Gauge symmetry enhancement in the Hamiltonian formalism

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We study the Hamiltonian structure of the gauge symmetry enhancement in the enlarged CP(N) model coupled with a $U(2)$ Chern-Simons term, which contains a free parameter governing explicit symmetry breaking and symmetry enhancement. After giving a general discussion of the geometry of constrained phase space suitable for the enhancement, we explicitly perform the Dirac analysis of our model and compute the Dirac brackets for the symmetry enhanced and broken cases. We also discuss some related issues.

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I. INTRODUCTION

It is well known that the nonlinear sigma models exhibit many interesting physical properties in the large-N limit [1]. One of them is the phenomenon of dynamical generation of a gauge boson in the CP(N) model [2], where the auxiliary $U(1)$ gauge field becomes dynamical through the radiative corrections [3]. Recently, some new properties have been explored in relation with this phenomenon. In particular, in Ref. [4] the issue of dynamical generation of a gauge boson has been analyzed in the context of an enlarged CP(N) model in lower dimensions. In this model, two complex projective spaces with different coupling constants couple with each other through interactions which preserve the exchange of the two spaces. In addition to the two auxiliary $U(1)$ gauge fields [corresponding to the diagonal a_μ and b_μ fields of Eq. (2.4) below] which represent each complex projective space, one extra auxiliary complex gauge field [the off-diagonal c_μ field of Eq. (2.4)] is introduced to couple the two spaces in the way which preserves the exchange symmetry. It turns out that when the two coupling constants are equal [which corresponds to the case of $r=1$ of Eq. (2.3)], the classical enlarged model becomes the nonlinear sigma model with the target space of the Grassmannian manifold [5]. It was shown in Ref. [4] that the additional gauge field, c_μ, also becomes dynamical through radiative corrections. Moreover, in the self-dual limit where the two running coupling constants become equal, it becomes massless and combines with the two $U(1)$ gauge fields to yield the $U(2)$ Yang-Mills theory. That is, the gauge symmetry enhancement has occurred in the self-dual limit. Away from this limit, the complex gauge field becomes massive and the symmetry remains to be $U(1)\times U(1)$.

The parameter r could be understood as an explicit gauge symmetry breaking parameter from $U(2)$ to $U(1)\times U(1)$, with the mass of the c_μ field being induced radiatively through the loop corrections when the symmetry is broken. This could provide a scheme of generating mass of the gauge bosons. Therefore, it would be worthwhile to study the enlarged CP(N) model from different aspects. In this paper, we study this model in the Hamiltonian formulation. We first recall that the gauge symmetry is realized as the Gauss law type of constraints in the Hamiltonian method. In the enlarged model of Ref. [4], the original gauge fields are auxiliary fields which become dynamical through the quantum corrections. From the Hamiltonian point of view, these auxiliary fields could be completely eliminated through the equations of motion from the beginning, and the Gauss law constraints could be only implicitly realized. However, in order to see the structure of gauge symmetry more explicitly, we couple the enlarged CP(N) model with some external gauge fields, which we choose to be described by the $U(2)$ Chern-Simons term. Then, we perform the Dirac analysis [6] of the resulting theory. The theory has both first- and second-class constraints, and it is found that for $r=1$ the Gauss constraints satisfy $U(2)$ symmetry algebra, whereas for $r\neq 1$ only $U(1)\times U(1)$ algebra. What happens is that two of the first-class constraints generating the gauge symmetry become second-class constraints away from the self-dual limit, reducing the resulting gauge symmetry.

However, it turns out that a smooth extrapolation from the $U(1)\times U(1)$ to $U(2)$ gauge symmetry algebra is not possible in the Dirac analysis. The reason is that in the Dirac method we have to compute the inverse of the Dirac matrix which is constructed with second-class constraints only. This inverse matrix with parameter r becomes singular if we take

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the limit of $r \to 1$, because two of the constraints change from second class into first class. When this happens, the Dirac matrix becomes degenerate and the inverse does not exist. From a physical point of view, this singular behavior could be associated with the second-order phase transition which one encounters in going to the limit $r=1$ [4].

The organization of the paper is as follows. In Sec. II, we define the enlarged CP(N) model coupled with the Chern-Simons term and perform the canonical analysis. In Sec. III, we give a somewhat general discussion of the geometry of the constrained phase space suited for gauge symmetry enhancement. In Sec. IV, we give an explicit computation of the Dirac bracket in the case of $r=1$ and $r\neq 1$ separately. Section V contains conclusion and discussions.

II. THE MODEL

We start from the Lagrangian written in terms of the $N \times 2$ matrix ψ such that

$$\mathcal{L}=\frac{1}{g^2}\mathrm{tr}[(D_\mu\psi)^\dagger(D^\mu\psi)-\lambda(\psi^\dagger\psi-R)]+\mathcal{L}_{\mathrm{cs}}, \quad (2.1)$$

where the field, ψ, is made of two complex N vectors ψ_1 and ψ_2 such that

$$\psi=[\psi_1,\psi_2], \quad \psi^\dagger=\begin{bmatrix}\psi_1^\dagger\\ \psi_2^\dagger\end{bmatrix}, \quad (2.2)$$

and the Hermitian 2×2 matrix λ is a Lagrange multiplier. The 2×2 matrix R is given by

$$R=\begin{bmatrix} r & 0\\ 0 & r^{-1}\end{bmatrix}, \quad (2.3)$$

with a real positive r. We will also use the notation $R_{ab}=r_a\delta_{ab}$ $(a,b,\ldots=1,2)$ with $r_1=r, r_2=r^{-1}$. The covariant derivative is defined as $D_\mu\psi\equiv\partial_\mu\psi-\psi A_\mu$ with a 2×2 anti-Hermitian matrix gauge potential A_μ associated with the local $U(2)$ gauge transformations. The components of A_μ can be explicitly written as follows:

$$A_\mu=-i\begin{bmatrix} a_\mu & c_\mu^*\\ c_\mu & b_\mu\end{bmatrix}. \quad (2.4)$$

$\mathcal{L}_{\mathrm{cs}}$ is the non-Abelian Chern-Simons gauge action given by

$$\mathcal{L}_{\mathrm{cs}}=-\frac{\kappa}{2}\epsilon^{\mu\nu\rho}\mathrm{tr}\left(\partial_\mu A_\nu A_\rho+\frac{2}{3}A_\mu A_\nu A_\rho\right). \quad (2.5)$$

The kinetic term of the Lagrangian (2.1) is invariant under the local $U(2)$ transformation, while the matrix R with $r\neq 1$ explicitly breaks the $U(2)$ gauge symmetry down to $U(1)\times U(1)$. Thus, the symmetry of our model is $[SU(N)]_{\mathrm{global}}\times[U(2)]_{\mathrm{local}}$ for $r=1$, while $[SU(N)]_{\mathrm{global}}\times[U(1)\times U(1)]_{\mathrm{local}}$ for $r\neq 1$. Therefore, the parameter r could be regarded as an explicit symmetry breaking parameter.

Let us perform the canonical analysis using the Dirac method [6]. We first define the conjugate momenta of the ψ_a^α field by $\Pi_a^\alpha=\partial\mathcal{L}/\partial\dot{\psi}_a^\alpha$, which gives

$$\Pi_a^\alpha=\frac{1}{g^2}(\dot{\psi}_a^{\alpha\dagger}+A_{0ab}\psi_b^{\alpha\dagger}). \quad (2.6)$$

The indices $a,b\ldots$ represent the $U(2)$ indices 1 and 2, while Latin indices $\alpha,\beta\ldots$ represent the global $SU(N)$ indices of ψ_1 and ψ_2. We will occasionally omit the global $SU(N)$ indices, when the context is clear. Likewise, the conjugate momentum of the $\psi_a^{\alpha\dagger}$ field is given by

$$\Pi_a^{\alpha\dagger}=\frac{1}{g^2}(\dot{\psi}_a^\alpha-\psi_b^\alpha A_{0ba}). \quad (2.7)$$

The momentum for the Lagrangian multiplier field λ_{ab} is constrained to vanish,

$$\Pi_{ab}^\lambda=0. \quad (2.8)$$

The conjugate momentum P_{ab}^μ for the gauge field $A_{\mu ab}$ is given by

$$P_{iab}=\kappa\epsilon_{ij}A_{jba}, \quad P_{0ab}=0. \quad (2.9)$$

In the above, the indices $i,j,\ldots$ represent the spatial ones with 1 and 2. In the following analysis we will not treat the first equation as a constraint. Instead P_{iab} is removed from the beginning and replaced by $\kappa\epsilon_{ij}A_{jba}$ [7]. The second equation, together with Eq. (2.8), defines the primary constraint of the theory. The Poisson bracket is defined by

$$\{\psi_a^\alpha(x),\Pi_b^\beta(y)\}=\delta_{ab}\delta^{\alpha\beta}\delta(x-y),$$

$$\{\lambda_{ab}(x),\Pi_{cd}^\lambda(y)\}=\delta_{ac}\delta_{bd}\delta(x-y),$$

$$\{A_{0ab}(x),P_{0cd}(y)\}=\delta_{ac}\delta_{bd}\delta(x-y),$$

$$\{A_{iab}(x),A_{jcd}(y)\}=-\frac{1}{\kappa}\epsilon_{ij}\delta_{ad}\delta_{bc}\delta(x-y). \quad (2.10)$$

After a straightforward Dirac analysis, we find that the system is described by the canonical Hamiltonian given by

$$\mathcal{H}_0=g^2\Pi_a\Pi_a^\dagger+\frac{1}{g^2}(D_i\psi)_a^\dagger(D_i\psi)_a+\frac{1}{g^2}\lambda_{ab}(\psi_b^\dagger\psi_a-R_{ba})$$
$$+(\Pi_a\psi_b-\psi_a^\dagger\Pi_b^\dagger+\kappa F_{12ab})A_{0ba}, \quad (2.11)$$

where we denote $FG\equiv F^\alpha G^\alpha$ and F_{12ab} is the magnetic field given by

$$F_{12ab}=\partial_1 A_{2ab}-\partial_2 A_{1ab}+[A_1,A_2]_{ab}. \quad (2.12)$$

Including all secondary constraints, we find that the dynamics is governed by the following constraints:

$$C_{ab}^{(0)}=\Pi_{ab}^{\lambda}\approx 0,$$

$$C_{ab}^{(1)}=P_{ab}^{0}\approx 0,$$

$$C_{ab}^{(2)}=\psi_a^{\dagger}\psi_b-R_{ab}\approx 0,$$

$$C_{ab}^{(3)}=\Pi_a\psi_b-\psi_a^{\dagger}\Pi_b^{\dagger}+\kappa F_{12ab}\approx 0,$$

$$C_{ab}^{(4)}=\Pi_a\psi_b+\psi_a^{\dagger}\Pi_b^{\dagger}-\frac{1}{g^2}[A_0,R]_{ab}\approx 0. \tag{2.13}$$

One can check that the time evolution of the above constraints is closed with a total Hamiltonian $\mathcal{H}_T=\mathcal{H}_0+\Sigma_{u=0}^{4}\Lambda_{ab}^{(u)}C_{ab}^{(u)}$ using the relations (2.10).

To separate the constraints into first and second classes, we first calculate the commutation relations of Eqs. (2.13) to yield the nonvanishing Poisson brackets,

$$\{C_{ab}^{(1)}(x),C_{cd}^{(4)}(y)\}=\frac{1}{g^2}(r_c-r_d)\,\delta_{ad}\delta_{bc}\delta(x-y), \tag{2.14}$$

$$\{C_{ab}^{(2)}(x),C_{cd}^{(3)}(y)\}=(r_c-r_d)\,\delta_{ad}\delta_{bc}\delta(x-y), \tag{2.15}$$

$$\{C_{ab}^{(2)}(x),C_{cd}^{(4)}(y)\}=(r_a+r_b)\,\delta_{ad}\delta_{bc}\delta(x-y), \tag{2.16}$$

$$\{C_{ab}^{(3)}(x),C_{cd}^{(3)}(y)\}=(\delta_{bc}C_{ad}^{(3)}-\delta_{ad}C_{cb}^{(3)})\,\delta(x-y), \tag{2.17}$$

$$\{C_{ab}^{(3)}(x),C_{cd}^{(4)}(y)\}=\frac{1}{g^2}([A_0,R]_{ad}\delta_{bc}-[A_0,R]_{bc}\delta_{ad})\,\delta(x-y), \tag{2.18}$$

$$\{C_{ab}^{(4)}(x),C_{cd}^{(4)}(y)\}=\kappa(F_{12cb}\delta_{ad}-F_{12ad}\delta_{bc})\,\delta(x-y). \tag{2.19}$$

Note that Eq. (2.17) satisfies $U(2)$ Gauss law algebra. Nevertheless, $C_{12}^{(3)}$ and $C_{21}^{(3)}$ become second-class constraints for $r\neq 1$, because in this case the right-hand sides of Eqs. (2.15) and (2.18) are nonvanishing for $c\neq d$.

Before proceeding to the calculation of the Dirac brackets we briefly review in the next section the structure of the constrained phase space in a geometric language. This section is included mainly to fix our notations, conventions, and terminology.

III. GEOMETRY OF CONSTRAINED PHASE SPACE

A phase space can be described by a manifold Γ with a nondegenerate closed 2-form, Ω_{AB}. The capital Roman letters $(A,B,\dots)$ are used to represent collectively the indices of the phase-space coordinates. In our case $x^A=(\Pi_a^{\alpha},\psi_a^{\alpha},A_{iab},A_{0ab},P_{0ab},\lambda_{ab},\Pi_{ab}^{\lambda})$. The Poisson bracket structure on Γ is defined as follows. For any given two functions F, G,

$$\{F,G\}=\Omega^{AB}\partial_A F\partial_B G, \tag{3.1}$$

where Ω^{AB} denotes the inverse of Ω_{AB}.

If a theory is constrained by the constraints, $\overline{C^{\mu}}=0$, the space of physical interest will be the submanifold $\overline{\Gamma}$ consisting of all points of Γ satisfying the constraints. This constrained subspace inherits a closed 2-form, $\overline{\Omega_{AB}}$, from Ω_{AB} by restriction, i.e., for any two vector fields $\overline{X^A}$, $\overline{Y^B}$ tangent to $\overline{\Gamma}$ we define $\overline{\Omega_{AB}}$ by

$$\overline{\Omega_{AB}}\overline{X^A}\overline{Y^B}\equiv\Omega_{AB}\overline{X^A}\overline{Y^B}. \tag{3.2}$$

Let us divide the discussion in two cases.

(i) $\overline{\Omega_{AB}}$ is nondegenerate. In this case, $(\overline{\Gamma},\overline{\Omega_{AB}})$ is the reduced phase space and the reduced bracket structure can be defined as before, using the inverse of $\overline{\Omega_{AB}}$. For any two functions $\overline{F}$, $\overline{G}$ of $\overline{\Gamma}$ we define

$$\{\overline{F},\overline{G}\}_D=\overline{\Omega^{AB}}\partial_A\overline{F}\partial_B\overline{G}. \tag{3.3}$$

The condition for nondegeneracy of $\overline{\Omega_{AB}}$ can be stated as

$$\det\{\overline{C^{\mu}},\overline{C^{\nu}}\}\neq 0. \tag{3.4}$$

This condition, in turn, is equivalent to the fact that none of the vectors $\Omega^{AB}\partial_B\overline{C^{\mu}}$ is tangent to $\overline{\Gamma}$. In this case, the constraints $\overline{C^{\mu}}=0$ are said to form a second class and the resulting bracket structure on $\overline{\Gamma}$ is called the Dirac bracket to distinguish it from the original Poisson bracket, Eq. (3.1).

It is well known that $\overline{\Omega^{AB}}$, when regarded as a tensor field of Γ, both of whose indices are tangent to the submanifold $\overline{\Gamma}$, is related to Ω^{AB} as follows:

$$\overline{\Omega^{AB}}=\Omega^{AB}+\Theta_{\mu\nu}^{-1}\Omega^{AM}\partial_M\overline{C^{\mu}}\Omega^{BN}\partial_N\overline{C^{\nu}}, \tag{3.5}$$

where $\overline{\Theta^{\mu\nu}}\equiv\{\overline{C^{\mu}},\overline{C^{\nu}}\}$. In terms of the Poisson bracket, the Dirac bracket can be written as

$$\{F,G\}_D=\{F,G\}-\{F,\overline{C^{\mu}}\}\Theta_{\mu\nu}^{-1}\{\overline{C^{\nu}},G\}. \tag{3.6}$$

(ii) $\overline{\Omega_{AB}}$ is degenerate. The situation in this case is slightly more complicated because the inverse does not exist. Therefore, we cannot define the bracket structure on all of the functions of $\overline{\Gamma}$. However, $\overline{\Omega_{AB}}$ defines for us a nondegenerate closed 2-form on the quotient manifold of $\overline{\Gamma}$ where any two points of $\overline{\Gamma}$ are identified if they are related by a curve which lies along the degeneracy directions everywhere. In fact, $\overline{\Omega_{AB}}$ is the pull-back to $\overline{\Gamma}$ of a nondegenerate closed 2-form on the quotient space under the quotient map. We will interpret $\overline{\Omega_{AB}}$ in both ways, either as a degenerate 2-form on

$\bar{\Gamma}$ or as a nondegenerate 2-form on the quotient manifold. In this case, the quotient space together with a nondegenerate closed 2-form, $\bar{\Omega}_{AB}$, is the fully reduced phase space and one can define the bracket structure. Physically, the degeneracy directions represent gauge directions and the quotient space is the space of gauge orbits. Since gauge invariant functions can be identified with the functions on the quotient manifold, the fact that we have a well-defined bracket structure on the quotient space means that the bracket structure can be well defined only on gauge invariant functions on $\bar{\Gamma}$.

Degeneracies are in fact associated with the existence of the so-called first-class constraints. Let k^A be an arbitrary vector field on $\bar{\Gamma}$ which points in some degeneracy direction. Then, for all vector fields, t^B, tangent to $\bar{\Gamma}$,

$$0=\bar{\Omega}_{AB}k^At^B=\Omega_{AB}k^At^B, \tag{3.7}$$

which implies that

$$\Omega_{AB}k^A=\lambda_{\bar{\mu}}\partial_B C^{\bar{\mu}}=\partial_B(\lambda_{\bar{\mu}}C^{\bar{\mu}}) \tag{3.8}$$

for some nontrivial $\lambda_{\bar{\mu}}$. Such a linear combination of the constraints, $\lambda_{\bar{\mu}}C^{\bar{\mu}}$, is called a first-class constraint and its Poisson bracket with all other constraints vanishes, i.e.,

$$\{\lambda_{\bar{\mu}}C^{\bar{\mu}},C^{\bar{\nu}}\}=0. \tag{3.9}$$

Conversely, when $\Theta^{\overline{\mu\nu}}\equiv\{C^{\bar{\mu}},C^{\bar{\nu}}\}$ is degenerate, there exists a nontrivial $\lambda_{\bar{\mu}}$ such that $\lambda_{\bar{\mu}}\Theta^{\overline{\mu\nu}}=0$ and it can be shown that $\lambda_{\bar{\mu}}C^{\bar{\mu}}$ generates a degeneracy of $\bar{\Omega}_{AB}$. That is, $k^A=\Omega^{AB}\partial_B(\lambda_{\bar{\mu}}C^{\bar{\mu}})$ is tangent to $\bar{\Gamma}$ and $\bar{\Omega}_{AB}k^At^B=0$ for all t^B tangent to $\bar{\Gamma}$. Other linear combinations of the constraints independent of all first-class constraints belong to the second class. Therefore, in the degenerate case one can decompose the constraints into two classes, $(C^{\bar{\mu}})=(C^{\bar{a}},C^{\bar{i}})$, where $C^{\bar{a}}$ denotes the first-class constraints and $C^{\bar{i}}$ the second-class constraints, and they satisfy

$$\{C^{\bar{a}},C^{\bar{\mu}}\}=0,\quad \det\{C^{\bar{i}},C^{\bar{j}}\}\neq 0. \tag{3.10}$$

Unlike $\bar{\Omega}_{AB}$, which can be regarded either as a nondegenerate 2-form on the quotient manifold or as a degenerate 2-form on $\bar{\Gamma}$, $\bar{\Omega}^{AB}$ has a well-defined meaning only as a tensor field on the quotient space. In order to compare it with Ω^{AB}, we choose a gauge slice. Then, using this one-to-one map between the space of gauge orbits and the gauge slice, one obtains the corresponding nondegenerate closed 2-form and its inverse on the gauge slice. Note that the 2-form on the gauge slice obtained this way is just the induced 2-form from Ω_{AB} by restriction to the gauge slice. Therefore, one can obtain the relations between $\bar{\Omega}^{AB}$ and Ω^{AB} by treating the gauge slicing conditions as additional constraints. When these are included all constraints form a second class, as one can see from the fact that the induced 2-form on the gauge slice is nondegenerate. Let $G^{\bar{a}}=0$ represent a choice of gauge slice. For this to be a good choice of gauge slicing $W^{\overline{ab}}\equiv\{G^{\bar{a}},C^{\bar{b}}\}$ should be invertible. Then, from Eq. (3.6) one obtains, after a straightforward calculation,

$$\begin{aligned}\{F,G\}'_D\equiv&\bar{\Omega}^{AB}\partial_AF\partial_BG\\ =&\{F,G\}+W^{-1}_{\overline{am}}W^{-1}_{\overline{bn}}(\{G^{\bar{m}},G^{\bar{n}}\}\\ &+\{G^{\bar{m}},C^{\bar{i}}\}\{G^{\bar{n}},C^{\bar{j}}\}\Theta^{-1}_{\overline{ij}})\{C^{\bar{a}},F\}\{C^{\bar{b}},G\}\\ &+W^{-1}_{\overline{ab}}\{C^{\bar{a}},F\}\{G^{\bar{b}},G\}-W^{-1}_{\overline{ab}}\{G^{\bar{b}},F\}\{C^{\bar{a}},G\}\\ &+W^{-1}_{\overline{ab}}\{G^{\bar{b}},C^{\bar{i}}\}\Theta^{-1}_{\overline{ij}}\{C^{\bar{j}},F\}\{C^{\bar{a}},G\}\\ &-W^{-1}_{\overline{ab}}\{G^{\bar{b}},C^{\bar{i}}\}\Theta^{-1}_{\overline{ij}}\{C^{\bar{a}},F\}\{C^{\bar{j}},G\}\\ &+\Theta^{-1}_{\overline{ij}}\{C^{\bar{i}},F\}\{C^{\bar{j}},G\},\end{aligned} \tag{3.11}$$

where $\Theta^{\overline{ij}}=\{C^{\bar{i}},C^{\bar{j}}\}$. When the functions F,G are gauge invariant, the above equation reduces to the usual Dirac bracket constructed using the second-class constraints only.

From a geometric point of view what happens in our model can be explained as follows. The vector fields which are (Poisson-)generated by the nondiagonal part of $U(2)$ constraints point in fixed directions in Γ. When $r\neq 1$, they are not tangent to $\bar{\Gamma}$. As the parameter, r, approaches 1, the constraints change gradually and $\bar{\Gamma}$ becomes tangent to those vector fields at $r=1$. Initially second-class constraints become first class, the gauge symmetry being enlarged from $U(1)\times U(1)$ to $U(2)$.

IV. DIRAC BRACKETS

In this section, we explicitly construct the Dirac brackets (3.6) of our model. It turns out that transition from $r\neq 1$ to $r=1$ is singular and we have to carry out the cases of $r=1$ and $r\neq 1$ separately. The reason is that in the Dirac method we have to compute the inverse of the Dirac matrix $\Theta^{\overline{ij}}$ of Eq. (3.5), which is constructed with second-class constraints only. This inverse matrix becomes singular in the limit of $r\to 1$, because part of the constraints change from second class into first class in the limit, and the determinant of the Dirac matrix becomes zero.

A. $r=1$ case

For the case of $r=1$, we have $R_{ab}=\delta_{ab}$, and it is easy to infer from the constraints algebra (2.14)–(2.19) that only $C^{(2)}_{ab}$ and $C^{(4)}_{ab}$ are second-class constraints. All of the $C^{(3)}_{ab}$'s are the first-class constraints whose Gauss law satisfies the $U(2)$ algebra (2.17). $C^{(0)}_{ab}$ and $C^{(1)}_{ab}$ completely decouple from the theory and can be set equal to zero.

One can thus obtain the following Poisson bracket relations $\Theta^{\overline{ij}}=\{C^{\bar{i}},C^{\bar{j}}\}$ among the second-class constraints $C^{\bar{i}}=(C^{(2)}_{11},C^{(2)}_{12},C^{(2)}_{21},C^{(2)}_{22},C^{(4)}_{11},C^{(4)}_{12},C^{(4)}_{21},C^{(4)}_{22})$ $(\bar{i}=1,2,\ldots,8)$,

$$\Theta=\begin{bmatrix} O & M\\ -M^T & N\end{bmatrix}, \tag{4.1}$$

where

$$M=\begin{bmatrix} 2g_{11} & 0 & 0 & 0 \\ 0 & 0 & 2g_{11} & 0 \\ 0 & 2g_{11} & 0 & 0 \\ 0 & 0 & 0 & 2g_{11} \end{bmatrix},$$

$$N=\begin{bmatrix} 0 & -f_{12} & f_{21} & 0 \\ f_{12} & 0 & -\delta f & -f_{12} \\ -f_{21} & \delta f & 0 & f_{21} \\ 0 & f_{12} & -f_{21} & 0 \end{bmatrix}. \tag{4.2}$$

Here we have defined $g_{11}\equiv|\psi_1|^2=r$, $g_{22}\equiv|\psi_2|^2=r^{-1}$, $f_{ab}=\kappa F_{12ab}$, and $\delta f=f_{11}-f_{22}$. For $r=1$, we have $g_{11}=g_{22}=1$.

The inverse matrix of Θ is given by

$$\Theta^{-1}=\begin{bmatrix} M^{T-1}NM^{-1} & -M^{T-1} \\ M^{-1} & O \end{bmatrix}, \tag{4.3}$$

with

$$M^{-1}=\begin{bmatrix} \frac{1}{2g_{11}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2g_{11}} & 0 \\ 0 & \frac{1}{2g_{11}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2g_{11}} \end{bmatrix},$$

$$M^{T-1}NM^{-1}=\begin{bmatrix} 0 & \frac{f_{21}}{4g_{11}^2} & -\frac{f_{12}}{4g_{11}^2} & 0 \\ -\frac{f_{21}}{4g_{11}^2} & 0 & \frac{\delta f}{4g_{11}^2} & \frac{f_{21}}{4g_{11}^2} \\ \frac{f_{12}}{4g_{11}^2} & -\frac{\delta f}{4g_{11}^2} & 0 & -\frac{f_{12}}{4g_{11}^2} \\ 0 & -\frac{f_{21}}{4g_{11}^2} & \frac{f_{12}}{4g_{11}^2} & 0 \end{bmatrix}. \tag{4.4}$$

The Dirac brackets (3.6) are then given by

$$\{\psi_a^\alpha(x),\Pi_b^\beta(y)\}_D=\left(\delta_{ab}\delta^{\alpha\beta}-\frac{\psi_c^\alpha\psi_c^{\beta\dagger}}{2g_{11}}\delta_{a1}\delta_{b1}+(1\leftrightarrow 2)\right)\delta(x-y),$$

$$\{\psi_a^{\alpha\dagger}(x),\Pi_b^\beta(y)\}_D=\left(-\frac{\psi_1^{\alpha\dagger}\psi_1^{\beta\dagger}}{2g_{11}}\delta_{a1}\delta_{b1}-\frac{\psi_2^{\alpha\dagger}\psi_1^{\beta\dagger}}{2g_{11}}\delta_{a1}\delta_{b2}+(1\leftrightarrow 2)\right)\delta(x-y),$$

$$\{\Pi_a^\alpha(x),\Pi_b^\beta(y)\}_D=\left[\left(\frac{\Pi_1^\alpha\psi_1^{\beta\dagger}-\psi_1^{\alpha\dagger}\Pi_1^\beta}{2g_{11}}+\frac{\psi_2^{\alpha\dagger}\psi_1^{\beta\dagger}-\psi_1^{\alpha\dagger}\psi_2^{\beta\dagger}}{2g_{11}}\frac{f_{12}}{2g_{11}}\right)\delta_{a1}\delta_{b1}+\left(\frac{\Pi_2^\alpha\psi_1^{\beta\dagger}-\psi_2^{\alpha\dagger}\Pi_1^\beta}{2g_{11}}+\frac{\psi_1^{\alpha\dagger}\psi_1^{\beta\dagger}}{2g_{11}}\frac{f_{21}}{2g_{11}}-\frac{\psi_2^{\alpha\dagger}\psi_2^{\beta\dagger}}{2g_{11}}\frac{f_{12}}{2g_{11}}-\frac{\psi_2^{\alpha\dagger}\psi_1^{\beta\dagger}}{2g_{11}}\frac{\delta f}{2}\right)\delta_{a1}\delta_{b2}+(1\leftrightarrow 2)\right]\delta(x-y),$$

$$\{\Pi_a^\alpha(x),\Pi_b^{\beta\dagger}(y)\}_D=\left[\left(\frac{\Pi_c^\alpha\psi_c^\beta-\psi_c^{\alpha\dagger}\Pi_c^{\beta\dagger}}{2g_{11}}+\frac{\psi_2^{\alpha\dagger}\psi_1^{\beta\dagger}}{2g_{11}}\frac{f_{12}}{2g_{11}}+\frac{\psi_1^{\alpha\dagger}\psi_2^{\beta\dagger}}{2g_{11}}\frac{f_{21}}{2g_{11}}-\frac{\psi_2^{\alpha\dagger}\psi_2^\beta}{2g_{11}}\frac{\delta f}{2}\right)\delta_{a1}\delta_{b1}-\frac{\psi_c^{\alpha\dagger}\psi_c^\beta}{2g_{11}}\frac{f_{12}}{2g_{11}}\delta_{a1}\delta_{b2}+(1\leftrightarrow 2)\right]\delta(x-y),$$

$$\{\lambda_{ab}(x),\Pi_{cd}^\lambda(y)\}_D=\delta_{ac}\delta_{bd}\delta(x-y),$$

$$\{A_{0ab}(x),P_{0cd}(y)\}_D=\delta_{ac}\delta_{bd}\delta(x-y),$$

$$\{A_{iab}(x),A_{jcd}(y)\}_D=-\frac{1}{\kappa}\epsilon_{ij}\delta_{ad}\delta_{bc}\delta(x-y). \tag{4.5}$$

B. $r\neq 1$ case

In this case, we first note that two of the constraints $C_{12}^{(3)}$ and $C_{21}^{(3)}$ which were first-class in the case of $r=1$ become second-class, because the gauge symmetry is reduced to $U(1)\times U(1)$. This is evident from Eq. (2.14), whose right-hand side is nonvanishing for $r_c\neq r_d$. Therefore, we have all together 12 second-class constraints $(C_{12}^{(1)},C_{21}^{(1)},C_{11}^{(2)},C_{12}^{(2)},C_{21}^{(2)},C_{22}^{(2)},C_{12}^{(3)},C_{21}^{(3)},C_{11}^{(4)},C_{12}^{(4)},C_{21}^{(4)},C_{22}^{(4)})$. One could proceed to the computation of the Dirac bracket with these 12 constraints, which is quite involved. However, it greatly simplifies the computation if one observes that the constraints $C_{12}^{(4)}$ and $C_{21}^{(4)}$ can be eliminated from the list by solving them explicitly with the variables A_{0ab} $(a\neq b)$ given by

$$A_{0ab}=\frac{g^2}{r_b-r_a}(\Pi_a\psi_b+\psi_a^\dagger\Pi_b^\dagger)(a\neq b). \tag{4.6}$$

Then, from Eqs. (2.14)–(2.19), $C_{12}^{(1)}$ and $C_{21}^{(1)}$ commute with the rest of the costraints, and the number of second-class constraints become eight; $C^{\bar{i}}=(C_{11}^{(2)},C_{12}^{(2)},C_{21}^{(2)},C_{22}^{(2)},C_{12}^{(3)},C_{21}^{(3)},C_{11}^{(4)},C_{22}^{(4)})$ $(\bar{i}=1,\ldots,8)$.

We now find an 8×8 matrix $\Theta^{\bar{i}\bar{j}}=\{C^{\bar{i}},C^{\bar{j}}\}$ of the form

$$\Theta=\begin{bmatrix} O & M\\ -M^T & 0\end{bmatrix}, \tag{4.7}$$

where M is given by

$$M=\begin{bmatrix} 0 & 0 & 2g_{11} & 0\\ 0 & \delta g & 0 & 0\\ -\delta g & 0 & 0 & 0\\ 0 & 0 & 0 & 2g_{22}\end{bmatrix}, \tag{4.8}$$

with $\delta g=g_{11}-g_{22}$. The inverse matrix of Θ is given by

$$\Theta^{-1}=\begin{bmatrix} O & -(M^{-1})^T\\ M^{-1} & 0\end{bmatrix}, \tag{4.9}$$

with

$$M^{-1}=\begin{bmatrix} 0 & 0 & -\frac{1}{\delta g} & 0\\ 0 & \frac{1}{\delta g} & 0 & 0\\ \frac{1}{2g_{11}} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{2g_{22}}\end{bmatrix}. \tag{4.10}$$

The Dirac bracket is then given by

$$\{\psi_a^\alpha(x),\Pi_b^\beta(y)\}_D=\left[\delta_{ab}\delta^{\alpha\beta}+\left(-\frac{\psi_1^\alpha\psi_1^{\beta\dagger}}{2g_{11}}+\frac{\psi_2^\alpha\psi_2^{\beta\dagger}}{\delta g}\right)\delta_{a1}\delta_{b1}+(1\leftrightarrow 2)\right]\delta(x-y),$$

$$\{\psi_a^{\alpha\dagger}(x),\Pi_b^\beta(y)\}_D=\left[-\frac{\psi_1^{\alpha\dagger}\psi_1^{\beta\dagger}}{2g_{11}}\delta_{a1}\delta_{b1}+\frac{\psi_2^{\alpha\dagger}\psi_1^{\beta\dagger}}{\delta g}\delta_{a1}\delta_{b2}+(1\leftrightarrow 2)\right]\delta(x-y),$$

$$\{\Pi_a^\alpha(x),\Pi_b^\beta(y)\}_D=\left[\frac{\Pi_1^\alpha\psi_1^{\beta\dagger}-\psi_1^{\alpha\dagger}\Pi_1^\beta}{2g_{11}}\delta_{a1}\delta_{b1}+\frac{\Pi_2^\alpha\psi_1^{\beta\dagger}+\psi_2^{\alpha\dagger}\Pi_1^\beta}{\delta g}+(1\leftrightarrow 2)\right]\delta(x-y),$$

$$\{\Pi_a^\alpha(x),\Pi_b^{\beta\dagger}(y)\}_D=\left[\left(\frac{\Pi_1^\alpha\psi_1^\beta-\psi_1^{\alpha\dagger}\Pi_1^{\beta\dagger}}{2g_{11}}+\frac{\Pi_2^\alpha\psi_2^\beta-\psi_2^{\alpha\dagger}\Pi_2^{\beta\dagger}}{\delta g}\right)\delta_{a1}\delta_{b1}+(1\leftrightarrow 2)\right]\delta(x-y),$$

$$\{A_{iab}(x),A_{jcd}(y)\}_D=-\frac{1}{\kappa}\epsilon_{ij}\delta_{ad}\delta_{bc}\delta(x-y). \tag{4.11}$$

We note that not only is the structure of the constraints different from the $r=1$ case, but also $r\to 1$ is not defined in the above algebra (4.11).

V. CONCLUSION

We performed canonical analysis of the gauge symmetry enhancement in the enlarged CP(N) model coupled with the $U(2)$ Chern-Simons term. We discussed the transition between $r=1$ and $r\neq 1$ cases in terms of the degeneracy of the constrained phase-space geometry. We found that the conventional Dirac method does not allow a smooth extrapolation of the symmetry enhanced and broken phases. This was essentially due to the fact that the Dirac procedure requires an inverse of the Dirac matrix, which is constructed with second-class constraints only, and becomes singular when some of the second-class constraints become first class. Physically, a second-order phase transition occurring as the symmetry breaking parameter r approaches the critical value 1 could be responsible for the nonsmooth transition.

We conclude with a couple of remarks. We have computed the Dirac bracket of Eq. (3.6) without gauge fixing and thus are considering only gauge invariant functions which commute with the first-class constraints. Instead one could try to fix the gauge first, thereby rendering all the constraints second class, and then proceed to the Dirac bracket (3.11). This would involve technically more difficult steps; for example, in the case of $r=1$, we need four gauge-fixing conditions corresponding to the $U(2)$ gauge symmetry, which could be chosen as the Lorentz gauge. Then the matrix would become 16×16. For the gauge conditions corresponding to $U(1)\times U(1)$ in the case of $r\neq1$, we have to evaluate the inverse of 12×12. Finally, it would be interesting to perform other quantization methods of our model. For example, in the BRST-BFV method [8], which avoids the second-class constraints from the beginning by enlarging the phase space, the issue of the connection between $r=1$ and $r\neq1$ values could be reexamined.

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