Magnetic moment of the ρ meson in QCD light cone sum rules

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The magnetic moment μ of the ρ meson is studied in QCD light cone sum rules, and it is found that $\mu = (2.3 \pm 0.5)$, in units of $e/2m_\rho$. A comparison of our result on the magnetic moment of the ρ meson with the predictions of the other approaches is presented.

I. INTRODUCTION

QCD sum rules, which are based on the first principles of QCD [1], is a powerful tool in the investigation of the hadron physics. In this method physically measurable quantities of hadrons are connected with QCD parameters, where hadrons are represented by their interpolating quark current taken at large virtualities, and following that, the correlator of these quark currents is introduced. The main idea of the method is to calculate the correlator with the help of operator product expansion (OPE) in the framework of QCD (accounting for both perturbative and nonperturbative contributions) and then connect them with the phenomenological part. The physical quantities of interest are determined by matching these two representations of the correlator.

The QCD sum rule method is successfully applied to many problems of hadron physics (about the method see, for example, review papers [2–5], and references therein). One of the important static characteristic of hadrons is their magnetic moment. Magnetic moments of nucleons are calculated in the framework of the QCD sum rule method in [6,7], using the external field technique, and using the same approach magnetic moment of the ρ meson is calculated in [8].

Furthermore, it should be mentioned here that in [9] form factors of the ρ meson are calculated at intermediate momentum transfer by using the three-point QCD sum rules method, and then extrapolating these form factors to $Q^2 = 0$ (this point lies outside the applicability region of the method).

In this work, we present an independent calculation of the magnetic moment of the ρ meson in the framework of an alternative approach to the traditional QCD sum rules, i.e., QCD light cone sum rules (QLCSR) method.

Few words about this method are in order. The QLCSR method is based on OPE near light cone, which is an expansion over the twist of the operators rather than dimensions as in the traditional QCD sum rules. The nonperturbative dynamics encoded in the light cone wave functions determines the matrix elements of the nonlocal operators between the vacuum and the hadronic states (more about this method and its applications can be found in [5,10])

The QLCSR is successfully applied to a variety of prob-lems in hadron physics. For example, magnetic moments of the octet and decuplet baryons are calculated in [11] and [12], respectively, and magnetic moment of the nucleon is first obtained in QLCSR in [13].

The paper is organized as follows. In Sec. II, QLCSR for the ρ meson magnetic moment is obtained. In Sec. III, our numerical results and a comparison with the results of the other approaches is presented.

II. QLCSR FOR THE ρ MESON MAGNETIC MOMENT

In this section we calculate the ρ meson magnetic moment in QLCSR. We consider the following correlator of two vector currents in the external electromagnetic field

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{ipx} \langle 0|T\{j_\nu(x) j_\mu^\dagger(0)\}|0\rangle_\gamma, \quad (1)$$

where the subscript γ denotes the external electromagnetic field, $j_\nu(x) = \bar{u}\gamma_\nu d(x)$ is the vector current with the ρ meson quantum number.

First, let us calculate the phenomenological part of the correlator. By inserting a complete set of states between the currents in Eq. (1) with quantum numbers of the ρ meson, we obtain the following representation of the correlator:

$$\Pi_{\mu\nu} = \frac{\langle 0|j_\nu|\rho(p)\rangle\langle\rho(p)|\rho(p')\rangle_\gamma\langle\rho(p')|j_\mu^\dagger|0\rangle}{(p^2 - m_\rho^2)(p'^2 - m_\rho^2)} + \cdots, \quad (2)$$

where $p' = p + q$, q is the photon momentum and $\cdots$ describe higher states and continuum contributions. The matrix element $\langle 0|j_\nu|\rho(p)\rangle$ is determined as

$$\langle 0|j_\nu|\rho(p)\rangle = \frac{m_\rho^2}{g_\rho}\varepsilon_\nu(p). \quad (3)$$

Assuming parity and time-reversal invariance, the electromagnetic vertex of the ρ meson can be written in terms of three form factors [14]:

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$$\langle \rho(p,\varepsilon^r)|\rho(\rho',\varepsilon^{r'})\rangle_\gamma = -e\varepsilon^\rho(\varepsilon^r)^\alpha(\varepsilon^{r'})^\beta\Bigg\{ G_1(Q^2)g_{\alpha\beta}(p$$

$$+p')_\rho + G_2(Q^2)(q_\alpha g_{\rho\beta} - q_\beta g_{\rho\alpha})$$

$$-\frac{1}{2m_\rho^2}G_3(Q^2)q_\alpha q_\beta(p+p')_\rho\Bigg\},$$

(4)

where ε_ρ is the photon and $(\varepsilon^r)^\alpha$, $(\varepsilon^{r'})^\beta$ are the ρ meson vector polarizations. The Lorentz invariant form factors $G_i(Q^2)$ are related to the charge, magnetic, and quadropole form factors through the relations

$$F_C = G_1 + \frac{2}{3}\eta F_\mathcal{D},$$

$$F_M = G_2,$$

$$F_\mathcal{D} = G_1 - G_2 + (1+\eta)G_3,$$

(5)

where $\eta = Q^2/4m_\rho^2$ is a kinematical factor. At zero momentum transfer, these form factors are proportional to the usual static quantities of charge, magnetic, moment μ, and quadropole moment $\mathcal{D}$:

$$eF_C(0) = e,$$

$$eF_M(0) = 2m_\rho\mu,$$

$$eF_\mathcal{D}(0) = m_\rho^2\mathcal{D}.$$

(6)

Using Eqs. (2)–(4) and performing summation over polarizations of the ρ meson, for the phenomenological part of the correlator we get

$$\Pi_{\mu\nu} = e\frac{m_\rho^4}{g_\rho^2}\varepsilon^\rho\frac{1}{(m_\rho^2-p^2)[m_\rho^2-(p+q)^2]}\Bigg\{ G_1(Q^2)(p+p')_\rho\Bigg[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_\rho^2} - \frac{p'_\mu p'_\nu}{m_\rho^2} + \frac{p'_\mu p_\nu}{2m_\rho^4}(Q^2+2m_\rho^2)\Bigg] + G_2(q^2)\Bigg[q_\mu g_{\nu\rho}$$

$$-q_\nu g_{\mu\rho} - \frac{p_\nu}{m_\rho^2}\Big(q_\mu p_\rho - \frac{1}{2}Q^2 g_{\mu\rho}\Big) + \frac{p'_\mu}{m_\rho^2}\Big(q_\nu p'_\rho + \frac{1}{2}Q^2 g_{\nu\rho}\Big) - \frac{p'_\mu p_\nu p_\rho}{m_\rho^4}Q^2\Bigg] - \frac{1}{2m_\rho^2}G_3(Q^2)(p+p')_\rho\Bigg[q_\mu q_\nu$$

$$-\frac{p_\nu q_\mu}{m_\rho^2}\frac{1}{2}Q^2 + \frac{p'_\mu q_\nu}{m_\rho^2}\frac{1}{2}Q^2 - \frac{p_\nu p'_\mu}{m_\rho^4}\frac{1}{4}(Q^2)^2\Bigg]\Bigg\},$$

(7)

where $Q^2 = -q^2$. Throughout our analysis, only the values of the form factors at $Q^2 = 0$ are needed. Additionally, using $p' = p+q$ and $q\varepsilon = 0$, Eq. (7) can be simplified and final answer for the phenomenological part can be written as

$$\Pi_{\mu\nu} = e\frac{m_\rho^4}{g_\rho^2}\frac{\varepsilon^\rho}{[m_\rho^2-(p+q)^2](m_\rho^2-p^2)}\Bigg\{ 2p_\rho F_C(0)\Bigg[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_\rho^2} - \frac{p_\mu q_\nu}{m_\rho^2}\Bigg] + F_M(0)\Bigg[q_\mu g_{\nu\rho} - q_\nu g_{\mu\rho} + \frac{1}{m_\rho^2}p_\rho(p_\mu q_\nu - p_\nu q_\mu)\Bigg]$$

$$-[F_C(0) + F_\mathcal{D}(0)]\frac{p_\rho}{m_\rho^2}q_\nu q_\mu\Bigg\}.$$

(8)

In order to extract out the magnetic moment of the ρ meson from Eq. (8), we will choose the structure $(p\varepsilon)(p_\mu q_\nu - p_\nu q_\mu)$. Hence, the phenomenological part of the correlator for the above-mentioned structure can be written as

$$\Pi = \frac{m_\rho^2}{g_\rho^2}\frac{1}{(m_\rho^2-p^2)[m_\rho^2-(p+q)^2]}\mu,$$

(9)

where magnetic moment μ is given in units of $e/2m_\rho$.

Our next task is calculation of the correlator in Eq. (1) from the QCD side. The correlator receives perturbative and nonperturbative contributions. The perturbative part corresponds to on-shell photon emission from virtual quarks and it is described by the triangle diagram [see Fig. 1]. In order to calculate the nonperturbative contributions [see Fig. 2], we need the matrix elements of the nonlocal operators between

the vacuum and the photon states, i.e., $\langle\gamma(q)|\bar{q}(x)\Gamma_i(0)|0\rangle$, where Γ is an arbitrary Dirac matrix. In our calculations we take into account twist-2, twist-3 and twist-4 photon wave functions (more about the photon wave functions, see [15]). In what follows we present definitions whose wave functions give contribution only to the structure $(p\varepsilon)(p_\mu q_\nu - p_\nu q_\mu)$:
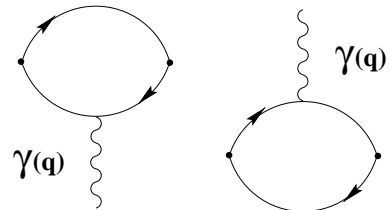

FIG. 1. Diagrams describing perturbative contribution to the correlator in Eq. (1).

$$\langle \gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = e e_q f_{3\gamma}\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx}\right)\int_0^1 du\, e^{iuqx}\psi^{(v)}(u),\tag{10}$$

$$\langle \gamma(q)|\bar{q}(x)\gamma_\mu \gamma_5 q(0)|0\rangle = -\frac{1}{4}e e_q f_{3\gamma}\epsilon_{\mu\alpha\beta\rho}\varepsilon^\alpha q^\beta x^\rho \int_0^1 du\, e^{iuqx}\psi^{(a)}(u),\tag{11}$$

$$\langle \gamma(q)|\bar{q}(x)\sigma_{\mu\nu} q(0)|0\rangle = -iee_q\langle \bar{q}q\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 du\, e^{iuqx}\left\{\chi\phi_\gamma(u) + \frac{x^2}{16}A(u)\right\}$$

$$-iee_q\langle \bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right) - x_\mu\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 du\, e^{iuqx}h_\gamma(u),\tag{12}$$

where $\phi_\gamma(u)$ is twist-2, $\psi^{(v)}(u)$ and $\psi^{(a)}(u)$ are twist-3, $A(u)$ and $h_\gamma(u)$ are twist-4 photon wave functions, respectively, and χ is the magnetic susceptibility. It should be noted here that there are several other functions $T_i(\alpha_i)$ and $\widetilde{S}(\alpha_i)$ (for their definitions, see [15]) that also give contribution to the above-mentioned structure. But their contributions are proportional to the quark mass (in our case u and d quark masses) and therefore irrelevant in the massless quark case.

After some effort, we get the following expression for the correlator from the QCD side in the x representation

$$\Pi_{\mu\nu} = e\int_0^1 du\int dx\, e^{i(p+uq)x}\varepsilon x(x_\mu q_\nu - x_\nu q_\mu)\left\{(e_d - e_u)\right.$$

$$\times\left[\frac{3}{4\pi^4 x^6} - f_{3\gamma}\frac{\psi^{(a)}(u)}{8\pi^2 x^4} + \frac{i}{2}f_{3\gamma}\frac{\psi^{(v)}(u)}{\pi^2(qx)x^4}\right.$$

$$\left.\left. - \frac{m_0^2}{384(qx)}\langle \bar{q}q\rangle^2 h_\gamma(u)\right] - \frac{(e_d + e_u)}{8(qx)}h_\gamma(u)\right\}.\tag{13}$$

Using Eq. (13) and after performing Fourier transformation, the result for the structure $(p\varepsilon)(q_\nu p_\mu - q_\mu q_\nu)$ can be obtained. The sum rules for the ρ meson can be obtained after applying double Borel transformation on the variables p^2 and $(p+q)^2$, which suppresses the continuum and higher states contributions (about this procedure, see [11,12,16,17], and references therein), and then matching both representations of the correlators.
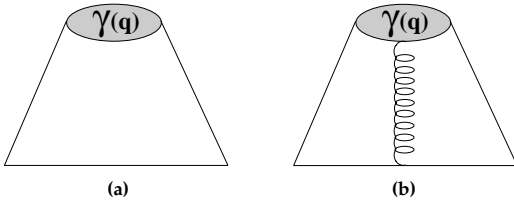

FIG. 2. Diagrams describing nonperturbative contribution to the correlator in Eq. (1). Here, Fig. 2(a) corresponds to the leading order contribution and Fig. 2(b) corresponds to the gluon correction to the correlator in Eq. (1). In these figures, the wavy line represents gluon and solid lines represent quark fields, respectively.

Finally, for the above-mentioned structure we get the following sum rule for the ρ meson magnetic moment:

$$\mu = \frac{g_\rho^2}{m_\rho^2}e^{m_\rho^2/M^2}(e_u - e_d)\left\{\frac{3}{8\pi^2}M^2 f_0(s_0/M^2) + \frac{f_{3\gamma}}{2}\psi^{(a)}(u_0)\right.$$

$$\left. - 2f_{3\gamma}\Psi^{(v)}(u_0)\right\},\tag{14}$$

where

$$\Psi^{(v)}(u) = \int_0^u \psi^{(v)}(v)dv,$$

and, the function

$$f_0(s_0/M^2) = 1 - e^{-s_0/M^2},$$

is used to subtract continuum contributions, and naturally, the Borel parameters M_1^2 and M_2^2 are set to be equal to each other, i.e., $M_1^2 = M_2^2 \equiv 2M^2$ since we are dealing with just a single meson, and hence

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} = \frac{1}{2}.$$

Note that, the last two terms in Eq. (13) disappear after double Borel transformation is performed.

The main reason why we choose the structure $(p\varepsilon)\times(q_\nu p_\mu - q_\mu q_\nu)$ is that the term proportional to the magnetic susceptibility χ does not give any contribution, and hence the main uncertainty coming from the definition of χ is absent in the sum rule.

III. NUMERICAL ANALYSIS

In this section we present our numerical analysis on the ρ meson magnetic moment. It follows from Eq. (14) that in order to perform further numerical analysis one needs to know the photon wave functions $\psi^{(a)}(u)$ and $\psi^{(v)}(u)$. The explicit expressions of the functions are
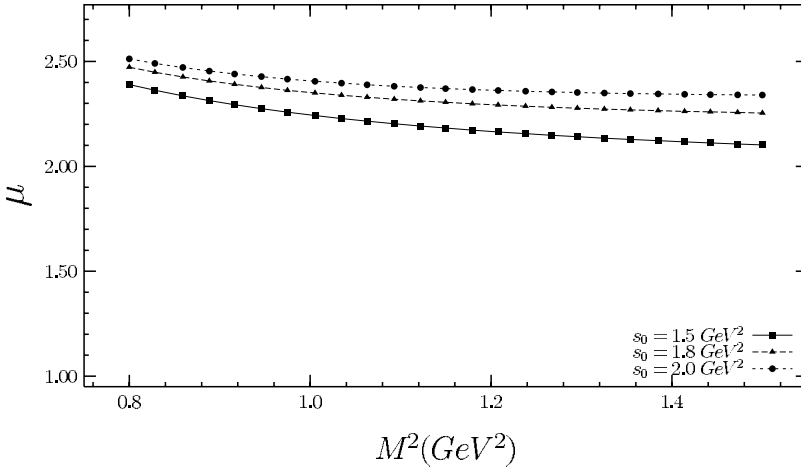
FIG. 3. The dependence of the magnetic moment of the ρ meson on the Borel parameter M^2, at three different values of the continuum threshold; $s_0 = 1.5$ GeV^2, $s_0 = 1.8$ GeV^2, $s_0 = 2.0$ GeV^2.

$$\psi^{(v)}(u) = 10u(1 - 3u + 2u^2) - \frac{15}{8}u(w_\gamma^A - 3w_\gamma^V)(1 - 10u$$
$$+ 30u^2 - 35u^3 + 14u^4),$$

$$\psi^{(a)}(u) = \frac{5}{2}\left[1 + \frac{9}{16}w_\gamma^V - \frac{3}{24}w_\gamma^A\right][1 - (2u - 1)^2][5(2u - 1)^2 - 1].$$

The values of the input parameters w_γ^V, w_γ^A, and $f_{3\gamma}$ are given in [15] to have the values $w_\gamma^V = (3.8 \pm 1.8)$, $w_\gamma^A = -(2.1 \pm 1.0)$, and $f_{3\gamma} = -(3.9 \pm 2.0) \times 10^{-3}$ GeV^{-2}. The remaining input parameters are $m_\rho = 0.77$ GeV and $g_\rho^2/4\pi = 1.27$.

In Fig. 3 we present the dependence of the magnetic moment on M^2 at three different values of the continuum threshold: $s_0 = 1.5$ GeV^2, $s_0 = 1.8$ GeV^2, and $s_0 = 2.0$ GeV^2. Note that M^2 in the sum rule is an auxiliary parameter and the physical quantities are expected to be independent of it. Therefore, one must look for a region of M^2 for which the magnetic moment μ be practically independent of it. The lower limit of M^2 is determined by the requirement that terms $\sim M^{-2n}(n > 1)$ remain subdominant. In other words, large power corrections must be absent in the sum rule. The upper bound of M^2 is determined by demanding that the contributions of the higher resonances and continuum are less than, for example, 30% of the total result. Our numerical calculation shows that these requirements are satisfied in the region 1.0 GeV$^2 \leqslant M^2 \leqslant 1.4$ GeV^2 and magnetic moment in this region is practically independent of M^2. We also see from this figure that as s_0 varies from $s_0 = 1.5$ GeV^2 to $s_0 = 2.0$ GeV^2, the magnetic moment of the ρ meson changes by an amount of approximately 10%. Therefore we can conclude that the result seems to be almost insensitive to the change in s_0 and M^2 in the above-mentioned region. The final result for the magnetic moment of the ρ meson turns out to be

$$\mu = 2.3 \pm 0.5,$$

in units of $(e/2m_\rho)$, where the error can be attributed to the variations in s_0, M^2, and uncertainties in the values of $f_{3\gamma}$, w_γ^V, and w_γ^A.

At the end, we would like to present a comparison of our result on the ρ meson magnetic moment with the ones existing in literature. In the Dyson-Schwinger based models, the ρ meson magnetic moment is estimated to have the value $\mu = 2.69$ [18], $2.5 \leqslant \mu \leqslant 3$ [19] in units of $(e/2m_\rho)$. Covariant and light front approaches with constituent quark model, both, predict $\mu = 2.23 \pm 0.13$ [20] and in light front formalism it is estimated to be $\mu = 1.83$ [21]. The magnetic moment of ρ meson was calculated long time ago in [22], by considering the low energy limit of the radiative amplitudes in conjunction with the amplitude calculated by the hard-pion technique and found that

$$\frac{16\pi^2\alpha^2 g_\rho^2}{m_\rho^2 \int ds\, \sigma_{e^+e^- \to n}} < \mu < 2.$$

The ρ meson magnetic moment was also calculated in the lattice QCD which predicted $\mu = 2.25(34)$ [23]. As has already been noted, the magnetic moment of the ρ meson in the framework of the traditional QCD sum rule in the presence of external field is calculated in [8] and it is obtained that $\mu = 1.5 \pm 0.3$. Our result is closer to the predictions of the works [20,23].

Finally, we would like to discuss briefly the question how to measure the magnetic moment of ρ meson in experiments. At present, even upper bounds for the magnetic and quadrupole moments of ρ meson are absent. The very short lifetime does not allow the use of vector-meson-electron scattering or spin procession technique [24] to measure the above-mentioned quantities.

An alternative method for determination of the multipole moments of particles based on soft photon emission off the hadrons was proposed in [25], since the photon carries information on higher multipoles of the emitting particles. The main idea of this work is that the amplitude for radiative process can be expressed as a power expansion in the photon energy w as follows:

$$M = \frac{A}{w} + Bw^0 + Cw + \cdots.$$

The electric charge contributes to the amplitude at order w^{-1} and the contribution coming from magnetic moment is proportional to w^0. Therefore, by measuring the cross section or decay width of the radiative process and neglecting terms linear in w, one can determine the magnetic moments of charged particles.

In [25,26], the possibility of measuring the magnetic moment of the charged ρ meson in radiative production and decays of such mesons is mentioned and it is claimed that combined angular and energy distributions of radiated photons is an efficient tool in measuring the magnetic moment of the charged ρ meson.

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