Chiral perturbation theory with Wilson-type fermions including a^2 effects: $N_f=2$ degenerate case

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We derive the quark mass dependence of m_π^2, $m_{\rm AWI}$, and f_π, using chiral perturbation theory, which includes the a^2 effect associated with the explicit chiral symmetry breaking of the Wilson-type fermions, in the case of the $N_f=2$ degenerate quarks. The distinct features of the results are (1) the additive renormalization for the mass parameter m_q in the Lagrangian, (2) $O(a)$ corrections to the chiral log ($m_q \log m_q$) term, (3) the existence of a more singular term $\log m_q$ generated by a^2 contributions, and (4) the existence of both $m_q \log m_q$ and $\log m_q$ terms in the quark mass from the axial Ward-Takahashi identity $m_{\rm AWI}$. By fitting the mass dependence of m_π^2 and $m_{\rm AWI}$, obtained by the CP-PACS Collaboration for $N_f=2$ full QCD simulations, we find that the data are consistently described by the derived formulas. Resumming the most singular terms $\log m_q$, we also derive modified formulas, which show a better control over the next-to-leading order correction.

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I. INTRODUCTION

One of the most serious systematic uncertainties in the current lattice QCD simulations is caused by chiral extrapolation. Because of the limitation of the current computational power, one cannot perform simulations directly at the physical light quark (up and down) mass. Instead, one has to perform simulations at several heavier quark masses and extrapolate the results to the physical quark mass point, using a polynomial (linear, quadratic, etc.) or the formula derived from chiral perturbation theory (ChPT) [1]. These extrapolations cause large systematic uncertainties, in particular in the case of full QCD simulations, where the lightest quark mass employed in the current QCD simulations is roughly half of the physical strange quark mass ($m_\pi/m_\rho \simeq 0.6$).

Recently, a more serious problem has been pointed out, in particular, for full QCD simulations with Wilson-type quarks: the expected chiral behavior predicted by the ChPT has not been observed. For example, the behavior of the pion mass m_π^2 as a function of quark mass m_q is given by

$$m_\pi^2 = A m_q \left[1 + \frac{A m_q}{16\pi^2 N_f f_\pi^2} \log(A m_q/\Lambda^2) \right], \tag{1}$$

where Λ is some scale parameter. Since the pion decay constant is experimentally known as $f_\pi = 93$ MeV, only A and Λ are unknown parameters. Unfortunately, such a two-parameter fit cannot explain the lattice data well, which look almost linear in the simulated range of quark masses. If one includes f_π as a free parameter, the best fit typically gives $f_\pi^2 \geqslant 5 \times (93\ \text{MeV})^2$ [2].

The most widely accepted interpretation for this discrepancy is that the simulated range of quark masses in the current simulations is still too heavy to apply ChPT. If this interpretation is true, the current lattice simulations with the (Wilson-type) dynamical quarks lose a large part of their power to predict properties of hadrons at the physical light quark masses.

In this paper, we investigate the theoretically more natural alternative that the explicit breaking of the chiral symmetry by the Wilson-type quark action modifies the formulas of ChPT at the finite lattice spacing. We first derive formulas in the modified chiral perturbation theory for the Wilson-type quark action, denoted by WChPT in this paper. Such attempts have been made before at the leading order [3] and the next-to-leading order [4]. At the leading order [3], WChPT predicts the existence of the parity-flavor breaking phase transition [5–7] for two-flavor QCD as long as massless pions appear at the critical quark mass. This analysis has also shown that the $O(a^2)$ chiral breaking term plays an essential role in generating the parity-flavor breaking phase transition, which is necessary to explain the existence of massless pions for the Wilson-type quark action [5–7]. In the next-to-leading order analysis [4], however, only the $O(a)$ breaking effects are included, and it is concluded that the effect of the chiral symmetry breaking can always be absorbed in the redefinition of the quark mass, so that all formulas in ChPT remain the same if one replaces the quark mass m_q with $m_q - m_c$, where m_c is the additive $O(a)$ counterterm for the quark mass. In Sec. II, we perform the next-to-leading order calculation in WChPT including $O(a^2)$ chiral symmetry breaking effects. To make the difference between WChPT and ChPT clear, we consider only the case of $N_f=2$ QCD with degenerate quark masses, and derive the formulas for the mass and decay constant of the pion as well as the axial Ward-Takahashi identity quark mass, as a function of the "quark mass" in the effective theory. In Sec. III, the derived formulas are applied to the data of the pion mass and the axial Ward-Takahashi identity quark mass calculated by the CP-PACS Collaboration [8]. We show that the data are consistent with the formulas. We have attempted the resummation of the most singular term, and have derived the modified formulas in Sec. IV. Our conclusions and discussion are given in Sec. V.

II. WILSON CHIRAL PERTURBATION THEORY

A. Derivation of effective Lagrangian

It is difficult to derive the effective chiral Lagrangian for mesons directly from lattice QCD with Wilson-type quarks using the symmetry, since the quark mass requires a counterterm m_c, which diverges as g^2/a near the continuum limit, so that $m_c a = O(1)$ and the conventional power counting of a fails. Therefore, following the proposal [3,4], we overcome this problem by first matching the lattice QCD to an effective continuumlike QCD including the scaling violations in higher dimensional local operators [9], and then match the latter to the effective Lagrangian for the Wilson chiral perturbation theory.

Close to the continuum limit, the lattice QCD can be described by an effective action in the continuum, which is expanded in powers of a as

$$S_{\rm eff} = S_0 + aS_1 + a^2 S_2 + \cdots, \tag{2}$$

where S_1 contains chiral noninvariant terms only, while S_2 contains chiral invariant as well as chiral noninvariant terms. By using the equation of motion and the redefinition of the quark field, quark mass, and coupling constant, only one term is relevant in S_1:

$$S_1 = a r_1 \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi + \cdots. \tag{3}$$

A similar analysis can be done for S_2 [10].

We now derive the effective Lagrangian of WChPT from $S_{\rm eff}$, using the symmetries of $S_{\rm eff}$ such as parity, axis interchange symmetry (rotational invariance in the continuum limit), and chiral symmetry. The last one is explicitly broken not only by the quark mass m but also by the breaking terms in S_1 and S_2, whose coefficients are denoted as $r_i (i = 1,2,3,\dots)$. One can make $S_{\rm eff}$ formally chiral invariant by transforming m and the r_i's to compensate the chiral variation of ψ and $\bar{\psi}$. For example, if one writes the quark mass term as

$$\bar{\psi} M P_R \psi + \bar{\psi} M^\dagger P_L \psi, \tag{4}$$

this term is invariant under

$$\psi \to (RP_R + LP_L)\psi, \quad \bar{\psi} \to \bar{\psi}(L^\dagger P_R + R^\dagger P_L), \tag{5}$$

$$M \to LMR^\dagger, \quad M^\dagger \to RM^\dagger L^\dagger, \tag{6}$$

where R and L are $\mathrm{SU}(N_f)$ chiral rotations. The usual mass term is recovered by setting $M = M^\dagger = m$. Similar transformations can be defined for the r_i's, but we do not give them explicitly since their details are irrelevant for later discussion. From this argument one concludes that the effective Lagrangian of the WChPT should have this (generalized) chiral $\mathrm{SU}(N_f)_R \otimes \mathrm{SU}(N_f)_L$ symmetry.

As mention in the Introduction, we consider the $N_f = 2$ case to make our argument simple and clear. In this case, the chiral field for the pseudoscalar mesons (pions) is given by

$$\Sigma(x) = \Sigma_0 \exp\left\{ i \sum_{a=1}^{3} \pi^a(x) t^a / f \right\}$$
$$= \Sigma_0 [\cos(\pi/f) + i \hat{\pi}^a t^a \sin(\pi/f)], \tag{7}$$

where $\pi^a(x)$ is the pion field, $t^a \equiv \sigma^a$ is the ordinary Pauli matrix, and f is the pion decay constant, whose experimental value is 93 MeV. The norm and the unit vector of the pion fields are given by $\pi^2 = \pi \cdot \pi = \Sigma_a \pi^a \pi^a$ and $\hat{\pi}^a = \pi^a/\pi$, respectively. As discussed in Ref. [3], the vacuum expectation value Σ_0 may have a complicated structure, leading to the spontaneous breaking of parity-flavor symmetry, but in this paper, we stay in the phase without this symmetry breaking, so that $\Sigma_0 = \mathbf{1}_{2\times 2}$. Under chiral rotation, this field is transformed as $\Sigma \to L \Sigma R^\dagger$. Under the transformation that $\pi \to -\pi$, called "parity" in this paper, $\Sigma \to \Sigma^\dagger$.

Using this field, we define the following naive operators for scalar (S), pseudoscalar (P), vector (V), and axial vector (A):

$$S^0 = \frac{1}{4}\mathrm{tr}(\Sigma + \Sigma^\dagger) = \cos(\pi/f), \quad S^a = \frac{1}{4}\mathrm{tr}\, t^a(\Sigma + \Sigma^\dagger) = 0, \tag{8}$$

$$P^0 = \frac{1}{4}\mathrm{tr}(\Sigma - \Sigma^\dagger) = 0,$$

$$P^a = \frac{1}{4}\mathrm{tr}\, t^a(\Sigma - \Sigma^\dagger) = i\hat{\pi}^a \sin(\pi/f), \tag{9}$$

$$L_\mu^0 = \frac{1}{2}\mathrm{tr}(\Sigma \partial_\mu \Sigma^\dagger) = 0, \quad L_\mu^a = \frac{1}{2}\mathrm{tr}\, t^a(\Sigma \partial_\mu \Sigma^\dagger), \tag{10}$$

$$R_\mu^0 = \frac{1}{2}\mathrm{tr}(\Sigma^\dagger \partial_\mu \Sigma) = 0, \quad R_\mu^a = \frac{1}{2}\mathrm{tr}\, t^a(\Sigma^\dagger \partial_\mu \Sigma), \tag{11}$$

$$V_\mu^0 = \frac{1}{2}(L_\mu^0 + R_\mu^0) = 0, \quad A_\mu^0 = \frac{1}{2}(L_\mu^0 - R_\mu^0) = 0, \tag{12}$$

$$V_\mu^a = \frac{1}{2}(L_\mu^a + R_\mu^a) = i e^{abc} \hat{\pi}^b \sin(\pi/f) \partial_\mu [\hat{\pi}^c \sin(\pi/f)], \tag{13}$$

$$A_\mu^a = \frac{1}{2}(L_\mu^a - R_\mu^a) = i\{\hat{\pi}^a \sin(\pi/f)\partial_\mu[\cos(\pi/f)]$$
$$- \cos(\pi/f)\partial_\mu[\hat{\pi}^a \sin(\pi/f)]\}, \tag{14}$$

where the superscripts 0 and a mean the flavor singlet and triplet, respectively. We also introduce left-handed (L) and right-handed (R) currents for later use. Due to the speciality of the $N_f = 2$ case, some of the above operators are identically zero. Here we do not consider the tensor (T) operator, which must contain two derivatives, since it does not contribute to the one-loop calculation in this paper.

Now we construct the effective Lagrangian, which must be invariant under parity, axis-interchange symmetry, and (generalized) chiral symmetry. In the one-loop calculation, which gives the main contribution at the next-to-leading order in chiral perturbation theory, it is enough for us to construct the effective Lagrangian up to order m, where m is the quark mass in the effective theory. On the other hand, we must include the $O(a^2)$ effect to realize massless pions at

$a\neq 0$ [3]. At the next-to-leading order, $O(m^2)$ counterterms (Gasser-Leutwyler coefficients) are also needed. We do not include, however, these terms in our effective Lagrangian, since we do not intend to determine them in this paper. Instead we introduce arbitrary scale parameters in the $\log(m)$ terms which appear in the one-loop integrals. Roughly speaking, we consider the situation that $1\gg a \geqslant m \simeq p^2 \geqslant a^2 \geqslant ma \simeq p^2 a \geqslant m^2 \simeq p^4 \simeq mp^2$, so that all terms up to ma or $p^2 a$ in this inequality will be included in the effective Lagrangian.

The chirally invariant contribution at the leading order, which has the least number of derivatives, is constructed from L_μ^a or R_μ^a as follows:

$$2\sum_{a=1}^{3} L_\mu^a L_\mu^a = 2\sum_{a=1}^{3} R_\mu^a R_\mu^a = \mathrm{tr}[\partial_\mu \Sigma^\dagger \partial_\mu \Sigma]$$
$$= 2\{\partial_\mu[\cos(\pi/f)]\partial_\mu[\cos(\pi/f)] + \partial_\mu[\hat{\pi}^a \sin(\pi/f)]\partial_\mu[\hat{\pi}^a \sin(\pi/f)]\}, \quad (15)$$

$$L_\mu^0 L_\mu^0 = R_\mu^0 R_\mu^0 = 0. \quad (16)$$

Note that the $R_\mu^a L_\mu^a$ term is prohibited by parity invariance. The chirally noninvariant parity-even term accompanied by one power of m, $r_1 = O(a)$ or $r_{i\geqslant 2} = O(a^2)$, is uniquely given by S^0. The chirally noninvariant terms whose coefficients include $r_1^2 = O(a^2)$ or $r_1\cdot m = O(ma)$ are given by $(S^0)^2$, $\Sigma_a (P^a)^2$, or $\mathrm{tr}(\Sigma+\Sigma^\dagger)^2$. For the $N_f=2$ case, however, the latter two terms are not independent, as evident from the expressions $\Sigma_{a=0}^3 (P^a)^2 = (S^0)^2 - 1$ and $\mathrm{tr}(\Sigma+\Sigma^\dagger)^2 \propto (S^0)^2$. An independent term at $O(ap^2)$ is given uniquely by $S_0\times \mathrm{tr}[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma]$, since $\mathrm{tr}[(\Sigma+\Sigma^\dagger)\partial_\mu\Sigma^\dagger\partial_\mu\Sigma]$ is not independent for SU(2).

Gathering all terms up to m, p^2, a^2 and $ma, p^2 a$, the effective Lagrangian becomes

$$L_{\mathrm{eff}} = \frac{f^2}{4}[1 + c_0(S^0-1)]\mathrm{tr}\{\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\} - c_1 S^0 + c_2 (S^0)^2, \quad (17)$$

where parameters c_0, c_1, and c_2 have leading m and a dependences as

$$c_0 = W_0 a + O(m), \quad (18)$$

$$c_1 = W_1 a + B_1 m, \quad (19)$$

$$c_2 = W_2 a^2 + V_2 ma + O(m^2). \quad (20)$$

Since c_0 is dimensionless and c_1 and c_2 have mass dimension 4, $W_0 \sim \Lambda[1+O(\Lambda a)]$, $W_1\sim\Lambda^5[1+O(\Lambda a)]$, $W_2 \sim \Lambda^6[1+O(\Lambda a)]$, $V_2\sim\Lambda^4[1+O(\Lambda a)]$, $B_1\sim\Lambda^3$, where Λ represents some mass scale of the theory such as Λ_{QCD}. The (subleading) a dependence of these parameters comes from the chiral breaking terms of $a^2 S_2$ in the effective action Eq. (2), which correspond to $r_{i\geqslant 2} = O(a^2)$ terms in c_0 and c_1, or $r_1\cdot r_{i\geqslant 2} = O(a^3)$ and $m\cdot r_{i\geqslant 2} = O(ma^2)$ terms in c_2. Chirally invariant parameters such as f receive $O(a^2)$ corrections from chirally invariant $O(a^2)$ terms in $a^2 S_2$. Note that $W_0, W_1, V_2 \sim O(a)$ if nonperturbatively $O(a, ma)$ improved fermions are employed for the lattice QCD action.

For later use, we define the operators in the effective theory, which correspond to the ones in QCD up to $O(a)$:

$$[S^0] = Z_S S^0\{1 + c_S(S^0-1)\}, \quad [P^a] = Z_P P^a\{1 + c_P(S^0-1)\}, \quad (21)$$

$$[V_\mu^0] = \tilde{c}_V \partial_\mu S^0, \quad [V_\mu^a] = Z_V V_\mu^a\{1 + c_V(S^0-1)\}, \quad (22)$$

$$[A_\mu^a] = Z_A\{A_\mu^a[1 + c_A(S^0-1)] + \tilde{c}_A \partial_\mu P^a\} \quad (23)$$

where $c_{S,P,V,A}$ and $\tilde{c}_{A,V}$ are $O(a)$ in general, or $O(a^2)$ if the lattice action and operators are nonperturbatively $O(a)$ and $O(ma)$ improved.

B. Next-to-leading order calculations

To perform the next-to-leading order (one-loop) calculation, we expand L_{eff} in terms of the pion field π^a as

$$L_{\mathrm{eff}} = \mathrm{const} + \frac{1}{2}\left[\partial_\mu\pi\cdot\partial_\mu\pi + \frac{c_1 - 2c_2}{f^2}\pi^2\right] + \frac{1}{6f^2}\left[(\pi\cdot\partial_\mu\pi)^2 - \left(1+\frac{3}{2}c_0\right)(\partial_\mu\pi\cdot\partial_\mu\pi)\pi^2\right] + \frac{(\pi^2)^2}{4!f^4}(8c_2 - c_1) \quad (24)$$

and the operators as

$$[S^0] = Z_S\left(1 - \frac{\pi^2}{2!f^2}\right)\left(1 - c_S\frac{\pi^2}{2!f^2}\right) = Z_S\left[1 - \frac{\pi^2}{2!f^2}(1+c_S)\right], \quad (25)$$

$$[P^a] = iZ_P\frac{\pi^a}{f}\left[1 - \frac{\pi^2}{3!f^2}(1+3c_P)\right], \quad (26)$$

$$[V_\mu^a] = iZ_V e^{abc}\frac{\pi^b\partial_\mu\pi^c}{f^2}\left(1 - \frac{\pi^2}{3!f^2}(1+3c_V)\right), \quad (27)$$

$$[V_\mu^0] = -\tilde{c}_V\frac{\pi\cdot\partial_\mu\pi}{f}\left(1 - \frac{\pi^2}{3!f^2}\right), \quad (28)$$

$$[A_\mu^a] = iZ_A\left[(1+\tilde{c}_A)\frac{\partial_\mu\pi^a}{f} - \frac{2\partial_\mu\pi^a\pi^2}{3f^3}\left(1 + \frac{3c_A + \tilde{c}_A}{4}\right) + \frac{2\pi^a\pi\cdot\partial_\mu\pi}{3f^3}\left(1 - \frac{\tilde{c}_A}{2}\right)\right]. \quad (29)$$

Using the pion propagator at the tree level, which is given by

$$\langle\pi^a(-p)\pi^b(p)\rangle_0 = \delta_{ab}\frac{1}{p^2 + m_0^2}, \quad (30)$$

$$m_0^2=\frac{c_1-2c_2}{f^2}, \tag{31}$$

we evaluate the loop integrals as usual:

$$\langle \pi^a(x)\pi^b(x)\rangle=\delta_{ab}I=\delta_{ab}\frac{m_0^2}{16\pi^2}\log\frac{m_0^2}{\Lambda^2}, \tag{32}$$

$$\langle \partial_\mu\pi^a(x)\partial_\nu\pi^b(x)\rangle=\delta_{ab}\frac{\delta_{\mu\nu}}{4}(-m_0^2 I), \tag{33}$$

where we introduce an arbitrary scale parameter Λ resulting after removal of the power divergences of loop integrals by the local counterterms. Therefore, although we use the same symbol, this Λ varies depending on physical observables.

The inverse pion propagator at the one-loop level is calculated as

$$\begin{aligned}L_{\rm eff}^{(2)}&=\frac{1}{2}(\partial_\mu\pi)^2\left\{1-\frac{I}{3f^2}\left(2+\frac{9c_0}{2}\right)\right\}\\&\quad+\frac{1}{2}\pi^2\left\{m_0^2\left(1-\frac{I}{6f^2}(1-9c_0)\right)+\frac{5c_2 I}{f^4}\right\}\\&=\frac{1}{2}[(\partial_\mu\pi_R)^2+m_\pi^2\pi_R^2],\end{aligned} \tag{34}$$

where

$$\pi=Z^{1/2}\pi_R, \tag{35}$$

$$Z=\left[1-\frac{I}{3f^2}\left(2+\frac{9c_0}{2}\right)\right]^{-1}, \tag{36}$$

$$m_\pi^2=m_0^2\left[1+\frac{m_0^2}{32\pi^2f^2}(1+6c_0)\log\frac{m_0^2}{\Lambda^2}+\frac{5c_2}{16\pi^2f^4}\log\frac{m_0^2}{\Lambda^2}\right]. \tag{37}$$

For the axial-vector current, we obtain

$$\begin{aligned}\langle[A_\mu^a](x)\pi_R^b(y)\rangle&=\delta_{ab}\frac{iZ_A}{f}\langle\partial_\mu\pi_R^a(x)\pi_R^b(y)\rangle_0 Z^{1/2}\left[(1+\tilde{c}_A)\right.\\&\quad\left.-\frac{I}{3f^2}\left(4+\frac{9c_A-3\tilde{c}_A}{2}\right)\right];\end{aligned} \tag{38}$$

therefore the decay constant at the one-loop order becomes

$$\begin{aligned}f_\pi&=\frac{iZ_A}{\sqrt{2}f^2}f(1+\tilde{c}_A)\left[1-\frac{m_0^2}{16\pi^2f^2}\left(1+\frac{3c_A}{2}-\frac{11\tilde{c}_A}{6}\right.\right.\\&\quad\left.\left.-\frac{3c_0}{4}\right)\log\frac{m_0^2}{\Lambda^2}\right].\end{aligned} \tag{39}$$

Taking $Z_A=-i\sqrt{2}f^2$, we have

$$f_\pi=f(1+\tilde{c}_A)\left[1-\frac{m_0^2}{16\pi^2f^2}(1+c_{f_\pi})\log\frac{m_0^2}{\Lambda^2}\right], \tag{40}$$

where $c_{f_\pi}=3c_A/2-11\tilde{c}_A/6-3c_0/4$. Note that f_π receives an $O(a)$ correction even in the chiral limit: $f_\pi=f(1+\tilde{c}_A)$.

Similarly, we have

$$\begin{aligned}\langle\partial_\mu[A_\mu^a](x)\pi_R^b(y)\rangle&=\langle\pi_R^a(x)\pi_R^b(y)\rangle_0\sqrt{2}fm_\pi^2Z^{1/2}\left[(1+\tilde{c}_A)\right.\\&\quad\left.-\frac{I}{3f^2}\left(4+\frac{9c_A-3\tilde{c}_A}{2}\right)\right],\end{aligned} \tag{41}$$

$$\begin{aligned}\langle[P^a](x)\pi_R^b(y)\rangle&=i\frac{Z_P}{f}\langle\pi_R^a(x)\pi_R^b(y)\rangle\\&\quad\times Z^{1/2}\left[1-\frac{5I}{3!f^2}(1+3c_P)\right].\end{aligned} \tag{42}$$

Then the partial conservation of axial-vector current (PCAC) quark mass $m_{\rm AWI}$ is given by

$$\begin{aligned}m_{\rm AWI}&=\frac{\langle\partial_\mu[A_\mu^a](x)\pi_R^b(y)\rangle}{\langle[P^a](x)\pi_R^b(y)\rangle}=\frac{\sqrt{2}f^2}{iZ_P}m_\pi^2(1+\tilde{c}_A)\\&\quad\times\left[1-\frac{m_0^2}{32\pi^2f^2}(1+3c_A-11\tilde{c}_A/3-5c_P)\log\frac{m_0^2}{\Lambda^2}\right]\\&=\frac{1+\tilde{c}_A}{2B_0}m_0^2\left[1+\frac{m_0^2c_{m_{\rm AWI}}+10c_2/f^2}{32\pi^2f^2}\log\frac{m_0^2}{\Lambda^2}\right],\end{aligned} \tag{43}$$

where $1/(2B_0)=\sqrt{2}f^2/(iZ_P)$ and $c_{m_{\rm AWI}}=6c_0-3c_A+11\tilde{c}_A/3+5c_P$.

Let us recall the leading m and a dependences of the parameters:

$$c_0=W_0a,\quad c_1=W_1a+B_1m,\quad c_2=W_2a^2+V_2ma, \tag{44}$$

$$c_P=W_Pa,\quad c_A=W_Aa,\quad \tilde{c}_A=\tilde{W}_Aa, \tag{45}$$

and then the pion mass at the tree level is written as

$$\begin{aligned}m_0^2&=\frac{c_1-2c_2}{f^2}=\frac{m(B_1-2V_2a)+aW_1-2a^2W_2}{f^2}\\&=A(m-m_c)\equiv Am_R\end{aligned} \tag{46}$$

where

TABLE I. Parameters of $N_f=2$ full QCD simulations by the CP-PACS Collaboration [8]. The scale a is fixed by $m_\rho=768.4$ MeV.

β	$L^3\times T$	c_{SW}	a (fm)	a^{-1} (GeV)	La (fm)	m_π/m_ρ
1.80	$12^3\times 24$	1.60	0.2150(22)	0.9178(94)	2.580(26)	0.55–0.81
1.95	$16^3\times 32$	1.53	0.1555(17)	1.269(14)	2.489(27)	0.58–0.80
2.10	$24^3\times 48$	1.47	0.1076(13)	1.834(22)	2.583(31)	0.58–0.81
2.20	$24^3\times 48$	1.44	0.0865(33)	2.281(87)	2.076(79)	0.63–0.80

$$A=\frac{B_1-2aV_2}{f^2},\quad m_c=-a\frac{W_1-2aW_2}{B_1-2aV_2},\quad m_R=m-m_c. \tag{47}$$

Here it is noted that $m_c=O(a)$ does not correspond to $1/(2K_c)$ in lattice QCD, since the $1/a$ contribution to the quark mass is already subtracted in m. Furthermore, for $m<m_c$, the pion would become tachyonic ($m_0^2<0$). As discussed in Ref. [3], however, as long as $c_2=W_2a^2+V_2m_ca=O(a^2)>0$, the parity-flavor symmetry breaking phase transition [5–7] occurs at $m=m_c=O(a)$, so that m_0^2 is always positive. In other words, the $O(a^2)$ contribution in c_2 is necessary for the consistency between the PCAC relation ($m_\pi^2\sim m_q$) and the absence of tachyons.[1]

We summarize the result of the one-loop calculation in terms of m_R and a:

$$m_\pi^2=Am_R\left[1+\frac{m_R(A+w_1a)}{32\pi^2f^2}\log\frac{Am_R}{\Lambda^2}+\frac{w_0a^2}{32\pi^2f^2}\log\frac{Am_R}{\Lambda_0^2}\right], \tag{48}$$

$$m_{\rm AWI}=A_0m_R\left[1+\frac{m_Rw_1^{\rm AWI}a}{32\pi^2f^2}\log\frac{Am_R}{\Lambda_{\rm AWI}^2}+\frac{w_0a^2}{32\pi^2f^2}\log\frac{Am_R}{\Lambda_0^2}\right], \tag{49}$$

$$f_\pi=f(1+\tilde{c}_A)\left[1-\frac{m_R(A+w_1^{\rm decay}a)}{16\pi^2f^2}\log\frac{Am_R}{\Lambda_{\rm decay}^2}\right] \tag{50}$$

where

$$w_1=6W_0+\frac{10V_2}{f^2},\quad w_0=\frac{10}{f^2}\left(\frac{m_cV_2}{a}+W_2\right), \tag{51}$$

$$w_1^{\rm AWI}=w_1-3W_A+\frac{11}{3}\tilde{W}_A+5W_P,\quad w_1^{\rm decay}=\frac{3}{2}W_A-\frac{11}{6}\tilde{W}_A-\frac{3}{4}W_0, \tag{52}$$

$$A_0=\frac{A(1+\tilde{c}_A)}{2B_0}\simeq 1+O(a). \tag{53}$$

[1]On the other hand, if $c_2<0$, no massless pion appears [3].

Note that here $m_c/a=O(1)$ and we recover the distinction among scale parameters (Λ, Λ_0, $\Lambda_{\rm AWI}$, or $\Lambda_{\rm decay}$).

These results reveal the following features of WChPT. In general, the chiral log terms ($m_R\log m_R$) receive $O(a)$ scaling violation. In addition to this, the a^2 contribution generates a $\log m_R$ term in m_π^2, which is more singular as a function of m_R than the usual chiral log term, $m_R\log m_R$. Furthermore, both the $m_R\log m_R$ and $\log m_R$ terms are generated in $m_{\rm AWI}$ by the scaling violations, $O(a)$ for the former and $O(a^2)$ for the latter. The coefficient of the $\log m_R$ term in $m_{\rm AWI}$ is the same as the one in m_π^2.

In the next section we employ the above formulas to fit the full QCD data obtained by the CP-PACS Collaboration [8].

III. ANALYSIS OF CP-PACS DATA

In this section, we apply the WChPT formulas to m_π^2 and $m_{\rm AWI}$ in the $N_f=2$ full QCD with the clover quark action [8].

A. Data sets and WChPT formulas

The CP-PACS Collaboration has performed large scale full QCD simulations with the renormalization group improved gauge action and $N_f=2$ (tadpole improved) clover quark action, at four different lattice spacings a and four different quark masses at each a, as summarized in Table I.

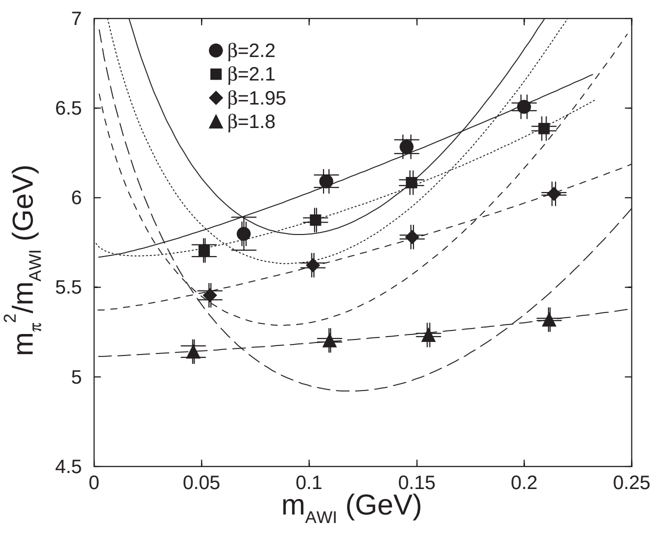

FIG. 1. The WChPT fits for m_π^2 and $m_{\rm AWI}$ at each β. Results are shown for $m_\pi^2/m_{\rm AWI}$ as a function of $m_{\rm AWI}$. For comparison the standard ChPT fits ($w_1=w_0=0$) are also included.

TABLE II. Parameters of the WChPT fit at each β.

β	K_c	A (GeV)	Λ (GeV)	$w_1 a$ (GeV)	$w_0 a^2$(GeV^2)	$w_1^{\rm AWI} a$ (GeV)	χ^2/DOF
1.80	0.147761(15)	5.114(28)	0.079(19)	−5.525(64)	0.206(22)	−0.560(74)	0.3
1.95	0.142160(19)	5.377(33)	0.193(51)	−5.162(74)	0.241(42)	−0.457(118)	0.3
2.10	0.139110(12)	5.807(14)	0.694(20)	−5.24(18)	0.417(50)	−1.15(27)	0.2
2.20	0.137691(23)	5.669(71)	0.128(88)	−5.15(20)	0.039(16)	−0.22(39)	0.7
			Resummed WChPT				
1.8	0.147562(15)	5.111(29)	0.067(12)	−4.862(46)	0.787(21)	0.124(15)	1.5
1.95	0.142009(7)	5.366(23)	0.132(15)	−4.538(52)	0.624(18)	0.310(32)	0.3
2.1	0.138959(13)	5.535(47)	0.131(71)	−4.79(14)	0.280(37)	0.181(49)	1.2
2.2	0.137657(36)	5.789(106)	0.391(82)	−4.63(12)	0.201(95)	0.195(96)	0.8

In Ref. [8] the data for m_π^2 and $m_{\rm AWI}$ were published. Unfortunately, the data for f_π at each quark mass are not available.

We define the quark mass m_R in the WChPT theory in terms of the hopping parameter K in lattice QCD as

$$m_R = Z_m\left(1 + b_m a\frac{m}{u_0}\right)\frac{m}{u_0}, \quad ma = \frac{1}{2K} - \frac{1}{2K_c}, \tag{54}$$

where K_c is the critical hopping parameter, and u_0 is the tadpole improvement factor, given by $u_0 = (1 - 0.8412/\beta)^{1/4}$. This m_R is identical to the renormalized VWI quark mass in Ref. [8]. By definition, $m_\pi^2 = 0$ at $m_R = 0$ in lattice QCD. We identify this m_R in lattice QCD with m_R in WChPT, since m_0^2, and therefore m_π^2, must vanish at $m_R = 0$ in WChPT. We also use the renormalized $m_{\rm AWI}$ defined as

$$m_{\rm AWI} = \frac{Z_A}{Z_P} m_{\rm AWI}^{\rm bare}. \tag{55}$$

We employ the following fitting forms for m_π^2 and $m_{\rm AWI}$:

$$m_\pi^2 = A m_R\left[1 + \frac{m_R A + m_R a w_1}{32\pi^2 f^2}\log\left(\frac{A m_R}{\Lambda^2}\right) + \frac{a^2 w_0}{32\pi^2 f^2}\log\left(\frac{A m_R}{\Lambda_0^2}\right)\right], \tag{56}$$

$$m_{\rm AWI} = A_0 m_R\left[1 + \frac{m_R a w_1^{\rm AWI}}{32\pi^2 f^2}\log\left(\frac{A m_R}{\Lambda_{\rm AWI}^2}\right) + \frac{a^2 w_0}{32\pi^2 f^2}\log\left(\frac{A m_R}{\Lambda_0^2}\right)\right]. \tag{57}$$

B. Results

We first fit the data at each a separately. Since there are only four data per observables at each a, it is impossible to fit an individual observable, m_π^2 or $m_{\rm AWI}$, as a function of m_R using Eq. (56) or Eq. (57), each of which contains four or more parameters. Therefore, we try to fit m_π^2 and $m_{\rm AWI}$ simultaneously. Since f cannot be determined without data of f_π, we fix $f = 93$ MeV.[2] Even in the simultaneous fit, the number of independent fitting parameters is still too large. Since theoretically $A_0 = 1$ in the continuum limit and the fit with $A_0 = 1$ becomes more stable, we fix $A_0 = 1$ in our fit. In order to reduce a number of parameters further, we set $\Lambda_{\rm AWI} = \Lambda_0 = \Lambda$, so we finally have six independent parameters K_c, A, Λ, w_1, $w_1^{\rm AWI}$, and w_0, for eight data points.

Figure 1 shows data and fits for $m_\pi^2/m_{\rm AWI}$ as a function of $m_{\rm AWI}$ at each a. For comparison, the results by the fit with standard chiral perturbation theory $(w_1 = w_0 = 0)$ are also given. It is manifest that the WChPT fits perform much better than the ChPT fits. The parameters extracted from the fits are given in Table II. Note, however, that χ^2 per degree of freedom (DOF) shown in the table has not been reliably estimated due to the correlation between m_π^2 and $m_{\rm AWI}$, which is not given in Ref. [8].

In Fig. 2, A, Λ, $w_1 a$, $w_1^{\rm AWI} a$, and $w_0 a^2$ are plotted as functions of a, together with K_c as a function of the bare gauge coupling constant g^2. While A, Λ, and $w_1 a$ are too scattered to be fitted, K_c, $w_0 a^2$, and $w_1^{\rm AWI} a$ may be fitted as

$$K_c = \frac{1}{8}\cdot\frac{1 + d_0(K_c)g^2 + d_1(K_c)g^4 + d_2(K_c)g^6}{1 + [d_0(K_c) - 0.02945]g^2}, \tag{58}$$

where 0.02945 is the one-loop coefficient [11] and

$$w_1^{\rm AWI} a = d_0(w_1)a, \quad w_0 a^2 = d_0(w_0)a^2. \tag{59}$$

Fit curves are also shown in Fig. 2, and the extracted parameters are given in column (a) of Table III.

To determine the a dependences of A, Λ, and $w_1 a$, we have fitted $m_\pi^2/m_{\rm AWI}$ as a function of both m_R and a, using the following formula derived from Eqs. (56),(57) with $\Lambda_{\rm AWI} = \Lambda$:

$$\frac{m_\pi^2}{m_{\rm AWI}} = \frac{A}{A_0}\left[1 + \frac{(A + \Delta w_1 a)m_R}{32\pi^2 f^2}\log\left(\frac{A m_R}{\Lambda^2}\right)\right], \tag{60}$$

[2] We have also performed the fit using measured values of f_π in the chiral limit at each β [8]. We found that the qualities of the two fits are similar.

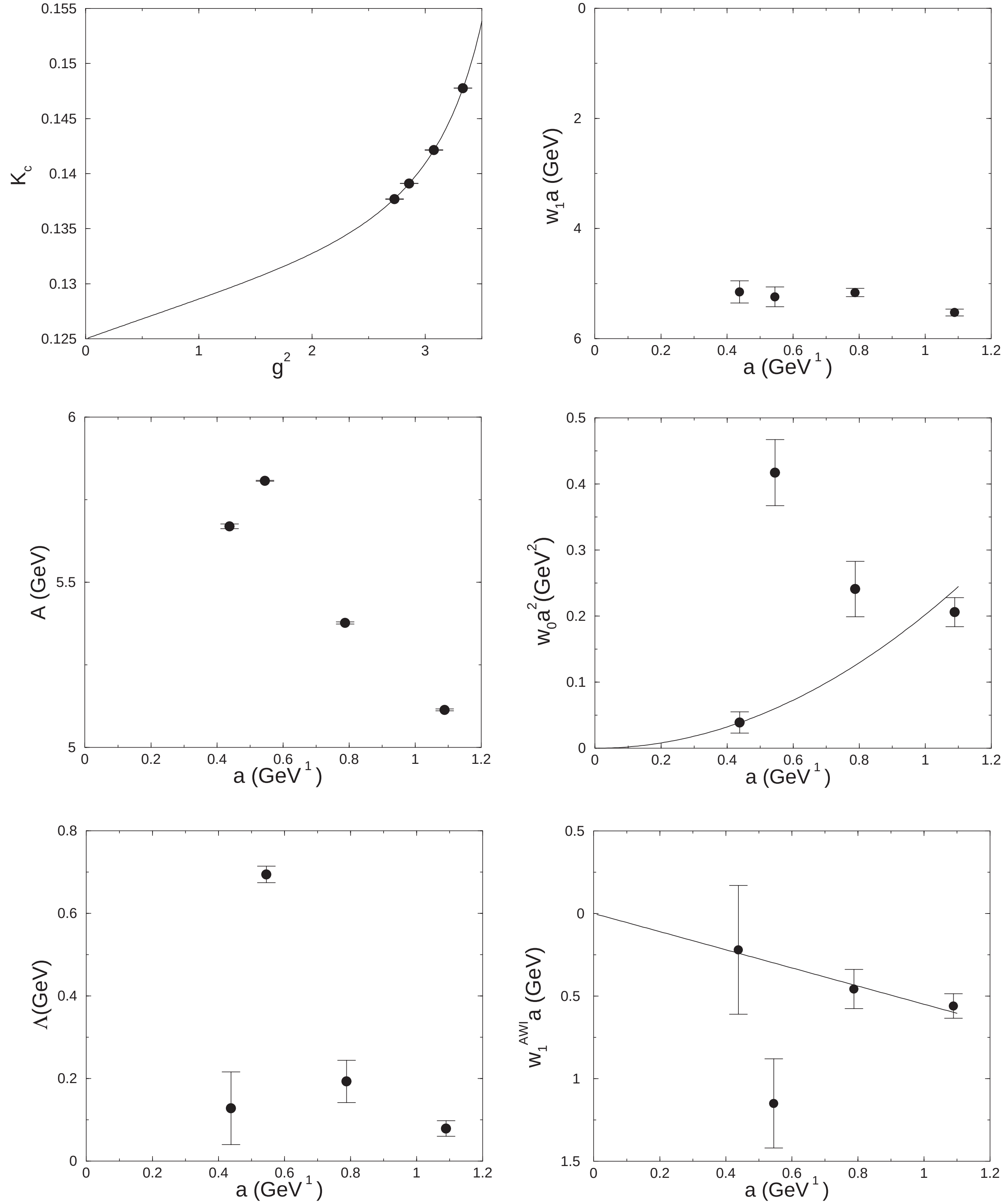

FIG. 2. The fit parameters as a function of a or g^2.

TABLE III. Continuum extrapolation of the WChPT fit parameters. (a) m_π^2 and $m_{\rm AWI}$ are fitted as a function of m_R at each a. Then parameters are fitted as a function of a. (b) $m_\pi^2/m_{\rm AWI}$ are fitted as a function of m_R and a.

(a)					(b) χ^2/DOF=1.3			
X	$d_0(X)$	$d_1(X)$	$d_2(X)$	χ^2/DOF	X	$d_0(X)$	$d_1(X)$	$d_2(X)$
K_c	−0.2127(10)	−0.008300(55)	0.000787(31)	3.6	A	8.087(97)	−1.002(29)	0.2672(29)
w_0	0.202(17)	0	0	20	Λ	1.196(35)	−0.8404(58)	0
$w_1^{\rm AWI}$	−0.549(61)	0	0	3.4	Δw_1	−1.62(25)	0	0
					A_0	−0.590(47)	0	0

where

$$A=d_0(A)[1+d_1(A)a+d_2(A)a^2], \quad A_0=1+d_0(A_0)a, \tag{61}$$

$$\Lambda=d_0(\Lambda)[1+d_1(\Lambda)a^2],$$

$$\Delta w_1=w_1-w_1^{\rm AWI}=d_0(\Delta w_1)a. \tag{62}$$

No $\log m_R$ term is presented in Eq. (60). Note, however, that the $\log m_{\rm AWI}$ term appears again if we replace m_R in the right-hand side of Eq. (60) with $m_{\rm AWI}$, due to the presence of the $\log m_R$ term in Eq. (57). With K_c fixed to the values in Table II, the fit works well, as shown in Fig. 3, and the fitted parameters are given in column (b) of Table III.

We roughly estimate the size of each parameter $B_1, V_2, W_{1,2,3}$ from the continuum extrapolations of A, w_1, w_0, and m_c. Since we cannot separate the $1/a$ contribution in $1/K_c$, however, m_c cannot be extracted. Therefore, we simply set $m_c=0$, giving that $W_1=2aW_2$; the leading contribution of W_1 vanishes. To reduce the number of parameters further, we set $W_0=0$. Then, extracting B_1, W_2, and V_2 as

$$B_1=f^2d_0(A)\equiv(\Lambda_{B_1})^3, \tag{63}$$

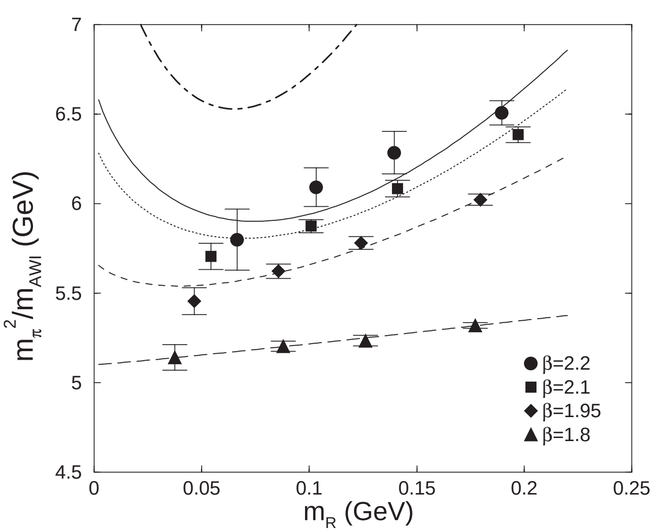

FIG. 3. The WChPT fits for $m_\pi^2/m_{\rm AWI}$ as a function of m_R and a. Results are shown for $m_\pi^2/m_{\rm AWI}$ as a function of m_R.

$$W_2=\frac{f^2d_0(w_0)}{10}\equiv(\Lambda_{W_2})^6, \tag{64}$$

$$V_2=\frac{f^2d_0(w_1)}{10}$$
$$=\frac{f^2[d_0(w_1^{\rm AWI})+d_0(\Delta w_1)]}{10}\equiv-(\Lambda_{V_2})^4, \tag{65}$$

we obtain $\Lambda_{B_1}=0.41$ GeV, $\Lambda_{W_2}=0.24$ GeV, and $\Lambda_{V_2}=0.21$ GeV. Then Λ_X takes a reasonable value, $\Lambda_X=0.2$–0.4 GeV. If $a\Lambda_X > m/\Lambda_X$, $O(a)$ terms become more important than m_R terms. With $\Lambda_X=0.2$–0.4 GeV, this condition at $a^{-1}=1$ GeV or $a^{-1}=2$ GeV corresponds to $m_R<40$–160 MeV or $m_R<20$–80 MeV, respectively.

C. Validity of (W)ChPT

We now estimate the relative size of the next-to-leading contribution to the leading contribution in WChPT for m_π^2:

$$R(\text{WChPT})=\frac{m_R(A+aw_1)+a^2w_0}{32\pi^2f^2}\log\left(\frac{Am_R}{\Lambda^2}\right) \tag{66}$$

for WChPT at finite a, where the parameters A, Λ, w_1, and w_0 depend on a. We plot R (WChPT) in Fig. 4 at $a(\text{GeV}^{-1})=0$, 0.44 ($\beta=2.2$), 0.55 ($\beta=2.1$), 0.79 ($\beta=1.95$), and 1.1 ($\beta=1.8$). While the one-loop contribution takes reasonable values, 10%–30%, at $0.1\ \text{GeV}<m_R<0.2$ GeV for all a, the contribution from $\log m_R$ in WChPT diverges as $m_R\to 0$. This might invalidate WChPT in the chiral limit. We will consider this problem in the next section.

IV. RESUMMATION OF $\log m_R$ TERMS

As evident from the analysis in the previous subsection, the $\log m_R$ contribution becomes larger and larger toward the chiral limit, so that we cannot neglect "higher order" term such as $(\log m_R)^n (n=2,3,\dots)$. We must perform a resummation of the $\log m_R$ term at all orders. Since it is possible in principle but difficult in practice to calculate the $(\log m_R)^n$ contribution at n-loop order, we derive resummed formulas from a different point of view.

As discussed in Refs. [5–7], the massless pion corresponds to the inverse of the divergent correlation length at

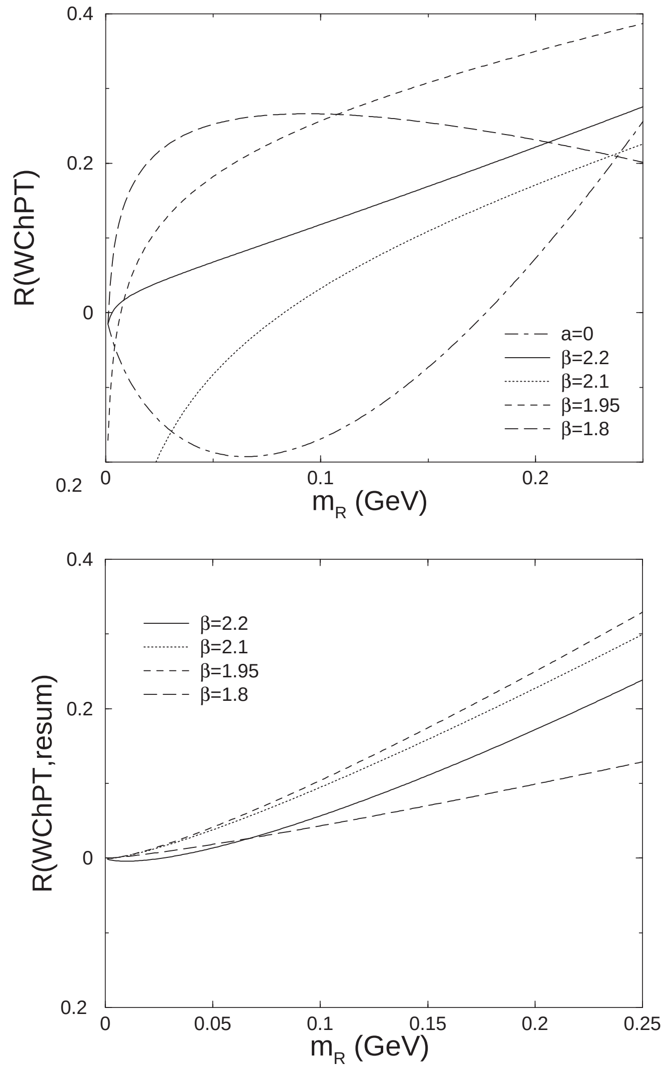

FIG. 4. (a) The relative size of the next-to-leading contribution to the leading one in WChPT as a function of the quark mass m_R at $\beta = 1.8$, 1.95, 2.1, and 2.2, together with the one in the continuum limit (ChPT). (b) Same quantities in the resummed WChPT.

the second order phase transition point. Since the effective theory which describes this phase transition is some four-dimensional scalar (pion) theory with rather complicated interactions,[3] the phase transition has a mean-field critical exponent with possible log corrections. In particular, the pion mass, the inverse of the correlation length, should behave near the critical point as

$$m_\pi^2 = C m_R \left\{ \log\left(\frac{m_R}{D} \right) \right\}^{\nu'} + \cdots, \tag{67}$$

where $\cdots$ represent less singular contributions. If we expand

$$\begin{aligned} \left\{ \log\left(\frac{m_R}{D} \right) \right\}^{\nu'} &= \left\{ \log\left(\frac{\Lambda_0^2}{AD} \right) + \log\left(\frac{A m_R}{\Lambda_0^2} \right) \right\}^{\nu'} \\ &= X^{\nu'} \sum_{n=0}^{\infty} \frac{\nu' !}{(\nu' - n)! n!} \left(\frac{Y}{X} \right)^n \\ &= X^{\nu'} \left(1 + \nu' \frac{Y}{X} + \cdots \right), \end{aligned} \tag{68}$$

where

$$X = \log\left(\frac{\Lambda_0^2}{AD} \right), \tag{69}$$

$$Y = \log\left(\frac{A m_R}{\Lambda_0^2} \right), \tag{70}$$

the formula at the next-to-leading order in WChPT, Eq. (56), is recovered, with the identification that

$$\frac{\nu'}{X} = \frac{a^2 w_0}{32\pi^2 f^2}, \quad C X^{\nu'} = A. \tag{71}$$

To determine ν' and X separately, the explicit calculation in WChPT at two-loops or more orders is necessary. This will be considered in future investigations.

We have finally obtained the following resummed formulas for m_π^2 and $m_{\rm AWI}$:

$$\begin{aligned} m_\pi^2 = {} & A m_R \left\{ \log\left(\frac{m_R}{\Lambda_0} \right) \right\}^{a^2 w_0 / 32\pi^2 f^2} \\ & \times \left[1 + \frac{m_R A + m_R a w_1}{32\pi^2 f^2} \log\left(\frac{A m_R}{\Lambda^2} \right) \right], \end{aligned} \tag{72}$$

$$\begin{aligned} m_{\rm AWI} = {} & A_0 m_R \left\{ \log\left(\frac{m_R}{\Lambda_0} \right) \right\}^{a^2 w_0 / 32\pi^2 f^2} \\ & \times \left[1 + \frac{m_R a w_1^{\rm AWI}}{32\pi^2 f^2} \log\left(\frac{A m_R}{\Lambda_{\rm AWI}^2} \right) \right], \end{aligned} \tag{73}$$

where A, Λ_0, and ω_0 may be different from those in Eqs. (56),(57). It is better to use these formulas instead of the previous ones, Eqs. (56),(57), in future investigations. Equation (60) remains the same.

As a trial, we use these formulas with $A_0 = 1$, $\Lambda_{\rm AWI} = \Lambda$, and $\Lambda_0 = 1$ GeV, in order to fit m_π^2 and $m_{\rm AWI}$ simultaneously, at each a. The quality of the fit is as good as the previous one, and the fitting parameters are compiled in the end of Table II. In addition, the next-to-leading contribution, the second term in Eq. (73), vanishes toward $m_R = 0$ as shown in Fig. 4, where R (WChPT) in the previous subsection, which is now modified as

[3]Indeed, our WChPT is an approximation of this effective theory.

$$R(\text{WChPT,resum}) = \frac{m_R A + m_R a w_1}{32\pi^2 f^2} \log\left(\frac{A m_R}{\Lambda^2}\right), \quad (74)$$

is plotted at $\beta = 1.8$, 1.95, 2.1, and 2.2.

V. CONCLUSIONS AND DISCUSSIONS

In this paper we have derived the effective chiral Lagrangian which includes the a^2 effect of the Wilson-type quark action in the case of the $N_f = 2$ degenerate quarks. Using this effective Lagrangian the quark mass (m_R) dependences of m_π^2, m_{AWI}, and f_π have been calculated at the one-loop level. We have then simultaneously fitted m_π^2 and m_{AWI}, obtained by the CP-PACS Collaboration for $N_f = 2$ full QCD simulations, using the WChPT formula, and have found that the data are consistently described. We have attempted the continuum extrapolation of the WChPT formula.

Compared to standard ChPT, several distinct features such as the additive mass renormalization, $O(a)$ corrections to the chiral log ($m_R \log m_R$) term, a more singular term ($\log m_R$) generated by $O(a^2)$ contributions, and the presence of both $m_R \log m_R$ and $\log m_R$ terms in m_{AWI}, lead to success for the WChPT formula in describing the CP-PACS data. Although an ambiguity in the definition of K_c caused by the additive mass renormalization can be avoided by the use of m_{AWI}, the last feature, the existence of both $m_R \log m_R$ and $\log m_R$ terms in m_{AWI}, makes the WChPT formula different from that in ChPT. The large $O(a)$ correction to the $m_R \log m_R$ term plays an essential role in describing the actual data, although more or fewer others have some contributions. We have also derived the formula after resumming the $\log m_R$ terms, using the fact that the mean-field critical exponent receives the log correction.

Because of the limitation of available data, our WChPT analysis is far from complete. Therefore it is important to refine the analysis by taking the correlation between m_π^2 and m_{AWI} into account and including f_π data in the simultaneous fit, in order to establish the validity of WChPT. Reanalyses of other full QCD data have to be done of course. It is also urgent to derive the WChPT formula for other cases [12] such as the quench/partially quench cases, the $N_f = 3$ nondegenerate case, vector mesons and baryons, and heavy-light mesons.

Once the validity of the WChPT to describe lattice QCD data is established, instead of thinking that the quark masses in the current full QCD simulations are too heavy for the ChPT to apply, we may say that some (but not all) lattice data are well described by the (Wilson) chiral perturbation theory, by which errors associated with the chiral extrapolation may be well controlled [13].

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