Can we live on a D-brane? Effective theory on a self-gravitating D-brane

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We consider a D-brane coupled with gravity in type IIB supergravity on S^5 and derive the effective theory on the D-brane in two different ways: that is, holographic and geometrical projection methods. We find that the effective equations on the brane obtained by these methods coincide. The theory on the D-brane described by the Born-Infeld action is not like Einstein-Maxwell theory in the lower order of the gradient expansion; i.e., the Maxwell field does not appear in the theory. Thus a careful analysis and statement for cosmology on a self-gravitating D-brane should be demanded in realistic models.

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I. INTRODUCTION

The discovery of the D-brane in string theory has altered the notion of extra dimensions dramatically. The gauge fields are confined on the brane and only gravity can propagate in the whole higher dimensional spacetime. Inspired by this possibility, Randall and Sundrum constructed a model where the size of the extra dimension can be infinite [1]. We need no longer a compatification of the extra dimension. Since then, the concept of a braneworld has been explored intensively in cosmology and gravity [2].

Although the braneworld is motivated by the D-brane, the connection between the D-brane and the braneworld in the Randall-Sundrum model is quite uncertain. In the Randall-Sundrum model, gravitational interactions play an essential role. A self-gravity of the brane is essential to localize massless gravitons on the brane. As for the confinement of the standard model particle, their model completely relies on the idea of the D-brane. However, the self-gravity of the D-brane is not clearly understood. Although aspects of the D-brane in supergravity are known, they are derived only from dual pictures of the probe D-brane. Related to this issue, the D-braneworld has been discussed with possible assumptions [3]: the brane action is supposed to be of Born-Infeld type [4] and the bulk field is only the cosmological constant. Adopting the Born-Infeld action as the braneworld action, the matter fields are automatically included, although we have assumed so far that the brane action is the Nambu-Goto plus four-dimensional matter action. This is an important advantage because the matter contribution to the braneworld is uniquely determined. What we want to do in the current paper is deriving an effective theory on the gravitating D-brane in a more realistic superstring theory.

There are several ways to derive an effective theory on a probe D-brane. Recently, Sato and Tsuchiya showed that the effective action for a probe D-brane can be derived by calculating the classical on-shell action in type IIB supergravity [5]. They calculated classical on-shell actions by solving the Hamilton-Jacobi equation. Then the Born-Infeld action is shown to be a solution. Their analysis opens up the new possibility of deriving an effective theory for a gravitating D-brane. A similar situation appears in the context of AdS conformal field theory (CFT) correspondence. From AdS/CFT correspondence, one can derive the generating function for boundary CFT by calculating the classical action in AdS supergravity. If one introduces a cutoff brane in the AdS spacetime, a coupling of gravity to CFT emerges as a consequence of the breaking of conformal invariance by the cutoff brane. An interesting point is that this effective theory for CFT coupled with gravity is nothing but the effective theory for a Randall-Sundrum braneworld, which is a gravitating (cutoff) brane in AdS spacetime. Thus one may expect that the introduction of a cutoff brane in calculations of classical on-shell actions in supergravity would provide us with an effective theory for a dual quantum field theory with the coupling of gravity and/or an effective theory for a self-gravitating braneworld. If we adopt this point of view in the calculation of on-shell actions in type IIB supergravity, we might be able to obtain an effective theory for the D-brane coupled with gravity and/or a self-gravitating D-braneworld.

We will derive an effective theory by two methods as in the case for a cutoff brane in AdS spacetime and compare them to each other. In the first method we will use the holographic conjecture in the braneworld [6–11] and solve the Hamilton-Jacobi equation, whose solution will play the role of the counterterms. Then, adopting an AdS/CFT like correspondence, we can derive an effective theory for the D-brane with the coupling of the gravity. The second one is the geometrical approach developed in Ref. [12] (see also Refs. [13–16]); we can obtain the gravitational equation on the gravitating brane by projecting the five-dimensional variables onto the brane, while it is not closed in four dimensions. Indeed, the projected bulk Weyl tensor appears as a source term. In the vacuum bulk case the bulk Weyl tensor will be negligible in the low energy limit while this will not be in the case when nontrivial bulk fields exist [9,16–18]. In the holographic point of view, only a part of the bulk Weyl tensor behaves like conformal field theory on the boundary [9,16,17]. To identify the CFT part of the bulk Weyl tensor, we must solve the bulk matter and gravitational fields. After that we can derive an effective theory for a gravitating brane. We expect that almost the same result will be obtained in the holographic approach.

We will work in type IIB supergravity on S^5 because the

AdS/CFT correspondence was originally formulated between super Yang-Mills theory and type IIB supergravity aided by D-branes [19]. For simplicity, however, we will turn off several fields in the course of the calculation. The rest of this paper is composed of two main parts. In Sec. II, we will adopt the holographic method. We first describe the strategy and then obtain the solution to the Hamilton-Jacobi equation. Finally we see the effective theory on the D-brane with the coupling to gravity. In Sec. III, we will solve the bulk in the long-wave approximation and try to get the effective equation on the gravitating D-brane in the geometrical approach. To make this procedure work well, we will put a specific ansatz on a zeroth order solution. That is, we need an analytical background solution in order to solve the next order equations. In Sec. IV, we will give a discussion. Therein we will compare the results obtained in each method and present their interpretation.

II. HOLOGRAPHIC APPROACH

We will derive the effective action for a gravitating D-brane using the AdS/CFT correspondence. See Ref. [10] for the study of holography on *probe* D-branes. This section is organized as follows. We begin with the Hamilton-Jacobi equation in Sec. II A and give its solution in Sec. II B. Then we derive the effective theory on the D-brane following the braneworld AdS/CFT correspondence in Sec. II C. We will consider two cases where the D-brane is described by the Born-Infeld action and supposed to be done by the Nambu-Goto action.

A. Type IIB supergravity on S^5 and Hamilton-Jacobi equation

We begin with the action for type IIB supergravity on S^5:

$$S=\frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\Big\{e^{-2\phi+(5/4)\rho}\Big[{}^{(5)}R+4(\nabla\phi)^2+\frac{5}{4}(\nabla\rho)^2 -5\nabla\phi\nabla\rho-\frac{1}{2}|H|^2\Big]-\frac{1}{2}e^{(5/4)\rho}[(\nabla\chi)^2+|\tilde{F}|^2+|\tilde{G}|^2] +e^{-2\phi+(3/4)\rho}R_{(S^5)}\Big\}, \tag{1}$$

where $H_{MNK}=(1/2)\partial_{[M}B_{NK]}$, $F_{MNK}=(1/2)\partial_{[M}C_{NK]}$, $G_{K_1K_2K_3K_4K_5}=(1/4!)\partial_{[K_1}D_{K_2K_3K_4K_5]}$, $\tilde{F}=F+\chi H$, and $\tilde{G}=G+C\wedge H$. $|A_q|^2=(1/q!)A_{K_1\cdots K_q}A^{K_1\cdots K_q}$. $M,N=0,1,2,3,4$ and hereafter we set $2\kappa^2=1$. For example, see Ref. [5] for the derivation.

Recently, Sato and Tsuchiya derived the Born-Infeld action for a probe D-brane as a solution to the Hamilton-Jacobi equation [5]. Since the effective action is obtained via the transition amplitude from the vacuum to the boundary state representing the probe D3-brane, it could be a classical counterterm. The solution to the Hamilton-Jacobi equation is considered in the classical limit of the Wheeler–DeWitt equation.

In this paper we will consider the self-gravitating D3-brane, not the probe one. Our purpose is to get the action for the gravitating D-brane where we can discuss the cosmology correctly. For this purpose we first write down the full expression of the Hamilton-Jacobi equation:

$$-\frac{e^{2\phi-(5/4)\rho}}{(\sqrt{-q})^2}\Bigg[\left(\frac{\delta S}{\delta q_{\mu\nu}}\right)^2+\frac{1}{2}\left(\frac{\delta S}{\delta\phi}\right)^2+\frac{1}{2}q_{\mu\nu}\frac{\delta S}{\delta q_{\mu\nu}}\frac{\delta S}{\delta\phi}+\frac{4}{5}\left(\frac{\delta S}{\delta\rho}\right)^2+\frac{\delta S}{\delta\phi}\frac{\delta S}{\delta\rho}+\left(\frac{\delta S}{\delta B_{\mu\nu}}-\chi\frac{\delta S}{\delta C_{\mu\nu}}-6C_{\alpha\beta}\frac{\delta S}{\delta D_{\mu\nu\alpha\beta}}\right)^2\Bigg] -e^{-2\phi+(5/4)\rho}\Big[{}^{(4)}R+4D^2\phi-\frac{5}{2}D^2\rho-4(D\phi)^2-\frac{15}{8}(D\rho)^2+5D\phi D\rho-\frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha}\Big]-e^{-2\phi+(3/4)\rho}R_{(S^5)} -e^{(5/4)\rho}\Big[-\frac{1}{2}(D\chi)^2-\frac{1}{12}\tilde{F}_{\mu\nu\alpha}\tilde{F}^{\mu\nu\alpha}\Big]-\frac{e^{-(5/4)\rho}}{(\sqrt{-q})^2}\Bigg[\frac{1}{2}\left(\frac{\delta S}{\delta\chi}\right)^2+\left(\frac{\delta S}{\delta C_{\mu\nu}}\right)^2+12\left(\frac{\delta S}{\delta D_{\mu\nu\alpha\beta}}\right)^2\Bigg]=0, \tag{2}$$

where $q_{\mu\nu}$ and D_μ are the induced metric on the D3-brane and its covariant derivative. $\mu,\nu=0,1,2,3$.

In Ref. [5], all fields were supposed to be constant and then it was shown that the Born-Infeld action with the Wess-Zumino terms is a solution up to full orders of α':

$$S_{\rm BI}{}^{(0)}=\alpha\int d^4x\sqrt{-q}e^{-2\phi+\rho} +\beta\int d^4xe^{-\phi}\sqrt{-\det(q_{\mu\nu}+B_{\mu\nu})} +\gamma\left(\int D+\int C\wedge B+\frac{1}{2}\int\chi B\wedge B\right), \tag{3}$$

where $\alpha^2=5R_{(S^5)}$ and $\beta^2=\gamma^2$. In our paper, on the other hand, we will not assume that these fields are constant. To solve the Hamilton-Jacobi equation, we will employ the gradient expansion scheme in the next sections.

B. Solution to the Hamilton-Jacobi equation

Let us solve the Hamilton-Jacobi equation using the gradient expansion scheme. The expansion parameter is $\epsilon=\ell^2/L^2$, where ℓ and L are the bulk curvature scale and the typical gradient scale on the brane, respectively. The solution is expanded as

$$S=S_0+S_1+S_2+\cdots. \tag{4}$$

For example, S_1 is expected to contain a linear combination of ${}^{(4)}R$, $B_{\mu\nu}B^{\mu\nu}$, $(D\phi)^2$, and so on.

1. Zeroth order

In the zeroth order the Hamilton-Jacobi equation becomes

$$-\frac{e^{2\phi-(5/4)\rho}}{(\sqrt{-q})^2}\left[\frac{\delta S_0}{\delta q_{\mu\nu}}\frac{\delta S_0}{\delta q_{\alpha\beta}}q_{\mu\alpha}q_{\nu\beta}+\frac{1}{2}\left(\frac{\delta S_0}{\delta\phi}\right)^2+\frac{1}{2}q_{\mu\nu}\frac{\delta S_0}{\delta q_{\mu\nu}}\frac{\delta S_0}{\delta\phi}+\frac{4}{5}\left(\frac{\delta S_0}{\delta\rho}\right)^2+\frac{\delta S_0}{\delta\phi}\frac{\delta S_0}{\delta\rho}\right]-e^{-2\phi+(3/4)\rho}R_{(S^5)}-12\frac{e^{-(5/4)\rho}}{(\sqrt{-q})^2}\left(\frac{\delta S_0}{\delta D_{\mu\nu\alpha\beta}}\right)^2=0. \tag{5}$$

It is easy to see that the solution can be written as

$$S_0=\int d^4x\sqrt{-q}\left[\alpha_0e^{-2\phi+\rho}+\beta_0e^{-\phi}+\frac{\gamma_0}{24}\epsilon^{\mu\nu\alpha\beta}D_{\mu\nu\alpha\beta}\right]. \tag{6}$$

Substituting the above into Eq. (5), we have an equation for α_0, β_0, and γ_0:

$$\left[\frac{1}{5}\alpha_0^2-R_{(S^5)}\right]e^{-2\phi+(3/4)\rho}-\frac{1}{2}(\beta_0^2-\gamma_0^2)e^{-(5/4)\rho}=0, \tag{7}$$

from which we find

$$\alpha_0^2=5R_{(S^5)}\quad\text{and}\quad\beta_0^2=\gamma_0^2. \tag{8}$$

2. First order

In the first order the Hamilton-Jacobi equation becomes

$$-\frac{e^{2\phi-(5/4)\rho}}{(\sqrt{-q})^2}\left[2\frac{\delta S_0}{\delta q_{\mu\nu}}\frac{\delta S_1}{\delta q_{\alpha\beta}}q_{\mu\alpha}q_{\nu\beta}+\left(\frac{\delta S_0}{\delta\phi}+\frac{1}{2}\frac{\delta S_0}{\delta q_{\mu\nu}}q_{\mu\nu}+\frac{\delta S_0}{\delta\rho}\right)\frac{\delta S_1}{\delta\phi}+\frac{1}{2}\frac{\delta S_0}{\delta\phi}\frac{\delta S_1}{\delta q_{\mu\nu}}q_{\mu\nu}+\left(\frac{8}{5}\frac{\delta S_0}{\delta\rho}+\frac{\delta S_0}{\delta\phi}\right)\frac{\delta S_1}{\delta\rho}+\left(\frac{\delta S_1}{\delta B_{\mu\nu}}-\chi\frac{\delta S_1}{\delta C_{\mu\nu}}-6C_{\alpha\beta}\frac{\delta S_0}{\delta D_{\mu\nu\alpha\beta}}\right)^2\right]-e^{-2\phi+(5/4)\rho}\left[{}^{(4)}R+4D^2\phi-\frac{5}{2}D^2\rho-4(D\phi)^2-\frac{15}{8}(D\rho)^2+5D\phi D\rho-\frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha}\right]+e^{(5/4)\rho}\left[\frac{1}{2}(D\chi)^2+\frac{1}{12}\tilde{F}_{\mu\nu\alpha}\tilde{F}^{\mu\nu\alpha}\right]-e^{-(5/4)\rho}\left(\frac{1}{\sqrt{-q}}\frac{\delta S_1}{\delta C_{\mu\nu}}\right)^2=0. \tag{9}$$

For simplicity we set $H_{\mu\nu\alpha}=0$ and $\tilde{F}_{\mu\nu\alpha}=0$. Thus $B_{\mu\nu}$ and $C_{\mu\nu}$ are closed, and then written as the vector potentials. We will also set $C_{\mu\nu}=0$ at the end of the calculations. Using the solution of S_0, Eq. (9) becomes

$$-\frac{e^{2\phi-(5/4)\rho}}{\sqrt{-q}}\left[\beta_0e^{-\phi}\left(\frac{1}{2}q_{\mu\nu}\frac{\delta S_1}{\delta q_{\mu\nu}}-\frac{\delta S_1}{\delta\rho}\right)-\frac{2}{5}\alpha_0e^{-2\phi+\rho}\frac{\delta S_1}{\delta\rho}+\frac{1}{\sqrt{-q}}\left(\frac{\delta S_1}{\delta B_{\mu\nu}}-\chi\frac{\delta S_1}{\delta C_{\mu\nu}}-6C_{\alpha\beta}\frac{\delta S_0}{\delta D_{\mu\nu\alpha\beta}}\right)^2\right]-e^{-2\phi+(5/4)\rho}\left[{}^{(4)}R+4D^2\phi-\frac{5}{2}D^2\rho-4(D\phi)^2-\frac{15}{8}(D\rho)^2+5D\phi D\rho\right]+\frac{1}{2}e^{(5/4)\rho}(D\chi)^2-e^{-(5/4)\rho}\left(\frac{1}{\sqrt{-q}}\frac{\delta S_1}{\delta C_{\mu\nu}}\right)^2=0. \tag{10}$$

Here, remember that the AdS/CFT correspondence will hold in the limit of

$$\alpha_0\to0, \tag{11}$$

that is, the AdS and S^5 curvature radii are much longer than the string length. In this limit we can see that the solution for S_1 is given by

$$S_1=\frac{1}{\beta_0}\int d^4x\sqrt{-q}e^{-3\phi+(5/2)\rho}\left[\frac{1}{2}{}^{(4)}R+4(D\phi)^2+\frac{35}{16}(D\rho)^2-\frac{25}{4}D\phi D\rho\right]-\frac{1}{4\beta_0}\int d^4x\sqrt{-q}e^{-\phi+(5/2)\rho}(D\chi)^2+\frac{\gamma_0}{4}\int d^4x\sqrt{-q}e^{-\phi}B_{\mu\nu}B^{\mu\nu}+\frac{\gamma_0}{4}\int d^4x\sqrt{-q}\epsilon^{\mu\nu\alpha\beta}\left[B_{\mu\nu}C_{\alpha\beta}+\frac{\chi}{2}B_{\mu\nu}B_{\alpha\beta}\right]. \tag{12}$$

Hereafter we will consider the limit of $\alpha_0=0$ and $R_{(S^5)}=0$.

3. Second order

Next we consider the second order. The Hamilton-Jacobi equation is

$$\frac{1}{\sqrt{-q}}\left[\frac{1}{2}q_{\mu\nu}\frac{\delta S_2}{\delta q_{\mu\nu}}-\frac{\delta S_2}{\delta\rho}+B_{\mu\nu}\frac{\delta S_2}{\delta B_{\mu\nu}}\right]=-\frac{e^{\phi}}{\beta_0(\sqrt{-q})^2}\left[\frac{\delta S_1}{\delta q_{\mu\nu}}\frac{\delta S_1}{\delta q_{\alpha\beta}}q_{\mu\alpha}q_{\nu\beta}+\frac{1}{2}\left(\frac{\delta S_1}{\delta\phi}\right)^2+\frac{1}{2}q_{\mu\nu}\frac{\delta S_1}{\delta q_{\mu\nu}}\frac{\delta S_1}{\delta\phi}+\frac{4}{5}\left(\frac{\delta S_1}{\delta\rho}\right)^2+\frac{\delta S_1}{\delta\phi}\frac{\delta S_1}{\delta\rho}\right]-\frac{e^{-\phi}}{2\beta_0}\left(\frac{1}{\sqrt{-q}}\frac{\delta S_1}{\delta\chi}\right)^2$$
$$=-\frac{e^{-5\phi+5\rho}}{4\beta_0^3}\left({}^{(4)}R_{\mu\nu}{}^{(4)}R^{\mu\nu}+\frac{1}{2}{}^{(4)}R^2\right)+\frac{1}{2\beta_0}e^{-3\phi+(5/2)\rho}\left[{}^{(4)}R^{\mu\nu}(B^2)_{\mu\nu}-\frac{1}{4}{}^{(4)}R\,\mathrm{Tr}(B^2)\right]-\frac{3\beta_0e^{-\phi}}{8}\left[\mathrm{Tr}(B^4)-\frac{1}{4}[\mathrm{Tr}(B^2)]^2\right]+\cdots, \tag{13}$$

where $(B^2)_{\mu\nu}=B_\mu{}^\alpha B_{\alpha\nu}$ and $\mathrm{Tr}(B^2)=B_{\mu\nu}B^{\nu\mu}$.

As we shall see soon, ${}^{(4)}R_{\mu\nu}=O(B^2)$ and ${}^{(4)}R=O(B^4)$ will hold. Bearing this in mind, S_2 can be evaluated as

$$S_2=\frac{1}{20\beta_0^3}\int d^4x\sqrt{-q}e^{-5\phi+5\rho}\,{}^{(4)}R_{\mu\nu}{}^{(4)}R^{\mu\nu}+\frac{1}{2\beta_0}\int d^4x\sqrt{-q}e^{-3\phi+(5/2)\rho}\,{}^{(4)}R^{\mu\nu}\overset{(1)}{T}_{\mu\nu}-\frac{\beta_0}{8}\int d^4x\sqrt{-q}e^{-\phi}\left[\mathrm{Tr}(B^4)-\frac{1}{4}[\mathrm{Tr}(B^2)]^2\right], \tag{14}$$

where we set $\beta_0=\gamma_0$ so that the Born-Infeld action is realized for the flat D-brane with the constant field $B_{\mu\nu}$. We also defined

$$\overset{(1)}{T}_{\mu\nu}=-(B^2)_{\mu\nu}+\frac{1}{4}q_{\mu\nu}\mathrm{Tr}(B^2). \tag{15}$$

4. Summary

The total solution to the Hamilton-Jacobi equation is summarized by

$$S_{\mathrm{ct}}=-(S_0+S_1+S_2+\cdots)=-(\tilde{S}_{\mathrm{BI}}+\tilde{S}_{\mathrm{EH}}+\tilde{S}_{\mathrm{WZ}}+\tilde{S}_2), \tag{16}$$

where

$$\tilde{S}_{\mathrm{BI}}=\beta_0\int d^4x\sqrt{-q}e^{-\phi}\left\{1+\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\frac{1}{8}\left[\mathrm{Tr}(B^4)-\frac{1}{4}[\mathrm{Tr}(B^2)]^2\right]\right\}, \tag{17}$$

$$\tilde{S}_{\mathrm{EH}}=\frac{1}{\beta_0}\int d^4x\sqrt{-q}\left\{e^{-3\phi+(5/2)\rho}\left[\frac{1}{2}{}^{(4)}R+4(D\phi)^2+\frac{35}{16}(D\rho)^2-\frac{25}{4}D\phi D\rho\right]-\frac{1}{4}e^{-\phi+(5/2)\rho}(D\chi)^2\right\}, \tag{18}$$

$$\tilde{S}_{\mathrm{WZ}}=\beta_0\int d^4x\sqrt{-q}\epsilon^{\mu\nu\alpha\beta}\left[\frac{1}{24}D_{\mu\nu\alpha\beta}+\frac{1}{4}B_{\mu\nu}C_{\alpha\beta}+\frac{\chi}{8}B_{\mu\nu}B_{\alpha\beta}\right], \tag{19}$$

and

$$\tilde{S}_2=\frac{1}{20\beta_0^3}\int d^4x\sqrt{-q}e^{-5\phi+5\rho}\,{}^{(4)}R_{\mu\nu}{}^{(4)}R^{\mu\nu}+\frac{1}{2\beta_0}\int d^4x\sqrt{-q}e^{-3\phi+(5/2)\rho}\,{}^{(4)}R^{\mu\nu}\overset{(1)}{T}_{\mu\nu}+\cdots. \tag{20}$$

Note that in the flat and constant field limit $\tilde{S}_{\mathrm{BI}}=S_{\mathrm{BI}}$ up to the current order. It is also noted that there is nontrivial coupling ${}^{(4)}R^{\mu\nu}\overset{(1)}{T}_{\mu\nu}$, and so on.

C. Effective equation on the D-brane

1. Strategy

In the braneworld the AdS/CFT correspondence may be formulated by the following partition functional argument [7]:

$$Z=\int\mathcal{D}g\,e^{iS_{\mathrm{bulk}}(g)+i(1/2)S_{\text{D-brane}}(q)+iS_{\mathrm{GH}}(q)}=\int\mathcal{D}q\,e^{(i/2)S_{\text{D-brane}}+iS_{\mathrm{ct}}}\langle e^{i\int d^4x q_{\mu\nu}T^{\mu\nu}}\rangle_{\mathrm{CFT}}, \tag{21}$$

where $S_{\rm ct}$ represents the counterterms which make the action finite and is given by the on-shell solution of the Hamilton-Jacobi equation, $S_{\rm ct}=-(S_0+S_1+S_2+\cdots)$. The variational principle implies

$$\frac{1}{\sqrt{-q}}\frac{\delta S_{\text{D-brane}}}{\delta q_{\mu\nu}}+2\frac{1}{\sqrt{-q}}\left(\frac{\delta S_{\rm ct}}{\delta q_{\mu\nu}}+\frac{\delta\Gamma_{\rm CFT}}{\delta q_{\mu\nu}}\right)=0. \tag{22}$$

To fix the first term we must specify the D-brane (cutoff brane) action. In this paper we consider two types of branes.

2. Born-Infeld membrane

First let us examine the case where the brane action is the Born-Infeld type:

$$S_{\text{D-brane}}=\beta\int d^4x e^{-\phi}\sqrt{-\det(q_{\mu\nu}+B_{\mu\nu})}. \tag{23}$$

The energy-momentum tensor of this brane becomes

$$T_{{\rm BI}\mu\nu}=\beta e^{-\phi}q_{\mu\nu}-\beta e^{-\phi}\overset{(1)}{T}_{\mu\nu}+\overset{(2)}{T}_{{\rm BI}\mu\nu} \tag{24}$$

and

$$\overset{(2)}{T}_{{\rm BI}\mu\nu}=\beta e^{-\phi}\left\{-\frac{1}{4}{\rm Tr}(B^2)\left[(B^2)_{\mu\nu}-\frac{1}{8}q_{\mu\nu}{\rm Tr}(B^2)\right]+(B^4)_{\mu\nu}-\frac{1}{8}q_{\mu\nu}{\rm Tr}(B^4)\right\}. \tag{25}$$

Substituting Eq. (23) into Eq. (22), we can obtain the effective gravitational equation on the brane. At that time we set

$$\beta=2\beta_0, \tag{26}$$

so that the brane geometry could be four-dimensional Minkowski spacetime.

In the first order the effective equation becomes just the vacuum one:

$$\begin{aligned}{}^{(4)}G_{\mu\nu}&=\beta_0 e^{3\phi-(5/2)\rho}\left(-\frac{1}{2}\overset{(2)}{T}_{{\rm BI}\mu\nu}+2\frac{1}{\sqrt{-q}}\frac{\delta S_2}{\delta q_{\alpha\beta}}q_{\mu\alpha}q_{\nu\beta}\right)+T^{\rm CFT}_{\mu\nu}\\
&=-3(D_\mu D_\nu-q_{\mu\nu}D^2)\phi+\frac{5}{2}(D_\mu D_\nu-q_{\mu\nu}D^2)\rho+[D_\mu\phi D_\nu\phi-5q_{\mu\nu}(D\phi)^2]\\
&\quad+\frac{15}{8}\left[D_\mu\rho D_\nu\rho-\frac{13}{6}q_{\mu\nu}(D\rho)^2\right]+\frac{1}{2}e^{2\phi}\left[D_\mu\chi D_\nu\chi-\frac{1}{2}q_{\mu\nu}(D\chi)^2\right]\\
&\quad-\frac{5}{4}(D_\mu\rho D_\nu\phi+D_\mu\phi D_\nu\rho-7q_{\mu\nu}D\phi D\rho)+T^{\rm CFT}_{\mu\nu}+\cdots.\end{aligned} \tag{27}$$

We may naively expect that Einstein-Maxwell theory governs the physics on the D-brane described by the Born-Infeld action. However, the result is not the case. Since $\tilde{S}_{\rm BI}$ is the same as $S_{\rm BI}$ up to the order of $(B^4)_{\mu\nu}$, the first order Einstein equation does not have the source of the Maxwell field while the contribution from holographic CFT exists.

As a result, the gravitational equation up to the second order is given by

$$\begin{aligned}{}^{(4)}G_{\mu\nu}&=T^{\rm CFT}_{\mu\nu}-3(D_\mu D_\nu-q_{\mu\nu}D^2)\phi+\frac{5}{2}(D_\mu D_\nu-q_{\mu\nu}D^2)\rho+[D_\mu\phi D_\nu\phi-5q_{\mu\nu}(D\phi)^2]+\frac{15}{8}\left[D_\mu\rho D_\nu\rho-\frac{13}{6}g_{\mu\nu}(D\rho)^2\right]\\
&\quad+\frac{1}{4}e^{2\phi}\left[D_\mu\chi D_\nu\chi-\frac{1}{2}q_{\mu\nu}(D\chi)^2\right]-\frac{5}{4}(D_\mu\rho D_\nu\phi+D_\mu\phi D_\nu\rho-7q_{\mu\nu}D\phi D\rho)-\frac{1}{5\beta_0^2}e^{-2\phi+(5/2)\rho}\Big({}^{(4)}R^\alpha_\mu{}^{(4)}R_{\alpha\nu}\\
&\quad-\frac{1}{4}q_{\mu\nu}{}^{(4)}R_{\alpha\beta}R^{\alpha\beta}\Big)-\frac{1}{10\beta_0^2}e^{-2\phi+(5/2)\rho}D^2\,{}^{(4)}R_{\mu\nu}+{}^{(4)}R_{\mu\alpha}(B^2)^\alpha{}_\nu+{}^{(4)}R_{\nu\alpha}(B^2)^\alpha{}_\mu-\frac{1}{4}{}^{(4)}R_{\mu\nu}{\rm Tr}(B^2)\\
&\quad+{}^{(4)}R^{\alpha\beta}B_{\mu\alpha}B_{\beta\nu}-\frac{1}{2}q_{\mu\nu}{}^{(4)}R_{\alpha\beta}(B^2)^{\alpha\beta}-\frac{1}{2}D^2\overset{(1)}{T}_{\mu\nu}.\end{aligned} \tag{28}$$

In the above we have dropped the second order terms which couple to the scalar fields to keep the form compact.

3. Nambu-Goto membrane

For the comparison, it might be worth considering the brane described by the Nambu-Goto action

$$S_{\rm NG}=2\beta_0\int d^4x\sqrt{-q}e^{-\phi}. \tag{29}$$

At the first order, the effective equation becomes

$$ {}^{(4)}G_{\mu\nu}=\beta_0^2 e^{2\phi-(5/2)\rho}\overset{(1)}{T}_{\mu\nu}-3(D_\mu D_\nu-q_{\mu\nu}D^2)\phi+\frac{5}{2}(D_\mu D_\nu-q_{\mu\nu}D^2)\rho+[D_\mu\phi D_\nu\phi-5q_{\mu\nu}(D\phi)^2]+\frac{15}{8}\left[D_\mu\rho D_\nu\rho-\frac{13}{6}q_{\mu\nu}(D\rho)^2\right]+\frac{1}{4}e^{2\phi}\left[D_\mu\chi D_\nu\chi-\frac{1}{2}g_{\mu\nu}(D\chi)^2\right]-\frac{5}{4}(D_\mu\rho D_\nu\phi+D_\mu\phi D_\nu\rho-7q_{\mu\nu}D_\alpha\phi D^\alpha\rho)+T^{\rm CFT}_{\mu\nu}+\cdots. \tag{30} $$

Cancellation does not occur and Einstein-Maxwell-scalar theory is realized on the brane. This is also an unexpected result.

III. GEOMETRICAL APPROACH

In the previous section, we saw unexpected results for the effective theory on the brane. It seems that they originate from the fact that the Born-Infeld action is a solution to the Hamilton-Jacobi equation. In order to understand why we obtain such consequences, we will rederive the gravitational equation on the D-brane using the geometrical method [12,15] in this section. To do so we will solve the bulk spacetime in the long wave approximation and then obtain the effective theory in the low energy limit. We will use slightly different notation from the previous sections.

The rest of this section is organized as follows. In Sec. III A, we give a formulation of the geometrical approach and stress that we must solve the bulk fields and gravity somehow. Then we solve them in the long wave approximation up to leading order for the gravitational theory on the brane. Finally, we derive the gravitational equation on the D-branes described by the Born-Infeld action in Sec. III B.

A. Formulation

The full metric is written as

$$ ds^2=e^{2\varphi(x)}dy^2+q_{\mu\nu}(y,x)dx^\mu dx^\nu. \tag{31} $$

The induced metric on the brane is $h_{\mu\nu}(x)=q_{\mu\nu}(y_0,x)$, where we suppose that the brane is located at $y=y_0$.

In the geometrical approach, the gravitational equation on the brane is given by

$$ {}^{(4)}G_{\mu\nu}(h)=\frac{2}{3}\left[T_{\mu\nu}+h_{\mu\nu}\left(T_{yy}-\frac{1}{4}T\right)\right]+KK_{\mu\nu}-K_\mu^{\ \alpha}K_{\nu\alpha}-\frac{1}{2}(K^2-K_{\alpha\beta}K^{\alpha\beta})h_{\mu\nu}-E_{\mu\nu}, \tag{32} $$

where

$$ T_{MN}=-2(\nabla_M\nabla_N-g_{MN}\nabla^2)\phi+\frac{5}{4}(\nabla_M\nabla_N-g_{MN}\nabla^2)\rho+\frac{1}{2}e^{2\phi}\left[\nabla_M\chi\nabla_N\chi-\frac{1}{2}g_{MN}(\nabla\chi)^2\right]+\frac{5}{16}[\nabla_M\rho\nabla_N\rho-3g_{MN}(\nabla\rho)^2]-2g_{MN}(\nabla\phi)^2+\frac{5}{2}g_{MN}\nabla_K\phi\nabla^K\rho+\frac{1}{4}(H_{MKL}H_N^{\ \ KL}-g_{MN}|H|^2)+\frac{1}{4}e^{2\phi}(\tilde F_{MKL}\tilde F_N^{\ \ KL}-g_{MN}|\tilde F|^2)+\frac{1}{96}e^{2\phi}\tilde G_{MK_1K_2K_3K_4}\tilde G_N^{\ \ K_1K_2K_3K_4}, \tag{33} $$

and thus

$$ T_{\mu\nu}+h_{\mu\nu}\left(T_{yy}-\frac{1}{4}T\right)=-2(D_\mu D_\nu\phi-h_{\mu\nu}D^2\phi)+\frac{5}{4}(D_\mu D_\nu\rho-h_{\mu\nu}D^2\rho)+\frac{1}{2}e^{2\phi}\left[D_\mu\chi D_\nu\chi-\frac{5}{8}h_{\mu\nu}(D\chi)^2\right]+\frac{5}{16}\left[D_\mu\rho D_\nu\rho-\frac{5}{2}h_{\mu\nu}(D\rho)^2\right]-\frac{3}{2}h_{\mu\nu}(D\phi)^2+\frac{15}{8}h_{\mu\nu}D_\alpha\phi D^\alpha\rho-2(K_{\mu\nu}-h_{\mu\nu}K)\partial_y\phi+\frac{5}{4}(K_{\mu\nu}-h_{\mu\nu}K)\partial_y\rho+\frac{3}{16}e^{2\phi}h_{\mu\nu}(\partial_y\chi)^2-\frac{15}{32}h_{\mu\nu}(\partial_y\rho)^2-\frac{3}{2}h_{\mu\nu}(\partial_y\phi)^2+\frac{15}{8}h_{\mu\nu}\partial_y\phi\partial_y\rho+\frac{1}{2}\left(H_{y\mu\alpha}H_{y\nu}^{\ \ \ \alpha}-\frac{1}{16}h_{\mu\nu}H_{y\alpha\beta}H^{y\alpha\beta}\right)+\frac{1}{2}e^{2\phi}\left(\tilde F_{y\mu\alpha}\tilde F_{y\nu}^{\ \ \ \alpha}-\frac{1}{16}h_{\mu\nu}\tilde F_{y\alpha\beta}\tilde F^{y\alpha\beta}\right)+\frac{1}{24}e^{2\phi}\left(\tilde G_{\mu y\alpha_1\alpha_2\alpha_3}\tilde G_{\nu y}^{\ \ \ \alpha_1\alpha_2\alpha_3}-\frac{1}{16}h_{\mu\nu}\tilde G_{y\alpha_1\alpha_2\alpha_3\alpha_4}\tilde G^{y\alpha_1\alpha_2\alpha_3\alpha_4}\right). \tag{34} $$

$E_{\mu\nu}$ is the projected five-dimensional Weyl tensor defined by $E_{\mu\nu}={}^{(5)}C_{\mu M\nu N}n^M n^N$. It is obvious that the above equation is not closed in four dimensions. Moreover, when bulk fields exist, $E_{\mu\nu}$ is not negligible in the low energy limit [9,16,17]. Since the Born-Infeld action appears as a solution to the Hamilton-Jacobi equation, we guess that $E_{\mu\nu}$ contains a part of the Born-Infeld energy-momentum tensor.

As in the previous section, for simplicity, we turn off almost fields except the scalar fields $B_{\mu\nu}$ and $\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4}$.

To obtain a background solution that is consistent with the junction condition, we assume that the action for the brane is given by

$$S_{\rm brane}=2\beta\int d^4x e^{-\phi}\sqrt{-\det(h+B)}+2\beta\int d^4x\sqrt{-h}\epsilon^{\mu\nu\alpha\beta}\left[\frac{1}{4}B_{\mu\nu}C_{\alpha\beta}+\frac{\chi}{8}B_{\mu\nu}B_{\alpha\beta}+\frac{1}{24}D_{\mu\nu\alpha\beta}\right]. \tag{35}$$

The boundary conditions on the brane are brought about by the junction conditions:

$$\left[(K_{\mu\nu}-h_{\mu\nu}K)e^{\varphi}+\left(2\partial_y\phi-\frac{5}{4}\partial_y\rho\right)h_{\mu\nu}\right](y_0,x)=-\frac{1}{4}e^{\varphi+2\phi-(5/4)\rho}T^{\rm BI}_{\mu\nu}, \tag{36}$$

$$[4Ke^{\varphi}-8\partial_y\phi+5\partial_y\rho](y_0,x)=\beta e^{\varphi+\phi-(5/4)\rho}\left[1-\frac{1}{4}{\rm Tr}(B^2)+\frac{1}{32}[{\rm Tr}(B^2)]^2-\frac{1}{8}{\rm Tr}(B^4)+O(B^6)\right], \tag{37}$$

$$[-Ke^{\varphi}-\partial_y\rho+2\partial_y\phi](y_0,x)=0, \tag{38}$$

$$\partial_y\chi(y_0,x)=-\frac{1}{8}\beta e^{\varphi-(5/4)\rho}\epsilon^{\mu\nu\alpha\beta}B_{\mu\nu}B_{\alpha\beta}, \tag{39}$$

$$H_{y\mu\nu}(y_0,x)=-\beta e^{\varphi+\phi-(5/4)\rho}\left[B_{\mu\nu}-\frac{1}{4}{\rm Tr}(B^2)B_{\mu\nu}+(B^3)_{\mu\nu}+O(B^5)\right], \tag{40}$$

$$\tilde{F}_{y\mu\nu}(y_0,x)=-\frac{\beta}{2}e^{\varphi-(5/4)\rho}\epsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}, \tag{41}$$

and

$$\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4}(y_0,x)=-\beta e^{\varphi-(5/4)\rho}\epsilon_{\mu_1\mu_2\mu_3\mu_4}, \tag{42}$$

where

$$T^{\rm BI}_{\mu\nu}=2\beta e^{-\phi}\left\{h_{\mu\nu}-\overset{(1)}{T}_{\mu\nu}-\frac{1}{4}{\rm Tr}(B^2)\left[(B^2)_{\mu\nu}-\frac{1}{8}h_{\mu\nu}{\rm Tr}(B^2)\right]+(B^4)_{\mu\nu}-\frac{1}{8}h_{\mu\nu}{\rm Tr}(B^4)+O(B^6)\right\}. \tag{43}$$

Using these junction conditions, Eq. (32) becomes

$${}^{(4)}G_{\mu\nu}(h)=\frac{5}{6}\beta^2e^{2\phi-(5/2)\rho}\overset{(1)}{T}_{\mu\nu}-\frac{4}{3}(D_\mu D_\nu-h_{\mu\nu}D^2)\phi+\frac{5}{6}(D_\mu D_\nu-h_{\mu\nu}D^2)\rho+\frac{1}{3}e^{2\phi}\left[D_\mu\chi D_\nu\chi-\frac{5}{8}h_{\mu\nu}(D\chi)^2\right]+\frac{5}{24}\left[D_\mu\rho D_\nu\rho-\frac{5}{2}h_{\mu\nu}(D\rho)^2\right]-h_{\mu\nu}(D\phi)^2+\frac{5}{4}h_{\mu\nu}D_\alpha\phi D^\alpha\rho-E_{\mu\nu}+O(B^4). \tag{44}$$

The bulk "evolutional" equations are

$$\pounds_n K={}^{(4)}R-\left(T^\mu{}_\mu-\frac{4}{3}T\right)-K^2-D^2\varphi-(D\varphi)^2, \tag{45}$$

$$\pounds_n\tilde{K}^\mu{}_\nu={}^{(4)}\tilde{R}^\mu{}_\nu-(D^\mu D_\nu\varphi+D^\mu\varphi D_\nu\varphi)_{\rm traceless}-\left(T^\mu{}_\nu-\frac{1}{4}\delta^\mu{}_\nu T^\alpha{}_\alpha\right)-K\tilde{K}^\mu{}_\nu, \tag{46}$$

$$4\partial_y^2\phi-\frac{5}{2}\partial_y^2\rho-8(\partial_y\phi)^2-\frac{25}{8}(\partial_y\rho)^2+10\partial_y\phi\partial_y\rho-\frac{3}{2}e^{2\phi}(\partial_y\chi)^2+e^{\varphi}K\left(4\partial_y\phi-\frac{5}{2}\partial_y\rho\right)+\left[\frac{5}{2}e^{2\phi}|\tilde{G}|^2-D_\mu\varphi D^\mu\left(4\phi-\frac{5}{3}\rho\right)+4D^2\phi-\frac{5}{2}D^2\rho-8(D\phi)^2-\frac{25}{8}(D\rho)^2+10D_\alpha\phi D^\alpha\rho-\frac{3}{2}e^{2\phi}(D\chi)^2+2|H|^2+\frac{1}{2}e^{2\phi}|\tilde{F}|^2\right]e^{2\varphi}=0, \tag{47}$$

$$-\partial_y^2\phi+\partial_y^2\rho+2(\partial_y\phi)^2+\frac{5}{4}(\partial_y\rho)^2-\frac{13}{4}\partial_y\phi\partial_y\rho-e^{\varphi}K(\partial_y\phi-\partial_y\rho)+\Big[-e^{2\phi}|\tilde{G}|^2+D_\mu\varphi D^\mu(\phi-\rho)-D^2\phi+D^2\rho+2(D\phi)^2+\frac{5}{4}(D\rho)^2-\frac{13}{4}D_\alpha\phi D^\alpha\rho-\frac{1}{2}|H|^2-\frac{1}{2}e^{2\phi}|\tilde{F}|^2\Big]e^{2\varphi}=0, \tag{48}$$

$$\partial_y(e^{(5/4)\rho}\partial_y\chi)+D_\alpha(e^{(5/4)\rho}D^\alpha\chi)+e^{(5/4)\rho}e^{\varphi}K\partial_y\chi-e^{2\varphi+(5/4)\rho}D_\mu\chi D^\mu\varphi=0, \tag{49}$$

$$\partial_y(e^{-2\phi+(5/4)\rho}H^{y\mu\nu}+e^{(5/4)\rho}\chi\tilde{F}^{y\mu\nu})+e^{\varphi}K(e^{-2\phi+(5/4)\rho}H^{y\mu\nu}+e^{(5/4)\rho}\tilde{F}^{y\mu\nu})+\frac{1}{2}e^{(5/4)\rho}F_{y\alpha\beta}\tilde{G}^{\mu\nu y\alpha\beta}=0, \tag{50}$$

$$\partial_y(e^{(5/4)\rho}\tilde{F}^{y\mu\nu})+e^{\varphi+(5/4)\rho}K\tilde{F}^{y\mu\nu}-\frac{1}{2}e^{(5/4)\rho}H_{y\alpha\beta}\tilde{G}^{\mu\nu y\alpha\beta}=0, \tag{51}$$

and

$$\partial_y(e^{(5/4)\rho}\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4})=Ke^{(5/4)\rho}\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4}. \tag{52}$$

The Hamiltonian and momentum constraints are

$$-\frac{1}{2}\Big[{}^{(4)}R-\frac{3}{4}K^2+\tilde{K}^\alpha{}_\beta\tilde{K}^\beta{}_\alpha\Big]=T_{yy}e^{-2\varphi}, \tag{53}$$

and

$$D_\nu K^\nu{}_\mu-D_\mu K=T_{\mu y}e^{-\varphi}, \tag{54}$$

respectively. The constraints for $H_{y\mu\nu}$, $\tilde{F}_{y\mu\nu}$, and $\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4}$ are

$$D^\alpha(e^{-\varphi-2\phi+(5/4)\rho}H_{y\alpha\mu}+e^{-\varphi+(5/4)\rho}\chi\tilde{F}_{y\alpha\mu})=0, \tag{55}$$

$$D^\alpha(e^{\varphi+(5/4)\rho}\tilde{F}_{y\alpha\mu})=0, \tag{56}$$

and

$$D^\alpha(e^{-\varphi+(5/4)\rho}\tilde{G}_{y\alpha\mu_1\mu_2\mu_3})=0. \tag{57}$$

B. Solving of the bulk and effective theory

Let us solve the bulk equations (45)–(52) with the junction conditions (36)–(42) in the long wave approximation. The infinitesimal parameter of the expansion is $\epsilon=(\ell/L)^2$, where ℓ is the bulk curvature scale and L is the typical scale on the brane.

1. Zeroth order

In the zeroth order the evolutional equations are

$$e^{-\varphi}\partial_y\overset{(0)}{\tilde{K}}{}^\mu{}_\nu=-\overset{(0)}{K}\overset{(0)}{\tilde{K}}{}^\mu{}_\nu, \tag{58}$$

$$e^{-\varphi}\partial_y\overset{(0)}{K}=-\Big(T^\mu{}_\mu-\frac{4}{3}T\Big)^{(0)}-\overset{(0)}{K}{}^2, \tag{59}$$

$$4\partial_y^2\phi_0-\frac{5}{2}\partial_y^2\rho_0+e^{\varphi}\overset{(0)}{K}\Big(4\partial_y\phi_0-\frac{5}{2}\partial_y\rho_0\Big)-8(\partial_y\phi_0)^2-\frac{25}{8}(\partial_y\rho_0)^2+10\partial_y\phi_0\partial_y\rho_0+\frac{5}{2}e^{2\varphi+2\phi_0}|\tilde{G}|^2=0, \tag{60}$$

$$-\partial_y^2\phi_0+\partial_y^2\rho_0+e^{\varphi}\overset{(0)}{K}(-\partial_y\phi_0+\partial_y\rho_0)+2(\partial_y\phi_0)^2+\frac{5}{4}(\partial_y\rho_0)^2-\frac{13}{4}\partial_y\phi_0\partial_y\rho_0-e^{2\varphi+2\phi_0}|\tilde{G}|^2=0, \tag{61}$$

and

$$\partial_y(e^{(5/4)\rho_0}\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4})=\overset{(0)}{K}e^{(5/4)\rho_0}\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4}. \tag{62}$$

The constraint equations are

$$D^{\mu_1}(e^{-\varphi+(5/4)\rho_0}\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4})=0. \tag{63}$$

The junction conditions are

$$\overset{(0)}{\tilde{K}}{}^\mu{}_\nu(y_0,x)=0,\quad \overset{(0)}{K}(y_0,x)=-\beta e^{\phi_0-(5/4)\rho_0},$$
$$\partial_y\phi_0(y_0,x)=0,\quad \partial_y\rho_0(y_0,x)=\beta e^{\varphi+\phi_0-(5/4)\rho_0}, \tag{64}$$

and

$$\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4}(y_0,x)=-\beta e^{\varphi-(5/4)\rho_0}\epsilon_{\mu_1\mu_2\mu_3\mu_4}. \tag{65}$$

The background solution is easily found as [21]

$$\phi_0=\phi_0(x),\quad \rho_0=\frac{4}{5}\log(y/y_0)+\sigma_0(x),\quad \chi_0=\chi_0(x),$$

$$\overset{(0)}{C}{}_{\mu\nu}=\overset{(0)}{B}{}_{\mu\nu}=0,$$

and

$$\tilde{G}_{y\mu_1\mu_2\mu_3\mu_4}=-\beta e^{\varphi-(5/4)\rho_0}\epsilon_{\mu_1\mu_2\mu_3\mu_4}, \tag{66}$$

where

$$\varphi(x)=-\phi_0(x)+\frac{5}{4}\sigma_0(x)+\log(4/5y_0\beta). \quad (67)$$

The extrinsic curvature is given by

$$\overset{(0)}{K}{}^{\mu}{}_{\nu}=-\frac{1}{5y}e^{-\varphi(x)}\delta^{\mu}{}_{\nu}, \quad (68)$$

and then the metric becomes

$$\overset{(0)}{g}{}_{\mu\nu}=a^2(y)h_{\mu\nu}(x), \quad (69)$$

where

$$a(y)=\left(\frac{y}{y_0}\right)^{-1/5}. \quad (70)$$

The behavior of the background metric brings us a serious problem, that is, the four-dimensional gravity cannot be recovered on the brane. The minimum way to see this is the dimensional reduction from five to four dimensions:

$$\int d^5x\sqrt{-g}\,{}^{(5)}R\sim\int dy a^2(y)\int d^4x\sqrt{-h}\,{}^{(4)}R(h). \quad (71)$$

In the above $\int dy a^2(y)=\infty$ when we consider the infinite extra dimensions, which implies that the four-dimensional gravity cannot be recovered. This problem might be regarded as a sort of no-go theorem proposed by Maldacena [20]. The simple resolution to this problem is compactification and/or introduction of another brane. There may be another possibility that the bulk or brane action is modified via some quantum effects. We leave this issue for future study.

Note that adding a Wess-Zumino term $\int D$ in the brane action is essential to obtain solutions. There is no solution with a similar form when brane is supposed to be described only by the Nambu-Goto action. This fact is consistent with the result that the Nambu-Goto action alone cannot satisfy the Hamilton-Jacobi equation.

2. First order

Next we turn to the first order equations. The junction conditions become

$$\overset{(1)}{K}(y_0,x)=-\frac{1}{4}\beta e^{\phi_0-(5/4)\rho_0}\mathrm{Tr}(B^2), \quad (72)$$

$$\overset{(1)}{\tilde{K}}{}^{\mu}{}_{\nu}(y_0,x)=\frac{1}{2}\beta e^{\phi_0-(5/4)\rho_0}\overset{(1)}{T}{}^{\mu}{}_{\nu}, \quad (73)$$

$$\partial_y\phi_1(y_0,x)=-\frac{1}{4}\beta e^{\varphi+\phi_0-(5/4)\rho_0}\mathrm{Tr}(B^2), \quad (74)$$

and

$$\partial_y\rho_1(y_0,x)=-\frac{1}{4}\beta e^{\varphi+\phi_0-(5/4)\rho_0}\mathrm{Tr}(B^2). \quad (75)$$

For the gravitational equation on the brane, the key equations are the evolutional equation for the traceless part of the extrinsic curvature and the Hamiltonian constraint:

$$\partial_y\overset{(1)}{\tilde{K}}{}^{\mu}{}_{\nu}={}^{(4)}\tilde{R}^{\mu}{}_{\nu}-(D^{\mu}D_{\nu}\varphi+D^{\mu}\varphi D_{\nu}\varphi)_{\mathrm{traceless}} -\left(T^{\mu}{}_{\nu}-\frac{1}{4}\delta^{\mu}{}_{\nu}T^{\alpha}{}_{\alpha}\right)^{(1)}-\overset{(0)}{K}\overset{(1)}{\tilde{K}}{}^{\mu}{}_{\nu}, \quad (76)$$

and

$$-\frac{1}{2}{}^{(4)}R+\frac{3}{4}\overset{(0)}{K}\overset{(1)}{K}=\overset{(1)}{T}_{yy}\,e^{-2\varphi}. \quad (77)$$

Since the right-hand side in Eq. (76) contains $\tilde{F}_{y\mu\nu}$ and $H_{y\mu\nu}$, we also need to solve their bulk equations

$$\partial_yX^{\mu\nu}+e^{\varphi}\overset{(0)}{K}X^{\mu\nu}+\frac{1}{2}e^{(5/4)\rho_0}F_{y\alpha\beta}\tilde{G}^{y\alpha\beta\mu\nu}=0 \quad (78)$$

and

$$\partial_y(e^{(5/4)\rho_0}\tilde{F}^{y\mu\nu})+e^{\varphi+(5/4)\rho_0}\overset{(0)}{K}\tilde{F}^{y\mu\nu}-\frac{1}{2}e^{(5/4)\rho_0}H_{y\alpha\beta}\tilde{G}^{y\alpha\beta\mu\nu}=0, \quad (79)$$

where $X^{\mu\nu}=e^{-2\phi_0+(5/4)\rho_0}H^{y\mu\nu}+e^{(5/4)\rho_0}\chi\tilde{F}^{y\mu\nu}$ and $(T^{\mu}_{\nu}-\frac{1}{4}\delta^{\mu}_{\nu}T^{\alpha}_{\alpha})^{(1)}$ is the first order part of $(T^{\mu}_{\nu}-\frac{1}{4}\delta^{\mu}_{\nu}T^{\alpha}_{\alpha})$. The solutions are easily found:

$$H_{y\mu\nu}(y,x)=-a(y)^9\beta e^{\varphi+\phi_0-(5/4)\sigma_0}B_{\mu\nu}(x) \quad (80)$$

and

$$\tilde{F}_{y\mu\nu}(y,x)=-a(y)^9\frac{\beta}{2}e^{\varphi-(5/4)\sigma_0}\hat{\epsilon}_{\mu\nu\rho\sigma}(h)B_{\alpha\beta}h^{\rho\alpha}h^{\sigma\beta}. \quad (81)$$

Let us derive the gravitational equation on the brane. From the Hamiltonian constraint on the brane, we first obtain

$${}^{(4)}\hat{R}(h)=-2\hat{D}^2\phi_0+\frac{6}{5}(D\hat{\phi}_0)^2+\frac{1}{2}e^{2\phi_0}(D\hat{\chi}_0)^2, \quad (82)$$

where $\hat{D}_{\mu}$ is the covariant derivative with respect to $h_{\mu\nu}$.

Substituting the above solutions into Eq. (76) and integrating over y, we obtain

$$\frac{1}{y_0}\overset{(1)}{\tilde{K}}{}^{\mu}{}_{\nu}(y,x)=\frac{5}{8}a^{-7}e^{\varphi}\Big[{}^{(4)}\hat{R}^{\mu}_{\nu}(h)+2\hat{D}^{\mu}\hat{D}_{\nu}\phi_0-\frac{5}{4}\hat{D}^{\mu}\hat{D}_{\nu}\rho_0-\frac{5}{16}\hat{D}^{\mu}\rho_0\hat{D}_{\nu}\rho_0-\frac{5}{4}\hat{D}^{\mu}\rho_0\hat{D}_{\nu}\phi_0-\frac{5}{4}\hat{D}^{\mu}\phi_0\hat{D}_{\nu}\rho_0$$
$$-\frac{1}{2}e^{2\phi_0}\hat{D}^{\mu}\chi_0\hat{D}_{\nu}\chi_0-\hat{D}^{\mu}\hat{D}_{\nu}\varphi-\hat{D}^{\mu}\varphi\hat{D}_{\nu}\varphi-a^{14}\beta^2e^{\varphi+2\phi_0-(5/2)\sigma_0}\hat{B}_{\mu}{}^{\alpha}\hat{B}_{\nu\alpha}\Big]_{\rm traceless}+a\chi^{\mu}_{\nu}(x), \tag{83}$$

where $\chi_{\mu\nu}(x)$ is the constant of integration. Together with the junction condition for K^{μ}_{ν} and the Hamiltonian constraint, we finally obtain the effective equation on the brane:

$${}^{(4)}G_{\mu\nu}(h)=-\left(\hat{D}_{\mu}\hat{D}_{\nu}+\frac{3}{4}h_{\mu\nu}\hat{D}^2\right)\left(2\phi_0-\frac{5}{4}\rho_0\right)+\frac{16}{5}\left(\hat{D}_{\mu}\rho_0\hat{D}_{\nu}\rho_0-\frac{7}{4}h_{\mu\nu}(\hat{D}\rho_0)^2\right)+\frac{1}{2}e^{2\phi_0}\left[\hat{D}_{\mu}\chi_0\hat{D}_{\nu}\chi_0-\frac{1}{2}h_{\mu\nu}(\hat{D}\chi_0)^2\right]$$
$$-(\hat{D}\phi_0)^2h_{\mu\nu}+\frac{5}{4}\hat{D}_{\alpha}\phi_0\hat{D}^{\alpha}\rho_0h_{\mu\nu}+\hat{D}_{\mu}\hat{D}_{\nu}\varphi-\frac{1}{4}h_{\mu\nu}(\hat{D}\varphi)^2+\hat{D}_{\mu}\varphi\hat{D}_{\nu}\varphi-\frac{1}{4}h_{\mu\nu}(\hat{D}\varphi)^2+\tilde{\chi}_{\mu\nu}(x). \tag{84}$$

This is the main result in this section. Although we can write φ in terms of ϕ_0 and ρ_0, we leave it from a pedagogical point of view. As in the previous section, it turns out again that the gravitational equation on the brane is *not* like Einstein-Maxwell theory. $\overset{(1)}{T}{}_{\mu\nu}$ is exactly canceled out.

Comparing Eq. (84) with Eq. (44), we find that the relation between $E_{\mu\nu}$ and $\tilde{\chi}_{\mu\nu}$ is

$$-E_{\mu\nu}=\tilde{\chi}_{\mu\nu}-\frac{5}{6}\beta^2e^{2\phi_0-(5/2)\rho_0}\overset{(1)}{T}{}_{\mu\nu}-\frac{5}{3}\left[\hat{D}_{\mu}\hat{D}_{\nu}\phi_0-\frac{1}{4}h_{\mu\nu}(D\hat{\phi}_0)^2\right]+\frac{5}{3}\left[\hat{D}_{\mu}\hat{D}_{\nu}\rho_0-\frac{1}{4}h_{\mu\nu}(\hat{D}\rho_0)^2\right]+\frac{1}{6}e^{2\phi}\Big[\hat{D}_{\mu}\chi_0\hat{D}_{\nu}\chi_0$$
$$-\frac{1}{4}h_{\mu\nu}(\hat{D}\chi_0)^2\Big]+\frac{5}{3}\left[\hat{D}_{\mu}\rho_0\hat{D}_{\nu}\rho_0-\frac{1}{4}h_{\mu\nu}(\hat{D}\rho_0)^2\right]+\hat{D}_{\mu}\phi_0\hat{D}_{\nu}\phi_0-\frac{1}{4}h_{\mu\nu}(\hat{D}\phi_0)^2-\frac{5}{4}\Big[\hat{D}_{\mu}\phi_0\hat{D}_{\nu}\rho_0+\hat{D}_{\mu}\rho_0\hat{D}_{\nu}\phi_0$$
$$-\frac{1}{2}h_{\mu\nu}\hat{D}_{\mu}\rho_0\hat{D}^{\mu}\phi_0\Big]. \tag{85}$$

We should again notice that the form of the brane action Eq. (35) is essential to have consistent solutions. Solutions for $H_{y\mu\nu}$ and $\tilde{F}_{y\mu\nu}$ in the bulk are automatically consistent with the boundary conditions derived from Eq. (35). We cannot find a consistent solution except for a trivial solution $B_{\mu\nu}=0$, if one chooses the brane action without a Born-Infeld term.

IV. DISCUSSION

In this paper we derived the effective theory on the D-brane described by the Born-Infeld action in type IIB supergravity. To bring out the essence we focused on gravity, the U(1) gauge field, and the scalars. We considered two different derivations by holographic and geometrical approaches. In both, it turns out that the effective theory is four-dimensional Einstein+scalar+holographic CFT (or integration of a constant) and that the Maxwell fields do not appear at the leading order. This is bad news for the D-braneworld scenario. Now we have a caution: careful consideration will be demanded in a realistic model.

Usually "the integration of constant" $\tilde{\chi}_{\mu\nu}$ in Eq. (84) is expected to correspond to the holographic CFT energy-momentum tensor [15,16]. Comparing Eq. (27) with Eq. (84), we can easily confirm that this is the case, that is, $\tilde{\chi}_{\mu\nu}$ is related to $T^{\rm CFT}_{\mu\nu}$ at the leading order. More precisely,

$$T^{\rm CFT}_{\mu\nu}=\tilde{\chi}_{\mu\nu}+\frac{5}{4}h_{\mu\nu}\Big[\hat{D}^2(-\phi_0+\rho_0)+3(\hat{D}\phi_0)^2$$
$$-\frac{11}{2}\hat{D}\phi_0D\hat{\rho}_0+\frac{5}{2}(\hat{D}\rho_0)^2\Big]$$
$$=\tilde{\chi}_{\mu\nu}-\frac{1}{4}h_{\mu\nu}J^{(\rho)}_{\rm CFT}, \tag{86}$$

where

$$J^{(\rho)}_{\rm CFT}=\frac{1}{\sqrt{-h}}\frac{\delta\Gamma_{\rm CFT}}{\delta\rho}. \tag{87}$$

In the above we have used the equations for the scalar fields that can be obtained through the variational principle of the action $S_{\text{D-brane}}+S_{\rm ct}+\Gamma_{\rm CFT}$ in the previous section:

$$\hat{D}^2(\phi_0-\rho_0)-3(\hat{D}\phi_0)^2-\frac{5}{2}(\hat{D}\rho_0)^2+\frac{11}{2}\hat{D}_{\mu}\phi_0\hat{D}^{\mu}\rho_0$$
$$=-\frac{1}{5}J^{\rho}_{\rm CFT}. \tag{88}$$

Thus we can confirm the desirable result here.

As stressed in Sec. III B 1, the current background solution is not like AdS spacetime and gravity cannot be confined on the brane at low energy without compactification. If we compactify the extra dimension, we must introduce another brane. In this case, the integration of the constant $\tilde{\chi}_{\mu\nu}$ is not the holographic CFT energy-momentum tensor but just the energy-momentum tensor on the brane. If the brane is the vacuum, $\tilde{\chi}_{\mu\nu}=0$. Then the effective theory is not like Einstein-Maxwell theory. But, if the Maxwell field lives on another brane, we can see that the field also appears on the D-brane.

In this paper we saw the drastic changes from the probe D-brane case when we take into account the self-gravity of the brane. Compared to the probe brane, the new ingredients are the junction conditions. The consistent solutions are extremely limited. In type IIB supergravity, indeed, we obtain a consistent bulk solution for the Born-Infeld action, but we do not for the Nambu-Goto one. This fact implies that one should be careful in connecting an effective action derived from AdS/CFT like correspondence to an effective action on a self-gravitating brane.

There are several remaining studies. The first is the higher order corrections and their meaning. In the holographic approach, we obtained the coupling between the curvature and the stress tensor of the gauge fields. A systematic analysis will be interesting. The second is about the localization of fermion fields on the D-brane. We hope that these issues will be addressed in the near future.

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