T-duality and Penrose limits of spatially homogeneous and inhomogeneous cosmologies

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The Penrose limits of inhomogeneous cosmologies admitting two Abelian Killing vectors and their Abelian *T* duals are found in general. The wave profiles of the resulting plane waves are given for particular solutions. Abelian and non-Abelian *T* dualities are used as solution generating techniques. Furthermore, it is found that, unlike the case of Abelian *T* duality, non-Abelian *T* duality and taking the Penrose limit are not commutative procedures.

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I. INTRODUCTION

The low energy limit of string or M theory admits a variety of cosmological solutions. In four dimensions, these string cosmologies differ from cosmologies derived from general relativity due to the presence of scalar fields and form fields (see, for example [1]). In analogy with standard cosmology, string cosmologies, as well, generically have an initial space-time singularity. Close to any singularity the low energy approximation breaks down and the full string or M theory is needed. However, in general it is not clear how to relate the solutions of the low energy limit to exact string solutions and if this is at all possible. Plane waves are known examples of exact classical string vacua [2]. This means that they are exact to all orders in the string tension α'. Recent developments in string or M theory have led to renewed interest in an argument by Penrose [3] showing that all space-times, locally, in the neighborhood of a null geodesic, have a plane wave as a limit [4,5]. Therefore, in the Penrose limit, any space-time can be related to an exact classical string vacuum. For some of the plane wave backgrounds descriptions in terms of (solvable) conformal field theories have been found, which determine the spectrum of string excitations and their scattering amplitudes [6]. Recently, superstrings in plane wave backgrounds have been also discussed [7].

Duality transformations relate different string backgrounds. The new solution leads to consistent string propagation if conformal invariance is preserved. Here, duality transformations to the lowest order in α' will be used as a generating technique to find new solutions to Einstein equations coupled to a dilaton and antisymmetric tensor field.

Abelian *T* duality allows us to transform backgrounds admitting at least one Abelian isometry into another background of this type. The transformation changes the metric, antisymmetric tensor field and the dilaton while keeping the Abelian isometry of the background [8–10]. Similarly, non-Abelian *T* duality transforms backgrounds with non-Abelian isometries. However, in this case the non-Abelian isometry might be lost during the transformation. Therefore backgrounds without any kind of symmetry might be related to ones admitting non-Abelian isometries [11–13]. With the extra fields being constant (or zero) general relativity is a particular solution of low energy string theory. Most of the solutions of general relativity admit some kind of Abelian or non-Abelian symmetries. Therefore using Abelian and/or non-Abelian *T* duality new solutions to string cosmology can be found. This has led already to a multitude of solutions [1]. However, in addition to finding new solutions of string cosmology it should be noted that these symmetries can also be used as solution generating techniques within standard general relativity.

Using *T*-duality transformations a given background can be connected to a variety of different string cosmologies. In the Penrose limit all of these reduce to a plane wave space-time. Therefore, it might be worthwhile to see if the resulting plane waves are connected by a *T*-duality transformation, or in other words, whether taking the Penrose limit and dualizing are commutative.

In the following, the Penrose limiting procedure and Abelian and non-Abelian *T* dualities are briefly reviewed. According to [3] any D dimensional metric in the neighborhood of a segment of a null geodesic containing no conjugate points can be written as [5]

$$ds^2=du\,dv+\alpha dv^2+\sum_i \beta_i dv\,dx^i+C_{ij}dx^i dx^j, \quad (1.1)$$

where α, β_i, and C_{ij} are functions of all coordinates and $i,j=1,2,\ldots,D-2$. Following Penrose the coordinates are rescaled by a constant factor $\Omega>0$,

$$u=\tilde{u}, \quad v=\Omega^2\tilde{v}, \quad x^i=\Omega\tilde{x}^i. \quad (1.2)$$

Taking the limit $\Omega\to 0$ of $d\tilde{s}^2/\Omega^2$ gives the behavior of the metric in the neighborhood of a null geodesic. In this case $\tilde{u}$ is an affine parameter. Güven [14] extended the Penrose limit to include other fields, such as gauge and scalar fields. In summary, for a scalar field, e.g., the dilaton ϕ, the antisymmetric tensor field $B=B_{MN}dX^M\wedge dX^N$, and the metric behavior in the Penrose limit is given by

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$$\hat{\phi}=\lim_{\Omega\to 0}\phi(\Omega),$$

$$\hat{B}=\lim_{\Omega\to 0}\Omega^{-2}B(\Omega),$$

$$d\hat{s^2}=\lim_{\Omega\to 0}\Omega^{-2}ds^2(\Omega), \tag{1.3}$$

where the argument Ω denotes the rescaling of variables (1.2).

Duality symmetries relate different string backgrounds. Abelian T duality is a symmetry with respect to an Abelian Killing direction. T dualities are derived from the two-dimensional σ-model action given by

$$S=\frac{1}{4\pi}\int d^2z\bigg\{\partial X^M[G_{MN}(X)+B_{MN}(X)]\bar{\partial}X^N + \frac{1}{2}R^{(2)}\phi(X)\bigg\}, \tag{1.4}$$

where $M,N=0,\dots,d$, $X^M\equiv(t,X^m)$ $(m=1,\dots,d)$ are the string coordinates, $R^{(2)}$ is the scalar curvature of the two-dimensional worldsheet, and G_{MN}, B_{MN}, and ϕ are functions of X. Choosing coordinates $\{x^\mu\}=\{x^0,x^a\}$ such that the Abelian isometry acts by translation of $x^0\equiv\theta$ and all background fields are independent of θ. The T-duality transformation is found by gauging the Abelian isometry and then introducing Lagrangian multipliers in order to keep the gauge connection flat. These Lagrangian multipliers are promoted to coordinates in the dual space-time. Dual and original quantities are related as follows [8,9]:

$$G'_{00}=\frac{1}{G_{00}},\quad G'_{0a}=\frac{B_{0a}}{G_{00}},\quad G'_{ab}=G_{ab}-\frac{G_{a0}G_{0b}+B_{a0}B_{0b}}{G_{00}}$$

$$B'_{0a}=\frac{G_{0a}}{G_{00}},\quad B'_{ab}=B_{ab}-\frac{G_{a0}B_{0b}+B_{a0}G_{0b}}{G_{00}}. \tag{1.5}$$

The dilaton is shifted to

$$\phi'=\phi-\log G_{00}. \tag{1.6}$$

In [11] a T-duality transformation for backgrounds with non-Abelian isometries was proposed. However, in [12] an example, namely, a Bianchi V cosmology was given for which this transformation does not lead to another consistent string background since the (low energy) β function equations are not satisfied. In [13] it was shown that in the case that the group of isometries of the background is not semisimple, which is the case for Bianchi V, a mixed gauge and gravitational anomaly is present. However, in [15] it was found that not all nonsemisimple groups lead to an anomaly. Non-Abelian duality transformations have been generalized to Poisson-Lie T duality which allows to find dual space-times even with respect to the nonsemisimple groups that were excluded for non-Abelian T duality [16]. However, here the focus will be on the standard non-Abelian T-duality procedure [11].

In general, it is not possible to write explicitly the gauge fixed action. Thus the dual fields cannot be presented in a closed form as it was possible in the Abelian case [cf. Eqs. (1.5) and (1.6)] [11].

In the following spatially homogeneous and simple inhomogeneous cosmologies will be investigated. The corresponding metrics admit three or two Killing vectors, respectively. Whereas the former admit non-Abelian isometries, the latter are Abelian. Therefore the structure is rich enough to apply Abelian and non-Abelian T dualities. The observable universe on large scales is well described by a Friedmann-Robertson-Walker universe which is a particular case of a spatially homogeneous universe. However, with a view to the question of initial conditions more general cosmologies deserve further study as well. The spatially homogeneous models were first classified by Bianchi into nine different types (cf. [17]). Bianchi models I–VII, locally rotationally symmetric (LRS) VIII and LRS IX have two-dimensional Abelian subgroups. Therefore these can be described in the same fashion as spatially inhomogeneous space-times admitting two Abelian Killing vectors.

II. ABELIAN T DUALITY OF G_2 COSMOLOGIES AND THE RADIAL PENROSE LIMIT

G_2 space-times admit two Abelian Killing vectors. Thus spatial homogeneity is broken along one spatial direction. In general these metrics can be written as [18]

$$ds^2=2e^{-M}dudv-\frac{2e^{-U}}{Z+\bar{Z}}(dx+iZdy)(dx-i\bar{Z}dy), \tag{2.1}$$

where M and U are real and Z is a complex function of the two null coordinates u and v. Therefore these space-times (2.1) are conveniently described in terms of a null tetrad.

Introducing coordinates $t=u-v, r=u+v$, say, makes the line element (2.1) similar to that of a cylindrical space-time. In that case, r could be interpreted as the radius of the cylinder. Geodesics in cylindrical space-times have been investigated in connection with nonsingular solutions in [19]. Although due to the presence of two Abelian Killing vectors there are two constants of motion in the set of geodesic equations the general solution is not straightforward to find and one has to specialize to certain types of geodesics. For radial geodesics the constants of motion are zero and explicit solutions can be found in closed form. Furthermore, the change to adapted null coordinates is not obvious. Therefore, in the following, only Penrose limits around radial null geodesics will be investigated.

The limiting procedure of Penrose [3] can be applied along a segment of a null geodesic without conjugate points. This means that the expansion of a congruence of neighboring null geodesics has to be finite. For geodesics with tangent vector parallel to $n^\mu=e^{M/2}\partial_u$ the expansion is given by $\mu+\bar{\mu}=e^{M/2}(e^{-U})_u/e^{-U}$ and equivalently for those with tangent vector parallel to $l^\mu=e^{M/2}\partial_v$ the expansion is given by $\rho+\bar{\rho}=-e^{M/2}(e^{-U})_v/e^{-U}$ [18]. Here μ and ρ are Newman-

Penrose spin coefficients. Therefore assuming that $e^{M/2}$ and e^{U} are bounded, the Penrose limit (1.3) of the metric (2.1) leads to a plane wave space-time with all functions just depending on one of the null coordinates, say u. However, in general the null coordinate will not be an affine parameter. Therefore in the following it is assumed that after taking the Penrose limit a new null coordinate $u=\int e^{-M(\tilde{u})}d\tilde{u}$ has been introduced. For plane waves, traveling in u direction, the only nonvanishing null tetrad component of the Weyl tensor is given by [18]

$$\Psi_4=\frac{Z_{uu}-U_uZ_u}{Z+\bar{Z}}-2\frac{(Z_u)^2}{(Z+\bar{Z})^2}. \tag{2.2}$$

The only nonvanishing tetrad component of the Ricci tensor is given by

$$\Phi_{22}=\frac{1}{4}\left[2U_{uu}-(U_u)^2-4\frac{Z_u\bar{Z}_u}{(Z+\bar{Z})^2}\right]. \tag{2.3}$$

In analogy with electromagnetism, Ψ_4 can be written as $\Psi_4=Ae^{i\alpha}$, where A is the amplitude and α is the polarization of the gravitational wave [18]. Therefore Ψ_4 determines the profile of the wave. It is interesting to note that the Brinkmann form of the metric can be read off from Ψ_4 and Φ_{22}. The Brinkmann form is given by

$$ds^2=2dudV+(h_{11}X^2+2h_{12}XY+h_{22}Y^2)du^2-dX^2-dY^2, \tag{2.4}$$

where h_{ij} are functions of u only. The Weyl and Ricci tensor components are given by [18]

$$\Psi_4=\tfrac{1}{2}(h_{11}-h_{22}+2ih_{12}),\quad \Phi_{22}=\tfrac{1}{2}(h_{11}+h_{22}). \tag{2.5}$$

Therefore calculating these quantities for the Einstein-Rosen form (2.1) allows to read off the profile of the gravitational wave, h_{ij}, in the Brinkmann form.

Assuming that the metric (2.1) describes a vacuum space-time, the following Brinkmann form for the resulting plane wave in the Penrose limit is obtained:

$$h_{11}=-\frac{\left[\left(\frac{2e^{-U}}{Z+\bar{Z}}\right)^{1/2}\right]_{uu}}{\left(\frac{2e^{-U}}{Z+\bar{Z}}\right)^{1/2}}-\frac{1}{4}\frac{[(Z-\bar{Z})_u]^2}{(Z+\bar{Z})^2}, \tag{2.6}$$

$$h_{12}=-\frac{i}{2}\frac{(Z-\bar{Z})_{uu}-U_u(Z-\bar{Z})_u}{Z+\bar{Z}}+i\frac{(Z_u)^2-(\bar{Z}_u)^2}{(Z+\bar{Z})^2}, \tag{2.7}$$

$$h_{22}=-\frac{\left[\left(\frac{Z+\bar{Z}}{2}e^{-U}\right)^{1/2}\right]_{uu}}{\left(\frac{Z+\bar{Z}}{2}e^{-U}\right)^{1/2}}+\frac{3}{4}\frac{[(Z-\bar{Z})_u]^2}{(Z+\bar{Z})^2}. \tag{2.8}$$

Using that $\Phi_{22}=0$ in vacuum it follows that $h_{11}=-h_{22}$.

The Abelian T-duality transformations (1.5) take a particularly simple form in terms of the functions U and Z when applied for $\phi=0$ and $B_{\mu\nu}=0$. The function M remains invariant under this duality transformation. T duality with respect to the Killing vector ∂_x results in

$$e^{-U'}=\frac{Z+\bar{Z}}{2},\quad Z'=e^{-U},$$

$$B'_{xy}=\frac{i}{2}(Z-\bar{Z}),\quad \phi'=-\ln\frac{2e^{-U}}{Z+\bar{Z}}. \tag{2.9}$$

The metric is diagonal and hence $h_{11}=\Psi_4+\Phi_{22}$ and $h_{22}=\Phi_{22}-\Psi_4$, which yields to

$$h_{11}=-\frac{\left[\left(\frac{Z+\bar{Z}}{2}e^{U}\right)^{1/2}\right]_{uu}}{\left(\frac{Z+\bar{Z}}{2}e^{U}\right)^{1/2}},\quad h_{22}=-\frac{\left[\left(\frac{Z+\bar{Z}}{2}e^{-U}\right)^{1/2}\right]_{uu}}{\left(\frac{Z+\bar{Z}}{2}e^{-U}\right)^{1/2}}. \tag{2.10}$$

Therefore if the seed metric (2.1) is diagonal then the wave profile in the direction orthogonal to the Killing direction along which the T-duality transformation is taken remains invariant.

T duality with respect to the Killing vector ∂_y results in

$$e^{-U'}=\frac{Z+\bar{Z}}{2Z\bar{Z}},\quad Z'=e^{U}$$

$$B'_{xy}=\frac{i}{2}\frac{Z-\bar{Z}}{Z\bar{Z}},\quad \phi'=-\ln\left(\frac{2e^{-U}}{Z+\bar{Z}}Z\bar{Z}\right). \tag{2.11}$$

The wave profile is given by

$$h_{11}=-\frac{\left[\left(\frac{Z+\bar{Z}}{2Z\bar{Z}}e^{-U}\right)^{1/2}\right]_{uu}}{\left(\frac{Z+\bar{Z}}{2Z\bar{Z}}e^{-U}\right)^{1/2}}\quad h_{22}=-\frac{\left[\left(\frac{Z+\bar{Z}}{2Z\bar{Z}}e^{U}\right)^{1/2}\right]_{uu}}{\left(\frac{Z+\bar{Z}}{2Z\bar{Z}}e^{U}\right)^{1/2}}. \tag{2.12}$$

Again it is found that in the case of a diagonal seed metric the wave profile stays invariant in the direction orthogonal to the Killing direction along which the T-duality transformation is taken.

The structure of the dual backgrounds (2.9) and (2.11) shows that the dual of a plane wave is again a plane wave. Since plane waves are exact classical string vacua this implies that Abelian T duality relates in this case two exact classical string vacua. Choosing a null geodesic in the (t,z)-plane where $u=t-z, v=t+z$, with z a longitudinal coordinate and t a timelike variable, the radial Penrose limit is found by the limiting procedure (1.3) for $u\rightarrow u, v\rightarrow\Omega^2 v$, $x\rightarrow\Omega x, y\rightarrow\Omega y$. Effectively this reduces all functions, i.e.,

TABLE I. Bianchi backgrounds and their duals. The second and third column give the functions U and Z of the Bianchi model in the first column. The last column denotes to which Bianchi model a given Bianchi model is related to using either the T-duality transformation (2.9) or (2.11). KS (open/closed) denotes the Kantowski-Sachs model with open or closed spatial sections. The last entry was already noted in [23]. Using string cosmologies with a dilaton and antisymmetric tensor field strength as seed backgrounds similar but different relations were found in [24].

Bianchi type	e^{-U}	Z	Relationship
II	a_1a_2	$\frac{a_2}{a_1}+iz$	$\mathrm{II}\overset{(2.9)}{\rightarrow}\mathrm{I}$
IV	$a_2a_3e^{2x}$	$\frac{a_2a_3}{a_2^2+a_3^2(f+x)^2}-i\frac{a_3^2(f+x)}{a_2^2+a_3^2(f+x)^2}$	$\mathrm{IV}\overset{(2.11)}{\rightarrow}\mathrm{VI}_{-1}$
V	$a_2a_3e^{2x}$	$\frac{a_3}{a_2}$	$\mathrm{V}\overset{(2.9)}{\rightarrow}\mathrm{VI}_{-1}$
			$\mathrm{V}\overset{(2.11)}{\rightarrow}\mathrm{VI}_{-1}$
VI_{-1}	a_1a_2	$\frac{a_2}{a_1}e^{2x}$	$\mathrm{VI}_{-1}\overset{(2.9)}{\rightarrow}\mathrm{V}$
LRS VIII	$a_1a_3\cosh y$	$\frac{a_1a_3\cosh y}{a_1^2\cosh^2 y+a_3^2\sinh^2 y}-i\frac{a_3^2\sinh y}{a_1^2\cosh^2 y+a_3^2\sinh^2 y}$	$\text{LRS VIII}\overset{(2.11)}{\rightarrow}\text{KS (open)}$
LRS IX	$a_1a_3\cos y$	$\frac{a_1a_3\cos y}{a_1^2\cos^2 y+a_3^2\sin^2 y}+i\frac{a_3^2\sin y}{a_1^2\cos^2 y+a_3^2\sin^2 y}$	$\text{LRS IX}\overset{(2.11)}{\rightarrow}\text{KS (closed)}$

$M(u,v)$, $Z(u,v)$, and $U(u,v)$ to functions of u only, which is equivalent to considering the limit $v\rightarrow 0$ [20]. Hence obtaining first the radial Penrose limit and then applying Abelian T duality yields the same as dualizing first and then obtaining the Penrose limit of the dual space-time.

Different spatially homogeneous backgrounds can be related to each other using the Abelian T-duality transformations (2.9) and (2.11). Therefore the Penrose limits of various Bianchi cosmologies are related by duality.

Isometries of spatially homogeneous metrics in four dimensions are described by three spacelike Killing vectors that form an algebra. In total there are nine different types originally classified by Bianchi (cf., e.g. [17]). They fall into two classes, A and B, according to whether the trace of the group structure constants vanishes or not. Bianchi types I, II, VI_{-1}, VII_0, VIII, and IX are of class A whereas Bianchi types III, IV, V, VI_h and VII_h are of class B.

Bianchi class A models can always be described by a diagonal metric in the invariant basis, i.e., $ds^2=dt^2-g_{ij}(t)\omega^i\omega^j$, where ω^i are the invariant basis one forms, satisfying $d\omega^i=\frac{1}{2}C^i_{jk}\omega^j\wedge\omega^k$ and C^i_{jk} are the group structure constants. Furthermore the metric is assumed to be of the form $g_{ij}=\mathrm{diag}(a_1^2(t),a_2^2(t),a_3^2(t))$. A Bianchi type-V background can also be described by a diagonal metric. However, for Bianchi type IV a nondiagonal metric is required. In order to investigate its behavior under the duality transformations (2.9) and (2.11) the ansatz of Harvey and Tsoubelis [21] was used. Namely, $\sigma^1=a_1\omega^1,\sigma^2=a_2\omega^2,\sigma^3=a_3f\omega^2+a_3\omega^3$, where a_i and f are functions of the timelike variable t only. σ^i are the basis one forms in the orthnormal frame, $ds^2=\eta_{\mu\nu}\sigma^\mu\sigma^\nu$, with $\eta_{\mu\nu}$ the Minkowski metric. Harvey and Tsoubelis found a solution, which, incidently, describes a plane wave, for $a_2=a_3$ [21]. No spatially homogeneous background was found when the T-duality transformations (2.9) and (2.11) were applied to backgrounds of Bianchi type $\mathrm{VI}_h/\mathrm{VII}_h$, namely, to the Lukash-type metric [22]. Bianchi models I–VII, LRS VIII, and LRS IX have two-dimensional Abelian subgroups. Therefore they can be written in the form of metric (2.1). The results are summarized in Table I.

Examples

The Kasner metric describes a homogeneous but anisotropic universe. Adapted to the G_2 symmetry the Kasner metric can be written as (see, for example [25])

$$ds^2=t^{(p^2-1)/2}(dt^2-dz^2)-t^{1+p}dx^2-t^{1-p}dy^2, \tag{2.13}$$

where p is a constant. Close to the initial singularity the metric (2.1) is well approximated by a Kasner metric with space-dependent Kasner exponents. In this case p becomes a function of z (cf., e.g. [26]). Introducing null coordinates $\tilde{u}=t-z,v=t+z$, taking the radial Penrose limit and finding an affine parameter u results in the following wave profiles:

$$h_{mm}=\kappa_m u^{-2}, \tag{2.14}$$

where $m=1,2$ and κ_m is constant, $\kappa_1=-\kappa_2=-p(1-p^2)/(p^2+1)^2$ for the seed metric (2.13), $\kappa_1=-(p+1)(p^2+p+2)/(p^2+1)^2$, $\kappa_2=\kappa_2^{(seed)}$ for the dual space-time (2.9), and $\kappa_1=\kappa_1^{(seed)}$, $\kappa_2=(p-1)(p^2-p+2)/(p^2+1)^2$ for the dual space-time (2.11). Hence in general, the wave profiles show a u^{-2} dependence. This was also found in the radial Penrose limit of the flat Friedmann-Robertson-Walker space-time and the near horizon limit of the fundamental string [5].

Models (2.1) for which Z is real or the imaginary part is subleading compared to the real one evolve at late times into the Doroshkevich-Zeldovich-Novikov (DZN) universe [27]. This is an anisotropic spatially homogeneous background with an effective null fluid due to gravitational waves. The DZN line element is given by

$$ds^2=e^{2t}(dt^2-dx^2)-t^{q+1}dy^2-t^{1-q}dz^2, \qquad (2.15)$$

where q is a constant. Choosing null coordinates $\tilde{u}=t-x$, $v=t+x$ taking the radial Penrose limit, finding the affine parameter u, the wave profiles h_{mm} are obtained as follows:

$$h_{mm}=\alpha_m u^{-2}(\ln u)^{-2}[\kappa_m+\ln u], \qquad (2.16)$$

where $m=1,2$ and α_m and κ_m are constant, $\alpha_1=(q+1)/2$, $\kappa_1=(1-q)/2$, and $\alpha_2=(1-q)/2$, $\kappa_2=(1+q)/2$ for the seed metric (2.15). $\alpha_1=-(q+1)/2$, $\kappa_1=(q+3)/2$ and $\alpha_2=\alpha_2^{(seed)}$, $\kappa_2=\kappa_2^{(seed)}$ for the dual space-time (2.9). $\alpha_1=\alpha_1^{(seed)}$, $\kappa_1=\kappa_1^{(seed)}$ and $\alpha_2=(q-1)/2, \kappa_2=(3-q)/2$ for the dual space-time (2.11).

There are a few known nonsingular solutions with G_2 symmetry (cf. [19,28,29]). Since in view of the *T*-duality transformations nondiagonal solutions are of particular interest, the nondiagonal solution given in [28,29] will be investigated. In the vacuum case, the line element can be written as, using the coordinates of [29],

$$ds^2=e^{a^2r^2}\cosh(2at)(dt^2-dr^2)-r^2\cosh(2at)d\varphi^2 - \frac{1}{\cosh(2at)}(dz+ar^2d\varphi)^2, \qquad (2.17)$$

where a is a constant. Whereas the *T*-duality transformation with respect to ∂_x leads to another nonsingular background, the *T*-duality transformation with respect to ∂_y leads to a singular background. In particular, the *T*-duality transformation (2.9) leads to

$$ds^2=\cosh(2at)[e^{a^2r^2}(dt^2-dr^2)-dz^2-r^2d\varphi^2],$$
$$\phi=\ln\cosh(2at), \quad B_{z\varphi}=ar^2. \qquad (2.18)$$

It is interesting to note that the application of Abelian *T* duality and the $SL(2,\mathbf{R})$ invariance of the axion and dilaton [30] shows that the solution (2.17) can be generated from a diagonal solution with a stiff perfect fluid as matter source, namely [31]

$$^{(E)}ds^2=e^{a^2r^2}(dt^2-dr^2)-dz^2-r^2d\varphi^2. \qquad (2.19)$$

This metric is obtained from Eq. (2.17) by first applying the *T* duality transformation (2.9) and then transforming to the conformally related Einstein frame $g_{\mu\nu}\to{}^{(E)}g_{\mu\nu}=e^{-\phi}g_{\mu\nu}$. The *T* duality transformation (2.9) also creates a nonvanishing antisymmetric tensor field $B_{z\varphi}$. However, using the $SL(2,\mathbf{R})$ invariance of the axion and dilaton [30] allows to reduce this solution to a pure dilaton solution which has effectively the energy-momentum tensor of a stiff perfect fluid (2.19). In this case, the symmetries of string cosmology have been used as solution generating techniques in standard relativity. This is possible due to the fact that a massless scalar field behaves as a stiff perfect fluid.

The Penrose limit and the resulting plane waves are found by introducing the null coordinates $u=t-r, v=t+r$. The final expressions are given in terms of the nonaffine parameter u. For the solution (2.17) the amplitudes are given by

$$h_{11}=a^2\frac{e^{-(a^2/2)u^2}}{\cosh^4(au)}\times[-3\cosh^2(au)-au\sinh(au)\cosh(au)+6],$$
$$h_{12}=a^2\frac{e^{-(a^2/2)u^2}}{\cosh^4(au)}[au\cosh(au)+6\sinh(au)], \qquad (2.20)$$

and $h_{22}=-h_{11}$. The amplitudes are regular everywhere. The radial Penrose limit of Eq. (2.18) results in a plane wave with profile

$$h_{11}=a^2\frac{e^{-(a^2/2)u^2}}{\cosh^4(au)}[au\sinh(au)\cosh(au)+\cosh^2(au)-3],$$
$$h_{22}=a^2\frac{e^{-(a^2/2)u^2}}{\cosh^4(au)}[au\cosh(au)+3\sinh(au)]\sinh(au). \qquad (2.21)$$

The wave profile is regular everywhere. The expressions for the amplitudes of the dual wave obtained from applying Abelian *T* duality with respect to ∂_y (2.11) are rather lengthy and are given in the Appendix. The different wave profiles are shown in Fig. 1. It is interesting to note that only h_{22} becomes singular for $u\to 0$ for the wave obtained from the *T*-duality transformation (2.11). Close to the singularity at $u=0$ h_{22} behaves as $h_{22}\sim u^{-2}$. This causes a strong curvature singularity to develop. In the approach to the singularity the string coupling $g^2=e^{\phi}$ diverges as u^{-2}. Therefore the expansion to lowest order in the string coupling is no longer valid.

III. NON-ABELIAN *T* DUALITY

The *T* dual with respect to a non-Abelian group of isometries is found by gauging the two-dimensional σ-model action, integrating over the introduced gauge fields, and gauge fixing the obtained action [11]. Before gauge-fixing this leads to a dual action of the form [11] in the notation of [12]

$$S'=S+\frac{1}{4\pi}\int d^2z(A^{\gamma}\overline{u}_{\gamma}+\overline{A}^{\delta}u_{\delta}+A^{\gamma}m_{\gamma\delta}\overline{A}^{\delta}), \qquad (3.1)$$

where

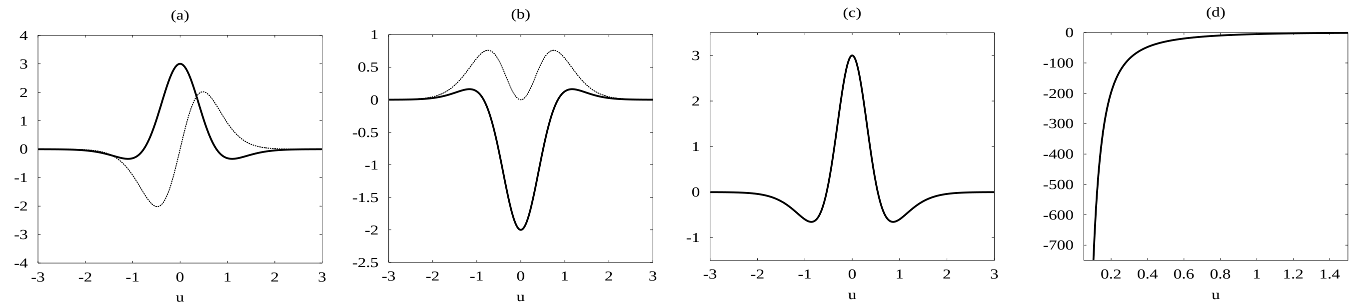

FIG. 1. (a) gives the wave profiles h_{11} (thick line) and h_{12} (thin line) of the plane wave in the radial Penrose limit of the nonsingular nondiagonal metric (2.20). (b) is the wave profile of the plane wave found by duality with respect to ∂_x (2.21) [h_{11} (thick line), h_{22} (thin line)]. (c) shows h_{11} and (d) h_{22} of the wave profile of the plane wave found by duality with respect to ∂_y as given in the Appendix (A1). $a=1$ for all figures.

$$u_\delta = -\partial \tilde{X}_\delta + \partial X^M (G_{MN}+B_{MN})\xi_\delta^N,$$

$$\overline{u}_\gamma = \overline{\partial}\tilde{X}_\gamma + \xi_\gamma^M (G_{MN}+B_{MN})\overline{\partial} X^N,$$

$$m_{\gamma\delta} = C^\lambda_{\gamma\delta}\tilde{X}_\lambda + \xi_\gamma^M (G_{MN}+B_{MN})\xi_\delta^N, \tag{3.2}$$

where greek indices are group indices and latin indices are target space-time indices. G_{MN} is the metric on the target space-time and B_{MN} is the antisymmetric tensor field. S is the original σ-model action (1.4). $\tilde{X}_\lambda$ are Lagrange multipliers introduced to keep the gauge connection flat. Depending on the gauge fixing they can become coordinates in the T-dual background.

To investigate whether taking the Penrose limit and dualizing the background space-time are commuting procedures we need to find the non-Abelian T dual of a plane wave background. The metric of a plane wave can always be written in the following form:

$$ds^2 = 2du dv - e^{-U}(e^V \cosh W dx^2 - 2\sinh W dx dy + e^{-V}\cosh W dy^2), \tag{3.3}$$

where U, V, and W are functions of u only. This metric admits five Killing vectors [18]

$$\xi_1 = \partial_x, \quad \xi_2 = \partial_y, \quad \xi_3 = \partial_v,$$

$$\xi_4 = x\partial_v + P_-(u)\partial_x + N(u)\partial_y,$$

$$\xi_5 = y\partial_v + P_+(u)\partial_y + N(u)\partial_x, \tag{3.4}$$

where $P_\pm(u) = \int e^{U\pm V}\cosh W du$, $N(u) = \int e^U \sinh W du$. All commutators vanish except for $[\xi_1,\xi_4]=\xi_3$ and $[\xi_2,\xi_5]=\xi_3$. With $[\xi_\alpha,\xi_\beta] = C^\mu_{\alpha\beta}\xi_\mu$, the only nonvanishing group structure constants are given by $C^3_{14}=1=C^3_{25}$. There are two semisimple subgroups, $\mathcal{G}_1=\{\xi_1,\xi_3,\xi_4\}$ and $\mathcal{G}_2=\{\xi_2,\xi_3,\xi_5\}$. In the following non-Abelian T duality with respect to the subgroup $\mathcal{G}_1$ will be considered. Furthermore, it will be assumed that the dilaton and the antisymmetric tensor field vanish, i.e., $\phi\equiv 0, B_{MN}\equiv 0$.

The first step to find the non-Abelian dual with respect to the subgroup $\mathcal{G}_1$, following the procedure of [11], is to calculate the matrix m. It is found that m is given by

$$m = \begin{pmatrix} G_{xx} & 0 & \tilde{X}_3 + G_{xx}P_- + G_{xy}N \\ 0 & 0 & 0 \\ -\tilde{X}_3 + P_- G_{xx} + N G_{xy} & 0 & G_{xx}P_-^2 + 2NP_- G_{xy} + G_{yy}N^2 \end{pmatrix}. \tag{3.5}$$

The null Killing vector ξ_3 leads to a singular part in the T-dual action. This yields a singular space-time that is singular everywhere if one tried to integrate over the gauge fields. Something similar happens in the case of Abelian T duality if the isometry has a fixed point [32]. In the case of the Euclidean two-dimensional black hole the horizon, on which the timelike Killing vector becomes null, is interchanged with a curvature singularity in the T-dual background [33]. It can also be seen in a straightforward manner in the example of the T dual of a two-dimensional plane [34],

$$ds^2 = dr^2 + r^2 d\theta^2. \tag{3.6}$$

The T dual with respect to the isometry $T=\partial_\theta$ is given by

$$ds^2 = dr^2 + r^{-2} d\theta^2. \tag{3.7}$$

The dilaton is given by $\phi = -\ln r^2$. The background becomes singular at $r=0$ which is exactly the point at which $T^2=0$. In the case of the plane wave space-time the Killing vector ξ_3 is null everywhere. Even though the other two Killing vectors of $\mathcal{G}_1$ are not null, the *T*-dual space-time is singular everywhere. Furthermore, the *T*-dual dilaton is given by [11,12]

$$\phi' = \phi - \log \det m, \tag{3.8}$$

which is singular everywhere in the *T*-dual background since $\det m = 0$.

Both non-Abelian subgroups, $\mathcal{G}_1$ and $\mathcal{G}_2$, of the group of motions of a simple plane wave space-time contain one null Killing vector. Therefore using the procedure of [11] to find the non-Abelian *T* dual of a pure plane wave results in a singular *T*-dual background. Nevertheless, since the effective metric is built out of G_{MN} and B_{MN}, taking a nonvanishing antisymmetric tensor field B_{MN} into account might lead to a *T*-dual background that is not singular everywhere. However, a constant *B* field is not enough since its Lie derivatives in the direction of the Killing vectors of the isometry group in general do not vanish. In that case, further terms have to be taken into account in the *T*-dual action (3.1) [35].

Another possibility to find non-Abelian *T* duals of a plane wave that are not singular everywhere arises if the plane wave space-time admits additional (non-null) isometries. For example, the WZW model of [36] admits an additional nonsemisimple group. Non-Abelian *T* duals with respect to these group have been found in [37,15]. In both cases it was found that non-Abelian *T*-duality transforms the original plane wave space-time into a background that is not a plane wave.

Other examples, of plane wave space-times with additional spacelike isometries are the solutions of [21] which admit Bianchi type IV. However, since the Bianchi IV is a nonsemisimple group it is not possible to use the procedure of [11]. In that case one would have to apply Poisson-Lie *T* duality to find an equivalent solution [16].

In the Penrose limit any space-time can be approximated around a null geodesic by a plane wave metric. If the resulting plane wave is such that it does not admit isometries in addition to the isometries of the plane wave [cf. Eq. (3.4)] then the non-Abelian *T* dual using the procedure of [11] is singular everywhere. The structure of the resulting plane wave depends on the particular null geodesic which was taken to obtain the Penrose limit and the symmetries of the original space-time. As was shown in [5] the number of linearly independent Killing vectors of the Penrose limit space-time is at least as large as $\max(n,2D-3)$, where n is the number of linear independent Killing vectors of the original space-time and D is the space-time dimension.

On the other hand, the non-Abelian *T* dual using the procedure of [11] of a cosmological space-time admitting spacelike Killing vectors is in general not singular everywhere. For spatially homogeneous backgrounds general expressions have been given for the non-Abelian *T* dual [12], as will be discussed below. Thus, taking the Penrose limit of this non-Abelian *T* dual results in a plane wave. Therefore, comparing this with the resulting singular space-time obtained from the non-Abelian *T* dual of a plane wave without additional isometries, it can be concluded that in this case taking the Penrose limit and taking the non-Abelian *T* dual are not commutative procedures.

If the resulting plane wave space-time in the Penrose limit does admit additional Killing vectors then one might consider the non-Abelian *T* dual with respect to these. In principle, it might be possible that for a particular case the non-Abelian *T* dual is again a plane wave and furthermore that taking the Penrose limit and taking the non-Abelian *T*-dual commutes. However, as mentioned above, the examples that have been found so far are such that the non-Abelian *T* dual of a plane wave is not a plane wave [37,15]. Assuming that these plane wave space-times with additional symmetries are the result of taking a particular Penrose limit of some space-time then this would be another example of the noncommutativity of taking the Penrose limit and taking the non-Abelian *T* dual.

As an example a vacuum Bianchi II cosmology will be considered. Its metric is given by

$$ds^2 = -dt^2 + a_1^2(dx - z\,dy)^2 + a_2^2 dy^2 + a_3^2 dz^2, \tag{3.9}$$

where $a_i = a_i(t)$ [38]. The only nonvanishing group structure constant is $C^1_{23} = 1$ [17].

In [12] the non-Abelian *T* duals of spatially homogeneous backgrounds have been found. The transformed metric, antisymmetric tensor field, and shifted dilaton are given by

$$\tilde{G} = (\gamma - \beta - \kappa)^{-1}\gamma(\gamma + \beta + \kappa)^{-1},$$

$$\tilde{B} = -(\gamma - \beta - \kappa)^{-1}(\beta + \kappa)(\gamma + \beta + \kappa)^{-1},$$

$$\tilde{\bar{\phi}} = \bar{\phi} - \log\det(\kappa + \gamma + \beta), \tag{3.10}$$

where κ is an antisymmetric matrix defined by $\kappa_{\alpha\beta} \equiv C^{\gamma}_{\alpha\beta}\tilde{X}_{\gamma}$. $\tilde{X}^{\lambda}$ are coordinates in the dual space-time. $\gamma_{\mu\nu}(t)$ is the metric in the invariant basis on hypersurfaces of constant time, $ds^2 = -dt^2 + \gamma_{\mu\nu}(t)\omega^{\mu}\omega^{\nu}$, and $\beta_{\mu\nu}(t)$ describes the antisymmetric tensor field in the synchronous frame $B = \beta_{\mu\nu}(t)\omega^{\mu}\wedge\omega^{\nu}$. Furthermore, $d\omega^a = \frac{1}{2}C^{\alpha}_{\mu\nu}\omega^{\mu}\wedge\omega^{\nu}$. For Bianchi type-A models $\gamma_{\mu\nu}(t)$ is diagonal, namely, $\gamma_{\mu\nu}(t) = \mathrm{diag}(a_1^2(t), a_2^2(t), a_3^2(t))$.

Applying the non-Abelian *T*-duality transformation (3.10) to the Bianchi II vacuum background (3.9) yields

$$ds^2 = -a_1^{-2}(d\eta^2 - dx^2) + t[(a_2a_3)^2 + x^2]^{-1} \times (a_3^2 dy^2 + a_2^2 dz^2),$$

$$\phi = -\ln[(a_1a_2a_3)^2 + a_1^2x^2], \quad B_{yz} = -\frac{x}{(a_2a_3)^2 + x^2}. \tag{3.11}$$

This metric is no longer of Bianchi type II. It admits two Abelian Killing vectors ∂_y, ∂_z. Introducing null coordinates $u = \eta - x, v = \eta + x$, the radial Penrose limit is found to be

$$d\hat{s}^2 = -\frac{1}{a_1^2}dudv + \left[(a_2a_3)^2 + \frac{u^2}{4}\right]^{-1}(a_3^2dy^2 + a_2^2dz^2),$$

$$\hat{\phi} = -\ln\left[(a_1a_2a_3)^2 + a_1^2\frac{u^2}{4}\right], \quad \hat{B}_{yz} = \frac{u}{2(a_2a_3)^2 + \frac{u^2}{2}}, \tag{3.12}$$

where $a_i = a_i(u)$.

Next we will consider the radial Penrose limit of the Bianchi II cosmology (3.9) written in the form (2.1). The metric (3.9) admits the following three Killing vectors [17]:

$$\xi_1 = \partial_x, \quad \xi_2 = \partial_y, \quad \xi_3 = \partial_z + y\partial_x. \tag{3.13}$$

Introducing null coordinates $u = \eta - z, v = \eta + z$, rescaling according to Eq. (1.2) and finding the limit $\lim_{\Omega \to 0}\Omega^{\Delta_\xi}\xi_\alpha(\Omega)$, where $\Delta_\xi \in \mathbf{R}$ [5], it turns out that, whereas ξ_1 and ξ_2 stay unchanged, ξ_3 becomes the null Killing vector ∂_v. In addition, there are the two Killing vectors ξ_4 and ξ_5. Hence, there are no additional isometries to the pure plane wave isometries (3.4). Therefore, the non-Abelian T dual can only be found with respect to one of the subgroups $\mathcal{G}_1$ or $\mathcal{G}_2$, respectively. As was shown above, this leads to a dual background that is singular everywhere. However, the Penrose limit of the non-Abelian T dual of the vacuum Bianchi II cosmology (3.12) only becomes singular locally. Thus, in this case, taking the Penrose limit and finding the non-Abelian T dual are not commutative procedures.

Finally, some comments on non-Abelian T duality as a solution generating technique will be made. In the last section it was shown that Abelian T duality can be used to connect solutions to general relativity of varying degree of generality. Basically, starting with one solution a more general solution was found. In general relativity the approach to the initial singularity is still an open question (for a recent account, see [39]) which is partly due to the fact that there are no known general solutions. The majority of known solutions admits some kind of symmetries. However, due to the nature of non-Abelian T duality most of the symmetries of the original space-time will be broken in the T-dual background. Therefore, one might use these transformations to generate very general solutions which admit, if at all, only few isometries. This will be discussed with the example of Bianchi VIII and IX as seed metrics. These are the most general spatially homogeneous metrics. Furthermore, their group structure is semisimple. The group structure constants are given by $C^1_{23} = \pm 1, C^2_{31} = 1, C^3_{12} = 1$, where the upper sign corresponds to Bianchi IX and the lower one to Bianchi VIII. The non-Abelian T-duality transformation (3.10) yields to

$$\tilde{G} = (a_1^2a_2^2a_3^2 + a_1^2x^2 + a_2^2y^2 + a_3^2z^2)^{-1} \times \begin{pmatrix} a_2^2a_3^2 + x^2 & \pm xy & \pm xz \\ \pm xy & a_1^2a_3^2 + y^2 & yz \\ \pm xz & yz & a_1^2a_2^2 + z^2 \end{pmatrix}, \tag{3.14}$$

$$\tilde{B} = (a_1^2a_2^2a_3^2 + a_1^2x^2 + a_2^2y^2 + a_3^2z^2)^{-1} \times \begin{pmatrix} 0 & a_3^2z - 2\epsilon xy & -a_2^2y - 2\epsilon xz \\ -a_3^2z + 2\epsilon xy & 0 & \pm a_1^2x \\ a_2^2y + 2\epsilon xz & \mp a_1^2x & 0 \end{pmatrix}, \tag{3.15}$$

$$\tilde{\phi} = -\log(a_1^2a_2^2a_3^2 + a_1^2x^2 + a_2^2y^2 + a_3^2z^2), \tag{3.16}$$

where $\epsilon = 1$ for Bianchi IX and $\epsilon = 0$ for Bianchi VIII. This background could be interpreted as an inhomogeneous generalization of a Bianchi I background. For small values of x, y, and z the spatial part imposes a small perturbation on a Bianchi I background. The vacuum Bianchi IX metric with three different scale factors is the Mixmaster model which shows chaotic behavior. However, the evolution of the scale-factors can be approximately described by a succession of Kasner epochs, each of them determined by a set of Kasner exponents $(\alpha_1, \alpha_2, \alpha_3)$ (cf., e.g. [40]). The Kasner metric is given by $ds^2 = -dt^2 + t^{2\alpha_1}dx^2 + t^{2\alpha_2}dy^2 + t^{2\alpha_3}dz^2$, and, in vacuum, the exponents satisfy $\Sigma_i\alpha_i = 1 = \Sigma_i\alpha_i^2$. Using such a solution in the expressions for the scale factors of the seed vacuum Bianchi IX metric one finds that the initial singularity persists in the T-dual background. Furthermore the metric is approximately diagonal, with the new scale factors being $1/a_i$. Hence there will be also Mixmaster oscillations in the dual background, though due to the presence of the scalar field and the antisymmetric tensor field these will cease after a finite number of oscillations [41]. Furthermore close to the singularity the T-dual universe enters into a strongly coupled regime, since the string coupling $g^2 = e^{\phi'}$ diverges for $t \to 0$.

The T-dual background is very inhomogeneous though it is also rather special, since the spatial dependence is completely fixed and does not allow for arbitrary constants, as it is the case for the scale factors $a_i(t)$.

IV. CONCLUSIONS

In the Penrose limit any space-time in the vicinity of a null geodesic can be approximated by a plane wave. Since plane waves are exact classical string vacua this might help to connect cosmological solutions to an underlying string vacuum. There are only very few known exact solutions that have a cosmological interpretation and these are very far away from describing our observable universe. Since plane waves are classical string vacua it makes sense to find a first quantized theory of a string propagating in these backgrounds. This has been studied in particular for singular backgrounds with wave profiles following a power law in the null coordinate [42]. Here it was found that this type of wave profile occurs for the radial Penrose limit of a Kasner universe, whose scale factors are following a power law in cosmic time. For space-times with more general functional behavior different types of evolution were found. In particular the wave profiles of the plane wave obtained in the radial Penrose limit of a nonsingular cosmological solution were determined. Here it might be interesting to study the first quantization of a string propagating in this background.

Low energy string theory admits a number of symmetries. These have been used to find more exact solutions, their corresponding Penrose limits and wave profiles. In addition relationships between different spatially homogeneous backgrounds have been found. This is interesting from the point of view that Abelian *T* duality and taking the radial Penrose limit are commuting procedures. Furthermore using Abelian *T* duality and the $SL(2,\mathbf{R})$ invariance of low energy string theory it was found that the nonsingular nondiagonal solution [28,29] can be reduced to a diagonal static solution [31]. This shows once more that the symmetries of low energy string theory can be used to learn more about solutions in general relativity.

The non-Abelian *T* dual of a vacuum plane wave space-time has been investigated in detail. It was found that if there are no additional isometries then dualizing with respect to one of the semisimple subgroups of isometries of the plane wave leads to a *T*-dual background that is singular everywhere. The reason for that is the presence of a null Killing vector in each subgroup. This is similar to what happens in Abelian *T* duality. If there are additional isometries one might find non-Abelian *T*-dual backgrounds that are not singular everywhere. In principle, it might also be possible that the non-Abelian *T* dual of a particular type of plane wave is again a plane wave and furthermore that, as in the Abelian case, taking the Penrose limit and taking the non-Abelian *T* dual do indeed commute. However, the examples known so far only relate plane wave space-times (with additional isometries) with backgrounds that do not describe plane waves. Here it might also be interesting to discuss these issues in the context of Poisson-Lie *T* duality.

The type of plane wave that is obtained in the Penrose limit depends on the null geodesic around which the Penrose limit is taken and on the isometries of the original space-time. For the class of backgrounds that lead in the Penrose limit to plane wave space-times with no additional isometries it was shown that the non-Abelian *T* dual is singular everywhere. Therefore for this class of solutions taking the Penrose limit and applying a non-Abelian *T*-duality transformation are not commutative procedures. This is in contrast to the case of Abelian *T*-duality where it was found [14] that, in general, the dualization procedure and taking the Penrose limit do commute.

Finally, the role of Abelian and non-Abelian *T* duality as solution generating techniques has been discussed. In particular, non-Abelian *T* duality was used to find more general inhomogeneous solutions which can be interpreted as inhomogeneous generalizations of a Bianchi I cosmology.

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APPENDIX: THE PROFILE OF THE DUAL WAVE OBTAINED FROM EQ. (2.17) WITH EQ. (2.11)

The dual wave profile resulting from applying Eq. (2.11) to the background (2.17) is given by

$$h_{11}=a^2e^{-(a^2/2)u^2}\left[\frac{a^5u^5\sinh(au)\cosh(au)-a^4u^4[\cosh^2(au)+3]+16a^2u^2\cosh^2(au)[2\ \cosh^2(au)-3]}{\cosh^4(au)[4\cosh^2(au)+a^2u^2]^2}\right.$$
$$\left.+\frac{-16au\sinh(au)\cosh^3(au)[\cosh^2(au)+6]+48\cosh^4(au)[2-\cosh^2(au)]}{\cosh^4(au)[4\cosh^2(au)+a^2u^2]^2}\right]$$
$$h_{22}=e^{-(a^2/2)u^2}\left[\frac{a^7u^7\sinh(au)\cosh(au)-3a^6u^6[\cosh^2(au)+1]+16a^4u^4\cosh^2(au)[\cosh^2(au)-4]}{u^2\cosh^4(au)[4\cosh^2(au)+a^2u^2]^2}\right.$$
$$\left.-\frac{16a^3u^3\sinh(au)\cosh^3(au)[8+\cosh^2(au)]80a^2u^2\cosh^6(au)}{u^2\cosh^4(au)[4\cosh^2(au)+a^2u^2]^2}-\frac{128au\sinh(au)\cosh^5(au)+128\cosh^6(au)}{u^2\cosh^4(au)[4\cosh^2(au)+a^2u^2]^2}\right]. \tag{A1}$$

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