R-parity violation in a supersymmetric GUT model and radiative neutrino masses

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Within the framework of an SU(5) SUSY GUT model, a mechanism which effectively induces R-parity-violating terms below the unification energy scale M_X is proposed. The model has matter fields $\bar{5}_{L(+)} + 10_{L(-)}$ and Higgs fields $H_{(-)}$ and $\bar{H}_{(+)}$ in addition to the ordinary Higgs fields $H_{(+)}$ and $\bar{H}_{(-)}$ which contribute to the Yukawa interactions, where $(\pm)$ denote the transformation properties under a discrete symmetry Z_2. The Z_2 symmetry is only broken by the μ term $\bar{H}_{(+)}H_{(-)}$ softly, so that the $\bar{5}_{(+)} \leftrightarrow \bar{H}_{(+)} \leftrightarrow \bar{H}_{(-)}$ mixing appears at $\mu < m_{SB}$, and R-parity violating terms $\bar{5}_L\bar{5}_L10_L$ are effectively induced from the Yukawa interactions $\bar{H}_{(-)}\bar{5}_{L(+)}10_{L(-)}$; i.e., the effective coupling constants λ_{ijk} of $\nu_{Li}e_{Lj}e^c_{Rk}$ and $\nu_{Li}d^c_{Rj}d_{Lk}$ are proportional to the mass matrices $(M^*_e)_{jk}$ and $(M^\dagger_d)_{jk}$, respectively. The parameter regions which are harmless for the proton decay are investigated. Possible forms of the radiatively induced neutrino mass matrix are also investigated.

I. INTRODUCTION

The origin of neutrino mass generation is still a mysterious problem in the unified understanding of quarks and leptons. As the origin, from the standpoint of a grand unification theory (GUT), currently, the idea of the so-call seesaw mechanism [1] is influential. On the other hand, an alternative idea that the neutrino masses are radiatively induced is still attractive. As an example of such a model, the Zee model [2] is well known. Regrettably, the original Zee model is not on the framework of GUT. A possible idea to embed the Zee model into GUT is to identify the Zee scalar h^+ as the slepton $\tilde{e}_R$ in an R-parity-violating supersymmetric (SUSY) model [3]. However, usually, it is accepted that SUSY models with R-parity violation are incompatible with a GUT scenario, because the R-parity-violating interactions induce proton decay [4,5]. By the way, there is another problem in a GUT scenario: i.e., how to give doublet-triplet splitting in SU(5) 5-plet Higgs fields. There are many ideas to solve this problem [6]. Although these mechanisms are very attractive, in the present paper, we will take another choice, that is, fine-tuning the parameters: we consider a possibility that a mechanism which provides the doublet-triplet splitting gives a suppression of the R-parity violating terms with baryon number violation while it gives visible contributions of the doublet component to the low energy phenomena (neutrino masses, lepton flavor processes, and so on) [4]. In the present paper, we will try to give an example of such a scenario.

In the present paper, in order to suppress the proton decay, a discrete symmetry Z_2 is introduced. The essential idea is as follows: we consider matter fields $\bar{5}_{L(+)} + 10_{L(-)}$ of SU(5)

and two types of SU(5) 5-plet and $\bar{5}$-plet Higgs fields $H_{(\pm)}$ and $\bar{H}_{(\pm)}$, where $(\pm)$ denote the transformation properties under a discrete symmetry Z_2 (we will call it "Z_2-parity" hereafter). The superpotential in the present model is given by

$$W = W_Y + W_H + W_{mix}, \qquad (1.1)$$

where W_Y denotes Yukawa interactions

$$W_Y = \sum_{i,j} (Y_u)_{ij} H_{(+)} 10_{L(-)i} 10_{L(-)j}$$

$$+ \sum_{i,j} (Y_d)_{ij} \bar{H}_{(-)} \bar{5}_{L(+)i} 10_{L(-)j}. \qquad (1.2)$$

Under the discrete symmetry Z_2, R-parity violating terms $\bar{5}_{L(+)}\bar{5}_{L(+)}10_{(-)}$ are exactly forbidden. The discrete symmetry Z_2 is softly violated only by the following μ terms:

$$W_H = \bar{H}_{(+)}(m_+ + g_+\Phi)H_{(+)} + \bar{H}_{(-)}(m_- + g_-\Phi)H_{(-)}$$

$$+ m_{SB}\bar{H}_{(+)}H_{(-)}, \qquad (1.3)$$

where Φ is an SU(5) 24-plet Higgs field with the vacuum expectation value (VEV) $\langle\Phi\rangle = v_{24}\text{diag}(2,2,2,-3,-3)$, and it has been introduced in order to give doublet-triplet splittings in the SU(5) 5- and $\bar{5}$-plets Higgs fields at an energy scale $\mu < M_X$ [M_X is an SU(5) unification scale]. The Z_2-parity is violated only by[1] the term $\bar{H}_{(+)}H_{(-)}$. Note that

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[1]The Z_2 symmetry can be softly violated not only by the term $\bar{H}_{(+)}H_{(-)}$, but also by terms $\bar{H}_{(-)}H_{(+)}$ and $\bar{5}_{L(+)1}H_{(-)}$. However, in the present scenario, the existence of $\bar{H}_{(+)}H_{(-)}$ is essential. The details are discussed in Appendix A.

$H_{(-)}$ and $\bar{H}_{(+)}$ in the m_{SB}-term do not contribute to the Yukawa interaction (1.2) directly, so that proton decay via the dimension-5 operator is suppressed in the limit of $m_{SB} \to 0$. (A similar idea, but without Z_2 symmetry, has been proposed by Babu and Barr [7].) The terms W_{mix} have been introduced in order to bring the $\bar{H}_{(+)} \leftrightarrow \bar{5}_{(+)}$ mixing:

$$W_{mix} = \sum_i \bar{5}_{L(+)i}(b_i m_5 + c_i g_5 \Phi) H_{(+)}, \qquad (1.4)$$

where $\Sigma_i |b_i|^2 = \Sigma_i |c_i|^2 = 1$. At the energy scale $\mu < M_X$, the terms $W_H + W_{mix}$ are effectively given by

$$
\begin{aligned}
W_H &+ W_{mix} \\
&= \sum_{a=2,3} m_+^{(a)} \left[\bar{H}_{(+)}^{(a)} \cos \alpha^{(a)} + \sum_i d_i \bar{5}_{L(+)i}^{(a)} \sin \alpha^{(a)} \right] H_{(+)}^{(a)} \\
&\quad + \sum_{a=2,3} m_-^{(a)} \bar{H}_{(-)}^{(a)} H_{(-)}^{(a)} + m_{SB} \sum_{a=2,3} \bar{H}_{(+)}^{(a)} H_{(-)}^{(a)}, \qquad (1.5)
\end{aligned}
$$

where $\Sigma_i |d_i|^2 = 1$, the index (a) denotes that the fields with (2) and (3) are doublet and triplet components of SU(5) $\to$ SU(2) $\times$ SU(3), respectively, and

$$m_+^{(2)} \cos \alpha^{(2)} = m_+ - 3 g_+ v_{24},$$

$$m_+^{(3)} \cos \alpha^{(3)} = m_+ + 2 g_+ v_{24}, \qquad (1.6)$$

$$m_+^{(2)} \sin \alpha^{(2)} d_i = m_5 b_i - 3 g_5 v_{24} c_i,$$

$$m_+^{(3)} \sin \alpha^{(3)} d_i = m_5 b_i + 2 g_5 v_{24} c_i, \qquad (1.7)$$

$$m_-^{(2)} = m_- - 3 g_- v_{24}, \quad m_-^{(3)} = m_- + 2 g_- v_{24}. \qquad (1.8)$$

Therefore, the m_{SB}-term together with the $m_+ \sin \alpha$-term induces the $\bar{H}_{(-)} \leftrightarrow \bar{5}_{L(+)}$ mixing, so that the R-parity violating terms $\bar{5}_L \bar{5}_L 10_L$ are generated from the Yukawa interactions $\bar{H}_{(-)} \bar{5}_{L(+)} 10_{L(-)}$. The coupling constants λ_{ijk} of $\bar{5}_i \bar{5}_j 10_k$ will be proportional to the charged lepton mass matrix $(M_e^*)_{jk}$ or down-quark mass matrix $(M_d^\dagger)_{jk}$. (The details are discussed in Sec. II.) As we demonstrate in Sec. II, we can show that the mixing $\bar{5}_{L(+)} \leftrightarrow \bar{H}_{(-)}$ is negligibly small for the colored sector, while it is sizable for the SU(2)-doublet sector.

The parameters in the present model need fine-tuning. For example, we will find that a large value of m_{SB} is not acceptable, because for such a large value of m_{SB} the proton decay due to the dimension five operator becomes visible. On the other hand, we will find that a smaller value of m_{SB} leads to a small bottom quark mass, so that a small value of m_{SB} is not acceptable. We will take $m_{SB} \sim 10^{14}$ GeV. In Sec. III, we will investigate the parameter regions which are harmless for the proton decay. In Sec. IV, we will investigate a possible form of the radiatively induced neutrino mass matrix due to the R-parity violation term $\bar{5}_L \bar{5}_L 10_L$. The radiatively induced neutrino mass matrix M_ν^{rad} will be expressed by the sum of two rank-1 matrices. On the other hand, we also have con-

tributions $M_{\tilde{\nu}}$ from VEVs $\langle \tilde{\nu}_i \rangle$ of the sneutrinos to the neutrino mass matrix M_ν. In Sec. V, a possible form of $M_\nu = M_{\tilde{\nu}} + M_\nu^{rad}$ is discussed from the phenomenological point of view. Finally, Sec. VI will be devoted to the summary.

II. $\bar{H}_{(-)}$-$\bar{5}_{(+)}$ MIXING

In order to suppress the proton decay, we want to take $m_+^{(2)} \sim M_W$ with a sizable $\alpha^{(2)}$, but $m_+^{(3)} \sim M_X$ with a negligibly small $\alpha^{(3)}$. However, from the relations (1.6) and (1.7), we obtain the relation

$$
\begin{aligned}
d_i \tan \alpha^{(3)} &= \frac{m_5 b_i + 2 g_5 v_{24} c_i}{m_+ + 2 g_+ v_{24}} \\
&= \frac{m_+^{(2)} \sin \alpha^{(2)} d_i + 5 g_5 v_{24} c_i}{m_+^{(2)} \cos \alpha^{(2)} + 5 g_+ v_{24}}. \qquad (2.1)
\end{aligned}
$$

The requirement $|\alpha^{(3)}| \lesssim M_W / M_X$ leads to the constraint $|g_5| \lesssim M_W / M_X$ for $|g_+| \sim 1$. We do not like to introduce such a small dimensionless parameter g_5. Therefore, for simplicity, we will put $g_5 = 0$ hereafter. Then, without loss of generality, we can put

$$\bar{5}'_{L(+)1} = \sum_i b_i \bar{5}_{L(+)i} \qquad (2.2)$$

where $\bar{5}'_{L(+)1}$ does not mean the observed first generation particle. (Hereafter, for convenience, we denote $\bar{5}'_{L(+)1}$ simply as $\bar{5}_{L(+)1}$.) The effective parameters $m_+^{(a)}$, $m_-^{(a)}$ and $\alpha^{(a)}$ are given as follows:

$$m_+^{(2)} = \sqrt{(m_+ - 3 g_+ v_{24})^2 + m_5^2},$$

$$m_+^{(3)} = \sqrt{(m_+ + 2 g_+ v_{24})^2 + m_5^2},$$

$$m_-^{(2)} = m_- - 3 g_- v_{24}, \quad m_-^{(3)} = m_- + 2 g_- v_{24}, \qquad (2.3)$$

$$\tan \alpha^{(2)} = \frac{m_5}{m_+ - 3 g_+ v_{24}} \simeq \frac{m_5}{m_+^{(2)}},$$

$$\tan \alpha^{(3)} = \frac{m_5}{m_+ + 2 g_+ v_{24}} \simeq \frac{m_5}{m_+^{(3)}}.$$

We will take

$$m_+^{(2)} \sim M_W, \quad m_+^{(3)} \sim M_X,$$

$$m_-^{(2)} \sim M_I, \quad m_-^{(3)} \sim M_X, \qquad (2.4)$$

$$\tan \alpha^{(2)} \sim \frac{m_5}{M_W}, \quad \tan \alpha^{(3)} \sim \frac{m_5}{M_X},$$

where $M_I \sim 10^{14}$ GeV and $m_5 \sim 10^1$ GeV as we state later. The mass matrix in the basis of $(\bar{H}_{(-)}, \bar{H}_{(+)}, \bar{5}_{L(+)1})$ and $(H_{(+)}, H_{(-)})$ is given by

$$M = \begin{pmatrix} 0 & m_- \\ m_+ \cos \alpha & m_{SB} \\ m_+ \sin \alpha & 0 \end{pmatrix}. \tag{2.5}$$

Here and hereafter, for simplicity, we drop the index (a). The mass matrix (2.5) is diagonalized as

$$\bar{U}^\dagger M U = D \equiv \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \\ 0 & 0 \end{pmatrix}, \tag{2.6}$$

where U and $\bar{U}$ are unitary operators which diagonalize $M^\dagger M$ and $M M^\dagger$ as

$$U^\dagger M^\dagger M U = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}, \tag{2.7}$$

and

$$\bar{U}^\dagger M M^\dagger \bar{U} = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.8}$$

respectively. Note that the matter field $\bar{5}'_{L1}$ is still massless, and also note that it is not in the eigenstate of the Z_2 parity.

The mixing matrix U is easily obtained from the diagonalization of

$$M^\dagger M = \begin{pmatrix} |m_+|^2 & m_{SB} m_+^* \cos \alpha \\ m_{SB}^* m_+ \cos \alpha & |m_{SB}|^2 + |m_-|^2 \end{pmatrix}. \tag{2.9}$$

For real m_1, m_{SB} and $m_\pm$, we obtain

$$U = \begin{pmatrix} \cos \theta_u & \sin \theta_u \\ -\sin \theta_u & \cos \theta_u \end{pmatrix}, \tag{2.10}$$

$$\tan 2\theta_u = \frac{2 m_{SB} m_+ \cos \alpha}{m_{SB}^2 + m_-^2 - m_+^2}, \tag{2.11}$$

$$m_1^2 = \frac{1}{2}(m_{SB}^2 + m_+^2 + m_-^2) - \frac{1}{2}Q, \tag{2.12}$$

$$m_2^2 = \frac{1}{2}(m_{SB}^2 + m_+^2 + m_-^2) + \frac{1}{2}Q, \tag{2.13}$$

where

$$Q = (m_{SB}^2 - m_+^2 + m_-^2)\cos 2\theta_u + 2 m_{SB} m_+ \cos \alpha \sin 2\theta_u. \tag{2.14}$$

When we define

$$A \equiv m_{SB}^2 - m_+^2 + m_-^2, \quad B \equiv 2 m_{SB} m_+ \cos \alpha, \tag{2.15}$$

$$\cos 2\theta_u = \frac{A}{\sqrt{A^2 + B^2}}, \quad \sin 2\theta_u = \frac{B}{\sqrt{A^2 + B^2}}, \tag{2.16}$$

the quantity Q is given by

$$Q = \sqrt{A^2 + B^2} = \sqrt{[m_{SB}^2 + (m_+ - m_-)^2][m_{SB}^2 + (m_+ + m_-)^2] - 4 m_{SB}^2 m_+^2 \sin^2 \alpha}. \tag{2.17}$$

The rotation $\bar{U}$ is also obtained from the diagonalization of

$$M M^\dagger = \begin{pmatrix} m_-^2 & m_{SB} m_- & 0 \\ m_{SB} m_- & m_{SB}^2 + m_+^2 \cos^2 \alpha & m_+^2 \cos \alpha \sin \alpha \\ 0 & m_+^2 \cos \alpha \sin \alpha & m_+^2 \sin^2 \alpha \end{pmatrix}. \tag{2.18}$$

The mixing matrix elements $\bar{U}_{i3}$ are easily obtain as follows:

$$\bar{U}_{13} = \frac{1}{N_3} m_{SB} \sin \alpha, \tag{2.19}$$

$$\bar{U}_{23} = -\frac{1}{N_3} m_- \sin \alpha, \tag{2.20}$$

$$\bar{U}_{33} = \frac{1}{N_3} m_- \cos \alpha, \tag{2.21}$$

where

$$N_3^2 = -m_-^2 + m_{SB}^2 \sin^2 \alpha. \tag{2.22}$$

Other matrix elements are obtained as follows: We express the mixing matrix $\bar{U}$ as

$$\bar{U} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}-s_{23}c_{12}s_{13} & c_{23}c_{12}-s_{23}s_{12}s_{13} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}c_{12}s_{13} & -s_{23}c_{12}-c_{23}s_{12}s_{13} & c_{23}c_{13} \end{pmatrix}, \tag{2.23}$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. Then, by comparing Eq. (2.23) with Eqs. (2.19)–(2.21), we obtain

$$s_{13} = \bar{U}_{13} = \frac{m_{SB}\sin \alpha}{\sqrt{m_-^2 + m_{SB}^2 \sin^2 \alpha}}, \quad c_{13} = \frac{1}{\sqrt{1+(m_{SB}/m_-)^2 \sin^2 \alpha}}, \tag{2.24}$$

$$s_{23} = \frac{\bar{U}_{23}}{c_{13}} = -\sin \alpha, \quad c_{23} = \cos \alpha. \tag{2.25}$$

By using the relation $(M^\dagger M)_{11} = \bar{U}_{11}\bar{U}_{11}(m_1')^2 + \bar{U}_{12}\bar{U}_{12}(m_2')^2$, the mixing angle θ_{12} is obtained as follows:

$$\cos 2\theta_{12} = \frac{1}{m_2^2 - m_1^2} \left[m_1^2 + m_2^2 - 2\frac{m_-^2}{c_{13}^2} \right] = \frac{1}{Q} (m_{SB}^2 + m_+^2 - m_-^2 - 2m_{SB}^2 \sin^2 \alpha)$$

$$= -\frac{m_-^2 - m_2^2 \cos 2\alpha - m_+^2}{\sqrt{m_-^4 + 2(m_2^2 - m_+^2)m_-^2 + m_2^4 + 2m_2^2 m_+^2 \cos 2\alpha + m_+^4}}. \tag{2.26}$$

Note that $\cos 2\theta_{12} \simeq -1$ for $m_-^2 \gg m_{SB}^2, m_+^2$, so that $\theta_{12} \simeq \pi/2$.

Since the physical fields $(\bar{H}_1, \bar{H}_2, \bar{5}_{L1}', \bar{5}_{L2}', \bar{5}_{L3}')$ are given by

$$\begin{pmatrix} \bar{H}_{(-)} \\ \bar{H}_{(+)} \\ \bar{5}_{L(+)1} \\ \bar{5}_{L(+)2} \\ \bar{5}_{L(+)3} \end{pmatrix} = \begin{pmatrix} \bar{U}_{11} & \bar{U}_{12} & \bar{U}_{13} & 0 & 0 \\ \bar{U}_{21} & \bar{U}_{22} & \bar{U}_{23} & 0 & 0 \\ \bar{U}_{31} & \bar{U}_{32} & \bar{U}_{33} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{H}_1 \\ \bar{H}_2 \\ \bar{5}_{L1}' \\ \bar{5}_{L2}' \\ \bar{5}_{L3}' \end{pmatrix}, \tag{2.27}$$

the Yukawa interactions $\bar{H}_{(-)}\bar{5}_{L(+)}10_{L(-)}$ lead to the effective Yukawa interactions at a low energy scale

$$(Y_d)_{ij}\bar{H}_1[\delta_{i1}(\bar{U}_{11}\bar{U}_{33} - \bar{U}_{13}\bar{U}_{31})\bar{5}_{L1}' + \bar{U}_{11}(\delta_{i2}\bar{5}_{L2}' + \delta_{i3}\bar{5}_{L3}')]10_{L(-)j}, \tag{2.28}$$

and the induced R-parity violating terms

$$(Y_d)_{ij}\bar{U}_{13}\bar{5}_{L1}'(\delta_{i1}\bar{U}_{33}\bar{5}_{L1}' + \delta_{i2}\bar{5}_{L2}' + \delta_{i3}\bar{5}_{L3}')10_{L(-)j}, \tag{2.29}$$

where we have assumed that $|m_1| \ll |m_2|$, i.e. the Higgs field surviving at a low energy scale is not $\bar{H}_2$, but $\bar{H}_1$.

The effective Yukawa interactions (2.28) give the fermion mass matrices

$$(M_e^*)_{ij} = \begin{cases} (\bar{U}_{11}^{(2)}\bar{U}_{33}^{(2)} - \bar{U}_{13}^{(2)}\bar{U}_{31}^{(2)})(Y_d)_{ij}v_d = \bar{U}_{22}^{(2)*}(Y_d)_{ij}v_d & \text{for } i=1, \\ \bar{U}_{11}^{(2)}(Y_d)_{ij}v_d & \text{for } i=2,3, \end{cases} \tag{2.30}$$

$$(M_d^\dagger)_{ij} = \begin{cases} (\bar{U}_{11}^{(2)}\bar{U}_{33}^{(3)} - \bar{U}_{13}^{(3)}\bar{U}_{31}^{(2)})(Y_d)_{ij}v_d & \text{for } i=1, \\ \bar{U}_{11}^{(2)}(Y_d)_{ij}v_d & \text{for } i=2,3, \end{cases} \tag{2.31}$$

where $v_d = \langle \bar{H}_1 \rangle$, and, in Eq. (2.30), we have used the general formula $U_{ik}U_{jl} - U_{il}U_{jk} = \varepsilon_{ijm}\varepsilon_{kln}U^*_{mn}$ for an arbitrary 3×3 unitary matrix U. Note that in the present model, the relation $M_d = M_e^T$ does not hold.

From the R-parity violating terms (2.29), we obtain coefficients $\lambda^{(2,2)}_{ijk}$, $\lambda^{(2,3)}_{ijk}$, $\lambda^{(3,2)}_{ijk}$ and $\lambda^{(3,3)}_{ijk}$, which are the coefficients of the interactions $(\nu_{L1}e_{Li} - e_{L1}\nu_{Li})e^c_{Rj}$, $(\nu_{L1}d^c_{Ri}d_{Lj} - e_{L1}d^c_{Ri}u_{Lj})$, $(d^c_{R1}e_{Li}u_{Lj} - d^c_{R1}\nu_{Li}d_{Lj})$, and $\varepsilon_{\alpha\beta\gamma}d^{c\alpha}_{R1}d^{c\beta}_{Ri}u^{c\gamma}_{Rj}$, respectively, as follows:

$$\lambda^{(2,2)}_{11j} = 0, \quad \lambda^{(2,2)}_{1ij} = \kappa(M^*_e)_{ij}/v_d \quad (i=2,3), \quad (2.32)$$

$$\lambda^{(2,3)}_{11j} = \frac{\kappa(M^\dagger_d)_{1j}/v_d}{1 - \xi\kappa\bar{U}^{(2)}_{31}/\bar{U}^{(3)}_{33}}, \quad \lambda^{(2,3)}_{1ij} = \kappa(M^\dagger_d)_{ij}/v_d \quad (i=2,3), \quad (2.33)$$

$$\lambda^{(3,2)}_{11j} = \frac{\xi\kappa(M^*_e)_{1j}/v_d}{1 - \kappa\bar{U}^{(2)}_{31}/\bar{U}^{(2)}_{33}}, \quad \lambda^{(3,2)}_{1ij} = \xi\kappa(M^*_e)_{ij}/v_d \quad (i=2,3), \quad (2.34)$$

$$\lambda^{(3,3)}_{11j} = 0, \quad \lambda^{(3,3)}_{1ij} = \xi\kappa(M^\dagger_d)_{ij}/v_d \quad (i=2,3), \quad (2.35)$$

where

$$\kappa = \frac{\bar{U}^{(2)}_{13}}{\bar{U}^{(2)}_{11}}, \quad \xi = \frac{\bar{U}^{(3)}_{13}}{\bar{U}^{(2)}_{13}}. \quad (2.36)$$

Note that the proton decay due to the exchange of squarks $\tilde{d}_i$ is forbidden in the limit of $\xi \to 0$, while the radiatively induced neutrino masses do not become zero even if $\xi \to 0$.

III. HOW TO SUPPRESS THE PROTON DECAY

First, we discuss the parameters in the doublet sector. We assume

$$(m^{(2)}_-)^2 \gg m^2_{SB} \gg (m^{(2)}_+)^2. \quad (3.1)$$

Then, we obtain the following approximate relations:

$$(m^{(2)}_1)^2 \simeq (m^{(2)}_+)^2 \left(1 - \frac{m^2_{SB}}{(m^{(2)}_-)^2}\cos^2\alpha^{(2)} \right) \sim M^2_W, \quad (3.2)$$

$$(m^{(2)}_2)^2 \simeq (m^{(2)}_-)^2 + m^2_{SB} \sim M^2_I, \quad (3.3)$$

$$\tan 2\theta^{(2)}_u = \frac{2m_{SB}m^{(2)}_+\cos\alpha^{(2)}}{(m^{(2)}_-)^2 + m^2_{SB} - (m^{(2)}_+)^2}$$

$$\simeq 2\frac{m_{SB}m^{(2)}_+}{(m^{(2)}_-)^2}\cos\alpha^{(2)}, \quad (3.4)$$

$$s^{(2)}_{13} \equiv \bar{U}^{(2)}_{13} = \frac{m_{SB}\sin\alpha^{(2)}}{\sqrt{(m^{(2)}_-)^2 + m^2_{SB}\sin^2\alpha^{(2)}}}$$

$$\simeq \frac{m_{SB}}{m^{(2)}_-}\sin\alpha^{(2)}, \quad (3.5)$$

$$s^{(2)}_{23} = -\sin\alpha^{(2)}, \quad c^{(2)}_{23} = \cos\alpha^{(2)}, \quad (3.6)$$

$$s^{(2)}_{12} \simeq 1 - \frac{1}{2}\left(\frac{m_{SB}}{m^{(2)}_-}\right)^2\cos^2\alpha^{(2)}, $$

$$c^{(2)}_{12} \simeq \frac{m_{SB}}{m^{(2)}_-}\cos\alpha^{(2)}. \quad (3.7)$$

Since the up-quark mass matrix M_u is given by $(M_u)_{ij} = c^{(2)}_u(Y_u)_{ij}v_u$, where $c^{(2)}_u = \cos\theta^{(2)}_u$ and $v_u = v\sin\beta$ ($v = 174$ GeV), the constraint [8] $\tan\beta > 1.5$ ($\sin\beta > 0.83$) in the conventional model from the perturbative calculability corresponds to the constraint

$$c^{(2)}_u\sin\beta > 0.83, \quad (3.8)$$

which is reasonably satisfied because the value of $c^{(2)}_u$ is given by

$$c^{(2)}_u \simeq \sqrt{1 - \left(\frac{m_{SB}m^{(2)}_+\cos\alpha^{(2)}}{(m^{(2)}_-)^2}\right)^2} \simeq 1. \quad (3.9)$$

On the other hand, since the down-quark mass matrix M_d is given by $(M_d)_{ij} = \bar{U}^{(2)}_{11}Y_{dij}v_d(i=2,3)$, where $v_d = v\cos\beta$ and

$$\bar{U}^{(2)}_{11} = c^{(2)}_{12}c^{(2)}_{13} \simeq \frac{m_{SB}}{m^{(2)}_-}\cos\alpha^{(2)}, \quad (3.10)$$

the constraint [8] $\tan\beta < 60$ (i.e. $\cos\beta > 0.017$) in the conventional model puts a constraint $(m_{SB}/m^{(2)}_-)\cos\alpha^{(2)} > 0.017/\cos\beta$, which leads to

$$\frac{m_{SB}}{m^{(2)}_-}\cos\alpha^{(2)} > 0.031, \quad (3.11)$$

where we have used the lower limit of $\tan\beta$, $\tan\beta \simeq 1.5$. A mass value of $\bar{H}_2$ smaller than $m^{(2)}_2 \sim 10^{13}$ GeV cannot be accepted because such a small value spoils the coincidence of the gauge coupling constants at $\mu = M_X$. From the relation (3.3), we must consider

$$m^{(2)}_- \simeq m^{(2)}_2 \gtrsim 10^{14} \text{ GeV}. \quad (3.12)$$

Therefore, a too-small value of m_{SB} is not acceptable in the present model,

$$m_{SB} \geq \frac{1}{\cos \alpha^{(2)}} \times 3 \times 10^{12} \ \text{GeV}. \qquad (3.13)$$

The parameter values in the triplet sector are sensitive to the proton decay. For example, the proton decay due to the exchange of squark $\tilde{d}$ is proportional to

$$\lambda_{1ij}^{(2,3)} \lambda_{1kl}^{(3,3)} \simeq \xi \kappa^2 \left(\frac{m_b}{v \cos \beta} \right)^2 |V_{ub}|^2, \qquad (3.14)$$

which must be smaller than $(M_{SUSY}/M_X)^2 \sim 10^{-26}$. Since $(m_b/v \cos \beta)^2 \sim 10^{-3}$, $|V_{ub}|^2 \sim 10^{-5}$ and

$$\xi = \frac{s_{13}^{(3)}}{s_{13}^{(2)}} \simeq \frac{m_-^{(2)} \sin \alpha^{(3)}}{m_-^{(3)} \sin \alpha^{(2)}} \sim \frac{M_I}{M_X} \frac{m_5/M_X}{m_5/M_W} \sim 10^{-16}, \qquad (3.15)$$

where we have used the relations (2.3) and the values $m_-^{(2)} \simeq M_I \sim 10^{14}$ GeV and $m_-^{(3)} \simeq M_X \sim 10^{16}$ GeV, we can estimate the value of Eq. (3.14) as

$$\lambda_{1ij}^{(2,3)} \lambda_{1kl}^{(3,3)} \sim \kappa^2 \times 10^{-24}. \qquad (3.16)$$

Since the parameter κ is given by

$$\kappa = \frac{U_{13}^{(2)}}{U_{11}^{(2)}} = \frac{\tan \theta_{13}^{(2)}}{c_{12}^{(2)}} \simeq \tan \alpha^{(2)} \simeq \frac{m_5}{m_+^{(2)}}, \qquad (3.17)$$

if we suppose $m_5 \sim 10$ GeV (i.e. $\kappa \sim 10^{-1}$), the proton decay due to the exchange of $\tilde{d}_3$ can barely be suppressed.

On the other hand, the proton decay [9] due to the dimension-5 operator is proportional to a factor

$$K \equiv \frac{1}{m_1^{(3)}} c_u^{(3)} \bar{U}_{11}^{(3)} + \frac{1}{m_2^{(3)}} s_u^{(3)} \bar{U}_{12}^{(3)}. \qquad (3.18)$$

The value of K takes a minimum at $m_-^{(3)} = m_+^{(3)}$. Therefore, we investigate the case

$$(m_+^{(3)})^2 = (m_-^{(3)})^2 \gg m_{SB}^2, \qquad (3.19)$$

which have already been assumed in the derivation of Eq. (3.15). Then, we can get the following approximate relations:

$$(m_1^{(3)})^2 \simeq (m_+^{(3)})^2 - m_{SB} m_+^{(3)},$$

$$(m_2^{(3)})^2 \simeq (m_+^{(3)})^2 + m_{SB} m_+^{(3)}, \qquad (3.20)$$

$$\tan 2\theta_u^{(3)} \simeq 2 \frac{m_+^{(3)}}{m_{SB}}, \quad \text{i.e.} \quad \cos 2\theta_u^{(3)} \simeq \frac{m_{SB}}{2m_+^{(3)}}, \qquad (3.21)$$

$$\bar{U}^{(3)} \simeq \begin{pmatrix} c_{12}^{(3)} & s_{12}^{(3)} & s_{13}^{(3)} \\ -s_{12}^{(3)} & c_{12}^{(3)} & s_{23}^{(3)} \\ s_{23}^{(3)} s_{12}^{(3)} & -s_{23}^{(3)} c_{12}^{(3)} & 1 \end{pmatrix}, \qquad (3.22)$$

where

$$\cos 2\theta_{12}^{(3)} \simeq \frac{m_{SB}}{2m_\pm^{(3)}}, \qquad (3.23)$$

$$s_{13}^{(3)} \simeq \frac{m_{SB}}{m_-^{(3)}} \sin \alpha^{(3)} \simeq \frac{m_{SB}}{m_-^{(3)}} \frac{m_5}{m_+^{(3)}} \sim 10^{-19}, \qquad (3.24)$$

$$s_{23}^{(3)} = -\sin \alpha^{(3)} \simeq -\frac{m_5}{m_+^{(3)}} \sim -10^{-15}. \qquad (3.25)$$

Therefore, the factor K is estimated as

$$K \simeq -\frac{m_{SB}}{(m_+^{(3)})^2} \sim -10^{-4} \frac{1}{M_X}. \qquad (3.26)$$

In order to suppress the proton decay due to the dimension-5 operator, it is better to take the value of m_{SB} as low as possible.

For example, the numerical values without approximation are as follows: for the input values

$$m_{SB} = 4 \times 10^{12} \ \text{GeV}, \quad m_5 = 2 \times 10^1 \ \text{GeV},$$

$$m_+^{(2)} = 2 \times 10^2 \ \text{GeV}, \quad m_-^{(2)} = 1 \times 10^{14} \ \text{GeV},$$

$$m_+^{(3)} = 5 \times 10^{16} \ \text{GeV}, \quad m_-^{(3)} = 5 \times 10^{16} \ \text{GeV}, \qquad (3.27)$$

$$\sin \alpha^{(2)} = 0.1, \quad \sin \alpha^{(3)} = 4 \times 10^{-16},$$

we obtain

$$m_1^{(2)} = -2 \times 10^2 \ \text{GeV}, \quad m_2^{(2)} = 1.0 \times 10^{14} \ \text{GeV}, \qquad (3.28)$$

$$U^{(2)} = \begin{pmatrix} 1 & 7.9 \times 10^{-14} \\ -7.9 \times 10^{-14} & 1 \end{pmatrix}, \qquad (3.29)$$

$$\bar{U}^{(2)} = \begin{pmatrix} 0.040 & 0.999 & 0.004 \\ -0.994 & 0.040 & -0.100 \\ -0.100 & 0.9 \times 10^{-16} & 0.995 \end{pmatrix}, \qquad (3.30)$$

$$m_1^{(3)} = -5.0 \times 10^{16} \ \text{GeV}, \quad m_2^{(3)} = 5.0 \times 10^{16} \ \text{GeV}, \qquad (3.31)$$

$$U^{(3)} = \begin{pmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{pmatrix}, \qquad (3.32)$$

$$\bar{U}^{(3)} = \begin{pmatrix} 0.707 & 0.707 & 3.2 \times 10^{-20} \\ -0.707 & 0.707 & -4.0 \times 10^{-16} \\ -2.8 \times 10^{-16} & 2.8 \times 10^{-16} & 1.000 \end{pmatrix}, \qquad (3.33)$$

$$\bar{U}_{11}^{(2)} = 0.040, \quad K = -3.2 \times 10^{-5}/M_X,$$

$$\kappa = 0.10, \quad \xi = 0.8 \times 10^{-17}. \qquad (3.34)$$

Therefore, these parameter values are harmless for the proton decay.

IV. RADIATIVELY INDUCED NEUTRINO MASS MATRIX

In a SUSY GUT scenario, there are many origins of the neutrino mass generations [10]. For example, the sneutrinos $\tilde{\nu}_{iL}$ can have VEVs $\langle \tilde{\nu} \rangle \neq 0$, and the neutrinos ν_{Li} thereby acquire their masses [11]. In the present model, there is an R-parity violating bilinear term $\bar{5}_{L(+)}H_{(+)}$, while there is no $\bar{H}_{(-)}H_{(+)}$ term (the so-called μ term). In the physical field basis [the basis on which the mass matrix (2.5) is diagonal], the so-called μ term, $m_1 \bar{H}_1 H_1$, appears, while the $\bar{5}_L H_1$ term is absent. Therefore, in the present model, the sneutrinos cannot have VEVs $\langle \tilde{\nu}_i \rangle$ at tree level. The VEVs $\langle \tilde{\nu}_i \rangle \neq 0$ will appear only through the renormalization group equation (RGE) effect. The contribution highly depends on an explicit model of the SUSY symmetry breaking. Since the purpose of the present paper is to investigate a general structure of the radiative neutrino masses, for the moment, we confine ourselves to discussing possible forms of the radiative neutrino mass matrix .

The radiative neutrino mass matrix M_ν^{rad} is given by

$$M_\nu^{rad} = M_\nu^e + M_\nu^d, \tag{4.1}$$

where M_ν^e is generated by the interactions $\nu_L e_L \tilde{e}_R^c$ and $\nu_L \tilde{e}_L e_R^c$ (i.e. by the charged lepton loop) and M_ν^d is generated by $\nu_L d_R^c \tilde{d}_L$ and $\nu_L \tilde{d}_R^c d_L$ (i.e. by the down-quark loop). We assume that the contributions from Zee-type diagrams due to $\bar{H}^+ \leftrightarrow \tilde{e}_R^+$ mixing is negligibly small because the term $\bar{H}\bar{H}10_L$ must be not $\bar{H}_1\bar{H}_1 10_L$, but $\bar{H}_1 \bar{H}_2 10_L$ (recall that only the field $\bar{H}_1$ has the VEV in the present model).

We consider the radiative diagram with $(\nu_L)_j \rightarrow (e_R)_l + (\tilde{e}_L^c)_n$ and $(e_L)_k + (\tilde{e}_L^c)_m \rightarrow (\nu_L^c)_i$ (see Fig. 1). The contributions $(M_\nu^e)_{ij}$ from the charged lepton loop are, except for the common factors, given as follows:

$$(M_\nu^e)_{ij} = (\lambda_{1km}\delta_{i1} - \lambda_{1im}\delta_{k1})(\lambda_{1jl}\delta_{n1} - \lambda_{1nl}\delta_{j1})$$
$$\times (M_e)_{kl}(\tilde{M}_{eLR}^2)_{nm} + (i \leftrightarrow j), \tag{4.2}$$

where M_e and $\tilde{M}_{eLR}^2$ are charged-lepton and charged-slepton-LR mass matrices, respectively. Here and hereafter, we will drop the common factor in $(M_\nu^{rad})_{ij}$, because we have an interest only in the relative structure of the matrix elements

$(M_\nu^{rad})_{ij}$. Since, as usual, we assume that the structure of $\tilde{M}_{eLR}^2$ is proportional to that of M_e, we obtain

$$(M_\nu^e)_{ij} = \lambda_{1im}\lambda_{1jl}(M_e)_{1l}(M_e)_{1m}$$
$$+ \delta_{i1}\delta_{j1}\lambda_{1km}\lambda_{1nl}(M_e)_{kl}(M_e)_{nm}$$
$$- \delta_{i1}\lambda_{1jl}\lambda_{1km}(M_e)_{1m}(M_e)_{kl}$$
$$- \delta_{j1}\lambda_{1il}\lambda_{1km}(M_e)_{1m}(M_e)_{kl}. \tag{4.3}$$

Since $\lambda_{1ij}^e \equiv \lambda_{1ij}^{(2,2)} = \kappa(1-\delta_{i1})(M_e^\dagger)_{ji}$ from the expression (2.32), we obtain the contribution from the charged lepton loop:

$$M_\nu^e = H_e^T S_1 H_e - S_1 H_e H_e - H_e^T H_e^T S_1 + S_1 \text{Tr}(H_e H_e), \tag{4.4}$$

where we have dropped the common factor κ, and the Hermitian matrix H_e and the rank-1 matrix S_1 are defined by

$$H_e = M_e M_e^\dagger, \tag{4.5}$$

$$S_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{4.6}$$

Similarly, we can obtain the contributions from the down-quark loop. From the expression (2.33), we denote $\lambda_{1ij}^d \equiv \lambda_{1ij}^{(2,3)}$ as

$$\lambda_{1ij}^d = \kappa[\rho\delta_{1i} + (1-\delta_{i1})](M_d^\dagger)_{ij}, \tag{4.7}$$

where

$$\rho = \frac{1}{1 - \xi\kappa U_{31}^{(2)}/U_{33}^{(3)}} \simeq 1. \tag{4.8}$$

Then, we obtain

$$M_\nu^d = H_d S_1 H_d^T - S_1 H_d^T H_d^T - H_d H_d S_1 + S_1 \text{Tr}(H_d H_d), \tag{4.9}$$

where

$$H_d = M_d^\dagger M_d. \tag{4.10}$$

Note that the result (4.9) is independent of the value of ρ.

The field $\bar{5}_{L(+)1}$ defined in Eq. (2.2) does not mean the observed first-generation field $(d^c, \nu, e)_L$ (and its SUSY
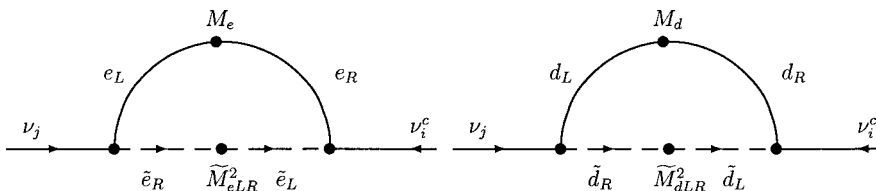

FIG. 1. Radiative generation of neutrino Majorana mass.

partner). The forms of M_ν^e and M_ν^d on the general basis are given by

$$M_\nu^e = H_e^T S H_e - S H_e H_e - H_e^T H_e^T S + S \, \mathrm{Tr}(H_e H_e), \tag{4.11}$$

$$M_\nu^d = H_d S H_d^T - S H_d^T H_d^T - H_d H_d S + S \, \mathrm{Tr}(H_d H_d), \tag{4.12}$$

where S is an arbitrary rank-1 matrix $S = U_5^T S_1 U_5$, which is given by the rebasing $\bar{5}_i \to \bar{5}_i' = (U_5^\dagger \bar{5})_i$.

It is convenient to investigate the form M_ν^{rad} on the basis on which the charged lepton mass matrix M_e is diagonal: $H_e = D_e^2 = \mathrm{diag}(m_{e1}^2, m_{e2}^2, m_{e3}^2) \equiv \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$. Then, the matrix M_ν^e is given by

$$M_\nu^e = \begin{pmatrix} S_{11}(m_{e2}^4 + m_{e3}^4) & S_{12}(m_{e3}^4 + m_{e1}^2 m_{e2}^2) & S_{13}(m_{e2}^4 + m_{e1}^2 m_{e3}^2) \\ S_{21}(m_{e3}^4 + m_{e1}^2 m_{e2}^2) & S_{22}(m_{e3}^4 + m_{e1}^4) & S_{23}(m_{e1}^4 + m_{e2}^2 m_{e3}^2) \\ S_{31}(m_{e2}^4 + m_{e1}^2 m_{e3}^2) & S_{32}(m_{e1}^4 + m_{e2}^2 m_{e3}^2) & S_{33}(m_{e1}^4 + m_{e2}^4) \end{pmatrix}. \tag{4.13}$$

On the basis with $H_e = D_e^2$, since the matrix H_d can be expressed as $H_d = U D_d^2 U^T$, where $U \equiv U_R^d$, the contribution M_ν^d is expressed as

$$M_\nu^d = U(M_\nu^d)' U^T, \tag{4.14}$$

$$(M_\nu^d)' = D_d^2 S' D_d^2 - S' D_d^4 - D_d^4 S' - S' \mathrm{Tr} \, D_d^4, \tag{4.15}$$

$$S' = U^\dagger S U^*. \tag{4.16}$$

Here, $(M_\nu^d)'$ is again given by an expression similar to Eq. (4.13),

$$(M_\nu^d)' = \begin{pmatrix} S_{11}'(m_{d2}^4 + m_{d3}^4) & S_{12}'(m_{d3}^4 + m_{d1}^2 m_{d2}^2) & S_{13}'(m_{d2}^4 + m_{d1}^2 m_{d3}^2) \\ S_{21}'(m_{d3}^4 + m_{d1}^2 m_{d2}^2) & S_{22}'(m_{d3}^4 + m_{d1}^4) & S_{23}'(m_{d1}^4 + m_{d2}^2 m_{d3}^2) \\ S_{31}'(m_{d2}^4 + m_{d1}^2 m_{d3}^2) & S_{32}'(m_{d1}^4 + m_{d2}^2 m_{d3}^2) & S_{33}'(m_{d1}^4 + m_{d2}^4) \end{pmatrix}. \tag{4.17}$$

The mass ratios $m_s^2/m_b^2 \simeq 7.02 \times 10^{-4}$ and $m_\mu^2/m_\tau^2 \simeq 3.43 \times 10^{-3}$ at $\mu = M_X$ are negligibly small compared with $\Delta m_{solar}^2 / \Delta m_{atm}^2 \sim 10^{-2}$, so that when we neglect the terms with m_{e1}^2/m_{e3}^2, m_{e2}^2/m_{e3}^2, m_{d1}^2/m_{d3}^2 and m_{d2}^2/m_{d3}^2, we can approximate Eqs. (4.13) and (4.17) as

$$M_\nu^e \simeq m_\tau^4 \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} = m_\tau^4 P S P,$$

$$(M_\nu^d)' \simeq m_b^4 \begin{pmatrix} S_{11}' & S_{12}' & 0 \\ S_{21}' & S_{22}' & 0 \\ 0 & 0 & 0 \end{pmatrix} = m_b^4 P S' P, \tag{4.18}$$

where P is defined as

$$P = \mathrm{diag}(1,1,0). \tag{4.19}$$

Therefore, we can express the neutrino mass matrix M_ν^{rad} in the following form:

$$M_\nu^{rad} = m_0(P S P + k U \cdot P U^\dagger S U^* P \cdot U^T), \tag{4.20}$$

where k is given by $k \simeq (m_b/m_\tau)^2$ and m_0 will be given later [in Eq. (4.22)]. The matrix S is a rank-1 matrix, so that PSP,

$U^\dagger S U^*$, and $U(P U^\dagger S U^* P) U^T$ are also rank-1 matrices. In other words, the radiative neutrino mass matrix M_ν^{rad} has the form which is described by two rank-1 matrices:

$$M_\nu^{rad} = m_0 \begin{pmatrix} g_1^2 & g_1 g_2 & 0 \\ g_2 g_1 & g_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ m_0 k \begin{pmatrix} f_1^2 & f_1 f_2 & f_1 f_3 \\ f_2 f_1 & f_2^2 & f_2 f_3 \\ f_3 f_1 & f_3 f_2 & f_3^2 \end{pmatrix}. \tag{4.21}$$

So far, we have not discussed the absolute magnitude of the neutrino mass matrix M_ν^{rad}. When we assume $m^2(\tilde{e}_R) \equiv m^2(\tilde{e}_{R3}) \simeq m^2(\tilde{e}_{R2}) \simeq m^2(\tilde{e}_{R1})$ and $m^2(\tilde{e}_L) \equiv m^2(\tilde{e}_{L3}) \simeq m^2(\tilde{e}_{L2}) \simeq m^2(\tilde{e}_{L1})$ and the rank-1 matrix S is normalized as $\mathrm{Tr}(S S^\dagger) = 1$, the coefficient m_0 in the expression (4.20) is given by

$$m_0 = \frac{1}{16\pi^2} \kappa^2 \frac{m_1^{(2)} m_\tau^4}{v^2} F(m^2(\tilde{e}_R), m^2(\tilde{e}_L)), \tag{4.22}$$

where

$$F(m_R^2, m_L^2) = \frac{1}{m_R^2 - m_L^2} \ln \frac{m_R^2}{m_L^2}. \qquad (4.23)$$

If $F(m^2(\tilde{e}_R), m^2(\tilde{e}_L)) \simeq F(m^2(\tilde{d}_R), m^2(\tilde{d}_L))$, the factor k is given by $k \simeq (m_b/m_\tau)^4 = 8.6$. However, in the present paper, we regard k as a free parameter. By using $1/16\pi^2 = 6.33 \times 10^{-3}$, $m_1^{(2)} \equiv m(H_1^{(2)}) = 2 \times 10^2$ GeV, $m_\tau(m_Z) = 1.75$ GeV, $v = 174$ GeV and $\tan\beta = 1.5$, we obtain

$$m_0 \simeq 1.9 \kappa^2 F \text{ eV}, \qquad (4.24)$$

where F is the value of $F(m^2(\tilde{e}_R), m^2(\tilde{e}_L))$ in the unit of TeV. If the neutrino mass matrix M_ν is dominated by the radiative mass terms M_ν^{rad} and we wish that the largest one of $m_{\nu i}$ is of the order of $\sqrt{\Delta m_{atm}^2} \simeq 0.05$ eV, the value $\kappa \sim 10^{-1}$ is favorable.

V. POSSIBLE FORM OF M_ν

The neutrino mass matrix in the present model is given by

$$M_\nu = M_\nu^{rad} + M_{\tilde{\nu}}. \qquad (5.1)$$

The contribution $M_{\tilde{\nu}}$ from $\langle \tilde{\nu}_i \rangle \neq 0$ is estimated as follows. Since the mass matrix for $(\nu_1, \nu_2, \nu_3, \tilde{W}^0)$ (except for the radiative masses) is given by

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2}gv_1 \\ 0 & 0 & 0 & \frac{1}{2}gv_2 \\ 0 & 0 & 0 & \frac{1}{2}gv_3 \\ \frac{1}{2}gv_1 & \frac{1}{2}gv_2 & \frac{1}{2}gv_3 & M_{\tilde{W}} \end{pmatrix}, \qquad (5.2)$$

where, for simplicity, we have dropped the elements for $\tilde{B}^0$, we obtain

$$M_{\tilde{\nu}} \simeq -\frac{1}{4}g^2 \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} (M_{\tilde{W}})^{-1} (v_1 \ v_2 \ v_3)$$

$$= -\frac{g^2}{4M_{\tilde{W}}} \begin{pmatrix} v_1^2 & v_1v_2 & v_1v_3 \\ v_1v_2 & v_2^2 & v_2v_3 \\ v_1v_3 & v_2v_3 & v_3^2 \end{pmatrix}, \qquad (5.3)$$

under the seesaw approximation. Note that the matrix $M_{\tilde{\nu}}$ is a rank-1 matrix.

As noted in Sec. IV, the sneutrinos in the present model cannot have nonzero VEVs at the tree level. The VEVs $\langle \tilde{\nu}_i \rangle \neq 0$ can appear only through the RGE effect. Therefore, the magnitudes of $v_i \equiv \langle \tilde{\nu}_i \rangle$ are highly dependent on a model of the SUSY breaking and the RGE effect. In the previous section, we have estimate the magnitudes of the radiative

masses, i.e. in Eq. (4.24). In the present paper, we are interested only in the form of the neutrino mass matrix M_ν. Therefore, we do not discuss the explicit symmetry breaking mechanism and the absolute magnitudes of $\langle \tilde{\nu}_i \rangle$.

Since the form of the flavor symmetry breaking in the present model is described by the rank-1 matrix S, it is likely that the structure of the rank-1 matrix $M_{\tilde{\nu}}$ is also given by the matrix S, i.e. $M_{\tilde{\nu}} = m_0 rS$, where the factor r denotes a relative ratio of $M_{\tilde{\nu}}$ to M_ν^{rad}. Then, the neutrino mass matrix M_ν is expressed as

$$M_\nu = m_0(rS + PSP + kU \cdot PU^\dagger SU^* P \cdot U^T), \qquad (5.4)$$

and

$$M_\nu = m_0 \begin{pmatrix} g_1^2(1+r) & g_1g_2(1+r) & g_1g_3r \\ g_2g_1(1+r) & g_2^2(1+r) & g_2g_3r \\ g_3g_1r & g_2g_3r & g_3^2r \end{pmatrix}$$

$$+ m_0 k \begin{pmatrix} f_1^2 & f_1f_2 & f_1f_3 \\ f_2f_1 & f_2^2 & f_2f_3 \\ f_3f_1 & f_3f_2 & f_3^2 \end{pmatrix} \qquad (5.5)$$

correspondingly to the expressions (4.20) and (4.21), respectively.

It is interesting to consider a case that the neutrino mass matrix M_ν is dominated by the radiative masses M_ν^{rad}. Or, we are also interested in a case with S which satisfies the relation $PSP = S$ [a case with $g_3 = 0$ in the expression (5.5)]. Then, since the neutrino mass matrix M_ν is still given by the form (4.20) [(4.21)], i.e. by the sum of two rank-1 matrices, it gives $\det M_\nu = 0$, so that one of the eigenvalues of M_ν is zero. Therefore, we can consider the following two cases:

$$m_{\nu 1} = 0, \ m_{\nu 2} = m_0\varepsilon, \ m_{\nu 3} = m_0, \qquad (5.6)$$

for a normal hierarchy model, and

$$m_{\nu 1} = \frac{1}{2}m_0(1-\varepsilon^2),$$

$$m_{\nu 2} = \frac{1}{2}m_0(1+\varepsilon^2),$$

$$m_{\nu 3} = 0, \qquad (5.7)$$

for an inverse hierarchy model, where

$$\varepsilon \simeq \sqrt{|R|}, \qquad (5.8)$$

$$R \equiv \frac{\Delta m_{solar}^2}{\Delta m_{atm}^2}. \qquad (5.9)$$

The inverse hierarchy case with the eigenvalues $m_{\nu 1} = -(1/2)m_0(1-\varepsilon^2)$, $m_{\nu 2} = (1/2)m_0(1+\varepsilon^2)$ and $m_{\nu 3} = 0$ is ruled out in the present model, because the case gives $\Sigma_i m_{\nu i} = m_0\varepsilon^2$, while the mass matrix (4.21) gives $\text{Tr} M_\nu = m_0(\Sigma_i g_i^2 + k\Sigma_i f_i^2)$, so that k must be negative to give a

small value of $\mathrm{Tr}\, M_\nu$. However, it is unlikely that the contributions M_ν^e and M_ν^d have opposite signs to each other.

In Appendix B, we will show that the form (4.20) can always contain parameter values which lead to a nearly bi-maximal mixing

$$
U_\nu = \begin{pmatrix}
c_{12}\sqrt{1-\varepsilon_{13}^2} & s_{12}\sqrt{1-\varepsilon_{13}^2} & \varepsilon_{13} \\[2mm]
-\dfrac{s_{12}-c_{12}\varepsilon_{13}}{\sqrt{2}} & \dfrac{c_{12}+s_{12}\varepsilon_{13}}{\sqrt{2}} & -\dfrac{\sqrt{1-\varepsilon_{13}^2}}{\sqrt{2}} \\[3mm]
-\dfrac{s_{12}+c_{12}\varepsilon_{13}}{\sqrt{2}} & \dfrac{c_{12}-s_{12}\varepsilon_{13}}{\sqrt{2}} & \dfrac{\sqrt{1-\varepsilon_{13}^2}}{\sqrt{2}}
\end{pmatrix},
\tag{5.10}
$$

where $c_{12}\sim s_{13}$ and $\varepsilon_{12}^2\ll 1$, and to the ratio $R\equiv\Delta m_{21}^2/\Delta m_{32}^2\sim 10^{-2}$. However, even if we assume the dominance of M_ν^{rad} in M_ν, since the expression (4.20) has many free parameters, we cannot give any predictions for the neutrino phenomenology unless we put a further ansatz for the flavor symmetry.

In Appendix C, we demonstrate a simple example of S which satisfies the relation $PSP=S$. The model can lead to a successful description of the observed neutrino masses and mixings [12–14]. However, this is merely one of the examples. The systematical search for the explicit form M_ν and a possible flavor symmetry will be a future task.

VI. SUMMARY

In conclusion, within the framework of an SU(5) SUSY GUT model, we have proposed a mechanism which effectively induces R-parity-violating terms at $\mu<m_{SB}$. In our model, those terms with lepton number violation are large enough to generate neutrino Majorana masses while those with baryon number violation are strongly suppressed so that the experimental bound of proton decay is evaded. This is related with doublet-triplet splitting. We have matter fields $\bar{5}_{L(+)}+10_{L(-)}$ and two types of Higgs fields $H_{(\pm)}$ and $\bar{H}_{(\pm)}$, where $(\pm)$ denote the transformation properties under a discrete symmetry Z_2. The Higgs fields $H_{(+)}$ and $\bar{H}_{(-)}$ couple to $10_{L(-)}10_{L(-)}$ and $\bar{5}_{L(+)}10_{L(-)}$, respectively, to make the Yukawa interactions. The Z_2 symmetry is only broken by the μ term, $m_{SB}\bar{H}_{(+)}H_{(-)}$, so that the $\bar{H}_{(-)}\leftrightarrow\bar{5}_{(+)}$ mixing is effectively induced at $\mu<m_{SB}$. Because of the heaviness of the color triplet components of the Higgs fields, the mixing is sizable in the $SU(2)_L$ doublet sector, while it is negligibly small in the $SU(3)_c$ triplet sector.

Whether the model is harmless or not for proton decay is highly sensitive to the choice of the parameter values, especially, m_{SB} and m_5. A smaller value of m_{SB} gives a lighter mass for the massive Higgs fields H_2 (another one, H_1, corresponds to the Higgs field in the conventional model), so that the case spoils the unification of the gauge coupling constants at $\mu=M_X$. On the other hand, a large value of m_{SB} induces the proton decay due to the dimension-5 operator. We have taken $m_{SB}\sim 10^{14}$ GeV. Also, a large value of m_5

induces the proton decay due to the exchange of squark $\tilde{d}$. We have taken $m_5\sim 10^1$ GeV. Those parameter values can give a reasonable magnitude of the neutrino mass. However, the choice of such a small m_5 gives a small mixing between $\bar{H}_{(+)}$ and $\bar{5}_{(+)}$, so that the case gives $\bar{U}_{22}^{(2)}\simeq\bar{U}_{11}^{(2)}$ and $\bar{U}_{11}^{(2)}\bar{U}_{33}^{(2)}-\bar{U}_{13}^{(2)}\bar{U}_{31}^{(2)}\simeq\bar{U}_{11}^{(2)}\bar{U}_{33}^{(3)}-\bar{U}_{13}^{(3)}\bar{U}_{31}^{(2)}$. Therefore, the case with $|\alpha^{(2)}|\ll 1$ cannot give a sizable deviation from $M_d^T=M_e$. However, this is critical for each parameter value. The details are dependent on the explicit model, i.e. on the choice of the forms S and $U\equiv U_R^d$. A further careful study based on an explicit model will be required.

Anyhow, if the present scenario is working, the proton decay will be observed in the near future, because possible parameter values are in critical ranges for the proton decay in order to explain the quark and lepton (charged lepton and neutrino) masses and mixings.

The present model leads to a radiatively induced neutrino mass matrix M_ν^{rad} which is given by the sum of two rank-1 matrices as shown in Eq. (4.20). The "two" is originated in the two contributions from charged lepton loop and down-quark loop. The reason that each contribution takes a rank-1 matrix form is because the mixing of the matter fields $\bar{5}_{L(+)i}$ ($i=1,2,3$) with the Higgs field $\bar{H}_{(-)}$ takes place only for a linear combination $\Sigma b_i\bar{5}_{L(+)i}$. The contribution $M_{\tilde\nu}$ from $\langle\tilde\nu_i\rangle\neq 0$ is also expressed by a rank-1 matrix. Then, the general form of M_ν is given by the expression (5.4) [(5.5)].

We have investigated an interesting case that the form of $M_{\tilde\nu}$ is given by a rank-1 matrix S, especially, the case with $PSP=P$. Then, the neutrino mass matrix M_ν is given by the form (4.20). Since the two rank-1 matrix model generally gives $\det M_\nu=0$, one of the eigenvalues has to be zero, so that we can consider two types of the mass hierarchy: the normal hierarchy with $D_\nu=m_0\mathrm{diag}(0,\varepsilon,1)$, and the inverse hierarchy with $D_\nu=(1/2)m_0\mathrm{diag}(1-\varepsilon^2,1+\varepsilon^2,0)$, where $\varepsilon^2\simeq\Delta m_{solar}^2/\Delta m_{atm}^2$. The case of the inverse hierarchy with $D_\nu=(1/2)m_0\mathrm{diag}(\varepsilon^2-1,\varepsilon^2+1,0)$ is ruled out. However, even if we assume that the observed neutrino masses and mixings are dominantly described by the radiative neutrino mass matrix (4.20) (or S satisfies $PSP=S$), we cannot yet give an explicit predictions unless we assume a further ansatz for the flavor symmetry, because we have many free parameters in the rank-1 matrix S and the unitary matrix $U\equiv U_R^d$. For a flavor symmetry in the neutrino mass matrix M_ν, we have known that a $2\leftrightarrow 3$ permutation symmetry is promising [15]. For example, a successful example given in Appendix C satisfies the $2\leftrightarrow 3$ symmetry. Moreover, a possibility that the $2\leftrightarrow 3$ symmetry can be applicable to the unified description of quark and charged lepton mass matrices has been pointed out [16]. However, since the purpose of the present paper is to give an R-parity violation mechanism [and the radiative neutrino mass matrix M_ν^{rad} within the framework of an SU(5) SUSY GUT without any concern for proton decay], we have not discussed the details further. It will be our next task to determine what flavor symmetry is reasonable.

Nevertheless, the present model will be worth noticing. In the present model, the coupling constants λ_{ijk} of $\nu_{Li}e_{Lj}e_{Rk}^c$

and $\nu_{Li}d^c_{Rj}d_{Lk}$ are proportional to the mass matrices $(M^*_e)_{jk}$ and $(M^\dagger_d)_{jk}$, respectively. The model will give fruitful phenomenology in flavor violating processes.

APPENDIX A: GENERAL FORM OF THE MASS MATRIX M

The Z_2 symmetry can softly be violated not only by the terms $\bar{H}_{(+)}H_{(-)}$, but also by terms $\bar{H}_{(-)}H_{(+)}$ and $\bar{5}_{L(+)i}H_{(-)}$. The mass matrix M given in Eq. (2.5) is generally represented by

$$M = \begin{pmatrix} m^{-+}_{SB} & m_- \\ m_+c_\alpha & m^{+-}_{SB}c_\beta \\ m_+s_\alpha & m^{+-}_{SB}s_\beta \end{pmatrix}, \tag{A1}$$

where $s_\alpha = \sin\alpha$, $c_\alpha = \cos\alpha$, and so on.

When we define a rotation

$$R_\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\beta & -s_\beta \\ 0 & s_\beta & c_\beta \end{pmatrix}, \tag{A2}$$

we obtain

$$R^T_\beta M = \begin{pmatrix} m^{-+}_{SB} & m_- \\ m_+\cos(\alpha-\beta) & m^{+-}_{SB} \\ m_+\sin(\alpha-\beta) & 0 \end{pmatrix}. \tag{A3}$$

Therefore, the general form (A1) can always be reduced into the form with $M_{32}=0$. Of course, the mixing angle between $H_{(+)}$ and $\bar{5}_{(+)1}$ in the model with $M_{32}=0$ is modified by the parameter of $\bar{5}_{L(+)}H_{(-)}$. However, it is not essential in the present model.

On the other hand, it is essential whether the Z_2 symmetry is broken by $\bar{H}_{(+)}H_{(-)}$ or $\bar{H}_{(-)}H_{(+)}$. First, let us see the case where the symmetry is broken only by $\bar{H}_{(-)}H_{(+)}$,

$$M = \begin{pmatrix} m_{SB} & m_- \\ m_+c_\alpha & 0 \\ m_+s_\alpha & 0 \end{pmatrix}. \tag{A4}$$

When we define

$$R_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{pmatrix}, \tag{A5}$$

we obtain

$$R^T_\alpha M = \begin{pmatrix} m_{SB} & m_- \\ m_+ & 0 \\ 0 & 0 \end{pmatrix}. \tag{A6}$$

The mixing matrix $\bar{U}$ among $(\bar{H}_{(-)},\bar{H}_{(+)},\bar{5}_{(+)1})$ is given by

$$\bar{U} = R_\alpha R_\theta = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\alpha s_\theta & c_\alpha c_\theta & -s_\alpha \\ -s_\alpha s_\theta & s_\alpha c_\theta & c_\alpha \end{pmatrix}, \tag{A7}$$

where

$$R_\theta = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A8}$$

with

$$\tan 2\theta = \frac{2m_{SB}m_+}{m^2_+ - m^2_- - m^2_{SB}}, \tag{A9}$$

because of

$$R^T_\alpha MM^T R_\alpha = \begin{pmatrix} m^2_{SB}+m^2_- & m_{SB}m_+ & 0 \\ m_{SB}m_+ & m^2_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{A10}$$

As we have shown in Eqs. (2.32)–(2.35), the coefficients λ_{1ij} of the R-parity violating terms $\bar{5}_{L1}\bar{5}_{Li}10_{L(-)j}$ are proportional to the factor $\kappa = \bar{U}^{(2)}_{13}/\bar{U}^{(2)}_{11}$. As seen in Eq. (A7), the case (A4) leads to $\bar{U}_{13}=0$, so that we cannot obtain the effective R-parity violating terms $\bar{5}_L\bar{5}_L10_{L(-)}$.

Of course, although we can obtain the effective R-parity violating terms in the general case with $M_{11}\neq0$ and $M_{22}\neq0$, the essential term to derive the effective R-parity violating term is not $\bar{H}_{(-)}H_{(+)}$, but $\bar{H}_{(+)}H_{(-)}$. In the present paper, in order to make the essential line of the scenario clear, we have confined ourselves to investigating only the case with $M_{22}\neq0$. Also note that the case without the $\bar{H}_{(-)}H_{(+)}$ term leads to $\langle\tilde{\nu}_i\rangle=0$ at tree level.

APPENDIX B: TWO RANK-1 MATRIX MODEL AND NEARLY BIMAXIMAL MIXING

In this appendix, we investigate the constraint on the two rank-1 matrix model with the form

$$M_\nu = \begin{pmatrix} f^2_1 & f_1f_2 & f_1f_3 \\ f_2f_1 & f^2_2 & f_2f_3 \\ f_3f_1 & f_3f_2 & f^2_3 \end{pmatrix} + \begin{pmatrix} g^2_1 & g_1g_2 & 0 \\ g_2g_1 & g^2_2 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\tag{B1}$$

which leads to a nearly bimaximal mixing

$$U_\nu = \begin{pmatrix} c_{12}\sqrt{1-\varepsilon_{13}^2} & s_{12}\sqrt{1-\varepsilon_{13}^2} & \varepsilon_{13} \\ -\dfrac{s_{12}-c_{12}\varepsilon_{13}}{\sqrt{2}} & \dfrac{c_{12}+s_{12}\varepsilon_{13}}{\sqrt{2}} & -\dfrac{\sqrt{1-\varepsilon_{13}^2}}{\sqrt{2}} \\ -\dfrac{s_{12}+c_{12}\varepsilon_{13}}{\sqrt{2}} & \dfrac{c_{12}-s_{12}\varepsilon_{13}}{\sqrt{2}} & \dfrac{\sqrt{1-\varepsilon_{13}^2}}{\sqrt{2}} \end{pmatrix}.$$

(B2)

First, we investigate a general form of M_ν which gives the neutrino mixing (B2) as follows:

$$M_\nu = U_\nu D_\nu U_\nu^T \equiv \begin{pmatrix} a & d_2 & d_3 \\ d_2 & b_2 & c \\ d_3 & c & b_3 \end{pmatrix},$$

(B3)

where $D_\nu \equiv \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, and

$$a = c_{12}^2(1-\varepsilon_{13}^2)m_{\nu 1} + s_{12}^2(1-\varepsilon_{13}^2)m_{\nu 2} + \varepsilon_{13}^2 m_{\nu 3},$$

(B4)

$$b_2 = \frac{1}{2}[(s_{12}-c_{12}\varepsilon_{13})^2 m_{\nu 1}$$
$$+ (c_{12}+s_{12}\varepsilon_{13})^2 m_{\nu 2} + (1-\varepsilon_{13}^2)m_{\nu 3}],$$

(B5)

$$b_3 = \frac{1}{2}[(s_{12}+c_{12}\varepsilon_{13})^2 m_{\nu 1}$$
$$+ (c_{12}-s_{12}\varepsilon_{13})^2 m_{\nu 2} + (1-\varepsilon_{13}^2)m_{\nu 3}],$$

(B6)

$$c = \frac{1}{2}[(s_{12}^2 - c_{12}^2\varepsilon_{13}^2)m_{\nu 1}$$
$$+ (c_{12}^2 - s_{12}^2\varepsilon_{13}^2)m_{\nu 2} - (1-\varepsilon_{13}^2)m_{\nu 3}],$$

(B7)

$$d_2 = -\frac{1}{\sqrt{2}}\sqrt{1-\varepsilon_{13}^2}[c_{12}(s_{12}-c_{12}\varepsilon_{13})m_{\nu 1}$$
$$+ s_{12}(c_{12}+s_{12}\varepsilon_{13})m_{\nu 2} + \varepsilon_{13}m_{\nu 3}],$$

(B8)

$$d_3 = -\frac{1}{\sqrt{2}}\sqrt{1-\varepsilon_{13}^2}[c_{12}(s_{12}+c_{12}\varepsilon_{13})m_{\nu 1}$$
$$+ s_{12}(c_{12}-s_{12}\varepsilon_{13})m_{\nu 2} + \varepsilon_{13}m_{\nu 3}].$$

(B9)

Next, we rewrite the expression (B3) into the expression (B1):

$$M_\nu = \begin{pmatrix} \alpha & \delta & d_3 \\ \delta & \beta & c \\ d_3 & c & b_3 \end{pmatrix} + \begin{pmatrix} a-\alpha & d_2-\delta & 0 \\ d_2-\delta & b_2-\beta & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(B10)

where α, β and δ must satisfy the relations

$$\beta = \frac{c^2}{b_3}, \quad \alpha = \frac{d_3^2}{b_3}, \quad \delta = \pm\frac{cd_3}{b_3},$$

(B11)

since the first term of Eq. (B10) is a rank-1 matrix. In order that the second term is a rank-1 matrix, the following relation must be satisfied:

$$(d_2-\delta)^2 = (a-\alpha)(b_2-\beta),$$

(B12)

which leads to the constraint

$$0 = a(b_2b_3-c^2) - (b_2d_3^2 + b_3d_2^2 - 2cd_2d_3)$$
$$= m_{\nu 1}m_{\nu 2}m_{\nu 3}[c_{12}^2 - s_{12}^2(1-2\varepsilon_{13}^2)],$$

(B13)

for the case $\delta = +cd_3/b_3$. In the two rank-1 matrix model, since $m_{\nu 1}m_{\nu 2}m_{\nu 3}=0$, the constraint (B13) [therefore, Eq. (B12)] is always satisfied. This means that the two rank-1 matrix model (B1) always has the parameter values which give the nearly bimaximal mixing (B2).

APPENDIX C: AN EXAMPLE OF M_ν

We demonstrate an example of the mass matrix (4.20). We take a simple form of the rank-1 matrix S which satisfies $PSP = P$,

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(C1)

and assume the following form of the neutrino mass matrix M_ν:

$$M_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + km_0 \begin{pmatrix} a^2 & a & a \\ a & 1 & 1 \\ a & 1 & 1 \end{pmatrix}.$$

(C2)

The mass matrix (C2) gives the maximal mixing

$$\sin^2 2\theta_{23} \equiv 4U_{\nu 23}^2 U_{\nu 33}^2 = 1$$

(C3)

and

$$U_{\nu 13} = 0.$$

(C4)

For $k \simeq 1/2$ and $a^2 \simeq 0$, the mass matrix (C2) leads to a nearly bimaximal mixing

$$U_\nu = \begin{pmatrix} c & s & 0 \\ -\dfrac{1}{\sqrt{2}}s & \dfrac{1}{\sqrt{2}}c & -\dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}}s & \dfrac{1}{\sqrt{2}}c & \dfrac{1}{\sqrt{2}} \end{pmatrix},$$

(C5)

where $s = \sin\theta$ and $c = \cos\theta$. When we put

$$k = \frac{1}{2}(1+x),$$

(C6)

we obtain

$$m_{\nu 1} = \frac{1}{4} m_0 (4 + 2x + a^2 + a^2 x$$
$$- \sqrt{4x^2 + 8a^2 + 12a^2 x + a^4 + 4a^2 x^2 + 2a^4 x + a^4 x^2}),$$

$$m_{\nu 2} = \frac{1}{4} m_0 (4 + 2x + a^2 + a^2 x$$
$$+ \sqrt{4x^2 + 8a^2 + 12a^2 x + a^4 + 4a^2 x^2 + 2a^4 x + a^4 x^2}),$$

$$m_{\nu 3} = 0, \tag{C7}$$

$$R \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \simeq - \frac{2\left(1 + \frac{1}{2}x\right)\sqrt{x^2 + 2a^2}}{\left(1 + \frac{1}{2}x + \frac{1}{2}\sqrt{x^2 + 2a^2}\right)}, \tag{C8}$$

$$\tan^2 \theta_{solar} \equiv \frac{U_{\nu 12}^2}{U_{\nu 11}^2} \simeq \frac{\sqrt{2a^2 + x^2} - x}{\sqrt{2a^2 + x^2} + x}. \tag{C9}$$

The values $a/x = 1$, $\sqrt{2}$, $\sqrt{3}$ and 2 give $\tan^2 \theta_{solar} = 0.27$, 0.38, 0.45 and 0.5 ($\theta_{solar} = 27°$, $32°$, $34°$ and $35°$), respectively. In order to fit the observed value [12–14]

$$R_{obs} = \frac{6.9 \times 10^{-5} \text{ eV}^2}{2.5 \times 10^{-3} \text{ eV}^2} = 2.76 \times 10^{-2}. \tag{C10}$$

For example, for the case $a/x = 2$, by taking $a = 2x = 0.0092$, we obtain the following numerical results:

$$m_{\nu 1} = 0.9954 m_0, \quad m_{\nu 2} = 1.0092 m_0, \quad m_{\nu 3} = 0, \tag{C11}$$

$$U_\nu = \begin{pmatrix} -0.8152 & 0.5791 & 0 \\ 0.4095 & 0.5765 & -0.7071 \\ 0.4095 & 0.5765 & 0.7071 \end{pmatrix}$$

$$\simeq \begin{pmatrix} -\dfrac{2}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} & 0 \\ \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}, \tag{C12}$$

$$\tan^2 \theta_{solar} = 0.505, \tag{C13}$$

together with $R = 0.0272$ and $\sin^2 2\theta_{atm} = 1$.

A simple example of the mixing matrix $U \equiv U_R^d$ which leads to the second term of the expression (C2) from the form (C1) is, for example, given by

$$U = \begin{pmatrix} 0 & s_\alpha & c_\alpha \\ -\dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} c_\alpha & \dfrac{1}{\sqrt{2}} s_\alpha \\ \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} c_\alpha & \dfrac{1}{\sqrt{2}} s_\alpha \end{pmatrix}, \tag{C14}$$

which lead to

$$U^T S U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_\alpha^2 & s_\alpha c_\alpha \\ 0 & s_\alpha c_\alpha & c_\alpha^2 \end{pmatrix}, \tag{C15}$$

so that

$$U \cdot P U^T S U P \cdot U^T$$

$$= U \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_\alpha^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot U^T$$

$$= \frac{s_\alpha^2}{2} \begin{pmatrix} 2s_\alpha^2 & -\sqrt{2} s_\alpha c_\alpha & -\sqrt{2} s_\alpha c_\alpha \\ -\sqrt{2} s_\alpha c_\alpha & c_\alpha^2 & c_\alpha^2 \\ -\sqrt{2} s_\alpha c_\alpha & c_\alpha^2 & c_\alpha^2 \end{pmatrix}. \tag{C16}$$

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