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One-loop effective action for $\mathcal{N}=4$ SYM theory in the hypermultiplet sector: Leading low-energy approximation and beyond

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We develop the derivative expansion of the one-loop $\mathcal{N}=4$ super Yang-Mills (SYM) effective action depending both on the $\mathcal{N}=2$ vector multiplet and on hypermultiplet background fields. Beginning with the formulation of $\mathcal{N}=4$ SYM theory in terms of $\mathcal{N}=1$ superfields, we construct the one-loop effective action with the help of superfield functional determinants and calculate this effective action in $\mathcal{N}=1$ superfield form using the approximation of constant Abelian strength F_{mn} and corresponding constant hypermultiplet fields. Then we show that the terms in the supercovariant derivative expansion of the effective action can be rewritten in terms of $\mathcal{N}=2$ superfields. As a result, we get a new derivation of the complete $\mathcal{N}=4$ supersymmetric low-energy effective action obtained by Buchbinder and Ivanov and find subleading corrections to it.

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I. INTRODUCTION

The $\mathcal{N}=4$ super Yang-Mills (SYM) theory has attracted much attention due to the remarkable properties that allow us to clarify profound questions concerning quantum dynamics in supersymmetric field models and their links with string or brane theory. The maximally extended rigid supersymmetry of the $\mathcal{N}=4$ SYM theory imposes strong restrictions on quantum dynamics. As a result, the quantities characterizing the theory in the quantum domain can be exactly found or studied in great detail (see, e.g., [1–5]).

This is the first paper in a series in which an attempt is made to calculate the one-loop low-energy effective action in the quantum gauge $\mathcal{N}=4$ SYM theory, depending on all fields of the $\mathcal{N}=4$ vector multiplet. Unfortunately, a manifestly supersymmetric formulation for $\mathcal{N}=4$ Yang-Mills theory is still unknown. At present, the best, most symmetric and adequate, description of $\mathcal{N}=4$ vector multiplet dynamics is given in terms of unconstrained harmonic $\mathcal{N}=2$ superfields. From this point of view, the $\mathcal{N}=4$ SYM theory is a model of $\mathcal{N}=2$ SYM theory coupled to a hypermultiplet in the adjoint representation of the gauge group. It is well known that the exact low-energy quantum dynamics of $\mathcal{N}=4$ SYM theory in the $\mathcal{N}=2$ vector multiplet sector is controlled by the nonholomorphic effective potential $\mathcal{H}(\mathcal{W},\bar{\mathcal{W}})$,[1] depending on the $\mathcal{N}=2$ strengths $\mathcal{W},\bar{\mathcal{W}}$ (see Refs. [2,7–11]). The explicit form of the nonholomorphic potential for the $SU(N)$ gauge group spontaneously broken down to its maximal torus looks like

$$\mathcal{H}(\mathcal{W},\bar{\mathcal{W}})=c\sum_{I<J}\ln\left(\frac{\mathcal{W}^I-\mathcal{W}^J}{\Lambda}\right)\ln\left(\frac{\bar{\mathcal{W}}^I-\bar{\mathcal{W}}^J}{\Lambda}\right), \qquad (1)$$

where Λ is an arbitrary scale, $I,J=1,\dots,N$, and $c=1/(4\pi)^2$ (for more detail, see Ref. [11]). The expression (1) defines the exact low-energy effective potential in leading order in the external momentum expansion in the $\mathcal{N}=2$ gauge superfield sector [7,8]. We emphasize that the result (1) is so general that it can be obtained entirely on symmetry grounds from the requirements of scale independence and R invariance up to a numerical factor [7,12]. Moreover, the potential (1) gets neither perturbative quantum corrections beyond one loop nor instanton corrections [7,8] (see also the discussion of the nonholomorphic potential in $\mathcal{N}=2$ SYM theories [12–15]). All these properties are very important for understanding the low-energy quantum dynamics of $\mathcal{N}=4$ SYM theory in the Coulomb phase. In particular, the effective potential (1) provides the first subleading terms in the interaction between parallel D3-branes in superstring theory (see, e.g., [16]). It has been proposed that the full $\mathcal{N}=4$ SYM effective action, depending on proper invariants constructed from the arbitrary powers of the Abelian strength F_{mn} and obtained by summing up all the loop quantum corrections, should reproduce (within certain limits) the Born-Infeld action [17] ($\mathcal{N}=4$ SYM-supergravity correspondence). These nonlocal contributions have been expanded in a low-energy approximation and expressed as the sum of an infinite series of local terms. It is argued that these local expressions reproduce contributions to the Born-Infeld action if supersymmetry has to determine its structure. A dis-

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[1]The low-energy effective action in an arbitrary $\mathcal{N}=2$ SYM model can contain, in principle, a holomorphic effective potential [6] but it vanishes in $\mathcal{N}=4$ gauge theory.

cussion of this correspondence and its two-loop test are given in Ref. [18] (see also the consideration of various aspects of the analogous problems for a non-Abelian background in Refs. [19–21] and the general approach to calculating the higher-loop corrections in [22]).

In order to clarify the structure of the restrictions on the effective action, stipulated by $\mathcal{N}=4$ supersymmetry, and to gain a deeper understanding of the $\mathcal{N}=4$ SYM-supergravity correspondence, we have to find an effective action that is not only in the $\mathcal{N}=2$ vector multiplet sector but also depends on all the fields of the $\mathcal{N}=4$ vector multiplet (see the discussion in [23]). This problem remained unsettled for a long time. Recently, the complete leading part $\sim F^4$ to the exact low-energy effective action containing the dependence on both $\mathcal{N}=2$ gauge superfields and hypermultiplets has been discovered [24]. It has been shown that the algebraic restrictions imposed by hidden $\mathcal{N}=2$ supersymmetry on the structure of the low-energy effective action in the $\mathcal{N}=2$ harmonic superspace approach turn out to be so strong that they allow us to restore the dependence of the low-energy effective action on the hypermultiplets on the basis of the known nonholomorphic effective potential (1). As a result, the additional hypermultiplet-dependent contributions containing the on-shell $\mathcal{W},\bar{\mathcal{W}}$ and the hypermultiplet q^{ia} [26] superfields have been obtained in the form

$$\mathcal{L}_q = c\left\{(X-1)\frac{\ln(1-X)}{X} + [\mathrm{Li}_2(X)-1]\right\}, \tag{2}$$

$$X = -\frac{q^{ia}q_{ia}}{\mathcal{W}\bar{\mathcal{W}}},$$

where $\mathrm{Li}_2(X)$ is the Euler dilogarithm function and c is the same constant as in Eq. (1) (see the details and notation in Refs. [5,24]). The effective Lagrangian (2), together with the nonholomorphic effective potential (1), determines the exact $\mathcal{N}=4$ supersymmetric low-energy effective potential in the theory under consideration.

The leading low-energy effective Lagrangian (2) was found in Ref. [24] on purely algebraic grounds. It would be extremely interesting to derive this Lagrangian and next-to-leading corrections in external momenta in the framework of quantum field theory (QFT). This problem seems to be very nontrivial since the expression (2) includes any powers of X and is singular at $\mathcal{W}=0$; therefore the result cannot be obtained by considering the Feynman diagrams with a fixed number of external hypermultiplet and gauge field legs. All such diagrams must be summed up. In a recent paper [27], the problem of computing the effective Lagrangian (2) was solved using covariant harmonic supergraph techniques [2,28]. The more general problem consists in the QFT or algebraic derivation of the subleading terms in the effective action, depending on all fields of the $\mathcal{N}=4$ supermultiplet, and representation of these terms in a completely $\mathcal{N}=4$ supersymmetric form. The present paper is just devoted to methods for solution of such a problem for the one-loop effective action. To be more precise, we discuss the construction of the derivative expansion of the one-loop effective Lagrangian $\mathcal{L}_{eff}$ depending on the $\mathcal{N}=2$ gauge background superfields, their spinor derivatives up to some order, and the hypermultiplet background superfields using the formulation of $\mathcal{N}=4$ SYM theory in terms of $\mathcal{N}=1$ superfields [29,30] and exploring derivative expansion techniques in $\mathcal{N}=1$ superspace [31] (see also [32]). It allows us to obtain the exact coefficients at various powers of the covariant spinor derivatives of the $\mathcal{N}=2$ superfield Abelian strength $\mathcal{W}$, corresponding to a constant space-time background that belongs to the Cartan subalgebra of the gauge group $SU(N)$ spontaneously broken down to $U(1)^{n-1}$ and the constant space-time background hypermultiplet q^{ia}:

$$\mathcal{W}| = \Phi = \text{const}, \quad D^i_\alpha \mathcal{W}| = \lambda^i_\alpha = \text{const}, \tag{3}$$

$$q^{ia}| = \text{const}, \quad D^i_{(\alpha}D_{\beta)i}\mathcal{W}| = F_{\alpha\beta} = \text{const},$$

$$D^{\alpha(i}D^{j)}_\alpha \mathcal{W}| = 0, \quad D^i_\alpha q^{aj}| = 0, \quad D^i_{\dot{\alpha}} q^{aj}| = 0,$$

where $\Phi = \mathrm{diag}(\Phi^1, \Phi^2, \ldots, \Phi^n)$, $\Sigma\Phi^I = 0$. This background is the simplest one allowing exact calculation of the one-loop effective action. We will show that in this case the $\mathcal{N}=1$ superspace effective action can be uniquely found on the basis of the effective action for a vanishing hypermultiplet [31,33] by means of a simple variable modification. Following this, the result obtained maintaining the complete hypermultiplet dependence is rewritten in a manifestly $\mathcal{N}=2$ supersymmetric form. For this purpose we use the same procedure as in [33] and natural prescriptions for reconstruction terms containing hypermultiplet derivatives. We emphasize that the background (3) is a special supersymmetric solution to the classical equations of motion of the $\mathcal{N}=1$ superfield model representing the $\mathcal{N}=4$ SYM theory in terms of $\mathcal{N}=1$ superfields, and therefore the effective action does not depend on the choice of the $\mathcal{N}=1$ superfield gauge fixing conditions we impose on the theory. Moreover, it can be shown that the background (3) is completely formulated in terms of $\mathcal{N}=2$ superfields, which provides the possibility of writing the effective action on this background in a manifestly $\mathcal{N}=2$ supersymmetric form. As long as we are interested in the $\mathcal{N}=2$ SYM effective action having special hypermultiplet matter fields and constructed on this background without any additional requirements except $\mathcal{N}=2$ supersymmetry and gauge invariance, we can be sure that it possesses the mentioned symmetry properties because the action is written in terms of $\mathcal{N}=2$ superfield strengths.

However, one should be extremely careful in respect of additional requirements like hidden $\mathcal{N}=2$ supersymmetry because of the background (3). As will be shown in Sec. II, this background is not form invariant under the hidden $\mathcal{N}=2$ supersymmetry transformations of $\mathcal{N}=4$ supersymmetry. Complete on-shell $\mathcal{N}=4$ supersymmetry involves transformations between the physical fields from the $\mathcal{N}=2$ vector multiplet and those from hypermultiplets. As a general rule higher-derivative additions to the actions are in general compatible with supersymmetry only if the transformation rules for the fields also receive higher-derivative corrections. The properties of the action obtained related to the hidden symmetry will be studied in a separate forthcoming work.

The paper is organized as follows. In the next section we recall the known properties of $\mathcal{N}=4$ SYM theory in the $\mathcal{N}=1$ and $\mathcal{N}=2$ formalism and discuss the background field quantization, including the choice of proper gauge fixing conditions. In Sec. III we describe the calculations leading to an exact one-loop $\mathcal{N}=1$ superfield effective action for the background (3). Section IV is devoted to the representation of this effective action in a manifestly $\mathcal{N}=2$ form and a discussion of the prescriptions necessary for obtaining such a form. In the Summary we formulate the final results and discuss unsolved problems.

II. MINIMAL FORMULATION OF $\mathcal{N}=4$ SYM THEORY IN $\mathcal{N}=1,2$ SUPERSPACES AND $\mathcal{N}=1$ SUPERSYMMETRIC BACKGROUND FIELD METHOD

A formulation of $\mathcal{N}=4$ SYM theory possessing off-shell manifestly $\mathcal{N}=4$ supersymmetry is unknown so far. Therefore the study of the concrete quantum aspects of this theory is usually based on its formulation either in terms of physical component fields (see, e.g., [34]), or in terms of $\mathcal{N}=1$ superspace (see, e.g., [29]), or in terms of $\mathcal{N}=2$ harmonic superspace [25,26]. In the first case, all four supersymmetries are hidden; in the second case, one of them is manifest and the other three are hidden; in the third case, two supersymmetries are manifest and the other two are hidden. It is worth pointing out that in all cases at least some of the supersymmetries are on shell. Taking into account that the presence of manifest symmetries simplifies the process of calculations in quantum theory, it is reasonable to consider that at present just the $\mathcal{N}=2$ harmonic superspace formulation is the best one for quantum $\mathcal{N}=4$ SYM theory. However, the formulation in terms of $\mathcal{N}=1$ superspace has its own positive features, basically due to the relatively simple structure of $\mathcal{N}=1$ superspace and the large accumulated experience of work with $\mathcal{N}=1$ supergraphs.

The $\mathcal{N}=4$ superfield description of the $\mathcal{N}=4$ vector multiplet can be realized with the help of on-shell $\mathcal{N}=4$ superfields W^{AB}, $A=1,\dots,4$ [35] satisfying the reality conditions

$$W^{AB}=\frac{1}{2}\varepsilon^{ABCD}W_{CD}, \quad W_{AB}=\overline{W}^{AB}$$

and the on-shell constraints

$$\bar{D}_{A\dot{\alpha}}W^{BC}=\frac{1}{3}\delta_A^{[B}\bar{D}_{E\dot{\alpha}}W^{EC]}, \quad D_\alpha^{(A}W^{B)C}=0.$$

All physical fields of the $\mathcal{N}=4$ vector multiplet are contained in the superfield W^{AB}. We point out also the attempts to develop an unconstrained formulation in the harmonic superspace approach [36], and discuss the integral invariants in $\mathcal{N}=4$ SYM theory that can be constructed from the field strength W^{AB}, which are integrals over fewer than the maximum number of odd coordinates but which are still manifestly supersymmetric (see [21]).

A. $\mathcal{N}=4$ SYM theory in $\mathcal{N}=1$ superspace

The physical field content of the superfield W^{AB} can be obtained by combining three $\mathcal{N}=1$ chiral superfields and one $\mathcal{N}=1$ vector multiplet superfield [29]. Then the six real scalars, which are the lowest components of the superfield W^{AB}, are represented by the three complex scalar components of the chiral $\mathcal{N}=1$ superfields Φ^i. The four Weyl fermions from W^{AB} are divided into three plus one. Three of them are considered as the spinor components of Φ^i and the fourth fermion is treated as a gaugino and constitutes, together with the real vector, the $\mathcal{N}=1$ vector multiplet superfield V. In such a description, the $SU(3)\otimes U(1)$ subgroup of the $SU(4)$ R-symmetry group is manifest, and the representations of $SU(4)$ are decomposed according to $\mathbf{6}\to\mathbf{3}+\bar{\mathbf{3}}$, $\mathbf{4}\to\mathbf{3}+\mathbf{1}$ so that the chiral superfields Φ^i transform in the $\mathbf{3}$ of $SU(3)$, the antichiral $\bar{\Phi}_i$ transform in the $\bar{\mathbf{3}}$, and the vector multiplet superfield is a singlet under $SU(3)$.

The action of the $\mathcal{N}=4$ SYM model is formulated in terms of $\mathcal{N}=1$ superspace as follows:

$$S=\frac{1}{g^2}\mathrm{tr}\left\{\int d^4x d^2\theta W^2+\int d^4x d^4\theta \bar{\Phi}_i \mathrm{e}^V\Phi^i\mathrm{e}^{-V} + \frac{1}{3!}\int d^4x d^2\theta i c_{ijk}\Phi^i[\Phi^j,\Phi^k] + \frac{1}{3!}\int d^4x d^2\bar{\theta} i c^{ijk}\bar{\Phi}_i[\bar{\Phi}_j,\bar{\Phi}_k]\right\}. \quad (4)$$

The notation and conventions correspond to those of Ref. [29]. All superfields here are taken in the adjoint representation of the gauge group. Both $\mathcal{N}=1$ SYM and the chiral superfield actions are superconformal invariants. In addition to the manifest $\mathcal{N}=1$ supersymmetry and $SU(3)$ symmetry on the $i,j,k,\dots$ indices of Φ and $\bar{\Phi}$, it has hidden global supersymmetry given by the transformations

$$\delta W_\alpha=-\epsilon_\alpha^i\bar{\nabla}^2\bar{\Phi}_{ci}+i\epsilon_i^{\dot{\alpha}}\nabla_{\alpha\dot{\alpha}}\Phi_c^i,$$
$$\delta\bar{W}_{\dot{\alpha}}=-\bar{\epsilon}_{\dot{\alpha}i}\nabla^2\Phi_c^i+i\epsilon^{\alpha i}\nabla_{\alpha\dot{\alpha}}\bar{\Phi}_{ci}, \quad (5)$$
$$\delta\Phi_c^i=\epsilon^{\alpha i}W_\alpha, \quad \delta\bar{\Phi}_{ci}=\bar{\epsilon}_i^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}.$$

The action (4) is also invariant under the transformations

$$\delta\Phi_c^i=c^{ijk}\bar{\nabla}^2(\bar{\chi}_j\bar{\Phi}_{ck})+i[\chi^j\bar{\Phi}_{cj},\Phi_c^i],$$
$$\delta\bar{\Phi}_{ci}=c_{ijk}\nabla^2(\chi^j\Phi_c^k)+i[\bar{\chi}_j\Phi_c^j,\bar{\Phi}_{ci}]. \quad (6)$$

Here the covariant spinor derivatives $\nabla_\alpha,\nabla_{\dot{\alpha}},\nabla^2$, and $\bar{\nabla}^2$ are defined in Ref. [29] and χ^i are the $\mathcal{N}=1$ superfield parameters forming the $SU(3)$ isospinor as well as Φ^i. These parameters include the central charge transformation parameters, supersymmetry transformation parameters, and internal symmetry parameters of $SU(4)/SU(3)$. The transformations (6) are given in terms of the background covariant superfields $\Phi_c=e^{\bar{\Omega}}\Phi e^{-\bar{\Omega}}$, $\bar{\Phi}_c=e^{-\Omega}\bar{\Phi}e^{\Omega}$ [29]. Further, we use only these covariant chiral superfields and the subscript c is omitted. It is convenient to introduce the new notation $\Phi^1=\Phi,\Phi^2=Q,\Phi^3=\tilde{Q}$ and rewrite the two last terms in Eq. (4) as follows:

$$i\int d^4xd^2\theta Q[\Phi,\tilde{Q}]+i\int d^4xd^2\overline{\theta Q}[\bar{\Phi},\bar{\tilde{Q}}],$$

which is the $\mathcal{N}=1$ form of the hypermultiplet and the lowest component of the chiral $\mathcal{N}=2$ field strength vector multiplet interaction for the $\mathcal{N}=4$ model.

If the gauge group is Abelian, we get a free model. In the non-Abelian case, the theory has a moduli space of vacua parametrized by the vacuum expectation values (VEV's) of the six real scalars. The manifold of vacua is determined by the conditions of vanishing scalar potential (F flatness plus D flatness) [37]. The solutions to the equations determining the vacuum structure of the theory can be classified according to the phase of the gauge theory they give rise to. In the pure Coulomb phase, each scalar field can have its specific nonvanishing VEV. As a result, the space of vacua is $\mathcal{M}=R^{6r}/\mathcal{S}_r$, where $\mathcal{S}_r$ is the Weyl group of permutations for r elements and the unbroken gauge group is $U(1)^r$. But when several VEV's coincide, some non-Abelian group $G\in SU(N)$ remains unbroken and some massless gauge bosons appear in the theory.

The fact that non-Abelian gauge theories are expected to describe a stack of coincident D-branes makes the task of writing an effective action much more complicated. A number of non-Abelian extensions of the Abelian super Born-Infeld theory were found [19,20]. However, there seems to be no obvious way of selecting out a unique action. Therefore the best available way of looking at this problem of constructing the effective action is to proceed order by order in the loop expansion.

B. $\mathcal{N}=4$ SYM theory in $\mathcal{N}=2$ harmonic superspace

From the $\mathcal{N}=2$ supersymmetry point of view, the $\mathcal{N}=4$ vector multiplet consists of an $\mathcal{N}=2$ vector multiplet and a hypermultiplet. Therefore the $\mathcal{N}=4$ SYM action can be treated as some special $\mathcal{N}=2$ supersymmetric theory, the action of which is the action for $\mathcal{N}=2$ SYM theory plus the action describing the hypermultiplet q^{ia} in the adjoint representation coupled to the $\mathcal{N}=2$ vector multiplet. Such a theory is formulated in $\mathcal{N}=2$ harmonic superspace [25,26]. The dynamic variables in this case are the real unconstrained analytic gauge superfield V^{++} and the complex unconstrained analytic superfield q^+. The harmonic gauge connection V^{++} serves as the potential of the $\mathcal{N}=2$ SYM theory and q^+ describes the hypermultipet. The action of the $\mathcal{N}=4$ SYM theory looks like

$$S[V^{++},q^+,\breve{q}^+]=\frac{1}{2g^2}\mathrm{tr}\int d^8z\mathcal{W}^2-\frac{1}{2g^2}\mathrm{tr}\times\int d\zeta^{-4}q^{+a}\mathcal{D}^{++}q_a^+. \qquad (7)$$

The corresponding equations of motion are

$$D^{++}q^{+a}+ig[V^{++},q^{+a}]=0, \qquad (8)$$
$$\mathcal{D}^{+\alpha}\mathcal{D}_\alpha^+\mathcal{W}=[q^{+a},q_a^+].$$

Here $a=1,2$ is the index of the rigid $SU(2)$ symmetry, $q_a^+=(q^+,\breve{q}^+)$, $q^{+a}=\varepsilon^{ab}q_b^+=(\breve{q}^+,-q^+)$, $\mathcal{W}$ is the strength of the $\mathcal{N}=2$ analytic gauge superfield V^{++} connection in the λ frame [25,26], g is a coupling constant, $d^8z=d^4xd^2\theta^+d^2\theta^-du$, $d\zeta^{-4}=d^4xd^2\theta^+d^2\overline{\theta^+}du$, and du is the measure of integration over the harmonic variables $u^{\pm i}$. The derivatives $D^+_{\alpha(\dot\alpha)}$ do not need a connection in the frame where G analyticity [25,26] is manifest. All other notation is given in Ref. [26]. Equations (8) present the $\mathcal{N}=4$ SYM field equations of motion written in terms of $\mathcal{N}=2$ superfields. The off-shell action (7) allows us to develop the manifest $\mathcal{N}=2$ supersymmetric quantization. Moreover, this action is invariant under hidden extra $\mathcal{N}=2$ supersymmetry transformations [26] which mix up $\mathcal{W},\overline{\mathcal{W}}$ with q_a^+. For our purpose, it is sufficient to point out that in the Abelian case the corresponding transformations of hidden $\mathcal{N}=2$ supersymmetry are defined only on shell and have the form

$$\delta\mathcal{W}=\frac{1}{2}\bar{\varepsilon}^{\dot\alpha a}\bar{D}^-_{\dot\alpha}q_a^+,\quad \delta\overline{\mathcal{W}}=\frac{1}{2}\varepsilon^{\alpha a}D^-_\alpha q_a^+, \qquad (9)$$

$$\delta q_a^+=\frac{1}{4}(\varepsilon_a^\alpha D_\alpha^+\mathcal{W}+\bar{\varepsilon}_a^{\dot\alpha}\bar{D}^+_{\dot\alpha}\overline{\mathcal{W}}),$$

$$\delta q_a^-=\frac{1}{4}(\varepsilon_a^\alpha D_\alpha^-\mathcal{W}+\bar{\varepsilon}_a^{\dot\alpha}\bar{D}^-_{\dot\alpha}\overline{\mathcal{W}}).$$

As a result, the model under consideration is $\mathcal{N}=4$ supersymmetric on shell.

The vacuum structure of the model (7) in the Abelian case is defined in terms of solutions to the following equations:

$$(\mathcal{D}^+)^2\mathcal{W}=(\bar{\mathcal{D}}^+)^2\overline{\mathcal{W}}=0,\ D^{++}q^{+a}=0, \qquad (10)$$

which are simple consequences of Eqs. (8) in the Abelian case. Equations (10) for physical components of the $\mathcal{N}=4$ vector multiplet determined by the expansion

$$q^+(\zeta,u)=f^i(x)u_i^++\theta^{+\alpha}\psi_\alpha(x)+\bar{\theta}^+_{\dot\alpha}\bar{\kappa}^{\dot\alpha}(x)+2i\theta^+\partial\!\!\!/\bar{\theta}^+f^i(x)u_i^-, \qquad (11)$$

$$\mathcal{W}=\phi(x)+\theta^{-\alpha}\lambda^+_\alpha(x)+\theta^{(+\alpha}\theta^{-\beta)}F_{\alpha\beta}(x)$$

look like

$$\partial\!\!\!/\psi=\partial\!\!\!/\bar{\kappa}=\Box f^i=\Box\phi=\partial\!\!\!/\lambda^i=\partial_mF_{mn}=0. \qquad (12)$$

The simplest solution to these equations of motion forms a set of constant background fields

$$f^i=\mathrm{const},\quad \psi=\mathrm{const},\quad \bar{\kappa}=\mathrm{const}, \qquad (13)$$

$$\phi=\text{const},\quad F_{mn}=\text{const},$$

which transform *linearly* through each other under the hidden $\mathcal{N}=2$ supersymmetry transformations (9):

$$\delta\phi=\frac{1}{2}\bar{\varepsilon}^{\dot\alpha a}\bar{\kappa}_{\dot\alpha a},\quad \delta\bar{\phi}=\frac{1}{2}\varepsilon^{\alpha a}\psi_{\alpha a},$$

$$\delta\psi_{\alpha a}=\frac{1}{2}\varepsilon_a^{\beta}F_{\alpha\beta},\quad \delta\bar{\kappa}_{\dot\alpha a}=\frac{1}{2}\bar{\varepsilon}_a^{\dot\beta}F_{\dot\alpha\dot\beta},$$

$$\delta f_a^i=\frac{1}{4}\varepsilon_a^{\alpha}\lambda_{\alpha}^i+\frac{1}{4}\bar{\varepsilon}_{a\dot\alpha}\bar{\lambda}^{i\dot\alpha},\quad \delta\lambda_{\alpha}^i=0,\quad \delta F_{\alpha\beta}=0. \tag{14}$$

The solution (13) is the simplest vacuum configuration carrying out a representation of $\mathcal{N}=4$ supersymmetry. We would calculate the $\mathcal{N}=4$ supersymmetric low-energy effective action in $\mathcal{N}=4$ SYM theory from this solution, if we had known how to do that.

It is instructive to compare the $\mathcal{N}=4$ supersymmetric backgrounds (13) and the background (3). The last background contains the components ϕ and F of the $\mathcal{N}=2$ vector multiplet when the components $\bar{\kappa}$ and ψ of the hypermultiplet are absent. As a result, the background (3) is not form invariant under the hidden $\mathcal{N}=2$ supersymmetry transformations (14) and therefore this background is not a representation of $\mathcal{N}=4$ supersymmetry. However, the background (3) is a representation of manifestly $\mathcal{N}=2$ supersymmetry. Therefore we can state that the effective action found on the background (3) within the $\mathcal{N}=1$ background field method will be manifestly $\mathcal{N}=2$ supersymmetric and gauge invariant but its properties under hidden $\mathcal{N}=2$ supersymmetry should be studied separately. To construct the deformed classical on-shell transformation as well as the complete $\mathcal{N}=4$ supersymmetric effective action that is invariant under this transformation, we can follow the approach developed in [24].

C. $\mathcal{N}=1$ background field quantization

For computation of the effective action we use the $\mathcal{N}=1$ superfield background field method (see, e.g., [29,30]) in combination with $\mathcal{N}=1$ superfield heat kernel techniques [30]. These methods for constructing the effective action in gauge field theories allow us to preserve a classical gauge invariance in the quantum theory and sum, in principle, an infinite set of Feynman diagrams to a single gauge invariant functional depending on the background fields. As we pointed out, the theory under consideration can be formulated either in terms of component fields, or in terms of $\mathcal{N}=1$ superfields, or in terms of $\mathcal{N}=2$ harmonic superfields. The evaluation of an effective action in component formulation is extremely complicated even within the background field method because of a very large number of interacting fields and the absence of manifest supersymmetry. The effective action can be studied within $\mathcal{N}=2$ harmonic superspace. The corresponding $\mathcal{N}=2$ background field method was proposed in Refs. [28]. The aspects of heat kernel techniques were considered in Refs. [38]. However, these techniques are undeveloped yet in many details and need to be extended for our goals. We take into account the problem of matrix operators mixing the $\mathcal{N}=2$ vector multiplet and hypermultiplet sectors. Of course, the effective action can be studied using harmonic supergraphs but we expect here to meet with the standard problem of how to organize the calculations in order to go beyond the leading low-energy approximation (see the calculations in leading low-energy approximation in, e.g., Refs. [11,18,27,28,38]. Therefore, we work within a formulation in terms of $\mathcal{N}=1$ superfields and use our experience to work with theories formulated in $\mathcal{N}=1$ superspace [22,30,31].

The background field method is based on splitting the fields into classical and quantum and imposing the gauge fixing conditions on quantum fields. Of course, as is often the case, the gauge fixing conditions can break some classical symmetries. (for detail see e.g. Refs. [39,40]).

We define one-loop effective action Γ depending on the background superfields (3) by a path integral over quantum fields in the standard form

$$e^{i\Gamma}=\int \mathcal{D}v\,\mathcal{D}\varphi\,\mathcal{D}c\,\mathcal{D}c'\,\mathcal{D}\bar{c}\,\mathcal{D}\bar{c}'\,e^{i(S_{(2)}+S_{\rm FP})}, \tag{15}$$

where $S_{(2)}$ is a quadratic in the quantum field part of the classical action, including a gauge fixing condition, and $S_{\rm FP}$ is the corresponding ghost action. A formal calculation of the above path integral leads to a functional determinant representation of the effective action [see Eq. (23)]. The main technical tool we use in this paper for the $\mathcal{N}=1$ superfield calculations is the background covariant gauge fixing multiparametric condition

$$S_{\rm GF}=-\frac{1}{\alpha g^2}\int d^8z(F^A\bar{F}^A+b^A\bar{b}^A); \tag{16}$$

here $b,\bar{b}$ are the Nielsen-Kallosh ghosts. We choose convenient gauge fixing conditions for the quantum superfields v and φ in the form

$$\bar{F}^A=\nabla^2v^A+\lambda\left[\frac{1}{\Box_+}\nabla^2\varphi^i,\bar{\Phi}_i\right]^A,$$

$$F^A=\bar{\nabla}^2v^A-\bar{\lambda}\left[\frac{1}{\Box_-}\bar{\nabla}^2\bar{\varphi}_i,\Phi^i\right]^A, \tag{17}$$

where $\alpha,\lambda,\bar{\lambda}$ are arbitrary numerical parameters and $\Box_+$, $\Box_-$ are the standard notation for Laplace-like operators in the $\mathcal{N}=1$ superspace. It is evident that the gauge fixing functions (17) are covariant under background gauge transformations. These gauge fixing functions (17) can be considered as a superfield form of the so-called R_ξ gauges (see Refs. [41,42]) which are usually used in spontaneously broken gauge theories. Since an Abelian background is a solution to classical equations of motion, we will not worry about the choice of gauge fixing parameters. Therefore it is convenient to take the gauge fixing which we call the Fermi-DeWitt gauge: $\alpha=\lambda=1$. Such a choice of the gauge parameters allows us to avoid the known problem [43] in the functional determinant method for calculating the mixed contribution which contains vector-chiral superfield propagators circulating along the loop.

We want to note once again that in gauge theories not all rigid symmetries of the classical action can be maintained manifestly in the quantum theory, even in the absence of anomalies. The issue here is that quantization requires gauge fixing and the latter, as can be shown, breaks some symmetries (breaking the classic conformal symmetry is discussed, e.g., in Ref. [40]). This is the known general situation (see, e.g., [39]). In our case, the gauge fixing (16) obviously also breaks rigid classic $\mathcal{N}=4$ symmetry (5), (6) since it is covariant only under $\mathcal{N}=1$ supersymmetry transformations. Therefore the effective action obtained should be invariant under the hidden transformations (5) deformed in some way. This deformation can, in principle, be computed by considering the Ward identities at each loop order but this nontrivial problem is beyond the purposes of this work.

After splitting each field into the background and quantum parts (i.e., $e^{V_{tot}}=e^{\Omega}e^{gv}e^{\bar{\Omega}}$, $\Phi\to\Phi+\varphi$, $\bar{\Phi}\to\bar{\Phi}+\bar{\varphi}$, $Q\to Q+q,\tilde{Q}\to\tilde{Q}+\tilde{q}$, $\bar{Q}\to\bar{Q}+\bar{q},\bar{\tilde{Q}}\to\bar{\tilde{Q}}+\bar{\tilde{q}}$), we can rewrite the quadratic part of the sum of the classical action (4) and gauge fixing action (16) in the form

$$S_{(2)}=-\frac{1}{2}\sum_{I<J}\int d^4x d^4\theta[\mathcal{F}^{IJ}\mathbf{H}_{IJ}\mathcal{F}^{\dagger IJ}+\bar{v}^{IJ}(O_V-M)_{IJ}v^{IJ}], \tag{18}$$

where $\mathcal{F}=(\bar{\varphi},\varphi,\bar{q},q,\bar{\tilde{q}},\tilde{q})$, $\mathcal{F}^{\dagger}=(\varphi,\bar{\varphi},q,\bar{q},\tilde{q},\bar{\tilde{q}})^T$,

$$M_{IJ}=(\bar{\Phi}_{IJ}\Phi_{IJ}+\bar{Q}_{IJ}Q_{IJ}+\bar{\tilde{Q}}_{IJ}\tilde{Q}_{IJ}), \tag{19}$$

$$O_V=\Box-iW^{\alpha}_{IJ}\nabla_{\alpha}-i\bar{W}^{\dot{\alpha}}_{IJ}\bar{\nabla}_{\dot{\alpha}},$$

and $W^{\alpha}_{IJ}=W^{\alpha}_I-W^{\alpha}_J$, $\bar{W}^{\dot{\alpha}}_{IJ}=\bar{W}^{\dot{\alpha}}_I-\bar{W}^{\dot{\alpha}}_J$ are the background field strengths belonging to the Cartan subalgebra and $\Phi_{IJ}=\Phi_I-\Phi_J$. The Weyl basis in the space of Hermitian traceless matrices from the algebra $su(N)$ was used in order to obtain Eq. (18). We consider the case of the gauge group $SU(N)$ broken down to the maximal torus $U(1)^{N-1}$. The constraint $I<J$ arises since the components of the quantum superfields that lie in the Cartan subalgebra do not interact with the background field and therefore they are completely decoupled. For details of using the Weyl basis to calculate the effective action, see, e.g., [11].

The operator $\mathbf{H}$ is a matrix depending on covariant derivatives and background fields. The explicit form of this matrix looks like

$$\begin{pmatrix}
G_+(\phi)\nabla^2\bar{\nabla}^2 & 0 & -\phi\bar{f}\frac{\nabla^2\bar{\nabla}^2}{\Box_+} & i\bar{v}\nabla^2 & -\phi f\frac{\nabla^2\bar{\nabla}^2}{\Box_+} & -i\bar{f}\nabla^2\\
0 & G_-(\phi)\bar{\nabla}^2\nabla^2 & iv\bar{\nabla}^2 & \bar{\phi}f\frac{\bar{\nabla}^2\nabla^2}{\Box_-} & -if\bar{\nabla}^2 & -\bar{\phi}v\frac{\bar{\nabla}^2\nabla^2}{\Box_-}\\
-f\bar{\phi}\frac{\nabla^2\bar{\nabla}^2}{\Box_+} & i\bar{v}\nabla^2 & G_+(f)\nabla^2\bar{\nabla}^2 & 0 & f\bar{v}\frac{\nabla^2\bar{\nabla}^2}{\Box_+} & i\bar{\phi}\nabla^2\\
-iv\bar{\nabla}^2 & -\bar{f}\phi\frac{\bar{\nabla}^2\nabla^2}{\Box_-} & 0 & G_-(f)\bar{\nabla}^2\nabla^2 & i\phi\bar{\nabla}^2 & -\bar{f}v\frac{\bar{\nabla}^2\nabla^2}{\Box_-}\\
-v\bar{\phi}\frac{\nabla^2\bar{\nabla}^2}{\Box_+} & \bar{f}\nabla^2 & -v\bar{f}\frac{\nabla^2\bar{\nabla}^2}{\Box_+} & -i\bar{\phi}\nabla^2 & G_+(v)\nabla^2\bar{\nabla}^2 & 0\\
if\bar{\nabla}^2 & -\bar{v}\phi\frac{\bar{\nabla}^2\nabla^2}{\Box_-} & -i\phi\bar{\nabla}^2 & -\bar{v}f\frac{\bar{\nabla}^2\nabla^2}{\Box_-} & 0 & G_-(v)\bar{\nabla}^2\nabla^2
\end{pmatrix}, \tag{20}$$

where the following notation is used

$$G_\pm(a)=1-\frac{(a\bar{a})}{\Box_\pm},\quad \phi=\Phi_{IJ},\quad \bar{\phi}=\bar{\Phi}_{IJ},$$

$$f=Q_{IJ},\quad \bar{f}=\bar{Q}_{IJ},\quad v=\tilde{Q}_{IJ},\quad \bar{v}=\bar{\tilde{Q}}_{IJ}$$

and $\Box_\pm$ means $\nabla^2\bar{\nabla}^2$ and $\bar{\nabla}^2\nabla^2$, respectively. In the space of chiral and antichiral superfields these operators act as follows:

$$\nabla^2\bar{\nabla}^2=\Box_+=\Box-i\bar{W}^{\dot{\alpha}}\bar{\nabla}_{\dot{\alpha}}-\frac{i}{2}(\bar{\nabla}\bar{W}),$$

$$\bar{\nabla}^2\nabla^2=\Box_-=\Box-iW^\alpha\nabla_\alpha-\frac{i}{2}(\nabla W).$$

It should be noted that, generally speaking, the second variation of the classical action leads to a 7×7 matrix operator. But the chosen gauge fixing condition (17) allows partial diagonalization to a $1\times1\oplus6\times6$ block matrix and separation of the kinetic operator for the vector fields. This gauge fixing condition eliminates the interaction vertices between quantum matter fields and quantum vector fields but generates new interaction vertexes between quantum chiral fields and ghosts.

Let us consider now the structure of a Faddeev-Popov ghost contribution to the one-loop effective action. The action of the Faddeev-Popov ghosts $S_{\rm FP}$ for the gauge fixing functions (17) has the form

$$S_{\rm FP}=\mathrm{tr}\int d^8z\left\{(\overline{c'}c-c'\bar{c})-\left(c'\left[\Phi^i,\frac{\lambda}{\Box_+}[\bar{c},\bar{\Phi}_i]\right]+\overline{c'}\left[\frac{\bar{\lambda}}{\Box_-}[c,\Phi^i],\bar{\Phi}_i\right]\right)\right\}. \quad (21)$$

It leads to the following contribution of the ghosts to the effective action:

$$\ln[\mathrm{Det}(\mathbf{H}_{\rm FP})]=2\sum_{I<J}\mathrm{Tr}\ln\begin{pmatrix}0 & \left(1-\frac{M}{\Box_+}\right)\nabla^2\bar{\nabla}^2\\ -\left(1-\frac{M}{\Box_-}\right)\bar{\nabla}^2\nabla^2 & 0\end{pmatrix}_{IJ}, \quad (22)$$

where M was defined in Eq. (19).

The final result of the integration in the path integral (15) over all quantum superfields is given by a formal representation for the one-loop effective action in terms of functional determinants

$$e^{i\Gamma}=\prod_{I<J}\mathrm{Det}^{-1}(O_V-M)\mathrm{Det}^{-1}(\mathbf{H})\mathrm{Det}^2(\mathbf{H}_{\rm FP}). \quad (23)$$

Since the strengths Φ and W_α belong to the Cartan subalgebra, only half of the roots should be taken into account during the integration over the quantum fields and the effective action looks like

$$\Gamma=\sum_{I<J}\Gamma_{IJ}.$$

Our next purpose is a computation of the above functional determinants.

III. EVALUATIONS OF SUPERFIELD FUNCTIONAL TRACES AND ONE-LOOP EFFECTIVE ACTION

In this section we present the basic steps of functional trace calculations for the operators, which make background-dependent contributions to the effective action (23). It is seen from Eq. (20) that in the absence of background superfields $Q,\tilde{Q}$ the matrix operator $\mathbf{H}$ includes only the background-dependent inverse propagators G_+,G_- and vertices for the background field Φ interacting with the quantum hypermultiplet. Such a situation has been studied in detail (see, e.g., Refs. [31,32,43,44]). It should be noted that the form of $\mathbf{H}$ containing dressed inverse propagators is directly related to the R_ξ gauge fixing conditions (17).

On the first stage we divide the matrix $\mathbf{H}$ into a sum of two matrices $\mathbf{H}=\mathbf{H}_\Box+\mathbf{H}_\nabla$ where the matrix $(\mathbf{H}_\Box)$ contains all blocks with $\nabla^2\bar{\nabla}^2,\bar{\nabla}^2\nabla^2$ and the matrix $\mathbf{H}_\nabla$ contains blocks with $\bar{\nabla}^2$ and ∇^2 only. Let us present the logarithm of the matrix $\ln(\mathbf{H})$ as follows:

$$\ln(\mathbf{H})=\ln(\mathbf{H}_\Box)+\ln(1-\mathbf{H}_\Box^{-1}\mathbf{H}_\nabla).$$

Using the known Frobenius formula for inversion of a block type matrix,

$$H=\begin{pmatrix}A & B\\ C & D\end{pmatrix},$$

$$H^{-1}=\begin{pmatrix}A^{-1}+A^{-1}BE^{-1}CA^{-1} & -A^{-1}BE^{-1}\\ -E^{-1}CA^{-1} & E^{-1},\end{pmatrix},$$

where $E=D-CA^{-1}B$, we get by direct calculation the inverse matrix for $\mathbf{H}_\Box$:

$$\mathbf{H}_{\Box}^{-1}=\begin{pmatrix}
g_+(\phi)\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0 & \frac{\phi\bar{f}}{\Box_{+M}}\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0 & \frac{\phi\bar{v}}{\Box_{+M}}\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0\\
0 & g_-(\phi)\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2} & 0 & \frac{\bar{\phi}f}{\Box_{-M}}\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2} & 0 & \frac{\bar{\phi}v}{\Box_{-M}}\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2}\\
\frac{f\bar{\phi}}{\Box_{+M}}\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0 & g_+(f)\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0 & \frac{f\bar{v}}{\Box_{+M}}\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0\\
0 & \frac{\bar{f}\phi}{\Box_{-M}}\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2} & 0 & g_-(f)\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2} & 0 & \frac{\bar{f}v}{\Box_{-M}}\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2}\\
\frac{v\bar{\phi}}{\Box_{+M}}\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0 & \frac{v\bar{f}}{\Box_{+M}}\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0 & g_+(v)\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2} & 0\\
0 & \frac{\bar{v}\phi}{\Box_{-M}}\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2} & 0 & \frac{\bar{v}f}{\Box_{-M}}\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2} & 0 & g_-(v)\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2}
\end{pmatrix}. \tag{24}$$

Here we have introduced the notation

$$g_{\pm}(\phi)=1+\frac{\phi\bar{\phi}}{\Box_{\pm M}},\quad \Box_{\pm M}=\Box_{\pm}+M.$$

One can note that the combination M [Eq. (19)] appeared naturally during the inversion procedure. Then we find the product $\mathbf{H}_{\Box}^{-1}\mathbf{H}_{\nabla}$ in a remarkably simple form:

$$\mathbf{H}_{\Box}^{-1}\mathbf{H}_{\nabla}=\begin{pmatrix}
0 & 0 & 0 & i\bar{v}\frac{\nabla^2}{\Box_-} & 0 & -i\bar{f}\frac{\nabla^2}{\Box_-}\\
0 & 0 & iv\frac{\bar{\nabla}^2}{\Box_+} & 0 & -if\frac{\bar{\nabla}^2}{\Box_+} & 0\\
0 & -i\bar{v}\frac{\nabla^2}{\Box_-} & 0 & 0 & 0 & i\bar{\phi}\frac{\nabla^2}{\Box_-}\\
-iv\frac{\bar{\nabla}^2}{\Box_+} & 0 & 0 & 0 & i\phi\frac{\bar{\nabla}^2}{\Box_+} & 0\\
0 & i\bar{f}\frac{\nabla^2}{\Box_-} & 0 & -i\bar{\phi}\frac{\nabla^2}{\Box_-} & 0 & 0\\
if\frac{\bar{\nabla}^2}{\Box_+} & 0 & -i\phi\frac{\bar{\nabla}^2}{\Box_+} & 0 & 0 & 0
\end{pmatrix}. \tag{25}$$

The next stage consists in matrix trace calculations. Let us expand $\mathrm{Tr}[\ln(1-\mathbf{H}_{\Box}^{-1}\mathbf{H}_{\nabla})]$ in a series in powers of $\mathbf{H}_{\Box}^{-1}\mathbf{H}_{\nabla}$. The nonzero matrix traces will have only even powers of the series, which are grouped into

$$\mathrm{Tr}_{6\times6}[\ln(1-\mathbf{H}_{\Box}^{-1}\mathbf{H}_{\nabla})]=\mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_+}\right)\frac{\nabla^2\bar{\nabla}^2}{\Box_+}\right]+\mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_-}\right)\frac{\bar{\nabla}^2\nabla^2}{\Box_-}\right], \tag{26}$$

where M was introduced in Eq. (19) and Tr means the functional trace. Also, we have to consider the matrix trace of $\ln(\mathbf{H}_{\Box})$. According to the above strategy, we write the matrix as a diagonal matrix plus the rest, i.e., $\mathbf{H}_{\Box}=\mathbf{H}_0+\mathbf{\Delta}$:

$$\mathrm{Tr}\ln(\mathbf{H}_{\Box})=\mathrm{Tr}\ln(\mathbf{H}_0)+\mathrm{Tr}\ln(1+\mathbf{H}_{\Box}^{-1}\mathbf{\Delta}), \tag{27}$$

where the matrix $\mathbf{H}_0$ contains only $\nabla^2\bar{\nabla}^2$ and $\bar{\nabla}^2\nabla^2$ at zero background fields $\Phi,Q,\tilde{Q}$ and therefore can be omitted. The matrix elements of $\mathbf{H}_{\Box}^{-1}\mathbf{\Delta}$ are blocks with chiral $\nabla^2\bar{\nabla}^2/\Box_+$ and antichiral $\bar{\nabla}^2\nabla^2/\Box_-$ projectors. After permutation of the lines and columns, the trace logarithm of the matrix $1+\mathbf{H}_0^{-1}\mathbf{\Delta}$ can be reorganized as follows:

$$\mathrm{Tr}_{6\times6}\ln(1+\mathbf{H}_0^{-1}\mathbf{\Delta}) = \mathrm{Tr}_{3\times3}\ln\left(1-\begin{pmatrix}(\phi\bar{\phi}) & (\phi\bar{f}) & (\phi\bar{v})\\ (f\bar{\phi}) & (f\bar{f}) & (f\bar{v})\\ (v\bar{\phi}) & (v\bar{f}) & (v\bar{v})\end{pmatrix}\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2}\right) + \left(\frac{\nabla^2\bar{\nabla}^2}{\Box_+^2}\to\frac{\bar{\nabla}^2\nabla^2}{\Box_-^2}\right). \tag{28}$$

A direct calculation of the matrix traces for the first terms in the Taylor series allows us to write the result as

$$\mathrm{Tr}_{6\times6}\ln(1+\mathbf{H}_0^{-1}\mathbf{\Delta}) = \mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_+}\right)\frac{\nabla^2\bar{\nabla}^2}{\Box_+}\right] + \mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_-}\right)\frac{\bar{\nabla}^2\nabla^2}{\Box_-}\right], \tag{29}$$

which together with Eq. (26) gives

$$\ln[\mathrm{Det}^{-1}(\mathbf{H})] = -2\,\mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_+}\right)\frac{\nabla^2\bar{\nabla}^2}{\Box_+}\right] - 2\,\mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_-}\right)\frac{\bar{\nabla}^2\nabla^2}{\Box_-}\right]. \tag{30}$$

The contribution of the Faddeev-Popov ghosts is determined by Eq. (22). Extracting and neglecting the expression

$$\ln\left(\begin{pmatrix}0 & \nabla^2\bar{\nabla}^2\\ -\bar{\nabla}^2\nabla^2 & 0\end{pmatrix}\right),$$

we obtain the ghost contribution to the effective action in the form

$$\ln[\mathrm{Det}^2(\mathbf{H}_{\mathrm{FP}})] = 2\,\mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_-}\right)\frac{\bar{\nabla}^2\nabla^2}{\Box_-}\right] + 2\,\mathrm{Tr}\left[\ln\left(1-\frac{M}{\Box_+}\right)\frac{\nabla^2\bar{\nabla}^2}{\Box_+}\right], \tag{31}$$

which is exactly Eq. (30) with the opposite sign. Therefore the second and third functional determinants in Eq. (23) cancel each other. This surprising cancellation between the contributions of ghost and chiral fields to the one-loop effective action in $\mathcal{N}=4$ SYM theory was first noted in [28] in the harmonic superspace approach. It should be especially pointed out that this result is correct only on a constant chiral superfield background.

Finally, due to the cancellation between Eqs. (31) and (30), the whole one-loop contribution to the effective action (23) has an extremely simple form and is determined only by the vector loop contribution

$$\Gamma = i\sum_{I<J}\mathrm{Tr}\ln(O_V-M)_{IJ}, \tag{32}$$

and all background superfield dependence is encoded in M. For the operator in the above relation, the power expansion over the Grassmann derivatives of the gauge field strength of the functional trace has already been calculated in different ways for models with one chiral background superfield (see [31–33] and references therein). As a result, we transformed a rather complicated problem with a hypermultiplet background to a known problem. The feature of the theory with the hypermultiplet background is its dependence on the combination (19) $M=(\bar{\Phi}\Phi+\bar{Q}Q+\bar{\tilde{Q}}\tilde{Q})$, which is invariant under the R-symmetry group of $\mathcal{N}=4$ supersymmetry. That allows us to apply the results obtained for $\mathcal{N}=1$ models to the case under consideration making the corresponding redefinition of the quantity M.

The functional trace (32) can be written as a power expansion of dimensionless combinations $\Psi,\bar{\Psi}$ of vector and hypermultiplet superfields, where

$$\bar{\Psi}^2=\frac{1}{M^2}\nabla^2W^2, \quad \Psi^2=\frac{1}{M^2}\bar{\nabla}^2\bar{W}^2. \tag{33}$$

In the constant field approximation this expansion is summed to the following expression for the whole one-loop effective action (see details in [33]):

$$\Gamma=\frac{1}{8\pi^2}\int d^8z\int_0^\infty dt\,te^{-t}\frac{W^2\bar{W}^2}{M^2}\omega(t\Psi,t\bar{\Psi}), \tag{34}$$

$$\omega(t\Psi,t\bar{\Psi})=\frac{\cosh(t\Psi)-1}{t^2\Psi^2}\,\frac{\cosh(t\bar{\Psi})-1}{t^2\bar{\Psi}^2}\times\frac{t^2(\Psi^2-\bar{\Psi}^2)}{\cosh(t\Psi)-\cosh(t\bar{\Psi})}.$$

Equation (34) is our central result. We see that the only difference between the $\mathcal{N}=4$ SYM effective actions with and without the hypermultiplet background is stipulated by the structure of the matrix M defined by Eq. (19). In component form, the closed relation for the one-loop effective action (34) has a natural Schwinger-type expansion over F^2/M^2 powers. The expansion does not include the F^6 term that is a property of $\mathcal{N}=4$ SYM theory [33,34]. The function ω defined in Eq. (34) (see [33]) has the following expansion:

$$\omega(x,y)=\frac{1}{2}+\frac{x^2y^2}{4\cdot5!}-\frac{5}{12\cdot7!}(x^4y^2+x^2y^4)+\frac{1}{34500}(x^2y^6+x^6y^2)+\frac{1}{86400}x^4y^4+\cdots. \tag{35}$$

Equation (35) allows us to expand the effective action (34) in series in powers of $\Psi^2,\bar{\Psi}^2$ as follows:

$$\Gamma=\Gamma_{(0)}+\Gamma_{(2)}+\Gamma_{(3)}+\cdots, \tag{36}$$

where the term $\Gamma_{(n)}$ contains the terms $c_{m,l}\Psi^{2m}\bar{\Psi}^{2l}$ with $m+l=n$. In the bosonic sector, this expansion corresponds to an expansion in powers of the strength F, namely, $\Gamma_{(n)}\sim F^{4+2n}/M^{2+2n}$, $M=(\Phi\bar{\Phi}+f^{ia}f_{ia})$, where $\Phi,\bar{\Phi}$, and f^{ia} are physical bosonic fields of the $\mathcal{N}=2$ vector multiplet and hypermultiplets.

IV. TRANSFORMATION OF THE $\mathcal{N}=1$ SUPERSYMMETRIC EFFECTIVE ACTION TO A MANIFESTLY $\mathcal{N}=2$ SUPERSYMMETRIC FORM

The effective action (34) and its expansion (36) are given in terms of $\mathcal{N}=1$ superfields. Our next purpose is to find a manifestly $\mathcal{N}=2$ form of each term in the expansion (36). To do that, we extract from M the $\mathcal{N}=1$ form of $X=-(\bar{Q}Q+\bar{\tilde{Q}}\tilde{Q})/\bar{\Phi}\Phi$, [which was defined in Eq. (2) in terms of $\mathcal{N}=2$ superfields], writing $M=\Phi\bar{\Phi}(1-X)$, and then expand the denominator $(1/M)^k$ from Eq. (34) in a power series in X. This expansion leads to the following form for a generic term of the series:

$$\int d^8z\frac{W^2\bar{W}^2}{(\Phi\bar{\Phi})^{2(m+l+k+1)}}(\nabla^2W^2)^m(\bar{\nabla}^2\bar{W}^2)^l \times[-(\bar{Q}Q+\bar{\tilde{Q}}\tilde{Q})]^k. \tag{37}$$

Further, using $\int d^{12}z=\int d^8z(\nabla_2)^2(\bar{\nabla}_2)^2$ and definitions of $\mathcal{N}=1$ projections for the $\mathcal{N}=2$ on-shell vector multiplet $\mathcal{W}|=\Phi,\nabla_{2\alpha}\mathcal{W}|=-W_\alpha,\nabla_2^2\mathcal{W}|=\bar{\nabla}^2\bar{\Phi}=0$, we can reconstruct the $\mathcal{N}=2$ form of the above generic term. It is worth pointing out that the reconstruction procedure has some off-shell ambiguity (see [45]) even for vanishing hypermultiplet fields, but this ambiguity is inessential on shell.

The derivative expansion (34) of the effective action contains the known nonholomorphic potential as a first term [see Eq. (42)]. It can be unambiguously rewritten in an $\mathcal{N}=2$ form, following from $\mathcal{N}=1$ calculations on the background (3). This unique term is automatically $\mathcal{N}=4$ supersymmetric since it does not contain the derivatives of the hypermultiplet and vector strengths. Recovering the other terms in the derivative expansion of the effective action is not so evident and needs special prescriptions.

The calculation of the above effective action was done on the constant background (3), but for recovering the $\mathcal{N}=2$ form such a background is insufficient. We must take into account the derivatives of the $\mathcal{N}=1$ hypermultiplet fields. The procedure for restoring the $\mathcal{N}=2$ supersymmetric expressions, based on the corresponding $\mathcal{N}=1$ reduction, always implies forming the $\mathcal{N}=2$ integral measure $\int d^{12}z=\int d^8z(\nabla_2)^2(\bar{\nabla}_2)^2$. Therefore, to get an integral over $\mathcal{N}=2$ superspace from an integral over $\mathcal{N}=1$ superspace, we must form the full derivatives $(\nabla_2)^2(\bar{\nabla}_2)^2$ in the initial $\mathcal{N}=1$ superspace integrand. In order to obtain such total derivatives in the integrand (37), we have to add all necessary $\nabla_{\alpha j}q^{ia}$ derivative-containing terms with specified numerical coefficients to the initial $\mathcal{N}=1$ superspace integrand by hand, since they did not appear in the process of computation. If we calculate the effective action in terms of $\mathcal{N}=1$ superfields not on the special background (3) but on the proper background (13), these terms, which are absent in Eq. (37), will be presented automatically.[2] Then the derivatives $\nabla_2^2\bar{\nabla}_2^2$ could be formed in the $\mathcal{N}=1$ superspace integrand and, as a result, we would obtain the integral over $\mathcal{N}=2$ superspace.

Further, we use evident enough assumptions about the properties of the effective action. The effective action is manifestly $\mathcal{N}=2$ supersymmetric and, hence, each term in its expansion in derivatives can be written as an integral over $\mathcal{N}=2$ superspace of a function depending on $\mathcal{N}=2$ superfield strengths, hypermultiplet superfields, and their spinor derivatives. It allows us to argue as follows. Using integration by parts in the integrals over $\mathcal{N}=2$ superspace for the effective action derivative expansion terms, we transfer all derivatives from hypermultiplets to the $\mathcal{N}=2$ superfield strengths and then one makes the reduction to $\mathcal{N}=1$ form. As a result, we see that all terms in the derivative expansion of $\mathcal{N}=2$ functionals can be written in a form similar to $\Gamma_{(n)}$ defined in Eq. (36), i.e., without derivatives of the hypermultiplet superfields. It means that we can act in reverse order beginning with the given $\mathcal{N}=1$ form and restoring the corresponding $\mathcal{N}=2$ form. Also, we take into account that the derivative expansion at vanishing hypermultiplet superfields is presented in terms of $\mathcal{N}=2$ superconformal scalars [33]:

$$\bar{\Psi}^2=\frac{1}{\bar{\mathcal{W}}^2}\nabla^4\ln\mathcal{W},\quad \Psi^2=\frac{1}{\mathcal{W}^2}\bar{\nabla}^4\ln\bar{\mathcal{W}}, \tag{38}$$

and will search for hypermultiplet dependence compatible with this property.

Further, we demonstrate how the use of the above prescription allows us to obtain the functionals $\Gamma_{(0)},\Gamma_{(2)},\Gamma_{(3)},\ldots$ [Eq. (36)] in terms of $\mathcal{N}=2$ superfields. Let us begin with the functional $\Gamma_{(0)}=[1/(4\pi)^2]\int d^8zW^2\bar{W}^2/M^2$ (which is $\sim F^4$) and rewrite it in the form Eq. (37) using $1/(1-X)^2=\Sigma_{k=0}^{\infty}(k+1)X^k$:

$$\frac{1}{(4\pi)^2}\int d^8z\left(\frac{W^2\bar{W}^2}{\Phi^2\bar{\Phi}^2}+\sum_{k=1}^{\infty}(k+1)\times\frac{W^2\bar{W}^2}{\Phi^{2+k}\bar{\Phi}^{2+k}}\cdot[-(\bar{Q}Q+\bar{\tilde{Q}}\tilde{Q})]^k\right). \tag{39}$$

[2]To get such an effective action in $\mathcal{N}=1$ formalism we have to carry out the calculations keeping the spinor derivatives of the background chiral superfields. The only example of these calculations was given within the Wess-Zumino model for finding the effective potential of auxiliary fields in Refs. [31,44]. In particular, such a potential for chiral $\mathcal{N}=1$ superfields of the $\mathcal{N}=2$ vector multiplet arises from the self-dual requirement for the $\mathcal{N}=4$ SYM effective action (see Ref. [46]).

It is natural to identify the quadratic combination $(\bar{Q}Q+\bar{\tilde{Q}}\tilde{Q})$ of $\mathcal{N}=1$ superfields with the $\mathcal{N}=1$ projection of the quadratic combination of Fayet-Sohnius hypermultiplets $q^{ia}q_{ia}$. Such an identification can be checked, e.g., by comparison of the component structures. Then we apply the relations

$$\nabla_2^2 \ln \mathcal{W}| = -\left(\frac{W^\alpha W_\alpha}{\Phi^2}\right)+\cdots,$$

$$\nabla_2^2 \frac{1}{\mathcal{W}^m}| = \frac{m(m+1)}{\Phi^m}\frac{W^\alpha W_\alpha}{\Phi^2}+\cdots, \tag{40}$$

where the ellipses mean the terms involving the derivatives of Φ which can be omitted in our on-shell analysis. Thus, the $\mathcal{N}=1$ integrand (39) can be written via $\mathcal{N}=2$ vector multiplet superfields and hypermultiplets as

$$\nabla_2^2 \ln \mathcal{W}\bar{\nabla}_2^2 \ln \bar{\mathcal{W}}+\sum_{k=1}^{\infty}\frac{1}{k^2(k+1)}\nabla_2^2\frac{1}{\mathcal{W}^k}\bar{\nabla}_2^2 \times\frac{1}{\bar{\mathcal{W}}^k}\cdot(-q^{ia}q_{ia})+\cdots, \tag{41}$$

where the ellipses mean all terms involving hypermultiplet derivatives of the form

$$\nabla_2^\alpha\frac{1}{\mathcal{W}^k}\nabla_{2\alpha}(-q^{ia}q_{ia})\bar{\nabla}_2^2\frac{1}{\bar{\mathcal{W}}^k},\ \frac{1}{\mathcal{W}^k}\nabla_2^2(-q^{ia}q_{ia})\bar{\nabla}_2^2\frac{1}{\bar{\mathcal{W}}^k},$$

which, according to the above prescriptions, should be added in order to obtain the full $\mathcal{N}=2$ integration measure $\nabla_2^2\bar{\nabla}_2^2$ in the integral over $\mathcal{N}=1$ superspace (40). As a result, the above prescriptions lead to the expression

$$\Gamma_{(0)}=\frac{1}{(4\pi)^2}\int d^{12}z\left(\ln\mathcal{W}\ln\bar{\mathcal{W}}+\sum_{k=1}^{\infty}\frac{1}{k^2(k+1)}X^k\right), \tag{42}$$

where $X=(-q^{ia}q_{ia}/\mathcal{W}\bar{\mathcal{W}})$ was defined in Eq. (2). The second term in Eq. (42) can be transformed to the form (2) using the power series for the Euler dilogarithm function and the relation $1/k^2(k+1)=1/k^2-1/k+1/(k+1)$. We see that the expression (42) is just the effective Lagrangian (2) found in [24,27]:

$$\Gamma_{(0)}=\frac{1}{(4\pi)^2}\int d^{12}z\left\{\ln\mathcal{W}\ln\bar{\mathcal{W}}+(X-1)\frac{\ln(1-X)}{X}+[\mathrm{Li}_2(X)-1]\right\}. \tag{43}$$

Thus, our $\mathcal{N}=1$ superfield approach automatically reproduces the complete low-energy effective action (2). All other terms in the expansion of the effective action (34) define the subleading higher derivative corrections to the low-energy effective action (2). Further, we present a few of these corrections in manifestly $N=2$ supersymmetric form.

The $\mathcal{N}=2$ form of the next term ($\sim F^8$) in the series (36) is reconstructed using Eq. (40) and the expansion of $(1/M)^6$ in X. Direct analysis, analogous to that in the previous case, leads to the following expression for $\Gamma_{(2)}$ in Eq. (36):

$$\Gamma_{(2)}=\frac{1}{2(4\pi)^2 5!}\int d^{12}z\,\mathbf{\Psi}^2\bar{\mathbf{\Psi}}^2\left(\frac{1}{(1-X)^2}+\frac{4}{(1-X)}+\frac{6X-4}{X^3}\ln(1-X)+4\frac{X-1}{X^2}\right). \tag{44}$$

The X-independent part of this term was given in [33]. Applying the same procedure to the third term ($\sim F^{10}$) in Eq. (36), one obtains

$$\Gamma_{(3)}=-\frac{5}{6(4\pi)^2}\int d^{12}z(\mathbf{\Psi}^4\bar{\mathbf{\Psi}}^2+\mathbf{\Psi}^2\bar{\mathbf{\Psi}}^4)\times\left(-\frac{1}{5!}+\frac{1}{7!}\frac{2X}{(1-X)^4}(56-116X+84X^2-21X^3)\right). \tag{45}$$

Thus, we have found the hypermultiplet-dependent terms complementary to $\Gamma_{(0)}$, $\Gamma_{(2)}$, and $\Gamma_{(3)}$ for the effective action obtained in [33] in the $\mathcal{N}=2$ vector multiplet sector. Clearly, every term in the expansion of the effective action (36) can be written in $\mathcal{N}=2$ supersymmetric form. For example, the X-dependent part of the fourth term ($\sim F^{12}$) in Eq. (36) contains two parts. The first one is

$$\Gamma_{(4_1)}=\frac{1}{(4\pi)^2}\frac{1}{17250}\int d^{12}z(\mathbf{\Psi}^2\bar{\mathbf{\Psi}}^6+\mathbf{\Psi}^6\bar{\mathbf{\Psi}}^2)\frac{12X}{(1-X)^6}\times(450-1545X+2284X^2-1779X^3+720X^4-120X^5) \tag{46}$$

and the second part is given as follows:

$$\Gamma_{(4_2)}=\frac{1}{5\cdot 6!}\frac{1}{(4\pi)^2}\int d^{12}z\,\mathbf{\Psi}^4\bar{\mathbf{\Psi}}^4\left(\frac{12(5X-4)}{X^5}\ln(1-X)-\frac{1}{5X^4(1-X)^6}(240-1620X+4610X^2-7120X^3+6363X^4-4878X^5+6135X^6-7560X^7+5670X^8-2268X^9+378X^{10})\right). \tag{47}$$

Since in the on-shell description the hypermultiplet superfields q_{ia} and superfield strengths $\mathcal{W},\bar{\mathcal{W}}$ are independent of the harmonic variables $u_i^{\pm}$, one can insert a harmonic integral $\int du$ into the expressions for $\Gamma_{(0)},\Gamma_{(2)},\Gamma_{(3)},\ldots$ and write the variables X as $X=(-2q^{+a}q_a^-/\mathcal{W}\bar{\mathcal{W}})$. This allows study of the variation of the effective action under the hidden supersymmetry transformations (9) using the harmonic superspace formalism.

Thus, we see that this $\mathcal{N}=2$ reconstruction procedure being applied to the effective action (34), which is written in terms of $\mathcal{N}=1$ superfields, can actually be realized for any terms in the expansion (36) by completing these terms with the corresponding complementary terms containing the hypermultiplet superfields.

V. SUMMARY

We have studied the one-loop effective action in $\mathcal{N}=4$ SYM theory, depending on $\mathcal{N}=2$ vector multiplet and hypermultiplet fields. The theory under consideration was formulated in $\mathcal{N}=1$ superspace and quantized in the framework of the background field method with the use of special gauge fixing conditions preserving manifest $\mathcal{N}=1$ supersymmetry. The effective action is given by superfield functional determinants. The concrete calculations of these determinants are done on a specific $\mathcal{N}=1$ superfield background corresponding to constant Abelian strength F_{mn} and constant hypermultiplet fields. We have proved that the effective action depending on all fields of the $\mathcal{N}=4$ vector multiplet is restored on the basis of calculations in only the $\mathcal{N}=2$ vector multiplet sector by a special change of the functional arguments [see Eqs. (32) and (34)].

We have examined the possibility of presenting the effective action obtained in a manifestly $\mathcal{N}=2$ supersymmetric form. Analyzing the effective action as an expansion in spinor covariant derivatives, we have shown that the terms of this expansion can be expressed via integrals over $\mathcal{N}=2$ superspace of the functions depending on $\mathcal{N}=2$ strengths, their spinor derivatives, and hypermultiplet superfields. As one of the results, we rederived the complete $\mathcal{N}=4$ supersymmetric low-energy effective action, which was discovered in [24]. All other terms in the derivative expansion of the effective action describe the next-to-leading corrections.

As we have already pointed out, the quantization procedure is noninvariant under the hidden $\mathcal{N}=2$ supersymmetry forming together with manifestly $\mathcal{N}=2$ supersymmetry a complete on-shell $\mathcal{N}=4$ supersymmetry of the classical action (7). Therefore, it is natural to expect that the higher-derivative corrections to the effective action can also be noninvariant. Moreover, the rigid classical symmetry transformations in quantum field theory can be deformed by quantum loop corrections. Hence, in the case under consideration one can expect that the classical hidden $\mathcal{N}=2$ supersymmetry transformations should get some quantum corrections and the effective action will be invariant under deformed hidden supersymmetry transformations. In forthcoming work we are going to study the structure of quantum deformations of the classical hidden $\mathcal{N}=2$ supersymmetry.

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