Majorana neutrino, the size of extra dimensions, and neutrinoless double beta decay

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The problem of the Majorana neutrino mass generated in the Arkani-Hamed–Dimopoulos–Dvali model with n extra spatial dimensions is discussed. Taking into account constraints on the neutrino masses coming from cosmological observations, it is possible to obtain lower limits on the size of the extra dimensions as large as 10^{-6} mm. In the case of $n=4$ it is easy to lower the fundamental scale of gravity from the Planck energy to the electroweak scale ~ 1 TeV without imposing any additional constraints. A link between the half-life of neutrinoless double beta decay and the size of extra dimensions is discussed.

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Theories and models with additional spatial dimensions have drawn much attention during recent years (see, e.g., [1] and references therein for a recent review). Such ideas have their source in the works of Kaluza and Klein, who showed in the 1920s [2,3] that adding a fifth spatial dimension allows us to unify Maxwell's and Einstein's theories, and obtain in this way a common description of electromagnetic and gravitational interactions. The theory of Kaluza and Klein had been proven to be wrong and therefore the idea of extra dimensions has been unpopular until the development of the string theory. The latter, which is now treated by many physicists as a very promising replacement for quantum field theory and a step in the right direction towards a theory of everything, requires for consistent formulation many more spatial dimensions than just three. What is more, new multidimensional objects, called branes, emerge from this theory in a natural way. Recent models suggest that our observable Universe could be embedded in such brane, which in turn floats in higher dimensional bulk, possibly interacting with fields that populate the bulk as well as with other branes. Such a setup gives completely new possibilities of solving many important issues, such as the problem of hierarchy among fundamental interactions and the smallness of the neutrino mass.

At present, there are two main approaches to the problem of extra dimensions. One of them, the Randall-Sundrum model, is based on five-dimensional background metric solutions [4,5]. This geometric approach allows us to solve the generalized Einstein equations. Another formalism, the Arkani-Hamed–Dimopoulos–Dvali (ADD) model ([6–8] and references therein), has more phenomenological bases, but explains in an easy way the observed weakness of gravitational force. It also solves the hierarchy problem by lowering the gravitational scale from the Planck energy in $(3+1)$ dimensions to the electroweak scale ~ 1 TeV in n additional dimensions. It assumes further that from all known interactions only gravity feels the extra dimensional space. The volume of extra dimensions, in which graviton can propagate, suppresses the observed strength of gravity. The whole standard model (SM) is confined to a 3-brane on which we live. Various modifications and improvements of the model introduce other branes, which exist parallel to ours, as well as other particles which can be found in the bulk. The possible interactions of our brane with these objects lead to a lot of interesting phenomena. To mention some examples from neutrino physics, one may find in the literature not only discussions of neutrino gravitational interactions [9], masses and mixing patterns [10], oscillations [11], sterile neutrinos [12], but also, e.g., unification schemes [13]. Among others a naturally small Majorana neutrino mass can be generated.

The first goal of the present work is to compare this mass with the newest theoretical and experimental limits, which allows us to set constraints on the sizes and number of extra dimensions. First, we take into account the limits on neutrino masses coming from astrophysical observations and obtain certain values of compactification radii of extra dimensions in the ADD model. The second goal is to establish a link between the physics of extra dimensions and theory of neutrinoless double beta decay. As is well known, the half-life of this exotic nuclear transition heavily depends on the masses and mixing of neutrinos, so the second source of constraints comes from the newest data from the neutrinoless double beta decay experiments.

In the ADD approach the standard model is localized on a three-dimensional brane which is embedded into a $(4+n)$-dimensional space-time. The n additional spacelike dimensions are often referred to as being *transverse* to our brane. This construction has its origin in the more general string theoretical setting with possibly more complicated geometry. For our purposes it is enough to assume that all the extra dimensions are characterized by a common size R, so that the volume V_n of the space of extra dimensions is proportional to R^n. One of the basic relations is the so-called reduction formula which can be obtained using, for example, the generalized Gauss law. It reads [14] $M_{Pl}^2 \sim R^n M_*^{2+n}$, where M_{Pl} is the Planck mass and M_* is the true scale of gravity, which we want to set somewhere around 1 TeV. The coordinates are denoted by $\{x^\mu, y^m\}$, where $\mu=0$–3 labels the ordinary space-time coordinates and $m=1$–n labels the extra dimensions. What is more, we identify $y \sim y+2\pi R$, that is, we compactify the extra dimensions on circles. From now on we will drop the indices μ and m for simplicity.

Assume that our brane has coordinates $\{x,0\}$ and that there is another brane located at $\{x', y_*\}$, separated from ours by the distance $r=|y_*|$ in the transverse dimensions.

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We assume further [14,15] that lepton number is conserved on our brane but maximally broken on the other one. The breaking occurs in a reaction where a particle χ with lepton number $L=2$ escapes the other brane into the bulk. This particle, called the *messenger*, may interact with our brane and transmit to us the information about lepton number breaking.

To be more specific, let us introduce a field $\phi_{L=2}$ located on the other brane, whose vaccum expectation value (vev) breaks the lepton number. What is more, it acts as a source for the bulk messenger field χ and "shines" it everywhere, in particular also on our brane. We introduce a lepton doublet $\mathcal{L}$ and a Higgs field $\mathcal{H}$ localized on our brane. They can interact with the messenger, and the interaction is given by the following action [15]:

$$M_*^{n-1}S^{int}\sim\int d^4x'\langle\phi\rangle\chi(x',y_*) + \int d^4x\,\alpha(lh^*)^2(x)\chi(x,0). \tag{1}$$

The first part of S^{int} describes the process which takes place on the other brane and the second one represents the interaction with SM particles, proportional to some numerical constant α. The strength of the shined χ is in a natural way suppressed by the distance r between branes, and therefore one can write for the messenger

$$\langle\chi\rangle=\langle\phi\rangle\Delta_n(r), \tag{2}$$

where $\Delta_n(r)$ is the n-dimensional propagator,

$$\Delta_n(r)=\frac{1}{-\partial_n^2+m_\chi^2}(r)=\int d^nk\frac{e^{ikr}}{k^2+m_\chi^2}.$$

The explicit form of the propagator reads

$$\Delta_2(r)\sim\begin{cases}-\log(rm_\chi) & (rm_\chi\ll 1)\\ e^{-rm_\chi}/\sqrt{rm_\chi} & (rm_\chi\gg 1),\end{cases}$$

$$\Delta_{n>2}(r)\sim\begin{cases}1/r^{n-2} & (rm_\chi\ll 1)\\ e^{-rm_\chi}/r^{n-2} & (rm_\chi\gg 1),\end{cases}$$

and one sees that it depends on the number of extra dimensions, the distance between branes, and the mass of the messenger. This feature will be used later. After substituting Eq. (2) into Eq. (1), writing the Higgs field in terms of its vev v, and identifying l with ν_L we arrive at a mass term of the Majorana form $m_{Maj}\nu_L^T\nu_L$ with the mass given approximately by

$$m_{Maj}\sim v^2\Delta(r)/M_*^{n-1}. \tag{3}$$

This is the original relation given by the authors in [15] and it is useful if one wants to set the M_* scale to a certain value, such as the electroweak scale. However, another approach is possible. Let us place the second brane as far as possible, that is, set $r=R$. Now, using the reduction formula one can rewrite Eq. (3) in the form

$$m_{Maj}\sim v^2R^{n(n-1)/(n+2)}M_{Pl}^{2(1-n)/(n+2)}\Delta_n(R) \tag{4}$$

and deduce the size R of extra dimensions imposing constraints on neutrino mass, coming from other sources, for example, cosmological observations, neutrinoless double beta decay, or neutrino oscillation experiments.

Recently reported cosmological observations [16,17] take into account, among others, redshifts of galaxies, the microwave background radiation, type Ia supernova behavior, and the Big Bang nucleosynthesis, and suggest the sum of neutrino masses of all flavors not to exceed few eV: $\Sigma_i(m_\nu)_i \lesssim 1$ eV.

Imposing this condition on Eq. (3) leads to a set of constraints for the possible size of extra dimensions. Putting in numbers: $v^2=(174\text{ GeV})^2\sim10^{22}\text{ eV}^2$, $M_{Pl}\sim10^{28}$ eV, $1\text{ eV}^{-1}\sim10^{-7}$ m, and using the explicit forms for the propagator, we obtain interesting bounds on R.

Two additional dimensions $(n=2)$. For light messenger particle we obtain bounds from above. If $Rm_\chi=0.1$, we get $R<1.8\times10^{-16}\text{ eV}^{-1}\sim10^{-20}$ mm. Similarly for $Rm_\chi=0.01$, the bound is even stronger $R<4.7\times10^{-18}\text{ eV}^{-1}\sim10^{-22}$ mm. If the messenger is heavy, due to exponent in the denominator of Δ, we obtain bounds from below. For $m_\chi=1$ keV, they turn out to be $R>1.5\times10^{-2}\text{ eV}^{-1}\sim1.5\times10^{-6}$ mm. Newton's law has been recently checked down to around 0.1 mm [18]. The distance of 10^{-6} mm is still out of range for the currently planned tabletop experiments, but R may be reached in the next generation of projects. It shows that this case may be promising. For heavier χ the limit goes down and takes the values for $m_\chi=1$ MeV, $R>1.15\times10^{-5}\text{ eV}^{-1}\sim10^{-9}$ mm and for $m_\chi=1$ GeV, $R>8\times10^{-9}\text{ eV}^{-1}\sim10^{-12}$ mm.

Three additional dimensions $(n=3)$. For light messenger, performing a similar analysis gives us $R<0.01$ mm. For heavy χ particle we can use the fact that Rm_χ must be at least equal to one, which implies $R<1.4$ mm. None of these cases is excluded and the distance 0.01 mm may be explored in not very far future.

Four and more additional dimensions $(n\geq4)$. The case of four extra dimensions and a light messenger is an exceptional one: $m_{Maj}\sim v^2R^2M_{Pl}^{-1}R^{-2}$ because the dependence on R is lost. Explicitly we obtain $m_{Maj}\sim10^{22}R^2(10^{28})^{-1}R^{-2}\text{ eV}=10^{-6}$ eV. One should stress that this result for the neutrino mass is in agreement with all experimental facts known nowadays. An arbitrary R implies also an arbitrary value of M_* which can be calculated from the reduction formula. For example, for $M_*=1$ TeV the size of extra dimensions needs to be $R\sim10^{-8}$ mm, which is quite reasonable. The case of heavy χ does not provide any additional constraints, namely, we get $Rm_\chi>-13.8$. Similarly for more than four dimensions we obtain $R>10^{-99}$ eV so all what we can say is only that these cases are not excluded.

It is desirable to compare the just obtained numbers with existing constraints on R coming from different sources than neutrino physics sources. One of the most restrictive bounds comes from supernova and neutron star data [19–23] and cosmological models [24,25]. Altogether, the newest limits are [21] $R<1.5\times10^{-7}$ mm for $(n=2)$, $R<2.6\times10^{-9}$ mm for $(n=3)$, $R<3.4\times10^{-10}$ mm for $(n=4)$. We note first that all of them present bounds from above, which means that for heavy messenger we may encounter a contradiction with the results from previous section.

For $n=2$ and a light messenger particle our constraints are significantly stronger, with difference of at least 13 orders of magnitude. Such striking result may either be considered as highly improbable or shed new light on the astrophysical and cosmological models used to derive the previous constraint. For a heavy χ, $m_\chi=1$ keV, we have obtained $R>10^{-6}$ mm while the previous results were $R<10^{-7}$ mm. It may seem that this case should be excluded. However, note that it is enough to lower the total mass of neutrinos by one order of magnitude $\Sigma m_\nu\leqslant0.1$ eV to get an agreement. Due to high uncertainty of neutrino absolute masses it sounds reasonable to treat this case on equal footing with the rest. Another way to overcome this problem is to increase the mass of χ to 10 keV and treat 1 keV as the absolute lower limit for it. For even heavier χ our results are complementary to previous bounds, placing R between 10^{-9}–10^{-7} mm and 10^{-12}–10^{-7} mm for $m_\chi=1$ MeV and $m_\chi=1$ GeV, respectively. In the case $n=4$ and light χ, $R<10^{-10}$ mm implies M_* to be of the order of 10–100 TeV, which still is low enough to solve the hierarchy problem of particle physics. The remaining cases do not provide any additional bounds to the previously known cases.

The presence of extra dimensions will surely influence the results of sensitive experiments. In this Brief Report we investigate the impact of extra dimensions on half-life of neutrinoless double beta decay.

Neutrinoless double beta decay $(0\nu2\beta)$ is a process in which a nucleus undergoes two simultaneous beta decays without emission of neutrinos, $A(Z,N)\rightarrow A(Z+2,N-2)+2e^-$. It requires neutrino to be a Majorana particle, so that two neutrinos emitted in beta decays annihilate with each other. It is readily seen that this process violates the lepton number by two units, thus is forbidden in the framework of SM. As a matter of fact, $0\nu2\beta$ has not been observed, but the nonobservability sets valuable constraints on the shape of nonstandard physics.

Ignoring the contributions from right-handed weak currents, the half-life of $0\nu2\beta$ can be written in the form [26] $(T_{1/2})^{-1}=\mathcal{M}\langle m_\nu\rangle^2/m_e^2$, where $\mathcal{M}$ is a nuclear matrix element which can be calculated within some nuclear model (such as the bag model or nonrelativistic quark model) and m_e is the electron mass. The so-called effective neutrino mass $\langle m_\nu\rangle$ is defined by the relation

$$\langle m_\nu\rangle=\sum_i |U_{ei}^2|m_i, \tag{5}$$

where U is the neutrino mixing matrix and m_i are neutrino mass eigenstates. One sees that it is possible to identify $\langle m_\nu\rangle$ with the ee entry of neutrino mass matrix in the flavor basis $\langle m_\nu\rangle=m_{ee}$, which is given exactly by the superposition of mass eigenstates from Eq. (5).

Since the mixing among neutrino mass states is not known exactly, we are forced to simplify our picture a little bit. Using results from the CHOOZ experiment [27,28] we get the constraint $|U_{e3}^2|<0.037-0.017$, depending on the mixing pattern considered (two- or three-neutrino mixing). It implies that the contribution coming from m_3 may be neglected. The remaining masses are nearly degenerate [29] and we can approximate $m_2\approx m_1$. Denoting the relative phase between the two flavors by ϕ_{12} we obtain $m_{ee}^2=[1-\sin^2(2\theta_{Solar})\sin^2(\phi_{12}/2)]m_1^2$. The currently favored large mixing angle solution for solar neutrino problem sets the value of $\sin^2(2\theta_{Solar})$ between 0.3 and 0.93 [30] which may introduce a huge uncertainty. Fortunately, in the final formula this uncertainty is heavily reduced. If we assume that CP symmetry is not violated, we have to set ϕ_{12} either to 0 or $\pi/2$. The latter case is of course more interesting and we will stick to it.

Under these conditions we are now legitimate to replace m_1 with m_{Maj} and link in this way the half-life of $0\nu2\beta$ with size of extra dimensions. We obtain

$$T_{1/2}^{th}\approx(1.17-1.8)T_{1/2}^{expt.}\langle m_\nu\rangle_{expt.}^2/m_{Maj}^2, \tag{6}$$

where m_{Maj} is given by the relation (4). The experimental values established by the IGEX Collaboration [31] are

$$T_{1/2}^{IGEX}>1.57\times10^{25}\ \text{y},\ \ \langle m_\nu\rangle_{IGEX}=(0.33-1.35)\ \ \text{eV}.$$

On the other hand, the Heidelberg-Moscow Collaboration [32] gave a best fit value for the effective neutrino mass $\langle m_\nu\rangle_{H-M}=0.39$ eV so in our calculations we set this parameter to 0.4 eV. Putting in numbers and expanding the formula (6) using Eq. (4) we arrive at the following relation:

$$T_{1/2}^{th}>\kappa10^{93n-150/(n+2)}R^{2n(1-n)/(n+2)}[\Delta_n(R)]^{-2}\ \ \text{y}. \tag{7}$$

Here, the uncertainty factor κ satisfies $0.74<\kappa<1.17$. As an example, we take a closer look at the two special cases: $n=2$ with heavy messenger and $n=4$ with light messenger. We would like to stress that the following discussion is valid only under our assumptions, i.e., we live in a brane world, generate neutrino masses in the ADD model, neglect the influence of m_3, and treat the remaining ones as nearly degenerate. Keeping this in mind, for the limiting case $n=2$, $m_\chi=10$ keV, formula (7) simplifies to $T_{1/2}^{th}>\kappa\times10^{13}\exp(2\times10^4R)$ y, with $R<1.5\times10^{-7}$ mm. The result is presented in Fig. 1 together with the bound from the experiment. One sees that with increasing size of extra dimensions, the half-life explodes exponentially. Therefore, the possibility of establishing $T_{1/2}$ in experiment will set precise constraints on physics of extra dimensions. It is visible from Fig. 1 that the allowed region lies between 1.39×10^{-7} mm and 1.5×10^{-7} mm, which implies $T_{1/2}>1.57\times10^{25}$ y and $T_{1/2}<(8-12)\times10^{25}$ y. The sensitivity of currently planned $0\nu2\beta$ experiments will allow us to probe this region and definitively confirm or exclude this case.

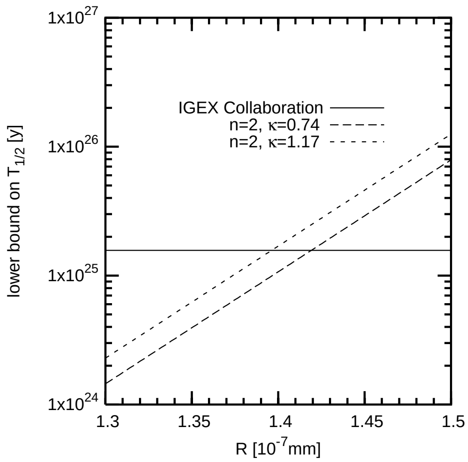

FIG. 1. Lower bounds on the half-life of neutrinoless double beta decay in the case of two additional spatial dimensions of size *R*. The mass of the messenger particle is $m_\chi = 10$ keV.

The second special case, $n=4$ with a light messenger particle, renders the value of $T_{1/2}$ up to $\sim 10^{37}$ y which, even if true, remains far beyond the sensitivity of current and currently planned experiments.

Theories with large additional dimensions have been intensively developed during past years. One of the main motivations for them is the theory of superstrings and neutrino physics. Using the ADD model we have discussed a new line of constraints on the radius of extra dimensions coming from neutrino masses. The most meaningful results were obtained for two and three transverse dimensions, but nothing excluded the possibilities that four or more additional dimensions exist.

In the present paper we have shown that there are two especially interesting cases allowed by the ADD model of extra dimensions. The number of additional transverse dimensions $n=2$ and mass of the messenger particle χ around 10 keV implies the size of the extra dimensions to be of the order of 10^{-7} mm, which gives the true scale of gravity $M_* \sim 1000$ TeV. We found that, in order to satisfy previously derived constraints, the minimal mass of the messenger should not be less than 10 keV. However, this value relies on the estimation $\Sigma m_\nu < 1$ eV, which in fact could be one or two orders of magnitude less. In such case the messenger may be lighter. The second interesting possibility, $n=4$ and light χ, suggests the mass of neutrino to be of the order of 10^{-6} eV and permits adjustments of R and M_* in wide range. Specifically, taking $R < 10^{-10}$ mm implies M_* to be of the order of 10–100 TeV, which is a reasonable value that solves the hierarchy problem. It is legitimate to state that all the constraints obtained here either improve the old one or are complementary to them.

One of the main drawbacks of extra dimensional theories is the difficulty of experimental verification. The best known bounds are the tabletop tests of Newton's law. Some proposals were given to link the presence of extra dimensions with behavior of supenovae, however the mechanism of supernova explosion is not known well enough to draw reasonable conclusions. In this paper we have outlined a method how to use the neutrinoless double beta decay for this purpose. Observation of this exotic nuclear process will tell us precisely which values of parameters are allowed. Till then, its nonobservability sets strict constraints, which can be combined with bounds from other sources.

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