Possible end of the universe in a finite future from dark energy with $w<-1$

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The occurrence of a big smash singularity which ends the universe in a finite time in the future is investigated in the context of superquintessence, i.e. dark energy with an effective equation of state parameter $w<-1$ and $\dot{H}>0$. The simplest toy model of superquintessence based on a single nonminimally coupled scalar field exhibits big smash solutions which are attractors in phase space.

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A new picture of the Universe has emerged in recent years: the data from the Boomerang [1] and MAXIMA [2] experiments confirm that we live in a spatially flat $(\Omega=1)$ universe, where $\Omega=\Omega^{(m)}+\Omega^{(q)}$ is the total energy density expressed in units of the critical density [3]. Baryonic and dark matter only account for $\Omega^{(m)}\simeq 0.3$ of the total energy density Ω, while the rest is due to a yet unknown form of *dark energy*. Studies of type Ia supernovae [4] and of radio galaxies [5] show that the present expansion of the Universe is accelerated, i.e., $\ddot{a}>0$, where $a(t)$ is the scale factor of the Friedmann-Robertson-Walker line element

$$ds^2=-dt^2+a^2(t)(dx^2+dy^2+dz^2) \tag{1}$$

describing our Universe in comoving coordinates (t,x,y,z). The Friedmann equation

$$\frac{\ddot{a}}{a}=-\frac{\kappa}{6}(\rho+3P), \tag{2}$$

where ρ and P are, respectively, the energy density and pressure of the source of gravity, shows that in order to have accelerated expansion the pressure of the dark energy dominating the dynamics must be negative, $P<-\rho/3$. A cosmological constant Λ as the explanation of dark energy is rejected by most cosmologists because of the cosmological constant problem [6] and of the cosmic coincidence problem. The vacuum energy density predicted by high energy physics with a Planck scale cutoff is wrong by 120 orders of magnitude (or 40 orders of magnitude if the cutoff is at the QCD scale). Solving the *cosmic coincidence problem* of why the dark energy came to dominate the dynamics only recently (at redshift $z\sim 1$) requires extreme fine tuning of Λ. One would rather have Λ exactly equal to zero due to some yet unknown mechanism than this extreme fine tuning, and it is preferable to explain the present cosmic acceleration with a dynamical vacuum energy called *quintessence*. Many models of quintessence, most of which based on scalar fields, have been proposed.

Observational efforts aim at determining the effective equation of state parameter of the universe $w\equiv P/\rho$. It was pointed out that the current data allow for [7,5,8,9], or even favor [10–15], values of this parameter in the $w<-1$ range. Were a value $w<-1$ to be confirmed by observations, it would be a most interesting finding, because such values cannot be explained by Einstein gravity with a minimally coupled scalar, if one assumes positivity of the energy density. In fact in such "canonical" models the scalar field has energy density and pressure

$$\rho=\frac{(\dot{\phi})^2}{2}+V(\phi), \quad P=\frac{(\dot{\phi})^2}{2}-V(\phi), \tag{3}$$

and an effective equation of state parameter

$$w=\frac{x-1}{x+1}, \tag{4}$$

where $x\equiv(\dot{\phi})^2/(2V)\geqslant 0$ if $V>0$, giving $-1\leqslant w\leqslant 1$ (the minimum of w being attained by de Sitter solutions). In these models one only obtains $w<-1$ by assuming $V<-(\dot{\phi})^2/2 \leqslant 0$, which in turn implies a negative energy density (3), and $\rho<0$ is hardly an acceptable proposition. Correspondingly, the Friedmann equation

$$\dot{H}=-\frac{\kappa}{2}(\rho+P) \tag{5}$$

tells us that, for a minimally coupled scalar in Einstein's gravity it is $\dot{H}=-\kappa(\dot{\phi})^2/2\leqslant 0$ (the extreme case $\dot{H}=0$ again corresponding to de Sitter solutions). A regime with $w<-1$ is associated to $\dot{H}>0$ and is called *superacceleration*; a form of dark energy capable of sustaining superacceleration was dubbed *superquintessence* [16] or *phantom energy* [10].

Models have been proposed to explain superacceleration regimes, including scalar fields nonminimally coupled to the Ricci curvature, actions with the "wrong" sign of the kinetic energy of the scalar, supergravity-inspired models with noncanonical kinetic energy terms and zero potential (*k-essence*), Brans-Dicke-like fields in scalar-tensor gravity, or stringy matter [17]. If the universe superaccelerates, its expansion becomes so fast that it risks ending its existence in a finite time, $a(t)\rightarrow\infty$ as $t\rightarrow t_0$ with t_0 finite. For solutions with this property (*big smash* or *big rip* solutions) [10,18,19,12,14] the energy density of superquintessence *increases* with time instead of redshifting away as the matter or radiation energy densities $\rho^{(m)}\propto a^{-3}$, $\rho^{(r)}\propto a^{-4}$, or as the

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energy density of "ordinary" quintessence. Big smash solutions occur in the theory of ordinary differential equations (ODEs) when the solutions of an ODE cannot be maximally extended to an infinite interval. An example of ODE which does not admit maximal extension of its solutions is

$$\frac{dy(x)}{dx}=Ay^2(x); \tag{6}$$

if $A>0$ the rate of change of the solution is fed by the increasing value of y itself in a positive feedback mechanism that makes $y(x)$ grow so fast that it explodes in a finite time, while this behavior is absent if $A\leqslant 0$. As we shall see, Eq. (6) is similar to the equation satisfied by the Hubble parameter of a superaccelerating universe.

A big smash can be avoided in certain models of superquintessence (e.g. [11]) but is a generic feature in other models [10]. Whether the big smash is unavoidable or not depends, of course, on the model adopted [20]: here we consider what is perhaps the simplest model of superquintessence, namely a single scalar field ϕ coupled nonminimally to the Ricci curvature, described by the action

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2}\left(\frac{1}{\kappa}-\xi\phi^2\right)-\frac{1}{2}g^{cd}\nabla_c\phi\nabla_d\phi-V(\phi)\right]+S^{(m)}, \tag{7}$$

where ξ is a dimensionless coupling constant and $S^{(m)}$ is the action for ordinary matter. Nonminimal coupling is introduced by renormalization even if it is absent at the classical level and is required in classical general relativity by the Einstein equivalence principle [21]. The field equations are

$$G_{ab}=\kappa T_{ab}[\phi], \tag{8}$$

$$T_{ab}[\phi]=\nabla_a\phi\nabla_b\phi-\frac{1}{2}g_{ab}\nabla^c\phi\nabla_c\phi-Vg_{ab}+\xi(g_{ab}\Box-\nabla_a\nabla_b)\times(\phi^2)+\xi G_{ab}\phi^2. \tag{9}$$

The gravitational coupling in Eq. (8) is the usual constant $\kappa=8\pi G$ and not the effective time-dependent coupling $\kappa_{eff}=\kappa(1-\kappa\xi\phi^2)^{-1}$ recurring in the literature and corresponding to a different way of writing the field equations [22]. Moreover, the scalar field stress-energy tensor (9) ("improved energy-momentum tensor" [23]) is covariantly conserved, $\nabla^bT_{ab}[\phi]=0$. In the metric (1), the field equations become

$$6[1-\xi(1-6\xi)\kappa\phi^2](\dot{H}+2H^2)-\kappa(6\xi-1)\dot{\phi}^2-4\kappa V+6\kappa\xi\phi\frac{dV}{d\phi}=0, \tag{10}$$

$$\frac{\kappa}{2}\dot{\phi}^2+6\xi\kappa H\phi\dot{\phi}-3H^2(1-\kappa\xi\phi^2)+\kappa V=0, \tag{11}$$

$$\ddot{\phi}+3H\dot{\phi}+\xi R\phi+\frac{dV}{d\phi}=0. \tag{12}$$

Only two of these equations are independent, as the Klein-Gordon equation (12) can be derived from the conservation equation $\dot{\rho}^{(\phi)}+3H(P^{(\phi)}+\rho^{(\phi)})=0$ when $\dot{\phi}\neq 0$, where

$$\rho^{(\phi)}=\frac{\dot{\phi}^2}{2}+V(\phi)+3\xi H\phi(H\phi+2\dot{\phi}) \tag{13}$$

and

$$P^{(\phi)}=\frac{\dot{\phi}^2}{2}-V(\phi)-\xi[4H\phi\dot{\phi}+2\dot{\phi}^2+2\phi\ddot{\phi}+(2\dot{H}+3H^2)\phi^2] \tag{14}$$

are, respectively, the effective energy density and pressure of the fluid equivalent to the nonminimally coupled scalar. Note that the Hamiltonian constraint (11) can be written as

$$H^2=\frac{\kappa}{3}\rho^{(\phi)}, \tag{15}$$

which is consistent with $\rho^{(\phi)}\geqslant 0$. This is not the case with other definitions of effective energy-momentum tensors $T_{ab}[\phi]$ used in the literature (see the discussion in Ref. [22]). In order to investigate the fate of the universe with respect to big smash solutions, we neglect the matter part of the action $S^{(m)}$; this assumption is justified by the fact that, when a superacceleration regime sets in, the superquintessence energy density $\rho^{(\phi)}$ quickly grows to dominate the matter energy density, which instead fades away.

Only the two variables H and ϕ are needed to describe the dynamics of the system (10)–(12), and the phase space is a two-dimensional manifold with a rather complex structure [24]. This is best seen by rewriting the field equations as [24]

$$\dot{\phi}=-6\xi H\phi\pm\frac{1}{2\kappa}\sqrt{\mathcal{F}(H,\phi)}, \tag{16}$$

$$\dot{H}=\left[3(2\xi-1)H^2+3\xi(6\xi-1)(4\xi-1)\kappa H^2\phi^2\mp\xi(6\xi-1)\sqrt{\mathcal{F}}H\phi+(1-2\xi)\kappa V-\kappa\xi\phi\frac{dV}{d\phi}\right]\frac{1}{1+\kappa\xi(6\xi-1)\phi^2}, \tag{17}$$

where

$$\mathcal{F}(H,\phi)=8\kappa^2\left[\frac{3H^2}{\kappa}-V(\phi)+3\xi(6\xi-1)H^2\phi^2\right]. \tag{18}$$

The appearance of the $\pm$ signs in Eqs. (16) and (17) requires a clarification: the phase space curved manifold is composed of two sheets, corresponding to the upper and lower sign, and there typically is a region forbidden to the dynamics corresponding to $\mathcal{F}(H,\phi)<0$; the two sheets join each other at the boundary of this forbidden region, which needs not be simply connected. Orbits of solutions lying in the "upper" sheet can switch to the "lower" sheet at these points, and vice versa [24]. Far from the forbidden region's boundary, the orbit of a solution is forced to stay in one sheet and cannot cross over to the other sheet without approaching the forbidden region and touching its boundary. Hence, for large values of H and ϕ only one sign in Eqs. (16) and (17) applies to each solution. Since $H(t)$ and $\phi(t)$ grow so quickly when the superacceleration regime sets in, it is meaningful to perform an asymptotic analysis for large values of these variables when searching for big smash solutions. We consider conformal coupling $\xi=1/6$, which is a stable infrared fixed point of the renormalization group [25], and consider as a toy model the potential

$$V(\phi)=\frac{m^2\phi^2}{2}+\lambda\phi^4, \tag{19}$$

where λ will be required to be negative. This does not harm the positivity of the energy density $\rho^{(\phi)}$ when $\xi\neq 0$. Equations (16) and (17) then reduce to

$$\dot{\phi}\simeq -H\phi\pm\sqrt{-2\lambda}\,\phi^2, \tag{20}$$

$$\dot{H}\simeq -2H^2+\mu\phi^2, \tag{21}$$

where $\mu\equiv\kappa m^2/6$. We look for big smash solutions of the form

$$a(t)=\frac{a_*}{|t-t_0|^{\alpha_\pm}}, \tag{22}$$

and

$$\phi(t)=\frac{\phi_*}{|t-t_0|^{\beta_\pm}}, \tag{23}$$

with $\alpha_\pm,\beta_\pm>0$, consistently with the approximation of large H and ϕ employed, and where t_0, a_* and ϕ_* are constants; t_0 will be approached from below. The substitution of Eqs. (22) and (23) into Eqs. (20) and (21) yields

$$\alpha_\pm=\frac{\pm\sqrt{-\lambda(2\mu+\lambda)}-(\mu+\lambda)}{\mu+4\lambda}, \tag{24}$$

$$\beta_\pm=1, \tag{25}$$

$$\phi_*^\pm=\pm\frac{1+\alpha_\pm}{\sqrt{-2\lambda}}. \tag{26}$$

By taking the positive sign in Eq. (24), one immediately sees that there are big smash solutions ($\alpha>0$) in the range of parameters

$$1<\frac{\mu}{|\lambda|}<4. \tag{27}$$

Next, one would like to know whether these big smash solutions are stable or if they disappear when perturbed. The equations for the perturbations δH and $\delta\phi$ are sufficiently involved to defy a direct analytical investigation. It is convenient, instead, to focus on the projection onto the (H,ϕ) plane of the two sheets composing the phase space. Assuming $\dot{H},\dot{\phi}\neq 0$, as is legitimate during the late stages of a superacceleration regime, one obtains the vector field

$$\frac{dH}{d\phi}=\frac{\dot{H}}{\dot{\phi}}=\frac{2u^2-\mu}{u\mp\sqrt{-2\lambda}}, \tag{28}$$

where $u\equiv H/\phi$. The identity

$$\frac{dH}{d\phi}=u+\phi\frac{du}{d\phi} \tag{29}$$

then yields

$$\frac{du}{d\phi}=\frac{u^2\pm\sqrt{-2\lambda}\,u-\mu}{u\mp\sqrt{-2\lambda}}\left(\frac{1}{\phi}\right). \tag{30}$$

It is straightforward to see that exact solutions of Eq. (30) with $u=\text{const}\neq\pm\sqrt{-2\lambda}$ exist and are given by

$$H=\frac{\mp\sqrt{-2\lambda}\pm\sqrt{-2\lambda+4\mu}}{2}\,\phi. \tag{31}$$

Equation (31) includes the big smash solutions (22)–(26). In fact, simple algebra shows that for the latter

$$u=\frac{\alpha_\pm}{\phi_*}=-\frac{\sqrt{4\mu-2\lambda}+\sqrt{-2\lambda}}{2}, \tag{32}$$

which reproduces a special case of Eq. (31).

Let us proceed to study the stability with respect to linear perturbations of the solutions of Eq. (30), which can be rewritten as

$$H=\gamma_i\phi, \tag{33}$$

where the constant γ_i can assume the values

$$\gamma_1=\frac{-\sqrt{-2\lambda}+\sqrt{4\mu-2\lambda}}{2}>0, \tag{34}$$

$$\gamma_2=\frac{-\sqrt{-2\lambda}-\sqrt{4\mu-2\lambda}}{2}<0, \tag{35}$$

$$\gamma_3=-\gamma_1<0, \tag{36}$$

$$\gamma_4=-\gamma_2>0 \tag{37}$$

[the big smash solutions (22)–(26) corresponding to γ_2]. The perturbations δu in

$$u(\phi)=u_0+\delta u=\gamma_i+\delta u(\phi) \tag{38}$$

satisfy the equation

$$\frac{d(\delta u)}{d\phi}=\frac{2u_0\delta u\pm\sqrt{-2\lambda}}{u_0\mp\sqrt{-2\lambda}}\,\frac{\delta u}{\phi} \tag{39}$$

which yields

$$\delta u = \epsilon \phi^{\delta_i}, \tag{40}$$

where ϵ is a constant and

$$\delta_i = \frac{2\gamma_i \pm \sqrt{-2\lambda}}{\gamma_i \mp \sqrt{-2\lambda}}. \tag{41}$$

There are the following possibilities for δ_i, corresponding to the four values of γ_i and to the upper and lower sign in Eq. (41):

$$\delta_{1a} = \frac{2\sqrt{4\mu - \lambda}}{-3\sqrt{-2\lambda} + \sqrt{4\mu - \lambda}}, \tag{42}$$

which is positive if $\mu/|\lambda| > 17/12 \simeq 1.4167$; this range of values of the parameters corresponds to growing perturbations and instability, while the finite range $0 < \mu/|\lambda| < 17/12$ corresponds to stability.

$$\delta_{1b} = 2\frac{\sqrt{4\mu - \lambda} - 2\sqrt{-2\lambda}}{\sqrt{-2\lambda} + \sqrt{4\mu - \lambda}} \tag{43}$$

is positive if $\mu/|\lambda| > 7/24 \simeq 0.2917$; this range corresponds to instability.

$$\delta_{2a} = \frac{2\sqrt{4\mu - \lambda}}{-2\sqrt{-2\lambda} - \sqrt{4\mu - \lambda}} < 0 \tag{44}$$

corresponds to the big smash solutions and to stability for any value of the parameters $\lambda < 0$ and μ; on the other hand, the possibility

$$\delta_{2b} = 2\frac{\sqrt{4\mu - \lambda} + 2\sqrt{-2\lambda}}{\sqrt{4\mu - \lambda} - \sqrt{-2\lambda}} > 0 \tag{45}$$

corresponds to instability. The remaining cases give the values of δ

$$\delta_{3a} = \delta_{1b}, \quad \delta_{3b} = \delta_{1a},$$

$$\delta_{4a} = \delta_{2b}, \quad \delta_{4b} = \delta_{2a} \tag{46}$$

already considered.

The big smash solutions are stable against linear perturbations and behave as attractors in phase space. Thus, there is a finite chance that a universe described by the model considered here end its existence in a finite time due to a big smash with infinite expansion, in which the energy density diverges instead of being diluted away and bound systems are gradually ripped apart [14].

A few considerations are in order: first, the value of the effective equation of state parameter w is still subject to uncertainty and a value $w < -1$ associated with superacceleration is not yet confirmed; second, even if such a value were supported by future experiments, it does not automatically imply that the universe will end in a big smash. Third, the toy model of superquintessence employed here should be generalized to more realistic models, which, however, are not constrained sufficiently well by the presently available data. Potentials used to model quintessence are usually different from Eq. (19), but the latter is a very common potential in scalar field cosmology and is easier to study as a toy model (it is difficult to reach conclusions about the existence of big smash solutions with other potentials, and even more difficult to perform a stability analysis). The presence and stability of big smash solutions in a finite future for a wide range of parameters leads one to regard a big smash as a generic feature of scalar field models of superquintessence that include a nonminimal coupling to gravity. This can be of the simple form described by the action (7), or of a more general form as in scalar-tensor theories, which have been known to contain smash solutions for a long time [26]. The action (7) can be explicitly reformulated as a scalar-tensor theory with a variable Brans-Dicke parameter

$$\omega(\varphi) = \frac{G\varphi}{4\xi(1 - G\varphi)}, \tag{47}$$

and

$$\varphi = \frac{1 - \kappa\xi\phi^2}{G}, \tag{48}$$

but more general forms of the coupling function $\omega(\phi)$ are possible. Since the field equations for the coupled variables H and ϕ in scalar-tensor gravity exhibit terms similar to the right-hand side of Eq. (6), solutions with explosive growth are possible in these theories.

The most stringent constraint on the theory of the nonminimally coupled scalar comes from Solar System experiments. The Brans-Dicke-like field φ mediates a long range force that is constrained by tests of general relativity. Since φ varies on a cosmological time scale, it is appropriate to approximate φ with its present value φ_0 and $\omega(\varphi) \simeq \omega(\varphi_0) \equiv \omega_0$. The lower bound on ω_0 is[1] $|\omega_0| > 500$ [29,30], which yields

$$|\omega_0| = \left| \frac{1 - \kappa\xi\phi_0^2}{4\kappa\xi\phi_0^2} \right| > 500. \tag{49}$$

Although the present day value of ϕ is unknown, a weak coupling regime in which $\kappa|\xi|\phi_0^2 \ll 1$ is plausible [27], given that typical values of ξ predicted by renormalization are of the order of 10^{-1}–10^{-2}, and that ϕ_0 cannot exceed the Planck mass $m_{pl} = G^{-1/2}$ by too much without causing fine-tuning problems in the parameters m and λ (the energy scale V must be below the Planck energy scale m_{pl}^4). In the weak coupling regime $\kappa|\xi|\phi_0^2 \ll 1$ and assuming that $\sqrt{\kappa}|\phi_0| \simeq 1$ (corresponding to $\phi_0 \simeq 0.2 m_{pl}$), one obtains $|\xi| < 5 \times 10^{-4}$ (this limit is weakened if $|\phi| \ll m_{pl}$). This constraint limits the amount of superacceleration that is present today (see, e.g. Ref. [28]), but it should be kept in mind that even a

[1]The more stringent bound $|\omega_0| > 3300$ [31] yields a constraint on $|\xi|$ of the same order of magnitude.

small amount of superacceleration can eventually lead to a big smash. Stringent limits on $|\xi|$ can be satisfied and the big smash will occur later: whether this amounts to fine tuning the value of ξ is determined by the still unknown value of the parameter w which quantifies the present amount of superacceleration (assuming that the universe really does superaccelerate, i.e. that $w<-1$). It is hoped that the observational determination of the value of w will soon clarify this issue.

To conclude, even if the departures of gravity from general relativity are small today in the Solar System, on a large scale they may have a catastrophic effect on the future of the universe. Usually, research on scalar-tensor cosmology has focussed on how scalar-tensor theories can depart from Einstein's gravity in the early universe and converge to it at later epochs. Here, the issue is rather the one of a cosmological solution of scalar-tensor gravity that is close to a general relativistic solution today (when superacceleration is still moderate and w close to -1), but will dramatically depart from it in the future. If the universe really superaccelerates, the concern about a big smash is legitimate. It is intriguing that observational data place our present Universe so close to the boundary $w=-1$ between the possibility of a big smash and certain evolution into infinite dilution in an infinite time.

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