming $c_\mu=\partial_\mu\sigma$ to be constant under the action of particle inertial frame transformations, one should observe that such an assumption implies that the imaginary part of the scalar component-field σ must be linear in the coordinates, $\sigma=c_\mu x^\mu$. As a matter of fact, a linear dependence on x^μ cannot be implemented by means of a SUSY-covariant constraint (i.e., SUSY-covariant derivatives acting on S), and, in that sense, the choice of a rigid $\partial_\mu\sigma$ breaks SUSY in exact analogy to the Lorentz-breaking scheme adopted. To better establish such a correspondence, one can consider the choice for constant $\partial_\mu\sigma$ to be accompanied by the requirement of a constant ψ (and a constant auxiliary field, F, as well[3]). On the other hand, the choice of a constant ψ requires ξ, the real part of s (that is not directly constrained by gauge invariance), to be also linear in the coordinates, $\xi=d_\mu x^\mu$. In this context, a (passive) SUSY-transformation keeps on equal footing all component fields as far as their space-time dependence is concerned.

In the next section, we provide the model with a nonpolynomial generalization, which brings about the possibility of understanding the 4D CS-like term as a first-order correction in a complete exponential scenario.

III. NONPOLYNOMIAL GENERALIZATION

Let us point out that the integration defined by means of the Grassmanian measure $d^2\bar\theta$ (or $d^2\theta$) can be represented by the action of a squared SUSY-covariant derivative (up to a normalization factor), $\bar D^2$ (or D^2), on the super-Lagrangian $W^a(D_aV)S+\text{H.c.}$, if one neglects boundary terms, and that the only sector of the superfield product $W(DV)S$ [or $\bar W(\bar D V)\bar S$] that admits a nonvanishing action under $\bar D^2$ (or D^2) is the factor DV (or $\bar D V$). Such a manipulation leads to the action $\int d^4x[d^2\theta\, W^a(\bar D^2 D_aV)S+d^2\bar\theta\,\bar W_{\dot a}(D^2\bar D^{\dot a}V)\bar S]$, and one can rewrite Eq. (2) through such a parametrization:

$$A=h\int d^4x\{d^2\theta[W^aW_aS]+d^2\bar\theta[\bar W_{\dot a}\bar W^{\dot a}\bar S]\}, \tag{11}$$

where a suitable dimensionless (perturbation) parameter h is inserted. We remark that such an inclusion does not spoil any power-counting renormalization property of the model. Moreover, as we aim at a SUSY version for a model hosting both the regular Maxwell kinetic term and the 4D CS-like term [14], we end up with the following combination:

$$\begin{aligned}A_{\text{MCS}}=&\frac{1}{4}\int d^4x\{d^2\theta[W^aW_a]+d^2\bar\theta[\bar W_{\dot a}\bar W^{\dot a}]\}\\&+\frac{h}{4}\int d^4x\{d^2\theta[W^aW_aS]+d^2\bar\theta[\bar W_{\dot a}\bar W^{\dot a}\bar S]\}.\end{aligned} \tag{12}$$

Such an expression induces a straightforward nonpolynomial generalization:

$$\begin{aligned}A_{\text{np}}=&\frac{1}{4}\int d^4x\{d^2\theta[W^aW_a\exp(hS)]\\&+d^2\bar\theta[\bar W_{\dot a}\bar W^{\dot a}\exp(h\bar S)]\},\end{aligned} \tag{13}$$

leaving room for a perturbative approach parametrized by orders of h. In fact, the action (13) includes a zero-order supersymmetric Maxwell theory, a first-order SUSY-extended 4D CS-like term [reproducing the action of the Eq. (9)], and higher-order contributions. In component-field parametrization, action (13) reads

$$\begin{aligned}A_{\text{n-p}}=&\frac{1}{4}\int d^4x\Bigg\{\exp(hs)\Bigg[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}-\frac{i}{2}\tilde F_{\mu\nu}F^{\mu\nu}\\&-2i\lambda^a\sigma^\mu{}_{a\dot a}\partial_\mu\bar\lambda^{\dot a}+4D^2+h[-2\sqrt{2}\lambda^a\psi_aD\\&+\lambda^a\lambda_aF-\sqrt{2}\lambda^a(\sigma^{\mu\nu})_a^bF_{\mu\nu}\psi_b]\\&-\frac{h^2}{2}\lambda^a\lambda_a\psi^b\psi_b\Bigg]+\text{H.c.}\Bigg\}.\end{aligned} \tag{14}$$

The exponential version brings about an expression of the form $-(i/8)\exp(hs)\tilde F_{\mu\nu}F^{\mu\nu}+\text{H.c.}$, demanding an integration by parts to reproduce a Chern-Simons-like term, $i\partial_\mu(s-s^*)\varepsilon^{\mu\alpha\beta\nu}F_{\alpha\beta}A_\nu$, as one expands the remaining exponential. One should also realize that a quartic fermion-fields coupling is already present at order h^2, even if the field equation for the auxiliary field D is not used to eliminate it. It is also interesting to observe how the background components s, ψ and F influence the gaugino physical mass.

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[1]The gauge invariance of action (2) will become clearly manifest in the next section, where we rephrase the supersymmetrization of the 4D Chern-Simons-like term in a formulation restricted to the chiral (antichiral for the H.c. counterpart) sector of superspace.

[2]We shall analyze the propagator structure for the gauge component-fields in a forthcoming communication. We anticipate that a constant ψ component-field configuration is compatible with the supersymmetry algebra.

[3]In fact, a constant auxiliary field F is singled out as a SUSY-invariant parameter, as far as one deals with a constant ψ.

IV. CONCLUDING COMMENTS

Working on the gauge-field sector of a system with a Lorentz breaking 4D-Chern-Simons-like term, we have been able to derive its minimal supersymmetric extension and a peculiar nonpolynomial generalization has been proposed that is compatible with $N=1$-SUSY. Focusing on the minimal SUSY-extension, one should already realize the presence of new couplings induced by the background (passive) superfield components. The assumption that the Lorentz breaking is implemented by means of a constant vector, regarded as a background input, finds its SUSY-counterpart in a set of requirements on the space-time dependence of each component-field of the background superfield, S. As discussed at the end of Sec. II, the question of the space-time dependence of the component fields accommodated in the background superfield, S, was clarified. Indeed, a scalar field, s, linearly dependent on x^{μ}, as well as a constant spinor field, ψ, arise in connection with gauge invariance, and these results indicate that, eventually, coupling terms are to be regarded as mass terms. A complete analysis of the propagator structure for the gauge supermultiplet, both in superspace and in component fields, is mandatory, including an interesting study of the gaugino (background-)induced mass. In terms of components, the explicit breaking of the Lorentz symmetry becomes manifest through the appearance of a gauge boson/gaugino mixed propagator induced by the action term that involves the gauge potential, the gaugino, and the background fermion according to Eq. (9). This is a rather peculiar point and, in deriving the full set of propagators, it will become clear whether the gauge field and its fermionic partner, λ, will share a common dispersion relation, for which the background-fermion condensate, $\bar{\psi}\psi$, contributes along with c_{μ}, the external vector responsible for the Lorentz breaking. The conditions to establish the causality and the unitarity of the model at the tree-approximation, as presented in Ref. [14], have now to be reassessed in view of the presence of the background-fermion condensate together with the c_{μ}-background vector. Therefore, besides considering the cases where c_{μ} is a timelike, a lightlike or a spacelike vector, conditions on the background-fermion condensate have to be properly set up in order that neither tachyons nor ghosts be present among the excitations corresponding to the poles of the propagators. We shall very soon report our efforts in this matter elsewhere.

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[1] S. Carroll, G. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).

[2] A.A. Andrianov, R. Soldati, and L. Sorbo, Phys. Rev. D **59**, 025002 (1999).

[3] R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999); M. Perez-Victoria, J. High Energy Phys. **04**, 032 (2001).

[4] F.R. Klinkhamer, Nucl. Phys. **B578**, 277 (2000).

[5] J.M. Chung and B.K. Chung, Phys. Rev. D **63**, 105015 (2001).

[6] C. Adam and F.R. Klinkhamer, Nucl. Phys. **B607**, 247 (2001).

[7] V.A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001); R. Lehnert, *ibid.* (to be published), gr-qc/0304013.

[8] V.A. Kostelecký, R. Lehnert, and M. Perry, ''Space-Time-Varying Constants and Lorentz Violation,'' astro-ph/0212003.

[9] V.A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); V.A. Kostelecký and R. Potting, Nucl. Phys. **B359**, 545 (1991); Phys. Lett. B **381**, 89 (1996); Phys. Rev. D **51**, 3923 (1995).

[10] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); *ibid.* **58**, 116002 (1998).

[11] M.S. Berger and V.A. Kostelecký, Phys. Rev. D **65**, 091701(R) (2002).

[12] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966); G.T. Zatsepin and V.A. Kuzmin, Pis'ma Zh. Eksp. Teor. Fiz. **4**, 114 (1966); Orfeu Bertolami, ''Threshold Effects and Lorentz Symmetry,'' hep-ph/0301191; J.W. Moffat, ''Spontaneous Violation of Lorentz Invariance and Ultrahigh-Energy Cosmic Rays,'' hep-th/0211167.

[13] D. Hutsemékers and H. Lamy, Astron. Astrophys. **332**, 410 (1998); **367**, 381 (2001); M. Goldhaber and V. Trimble, J. Astrophys. Astron. **17**, 17 (1996); V.B. Bezerra, H.J. Mosquera Cuesta, and C.N. Ferreira, Phys. Rev. D **67**, 084011 (2003).

[14] A.P. Baěta Scarpelli, H. Belich, J.L. Boldo, and J.A. Helayël-Neto, Phys. Rev. D **67**, 085021 (2003).

[15] H. Belich, M.M. Ferreira, Jr., J. A. Helayël-Neto, and M.T.D. Orlando, Phys. Rev. D **68**, 025005 (2003).

[16] H. Belich, M.M. Ferreira, Jr., J. A. Helayël-Neto, and M.T.D. Orlando, Phys. Rev. D **67**, 125011 (2003).