Chiral multiplets of heavy-light mesons

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The recent discovery of a narrow resonance in $D_s\pi^0$ by the BaBar Collaboration is consistent with the interpretation of a heavy $J^P(0^+,1^+)$ spin multiplet. This system is the parity partner of the ground state $(0^-,1^-)$ multiplet, which we argue is required in the implementation of $SU(3)_L\times SU(3)_R$ chiral symmetry in heavy-light meson systems. The $(0^+,1^+)\rightarrow(0^-,1^-)+\pi$ transition couplings satisfy a Goldberger-Treiman relation, $g_\pi=\Delta M/f_\pi$, where ΔM is the mass gap. The BaBar resonance fits the 0^+ state, with a kinematically blocked principal decay mode to $D+K$. The allowed $D_s+\pi$, $D_s+2\pi$, and electromagnetic transitions are computed from the full chiral theory and found to be suppressed, consistent with the narrowness of the state. This state establishes the chiral mass difference for all such heavy-quark chiral multiplets, and precise predictions exist for the analogous B_s and strange doubly heavy baryon states.

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I. INTRODUCTION

Recently, the BaBar Collaboration has reported the observation of a narrow resonance in $D_s^+\pi^0$ with a mass of 2317 MeV [1]. There is also a hint of a second state in $D_s\pi^0\gamma$ with a mass 2460 MeV. The mass difference between the $D_s^*(2317)$ and the well established lightest charm meson, $D_{u,d}$ is $\Delta M=452$ MeV. This is less than the kaon mass, thus kinematically forbidding the decay $D_s^*(2317)\rightarrow D_{u,d}+K$. In the present paper we will argue that both of these states are indeed members of the heavy $(0^+,1^+)$ spin multiplet, usually identified with the $j_\ell=1/2$, p-wave valence light-quark system.

Heavy-light systems involving a single valence light quark, heavy-light mesons, and baryons with two heavy quarks, can be viewed as a "tethered" constituent quark. In QCD the light-quark chiral symmetry is spontaneously broken and the symmetry is realized nonlinearly, usually described via chiral Lagrangians with nearly massless pions. Suppose we could somehow modify QCD to restore both the explicit and the spontaneously broken light-quark chiral symmetries while maintaining the confining properties of QCD. For example, a hypothetical four-fermion interaction involving only the light quarks could trigger such a phase change.

In the symmetric phase, the constituent quark would be expected to become massless—preserving the chiral symmetry. However, tethered constituent quark states cannot become massless, due to the presence of the heavy quark(s) which tether the light quark. In this case, the confined heavy-light hadrons *would be forced* to appear in parity-doubled bound states transforming as linear representations of the light-quark chiral symmetry. The heavy-light hadrons would be described by an effective field theory of these parity-doubled states. The light-hadron states would also appear in either massless chiral representations, like the constituent quark, or in massive parity-doubled representations. The pions would be forced into a linear chiral multiplet with scalar mesons.

As we adiabatically turn off the interactions modifying QCD and restore the spontaneously broken chiral phase, we would expect the effective field theory Lagrangians describing the light hadrons and the heavy-light hadrons to evolve smoothly, with the main effect being a shift in the scalar mass terms (this is irrespective of the order of the associated chiral phase transition; it is simply the flow of the Lagrangian parameters that matters to us). Linear Σ models have been used successfully to describe the physics of light hadrons since their introduction by Gell-Mann and Lévy [2] in 1960. For heavy-light systems, the effective field theory would require parity-doubled representations of the hadrons with the doubling degeneracies being lifted through couplings to the light-quark chiral fields. An essential feature of this dynamical symmetry breaking mechanism is the Goldberger-Treiman relation [3] between the mass splitting in chiral multiplets and the couplings to soft pions. We will use this analogy to construct a Σ model for the tethered constituent-quark states that invokes linear realizations of the light-quark chiral symmetry and a smooth interpolation to the chiral broken phase. Effective Lagrangians with both heavy-quark symmetry and linearly realized chiral symmetry have been constructed previously to generate a simplified dynamical model of the bound meson states of heavy and light quarks [4–6].

The main consequence is that this newly observed multiplet is, to a good approximation, the *chiral partner* of the $(0^-,1^-)$ ground state. Physically, this means that the two orthogonal linear combinations of meson fields, $D(0^+,1^+)+D(0^-,1^-)$ and $D(0^+,1^+)-D(0^-,1^-)$, have well defined transformation properties under $SU(3)_L\times SU(3)_R$, transforming as (approximately) pure (1,3) and (3,1), respectively. The parity doubling implies that the main decay transitions $(0^+,1^+)\rightarrow(0^-,1^-)+$"$\pi$," where "$\pi$" refers to any of the pseudoscalar octet mesons, are governed by a Goldberger-Treiman (GT) relation $g_\pi=\Delta M/f_\pi$, where ΔM is the 0^- - 0^+ mass difference, g_π is the $0^+\rightarrow 0^-\pi$ coupling con-

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stant, and f_π is the pion decay constant. ΔM represents a left-right transition, the analogue of the mass of the nucleon, which occurs in the successful GT relation $g_{NN\pi}=m_N/f_\pi$. The observed magnitude of $\Delta M \approx m_N/3$, the mass scale of a single constituent quark, is indicative of a connection with spontaneously broken chiral symmetry breaking. The $D_s(2317)$ would be the first clear observation of this more general phenomenon, which we expect to hold in all heavy-light mesons and doubly heavy baryons, and to yield correspondingly narrow states in the B_s mesons and the strange heavy-heavy-light baryons *ccs*, *cbs*, and *bbs*.

The GT relation implies that the expected rates for the nonstrange resonances to decay through pionic transitions are now determined precisely for all of the analogous systems, and we tabulate them. The BaBar resonances, however, interpreted as the 0^+ and 1^+ states, would have had principal GT transitions, decaying through a kaonic transition $D_s(2317)\rightarrow D_{u,d}+K$, but this is blocked by kinematics. It must therefore proceed through $SU(3)$ breaking effects, decaying by $D_s(2317)\rightarrow D_{u,d}+(\eta\rightarrow\pi^0)$, emitting a virtual η that then mixes with the π^0 through isospin violating effects. We compute its width and find it is indeed narrow. We also tabulate the widths for all analogous processes. We further show that electromagnetic transitions are indeed, and somewhat remarkably, suppressed, since they involve cancellations between the heavy and light magnetic moments. Again, we tabulate rates for the analogue systems. The overall picture of the chiral structure of the heavy-light systems works quite well.

In the next section we begin with the familiar fact that heavy-light mesons $H\sim\bar{Q}q$, containing one heavy quark Q and one light quark q, are subject to powerful symmetry constraints. The heavy-quark symmetry must apply in the limit $m_Q\rightarrow\infty$ [7–9], typically implying vanishing hyperfine splitting effects and leading to degenerate spin multiplets, as in the $(0^-,1^-)$ ground state of the system. In addition, the light-quark chiral symmetries of QCD must apply. Hence, in the limit $m_q\rightarrow 0$, where $q=(u,d,s)$, any Lagrangian must be invariant under the $SU(3)_L\times SU(3)_R$ chiral symmetry, broken by the light-quark mass matrix and electromagnetism. Together these heavy-quark (HQ) and chiral light-quark symmetries control the interactions of heavy-light (HL) mesons with pions and K mesons, etc. [10]. Presently we will not delve into chiral constituent-quark models. Rather, we write directly the chiral Lagrangian for the two heavy-quark multiplets $H\sim(0^-,1^-)$ and $H'\sim(0^+,1^+)$, implementing both HQ symmetry and chiral $SU(3)_L\times SU(3)_R$.

II. EFFECTIVE LAGRANGIANS

We begin in the limit in which the linear chiral symmetry is an exact symmetry of the vacuum. In this limit the heavy-light $(0^-,1^-)$ multiplet is degenerate with the $(0^+,1^+)$ multiplet. We must therefore introduce four independent heavy-meson fields; $H(H')$ are 0^- (0^+) scalars, while H_μ (H'_μ) are 1^- (1^+) vectors. Heavy-quark symmetry is implemented by constructing multiplets for a fixed four-velocity supersector v_μ. One heavy-spin multiplet consists of the 0^+ scalar together with the abnormal parity (1^+) vector as (H',H'^μ).

Under heavy-spin $O(4)=SU(2)_h\times SU(2)_l$ rotations the $(H,'H'^\mu)$ mix analogously to (H,H^μ), transforming as the **4** representation of $O(4)$ (the four-velocity label$_v$ and $SU(3)^i$ indices are understood):

$$\mathcal{H}'=(i\gamma^5H'+\gamma_\mu H'^\mu)\left(\frac{1+\not{v}}{2}\right). \tag{1}$$

The other multiplet consists of the usual 0^- scalar and a 1^- vector (H,H^μ):

$$\mathcal{H}=(i\gamma^5H+\gamma_\mu H^\mu)\left(\frac{1+\not{v}}{2}\right). \tag{2}$$

Note that we have the constraint $v_\mu H^\mu=0$. We have introduced the caligraphic $\mathcal{H}$ and $\mathcal{H}'$ with explicit projection factors. We have reversed the order of the heavy-spin projection and the field components because it is more convenient for writing manifestly chirally invariant operators this way. The field $\mathcal{H}'$ has overall odd parity, while $\mathcal{H}$ is even. With either of these fields a properly normalized kinetic term can be written as

$$-i\frac{1}{2}\mathrm{Tr}(\bar{\mathcal{H}}v\cdot\partial\mathcal{H}) \tag{3}$$

where the trace extends over Dirac and flavor indices (see the Appendix for the full normalization conventions).

To implement a linear chiral symmetry multiplet structure in the HL sector we construct left-handed and right-handed linear combinations of the heavy-spin multiplets. We define the following chiral combinations:

$$\mathcal{H}_L=\frac{1}{\sqrt{2}}(\mathcal{H}-i\mathcal{H}'),\quad \mathcal{H}_R=\frac{1}{\sqrt{2}}(\mathcal{H}+i\mathcal{H}'). \tag{4}$$

Under $SU(3)_L\times SU(3)_R$ these fields transform as $\mathcal{H}_R\sim(1,3)$ and $\mathcal{H}_L\sim(3,1)$.

To describe the light mesons we introduce the chiral field Σ, transforming as $(\bar{3},3)$ under $SU(3)_L\times SU(3)_R$. The usual linear Σ model Lagrangian is:

$$\mathcal{L}_L=\frac{1}{4}\mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma)+\kappa\,\mathrm{Tr}(\mathcal{M}_q\Sigma+\mathrm{H.c.})-V(\Sigma), \tag{5}$$

where $\mathcal{M}_q$ is the light-quark mass matrix, representing explicit $SU(3)_L\times SU(3)_R$ breaking, and

$$V(\Sigma)=-\frac{1}{4}\mu_0^2\mathrm{Tr}(\Sigma^\dagger\Sigma)+\frac{1}{8}\lambda_0\mathrm{Tr}(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma) -\Lambda_0(e^{i\theta}\det\Sigma+\mathrm{H.c.})+\cdots, \tag{6}$$

where we have included $U(1)_A$ breaking effects through a 't Hooft determinant term. The field Σ is a 3×3 complex matrix. The imaginary components of Σ are the 0^- nonet, including π,K,η,η', while the real components form a 0^+ nonet.

In the $\mathcal{M}_q=0$ limit and in the chiral symmetric phase $\langle\Sigma\rangle=0$, the 0^+ and 0^- octets are degenerate, forming a massive parity-doubled nonet. When $\mathcal{M}_q=0$ and the chiral symmetry is spontaneously broken $\langle\Sigma\rangle=I_3 f_\pi$, and the 0^+ nonet becomes heavy, while the 0^- octet becomes a set of massless Goldstone bosons, the η' receiving a nonzero mass from the 't Hooft determinant. In the spontaneously broken symmetry phase we can write

$$\Sigma=\xi\tilde{\sigma}\xi, \quad \xi=\exp(i\pi\cdot\lambda/2f_\pi), \tag{7}$$

where the 0^+ nonet field is

$$\tilde{\sigma}=\sqrt{\frac{2}{3}}\sigma I_3+\sigma^a\lambda^a \tag{8}$$

and $\langle\sigma\rangle=\sqrt{3/2}f_\pi$. If we then take the 0^+ nonet mass to infinity, holding f_π fixed, we can describe the octet of pseudoscalar mesons in the nonlinear Σ model:

$$\Sigma=f_\pi\exp(i\pi^a\cdot\lambda^a/f_\pi). \tag{9}$$

Note that $f_\pi=93.3$ MeV in this normalization. The nonlinear chiral Lagrangian takes the form

$$\mathcal{L}_L=\frac{1}{4}\mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma)+\frac{1}{2}\kappa\,\mathrm{Tr}(\mathcal{M}_q\Sigma+\mathrm{H.c.}), \tag{10}$$

where we restore the $SU(3)_L\times SU(3)_R$ symmetry breaking through the usual first order inclusion of $\mathcal{M}_q\neq 0$. The choice $\kappa=f_\pi m_\pi^2/(m_u+m_d)$ fits the meson masses, yielding the Gell-Mann–Okubo formula. The expansion of the mass term in the meson fields to quadratic order yields an isospin violating $\pi^0\eta$ mixing term that we will require later:

$$\mathcal{L}_L=\cdots+\frac{m_\pi^2(m_u-m_d)}{\sqrt{3}(m_u+m_d)}\pi^0\eta. \tag{11}$$

We now write an effective Lagrangian involving both the HL mesons and the Σ field, implementing HQ symmetry and chiral symmetry. The lowest order effective Lagrangian to first order in an expansion in the chiral field Σ and to zeroth order in $(1/m_Q)$ is [4]

$$\begin{aligned}\mathcal{L}_{LH}=&-i\frac{1}{2}\mathrm{Tr}(\overline{\mathcal{H}}_L v\cdot\partial\mathcal{H}_L)-i\frac{1}{2}\mathrm{Tr}(\overline{\mathcal{H}}_R v\cdot\partial\mathcal{H}_R)\\&-\frac{g_\pi}{4}[\mathrm{Tr}(\overline{\mathcal{H}_L}\Sigma^\dagger\mathcal{H}_R)+\mathrm{Tr}(\overline{\mathcal{H}_R}\Sigma\mathcal{H}_L)]-\Delta[\mathrm{Tr}(\overline{\mathcal{H}_L}\mathcal{H}_L)\\&+\mathrm{Tr}(\overline{\mathcal{H}_R}\mathcal{H}_R)]+i\frac{g_A}{2f_\pi}[\mathrm{Tr}(\overline{\mathcal{H}_L}\gamma^5(\partial\!\!\!/\Sigma^\dagger)\mathcal{H}_R)\\&-\mathrm{Tr}(\overline{\mathcal{H}_R}\gamma^5(\partial\!\!\!/\Sigma)\mathcal{H}_L)].\end{aligned} \tag{12}$$

Here we will neglect explicit $SU(3)_L\times SU(3)_R$ symmetry breaking effects which arise through $\mathcal{M}_q\neq 0$ corrections to the vacuum expectation value $\langle\Sigma\rangle$. The effects of quadratic terms of order $\Sigma^\dagger\Sigma/\Lambda_{QCD}$, which we have not displayed, are essential, however, to fit the D_s and $D_{u,d}$ mass differences, since the two-state mixing produces a level repulsion, decreasing the D_s mass relative to the $D_{u,d}$ at linear order in $\mathcal{M}_q$. These terms do arise with approximately the correct scale of coefficients in chiral quark models such as [4]. The Δ term in Eq. (12) can be "gauged away" by a reparameterization transformation on the fields, so we henceforth drop it.

Additional terms can be added at first order in $(1/m_Q)$ to accommodate the intramultiplet hyperfine mass splitting effects:

$$\begin{aligned}\mathcal{L}_{0,hyperfine}=&\frac{\Lambda_{QCD}^2}{12m_Q}[k_1\mathrm{Tr}(\overline{\mathcal{H}}_L\sigma_{\mu\nu}\mathcal{H}_L\sigma^{\mu\nu})\\&+k_2\mathrm{Tr}(\overline{\mathcal{H}}_R\sigma_{\mu\nu}\mathcal{H}_R\sigma^{\mu\nu})].\end{aligned} \tag{13}$$

Parity symmetry implies invariance under $L\leftrightarrow R$, and $\Sigma\leftrightarrow\Sigma^\dagger$; hence

TABLE I. The heavy-light spectrum compared to experiment. We report the difference between the excited state masses and the ground state (D or B) in each case. We have assumed that $\Delta M(m_c)=\Delta M(m_b)=\Delta M(\infty)=349$ MeV.

Charmed meson masses (MeV)			Bottom meson masses (MeV)		
	Model	Experiment		Model	Experiment
$D^{*0}-D^0$	142 [a]	142.12 ± 0.07	$B^{*0}-B^0$	46 [a]	45.78 ± 0.35
$D^{*+}-D^+$	141 [a]	140.64 ± 0.10	$B^{*+}-B^+$	46 [a]	45.78 ± 0.35
$D_s^{*+}-D_s^+$	144 [a]	143.8 ± 0.41	$B_s^{*+}-B_s^+$	47 [a]	47.0 ± 2.6
$D^0(0^+)-D^0$	349 [b]		$B^0(0^+)-B^0$	349 [b]	
$D^+(0^+)^-D^+$	349 [b]		$B^+(0^+)^-B^+$	349 [b]	
$D_s^+(0^+)^-D_s^+$	349 [a]	349 ± 1.3 [c]	$B_s^+(0^+)^-B_s^+$	349	
$D^0(1^+)-D^0(0^+)$	142		$B^0(1^+)-B^0(0^+)$	46	
$D^+(1^+)^-D^+(0^+)$	141		$B^+(1^+)^-B^+(0^+)$	46	
$D_s^+(1^+)^-D_s^+(0^+)$	144		$B_s^+(1^+)^-B_s^+(0^+)$	47	

[a]Experimental input to model parameter fit.
[b]$SU(3)$ symmetry limit result; $(m_{u,d}^*/m_s^*)\Delta M=255$ MeV is expected in chiral constituent models ([4], see text).
[c]BaBar result [1].

$$k \equiv k_1 = k_2. \tag{14}$$

There are additional terms of order $1/m_Q$, such as $\mathrm{Tr}[\overline{\mathcal{H}}_L(v\cdot\partial)^2\mathcal{H}_L]+(L\leftrightarrow R)$.

The hyperfine splitting effects to first order in Σ and first order in $1/m_Q$ are LR transition terms of the form

$$\mathcal{L}_{1,hyperfine}=\frac{k'\Lambda_{QCD}^2}{12m_Q f_\pi}[\mathrm{Tr}(\overline{\mathcal{H}}_L\sigma_{\mu\nu}\Sigma^\dagger\mathcal{H}_R\sigma^{\mu\nu})+\mathrm{H.c.}]. \tag{15}$$

Since these terms are overall second order effects we expect them to be small, $k'\ll k$.

We can perform redefinitions of the heavy fields to bring them into linear flavor $SU(3)$ representations in the parity eigenbasis:

$$\mathcal{H}_L=\frac{1}{\sqrt{2}}\xi^\dagger(\mathcal{H}-i\mathcal{H}'),\quad \mathcal{H}_R=\frac{1}{\sqrt{2}}\xi(\mathcal{H}+i\mathcal{H}'), \tag{16}$$

and the Lagrangian now takes the form

$$\begin{aligned}\mathcal{L}_{LH}=&-\frac{1}{2}\mathrm{Tr}[\overline{\mathcal{H}}v\cdot(i\partial+\mathcal{V})\mathcal{H}]-\frac{1}{2}\mathrm{Tr}[\overline{\mathcal{H}}'v\cdot(i\partial+\mathcal{V})\mathcal{H}']+i\frac{1}{2}G_A\mathrm{Tr}(\overline{\mathcal{H}}'v\cdot\mathcal{A}\mathcal{H})-i\frac{1}{2}G_A\mathrm{Tr}(\overline{\mathcal{H}}v\cdot\mathcal{A}\mathcal{H}')+\frac{g_\pi}{4}[\mathrm{Tr}(\overline{\mathcal{H}}'\tilde{\sigma}\mathcal{H}')\\&-\mathrm{Tr}(\overline{\mathcal{H}}\tilde{\sigma}\mathcal{H})]+\frac{g_A}{2f_\pi}[\mathrm{Tr}(\overline{\mathcal{H}}'\gamma^5\gamma_\mu\{\mathcal{A}^\mu,\tilde{\sigma}\}\mathcal{H}')-\mathrm{Tr}(\overline{\mathcal{H}}\gamma^5\gamma_\mu\{\mathcal{A}^\mu,\tilde{\sigma}\}\mathcal{H})]+\frac{g_A}{2f_\pi}\mathrm{Tr}[\overline{\mathcal{H}}'\gamma^5\gamma_\mu(\partial^\mu\tilde{\sigma}-i[\mathcal{V}^\mu,\tilde{\sigma}])\mathcal{H}]\\&+\frac{g_A}{2f_\pi}\mathrm{Tr}[\overline{\mathcal{H}}\gamma^5\gamma_\mu(\partial^\mu\tilde{\sigma}-i[\mathcal{V}^\mu,\tilde{\sigma}])\mathcal{H}']+\cdots,\end{aligned} \tag{17}$$

where

$$\mathcal{V}_\mu=\frac{1}{2}(\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger)=\frac{i}{8f_\pi^2}[\tilde{\pi},\partial_\mu\tilde{\pi}]+\cdots, \tag{18}$$

$$\mathcal{A}_\mu=i\frac{1}{2}(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger)=-\frac{1}{2f_\pi}\partial_\mu\tilde{\pi}+\cdots, \tag{19}$$

and $\tilde{\pi}=\sqrt{2/3}\,\eta'+\pi^a\lambda^a$. We have introduced a phenomenological parameter G_A, which is unity in the lowest order model of Eq. (12), but can differ from unity because of mixing with higher states and other effects in full QCD. The chiral Lagrangian of Eq. (17) obeys a nonlinearly realized chiral symmetry appropriate to the chiral broken phase, in addition to heavy-quark symmetry.

In the chiral symmetric phase the multiplets $\mathcal{H}'$ and $\mathcal{H}$ are degenerate in mass. In the broken phase, however, we have $\langle\tilde{\sigma}\rangle\approx f_\pi I_3$, and from Eq. (17) we learn that the physical mass of the $\mathcal{H}'$ state is elevated by the amount $+g_\pi f_\pi/2$, while the $\mathcal{H}$ state is depressed by $-g_\pi f_\pi/2$ (see the Appendix for normalization conventions). The mass difference is given by the Goldberger-Treiman relation

$$\Delta M=g_\pi f_\pi, \tag{20}$$

where ΔM is the mass difference between the multiplets that is now measured to be 349 MeV from the BaBar results (see Table I). This implies $g_\pi=3.73$. With the introduction of the phenomenological parameter G_A in Eq. (17), the true soft pion coupling constant $\tilde{g}_\pi$ differs from g_π and is given by the modified Goldberger-Treiman relation

$$\tilde{g}_\pi=G_A\Delta M/f_\pi. \tag{21}$$

Note that we can decouple the heavier field $\mathcal{H}'$ and the 0^+ nonet, yielding an effective chiral Lagrangian for the lower energy field $\mathcal{H}$ alone:

$$\mathcal{L}_{LH}=-\frac{1}{2}\mathrm{Tr}[\overline{\mathcal{H}}v\cdot(i\partial+\mathcal{V})\mathcal{H}]-g_A\mathrm{Tr}\,\overline{\mathcal{H}}\gamma^5\mathcal{A}\mathcal{H}. \tag{22}$$

This Lagrangian [11] contains relatively limited information, compared to Eq. (17), and will not apply on energy scales approaching ΔM above the ground state mass.

The hyperfine mass splitting effects now take the form

$$\begin{aligned}\mathcal{L}_{LH,hyperfine}=k\frac{\Lambda_{QCD}^2}{12m_Q}&[\mathrm{Tr}(\overline{\mathcal{H}}'\sigma_{\mu\nu}\mathcal{H}'\sigma^{\mu\nu})\\&+\mathrm{Tr}(\overline{\mathcal{H}}\sigma_{\mu\nu}\mathcal{H}\sigma^{\mu\nu})].\end{aligned} \tag{23}$$

We have assumed that $k'\ll k$ is negligible, as per the discussion of ordering the strengths of various terms. This implies that the hyperfine splitting within the heavy $(0^+,1^+)$ multiplet is identical to that in the ground state $(0^-,1^-)$ mesons.

III. SPECTRUM

From the Lagrangian of Eq. (17) we see that the chiral multiplet structure, together with HQ symmetry, controls the masses within the $(0^+,1^+)$ multiplet. The spin-weighted center of mass of any $(0^+,1^+)$ multiplet will have a universal $\Delta M(m_Q)$ above the corresponding spin-weighted ground state in all heavy-light systems. This is weakly dependent upon m_Q, and approaches a universal value $\Delta M(\infty)$ in the heavy-quark symmetry limit $m_Q\to\infty$.

The observed $D_s(0^+)$ resonance in the BaBar experiment measures $\Delta M(m_c)$. $\Delta M(m_c)$ is therefore determined by the mass difference of the $D_s(0^+,2317)$ and the ground state $D_s(0^-,1969)$ to be

$$\Delta M(m_c)=349 \ \text{MeV}. \tag{24}$$

A predicted value of $\Delta M(\infty)\approx 338$ MeV was obtained in [4] from a fit to the HL chiral constituent-quark model.

Using $\Delta M(m_c)$ we predict the $D_s(1^+)$ mass:

$$M(D_s(1^+))=2460 \ \text{MeV} \tag{25}$$

from the sum of the $D_s(1^-,2112)$ mass and $\Delta M(m_c)$. This is in good agreement with the hinted second resonance in $D_s\pi^0\gamma$ in the BaBar data.

In the nonstrange $D^{\pm}(0^+,1^+)$ and $D^0(0^+,1^+)$ multiplets the chiral mass gap is given by $\Delta M(m_c)$, subject to $SU(3)_L\times SU(3)_R$ explicit symmetry breaking corrections. Neglecting these symmetry breaking effects we would predict

$$M(D^{\pm}(0^+))=2217 \ \text{MeV},$$

$$M(D^{\pm}(1^+))=2358 \ \text{MeV},$$

$$M(D^0(0^+))=2212 \ \text{MeV},$$

$$M(D^0(1^+))=2355 \ \text{MeV}. \tag{26}$$

We caution, however, that in the chiral constituent-quark model of [4] we would obtain a mass gap of $(m^*_{u,d}/m^*_s)\Delta M(m_c)\approx 255$ MeV, reducing the above results by 94 MeV. Hence, the explicit $SU(3)$ breaking effects could be large and are model dependent.

There will be corrections of order Λ_{QCD}/m_c to the inferred value of the universal $\Delta M(\infty)$, from the center of mass. The B system will provide a better determination of the heavy-quark symmetry limit and the chiral mass gap $\Delta M(\infty)$. We have no prediction for these corrections at present so we take $\Delta M(m_b)=\Delta M(m_c)\pm 35$ MeV.

For the B_s system we have the established ground state mass of $M(B_s(0^-))=5370$ MeV and we likewise infer

$$M(B_s(1^-))=5417 \ \text{MeV}. \tag{27}$$

From this we predict

$$M(B_s(0^+))=5718\pm 35 \ \text{MeV},$$

$$M(B_s(1^+))=5765\pm 35 \ \text{MeV}. \tag{28}$$

In the nonstrange B system we therefore predict in the $SU(3)_L\times SU(3)_R$ limit

$$M(B^{\pm}(0^+))=M(B^0(0^+))=5627\pm 35 \ \text{MeV}. \tag{29}$$

The $M(B^{\pm}(1^-))$ and $M(B^0(1^-))$ masses must be inferred from heavy-quark symmetry. This is an intramultiplet hyperfine splitting, above the $M(B^{\pm}(0^+))=5627$ MeV ground state. In the B system it is reduced by a factor of $m_c/m_b=0.33$ relative to the corresponding $M(D(1^-))-M(D(0^-))=142$ MeV. Hence, we have

$$M(B^{\pm}(1^-))-M(B^{\pm}(0^-))=47 \ \text{MeV}. \tag{30}$$

We thus predict

$$M(B^{\pm}(1^+))=M(B^0(1^+))=5674\pm 35 \ \text{MeV}. \tag{31}$$

As in the nonstrange $D_{u,d}$ system, the explicit $SU(3)$ breaking effects may be analogously large.

The determination of ΔM is model dependent. In the constituent-quark model of [4] we argued that ΔM is linear in the constituent-quark mass, for small mass, and predicted a value of $\Delta M=338$ MeV. For large quark mass, a linear potential model would suggest that the intermultiplet splitting should scale as $\Delta M\sim(m_q)^{-1/3}$. These potential models tend to predict a larger ΔM than observed by the BaBar experiment. Since the intermultiplet splitting should vanish in the chiral symmetric phase, it is clear that a potential model is likely to overestimate the size of the actual splitting for smaller quark mass. Of course, it should also be stated that a valence quark picture for tethered quark states is likely to be an oversimplification.

IV. PIONIC TRANSITIONS

A. Intermultiplet transitions

The chiral structure of the theory controls the decays of the form $(0^+,1^+)\to(0^-,1^-)+\pi$. These decays between multiplets proceed through the axial coupling term:

$$+i\frac{1}{2}G_A\mathrm{Tr}(\bar{\mathcal{H}}'v\cdot\mathcal{A}\mathcal{H})-i\frac{1}{2}G_A\mathrm{Tr}(\bar{\mathcal{H}}v\cdot\mathcal{A}\mathcal{H}'). \tag{32}$$

All such transitions for HL mesons and doubly heavy baryons are governed by the same amplitude, and differ only in the phase space.

1. $D_{u,d}(0^+,1^+)\to D_{u,d}(0^-,1^-)+\pi$

The amplitudes $D_{u,d}(0^+,1^+)\to D_{u,d}(0^-,1^-)+\pi$ follow from writing Eq. (32) in component form. For example, the π^0 transition is

$$+i\frac{1}{2}G_A\mathrm{Tr}(\bar{\mathcal{H}}'v\cdot\mathcal{A}\mathcal{H})\to-\frac{iG_A}{2f_\pi}(\bar{D}'_\mu D^\mu-\bar{D}'D)v_\nu\partial^\nu\pi^0, \tag{33}$$

leading to the partial width

$$\Gamma(D_{u,d}(0^+,1^+)\to D_{u,d}(0^-,1^-)+\pi^0)=\frac{G_A^2(\Delta M)^2}{4\pi f_\pi^2}|\vec{p}_\pi|$$
$$=164G_A^2 \ \text{MeV}. \tag{34}$$

The charged pion rate differs by $\sqrt{2}$ in amplitude:

$$\Gamma(D_{u,d}(0^+,1^+)\to D_{u,d}(0^-,1^-)+\pi^+)=326G_A^2 \ \text{MeV}. \tag{35}$$

Thus the partial widths of the $D_{u,d}(0^+)$ and $D_{u,d}(1^+)$ are identical, and the total widths are 490 G_A^2 MeV. We expect $G_A\approx 1$. We caution, however, that these widths are proportional to ΔM and will be modified if there are significant $SU(3)$ breaking corrections to the nonstrange ΔM.

2. $B_{u,d}(0^+,1^+)\to B_{u,d}(0^-,1^-)+\pi$

We expect identical results for the $B_{u,d}(0^+,1^+)\to B_{u,d}(0^-,1^-)+\pi$ transitions:

$$\Gamma(B_{u,d}(0^+,1^+)\to B_{u,d}(0^-,1^-)+\pi^0)=164G_A^2\ \ \text{MeV}, \tag{36}$$

$$\Gamma(B_{u,d}(0^+,1^+)\to B_{u,d}(0^-,1^-)+\pi^+)=326G_A^2\ \ \text{MeV}. \tag{37}$$

As in the previous case, there are potentially significant $SU(3)$ breaking corrections to these widths, if there are correspondingly significant modifications to the nonstrange ΔM.

3. $D_s(0^+,1^+)\to D_s(0^-,1^-)+\pi^0$

The decay proceeds by the emission of a virtual η that then mixes with the π^0 through the light-meson chiral Lagrangian:

$$+i\frac{1}{2}G_A\mathrm{Tr}(\overline{\mathcal{H}}'v\cdot\mathcal{A}\mathcal{H})\to-\frac{iG_A}{f_\pi}(\overline{D_s}'_\mu D_s^\mu-\overline{D_s}'D_s)\times\left(\frac{-2}{\sqrt{3}}\right)v_\nu\partial^\nu\eta^0. \tag{38}$$

The amplitude for the decay is therefore

$$\frac{\Delta M}{2f_\pi}\delta_{\eta\pi^0}G_A,\qquad \delta_{\eta\pi^0}=\left(\frac{2m_\pi^2(m_u-m_d)}{(m_\eta^2-m_\pi^2)(m_u+m_d)}\right), \tag{39}$$

where $\delta_{\eta\pi^0}$ parameterizes the $\eta\pi^0$ mixing. $\delta_{\eta\pi^0}$ vanishes in the limit of nonet symmetry, as a contribution from the η' meson will exactly cancel the η contribution given above. Instanton effects, parameterized by the determinant term in the Σ potential in Eq. (6), break the nonet symmetry and generate a large contribution to the η' mass, suppressing the cancellation. The singlet axial current anomalies also signal a direct coupling of the singlet η' to gluons, which will modify the nonet coupling of the η' to hadrons, perhaps further suppressing the singlet contribution to the mixing with the π^0. In the following we include only the octet η contribution to the mixing with the π^0. Using $\delta_{\eta\pi^0}=(1/2)\times(1/43.7)$ [12,13], the widths are given by

$$\Gamma(D_s(0^+)\to D_s(0^-)+\pi^0)=(164\ \ \text{MeV})\,\delta^2_{\eta\pi^0}=21.5G_A^2\ \ \text{keV}, \tag{40}$$

$$\Gamma(D_s(1^+)\to D_s(1^-)+\pi^0)=(164\ \ \text{MeV})\,\delta^2_{\eta\pi^0}=21.5G_A^2\ \ \text{keV}. \tag{41}$$

4. $B_s(0^+,1^+)\to B_s(0^-,1^-)+\pi^0$

This case is identical to the D_s system discussed above:

$$\Gamma(B_s(0^+)\to B_s(0^-)+\pi^0)=(164\ \ \text{MeV})\,\delta^2_{\eta\pi^0}=21.5\ \ G_A^2\ \ \text{keV}, \tag{42}$$

$$\Gamma(B_s(1^+)\to B_s(1^-)+\pi^0)=(164\ \ \text{MeV})\,\delta^2_{\eta\pi^0}=21.5G_A^2\ \ \text{keV}. \tag{43}$$

5. $D_s(1^+)\to D_s(0^-)+2\pi$

The analysis of the 2π transitions is more complicated and involves effects from the 0^+ nonet. These effects are not expected to be cleanly separable from other resonances higher in the spectrum of the light-quark system. Nonetheless, these probably indicate the correct order of the effect, and we include them here because they are predicted consequences of the model.

In the model the decay can proceed via decay to a virtual $\tilde{\sigma}$, which then converts to the 2π state, $D_s(1^+)\to D_s(0^-)+(\sigma\to 2\pi)$. The relevant component of the $\tilde{\sigma}$ field, which is a 3×3 matrix, is the (33) component. This must then mix with the (11) and (22) components to produce the pions.

The relevant HL meson operator is the term from Eq. (17) of the form

$$\frac{g_A}{2f_\pi}\mathrm{Tr}(\overline{\mathcal{H}}'\gamma^5\gamma_\mu(\partial^\mu\tilde{\sigma}-i[\mathcal{V}^\mu,\tilde{\sigma}])\mathcal{H})+\cdots. \tag{44}$$

In component form this becomes

$$-\frac{ig_A}{f_\pi}D'_{s\mu}{}^\dagger\cdot D_s\left(\frac{\sqrt{2}}{\sqrt{3}}\partial^\mu\sigma^0-\frac{2}{\sqrt{3}}\partial^\mu\sigma^8\right). \tag{45}$$

The mixing of the σ^0 and σ^8 with 2π is controlled by the light-sector chiral Lagrangian of Eq. (5). Using the replacement $\Sigma\to\xi\tilde{\sigma}\xi$, Eq. (5) becomes

$$\frac{1}{4}\mathrm{Tr}(\partial\tilde{\sigma}-i[\mathcal{V}_\mu,\tilde{\sigma}])^2+\frac{1}{4}\mathrm{Tr}\{\mathcal{A}_\mu,\tilde{\sigma}\}^2+\cdots. \tag{46}$$

We shift $\sigma^0=\sqrt{3/2}f_\pi+\sigma$ and expand the currents to obtain the effective couplings to the 2π:

$$\to\frac{1}{f_\pi\sqrt{3}}[(\partial\vec{\pi})^2-m_\pi^2(\vec{\pi})^2][\sqrt{2}\sigma^0+\sigma^8]. \tag{47}$$

Putting this together gives the $D_s(1^+)\to D_s(0^-)+\pi^0\pi^0$ amplitude:

$$\frac{2g_A}{3f_\pi^2}\epsilon^\mu q_\mu(q^2-4m_\pi^2)\left[\frac{1}{q^2-m^2_{\sigma^0}}-\frac{1}{q^2-m^2_{\sigma^8}}\right], \tag{48}$$

where $q^2=(p_1+p_2)^2$ is the 2π system invariant mass. The $\pi^+\pi^-$ amplitude is $\sqrt{2}$ larger.

The resulting widths are controlled by $\Delta M(D_s(1^+)-D_s(0^-))=491$ MeV. The phase space integrals are extremely sensitive to scalar masses, varying by over an order of magnitude when the lighter singlet mass is varied over the range 0.8 to 1.2 GeV. We will give the values for $m_{\sigma^0}=1.0$ GeV with a heavier octet scalar at 1.5 GeV (note that the singlet-octet splitting is opposite for scalars and pseudoscalars). As in our discussion of the η-π^0 mixing, the coupling of the singlet meson to hadrons is also uncertain due to the mixing with gluons. Nevertheless, we give representative widths using nonet couplings and the scalar masses given above:

$$\Gamma(D_s(1^+)\to D_s(0^-)\pi^0\pi^0)=6.4g_A^2=2.3\ \text{keV},$$

$$\Gamma(D_s(1^+)\to D_s(0^-)\pi^+\pi^-)=5.4g_A^2=1.9\ \text{keV},$$

$$\Gamma(D_s(1^+)\to D_s(0^-)\pi\pi)=4.2\ \text{keV}, \tag{49}$$

where $g_A=0.6$ is used. The corresponding widths in the B_s system are significantly smaller due to the reduced phase space:

$$\Gamma(B_s(1^+)\to B_s(0^-)\pi^0\pi^0)=0.19g_A^2=0.07\ \text{keV},$$

$$\Gamma(B_s(1^+)\to B_s(0^-)\pi^+\pi^-)=0.13g_A^2=0.05\ \text{keV},$$

$$\Gamma(B_s(1^+)\to B_s(0^-)\pi\pi)=0.12\ \text{keV}. \tag{50}$$

6. $D_s(1^+)\to D_s(0^-)+3\pi$

This decay is allowed by the phase space and proceeds through ω-ϕ mixing. It is highly suppressed by chiral symmetry, and also has a strong OZI-rule suppression. We do not consider it further in the present paper.

B. Intramultiplet transitions

The chiral structure of the theory controls the intramultiplet decays of the form $D(1^\pm)\to D(0^\pm)+\pi$ (see, e.g., [14]). These decays within multiplets proceed through the g_A coupling term:

$$+\frac{g_A}{2f_\pi}[\mathrm{Tr}(\overline{\mathcal{H}}'\gamma^5\gamma_\mu\{\mathcal{A}^\mu,\tilde{\sigma}\}\mathcal{H}')-\mathrm{Tr}(\overline{\mathcal{H}}\gamma^5\gamma_\mu\{\mathcal{A}^\mu,\tilde{\sigma}\}\mathcal{H})] \tag{51}$$

with $\tilde{\sigma}\to f_\pi I_3$. Such transitions are relevant only for the charmed mesons. The intramultiplet hyperfine splitting in mass between the $1^\pm$ and $0^\pm$ states is too small in the B mesons (and even smaller in the *ccq*, *bcq*, and *bbq* baryons) to allow this decay.

The resulting decay widths are

$$\Gamma(D^{*+}(1^-)\to D^+(0^-)\pi^0)=181g_A^2\ \text{keV}=65.2\ \text{keV},$$

$$\Gamma(D^{*+}(1^-)\to D^0(0^-)\pi^+)=83g_A^2\ \text{keV}=30.1\ \text{keV}, \tag{52}$$

where $g_A=0.6$ was used. The identical widths are obtained for the $1^+\to 0^++\pi$ modes:

$$\Gamma(D^{*+}(1^+)\to D^+(0^+)\pi^0)=181g_A^2\ \text{keV}=65.2\ \text{keV},$$

$$\Gamma(D^{*+}(1^+)\to D^0(0^+)\pi^+)=83g_A^2\ \text{keV}=30.1\ \text{keV}. \tag{53}$$

V. ELECTROMAGNETIC TRANSITIONS

In the static limit, heavy-light mesons can be used to define the electromagnetic properties of the tethered constituent quark. In fact, it has sometimes been suggested that the constituent-quark mass be defined through the meson magnetic moment in this limit.

The $M1$ electromagnetic transitions govern the intramultiplet processes $1^\pm\to 0^\pm\gamma$, while the $E1$ transitions govern the intermultiplet processes $(1^+,0^+)\to(1^-,0^-)\gamma$. There are significant finite heavy-quark mass corrections particularly for the D_s system. We observe below that the $1^-\to 0^-\gamma$ $M1$ transition amplitude, and the three $E1$ transition amplitudes $1^+\to 1^-\gamma$, $1^+\to 0^-\gamma$, and $0^+\to 1^-\gamma$ receive a common overall coefficient $r_{\bar{Q}q}$. We find

$$r_{\bar{Q}q}=\left(1-\frac{m_q^* e_{\bar{Q}}}{m_{\bar{Q}}^* e_q}\right), \tag{54}$$

where m^* and e are the mass and charge of the constituent quarks. In the D_s system the anticharm quark has a charge of $-2/3$ and the strange quark charge $-1/3$ leading to a large suppression (see the Appendix of [15]):

$$r_{\bar{c}s}=\left(1-\frac{2m_s^*}{m_c^*}\right). \tag{55}$$

The D_d has a somewhat smaller suppression and the D_u an enhancement. In the B-meson system the situation is reversed as the $\bar{b}$ quark has charge $+1/3$ although the overall effects are much smaller due to the larger mass for the b quark.

We use the usual constituent-quark potential model to estimate the electromagnetic transition rates. For the $M1$ magnetic transitions $1^-\to 0^-\gamma$ the rate is given by

$$\Gamma_{\mathrm{M1}}(i\to f\gamma)=\frac{4\alpha}{3}\mu_{\bar{Q}q}^2k^3(2J_f+1)|\langle f|j_0(kr)|i\rangle|^2, \tag{56}$$

where the magnetic dipole moment is

$$\mu_{\bar{Q}q}=\frac{m_{\bar{Q}}^*e_q-m_q^*e_{\bar{Q}}}{2m_{\bar{Q}}^*m_q^*}=\frac{e_q}{2m_q^*}r_{\bar{Q}q} \tag{57}$$

and k is the photon energy.

The strength of the electric-dipole transitions is governed by the size of the radiator and the charges of the constituent quarks. The $E1$ transition rate is given by

$$\Gamma_{E1}(i\to f+\gamma)=\frac{4\alpha\langle e_{\mathrm{avg}}\rangle^2}{27}k^3(2J_f+1)|\langle f|r|i\rangle|^2\mathcal{S}_{if}, \tag{58}$$

where the mean charge is

$$\langle e_{\rm avg}\rangle = \frac{m_Q^* e_q - m_q^* e_{\bar{Q}}}{m_Q^* + m_q^*} = \frac{e_q m_Q^* r_{\bar{Q}q}}{m_Q^* + m_q^*}, \tag{59}$$

k is the photon energy, and the statistical factor $\mathcal{S}_{if}$ for $(i,f)=(0^+,1^-)$ is 1, for $(1^+,1^-)$ is 2/3, and for $(1^+,0^-)$ is 1.

To evaluate the factor $r_{\bar{Q}q}$ we use the constituent-quark masses:

$$m_u^* = m_d^* = 350\ \text{MeV},$$

$$m_s = 480\ \text{MeV},$$

$$m_c^* = [3M(J/\Psi) - M(\eta_c)]/4 = 1530\ \text{MeV},$$

$$m_b^* = [3M(\Upsilon) - M(\eta_b)]/4 = 4730\ \text{MeV}. \tag{60}$$

This in turn leads to the $r_{\bar{Q}q}$ factors:

$$r_{\bar{c}u} = 1.23, \quad r_{\bar{b}u} = 0.85,$$

$$r_{\bar{c}d} = 0.54, \quad r_{\bar{b}d} = 1.07,$$

$$r_{\bar{c}s} = 0.38, \quad r_{\bar{b}s} = 1.10. \tag{61}$$

With these $r_{\bar{Q}q}$ factors we can see the large cancellation between the light (d,s) quark moment and the charm quark moment. Using the measured total width of the D^{+*} to set the pionic transition, the partial rates for photonic decays in the D^{0*} and D^{+*} systems can be calculated. The uncertainty in the total width drops out for the ratio of partial widths:

$$\frac{\Gamma(D^{+*}\to D^+ + \gamma)}{\Gamma(D^{0*}\to D^0 + \gamma)} = \frac{1.6\pm 0.4}{27.4\pm 2.1} = 0.058\pm 0.015. \tag{62}$$

This implies

$$\left|\frac{\mu(D^{+*})}{\mu(D^{0*})}\right| = 0.24\pm 0.03\Big|_{expt} = \frac{1}{2}\left(\frac{r_{\bar{c}d}}{r_{\bar{c}u}}\right) = 0.22\Big|_{theory}. \tag{63}$$

In the HQ limit this ratio is 0.5. Hence the finite charm quark mass provides a large cancellation for the D^{+*} system. This suppression of the $M1$ transition will be even larger for the D_s^{+*} system as $r_{\bar{c}s} < r_{\bar{c}d}$. Rates for the allowed $M1$ transitions are given in Table II.

The same cancellation that appears for the $M1$ transition is operative for the $E1$ transitions. In the D_s system this greatly suppresses the rate for the $(0^+,1^+)\to(0^-,1^-)+\gamma$ allowed $E1$ transitions. The $E1$ transition rates and photon energies are also presented in Table II.

The observed ratio of branching fractions $(D_s(1^-)\to D_s(0^-)\pi^0)/\Gamma(D_s(1^-)\to D_s(0^-)\gamma) = 0.062\pm 0.026$ is large compared to our prediction of 0.018. This may indicate that $r_{\bar{c}s}$ is more suppressed than our estimate in Eq. (61). If we implement the experimental value for this ratio, then the $E1$ radiative transitions of Table II for the $\bar{c}s$ system should be reduced by a factor of ~ 3.

In the B system there is no suppression for the B_d and B_s transitions which are slightly enhanced by the $r_{\bar{b}q}$ factors. There is a small suppression in the B_u states. The resulting electromagnetic transition rates and photon energies for the narrow B states are presented in Table II.

For the $1^+\to 0^+\gamma$ $M1$ transition we define the coefficient $r'_{\bar{Q}q}$:

$$r'_{\bar{Q}q} = \left(1 + 3\frac{m_q^* e_{\bar{Q}}}{m_Q^* e_q}\right). \tag{64}$$

The decay rate is given by

$$\Gamma_{M1}(i\to f\gamma) = \frac{4\alpha}{3}\mu'^2_{\bar{Q}q} k^3 (2J_f+1)|\langle f|j_0(kr)|i\rangle|^2, \tag{65}$$

where the effective magnetic dipole moment $\mu'_{\bar{Q}q}$ is now

$$\mu'_{\bar{Q}q} = \frac{-m_Q^* e_q - 3m_q^* e_{\bar{Q}}}{6m_Q^* m_q^*} = -\frac{e_q}{6m_q^*} r'_{\bar{Q}q} \tag{66}$$

and k is the photon energy,

$$r'_{\bar{c}s} = 2.88, \quad r'_{\bar{b}s} = 0.70. \tag{67}$$

These decay rates are also included for the D_s and B_s systems in Table II.

Finally, we have ignored mixing between the two 1^+ p-wave mesons as the parity partner of the s-wave mesons has $j_\ell = 1/2$, which does not mix with the $j_\ell = 3/2$ state at leading order in the heavy-quark expansion. The total angular momentum of the light quark, j_ℓ, is conserved in the heavy-quark limit.

VI. DOUBLY HEAVY BARYONS

We will provide only a schematic discussion of the corresponding situation in the doubly heavy baryons and defer tabulating the detailed results. These systems provide interesting targets of opportunity in the spectroscopy of QCD, but are challenging to reconstruct. For some recent reviews and relevant information see [16]. A chiral constituent-quark model similar to [4] has also been developed for these systems [17].

There are four distinct doubly heavy baryon systems, each transforming as flavor $SU(3)$ triplets, $[cc]_{J=1}(u,d,s)$, $[bc]_{J=0}(u,d,s)$, $[bc]_{J=1}(u,d,s)$, and $[bb]_{J=1}(u,d,s)$. There is evidence for doubly charmed baryons in the SELEX data [18].

These systems are interesting because of heavy-quark symmetry, since the $[QQ]$ subsystem has a large mass, of order $\sim 2m_Q$, and forms, in the subsystem ground state, a tightly bound anticolor triplet combination, which can be viewed as a heavy spin-1 or spin-0 antiquark, $[QQ]\sim\overline{Q}'$. Hence, doubly heavy baryons can be viewed as ultraheavy

TABLE II. The predicted hadronic and electromagnetic transition rates for narrow $j_l^P = 1/2^-$ $(1S)$ and $j_l^P = 1/2^+$ $(1P)$ heavy-light states. ''Overlap'' is the reduced matrix element overlap integral; ''dependence'' refers to the sensitive model parameters, as defined in the text. We take $G_A = 1$ and extract g_A from a fit to the D^{+*} total width. Note that the $\overline{c}s$ transitions are sensitive to $r_{\overline{c}s}$; if we implement the observed ratio of branching fractions $[D_s(1^-)\rightarrow D_s(0^-)\pi^0]/\Gamma(D_s(1^-)\rightarrow D_s(0^-)\gamma) = 0.062 \pm 0.026$ then the $E1$ radiative transitions for the $\overline{c}s$ system should be reduced by a factor of ~ 3.

System	Transition	Q(keV)	Overlap	Dependence	Γ (keV)	Expt branching ratio
$(c\overline{u})$	$1^- \rightarrow 0^- + \gamma$	137	0.991	$r_{\overline{c}u}$	33.5	$(38.1 \pm 2.9)\%$
	$1^- \rightarrow 0^- + \pi^0$	137		g_A	43.6	$(61.9 \pm 2.9)\%$
	total				77.1	
$(c\overline{d})$	$1^- \rightarrow 0^- + \gamma$	136	0.991	$r_{\overline{c}d}$	1.63	$(1.6 \pm 0.4)\%$
	$1^- \rightarrow 0^- + \pi^0$	38		g_A	30.1	$(30.7 \pm 0.5)\%$
	$1^- \rightarrow 0^- + \pi^+$	39		g_A	65.1	$(67.7 \pm 0.5)\%$
	total				96.8	96 ± 22
$(c\overline{s})$	$1^- \rightarrow 0^- + \gamma$	138	0.992	$r_{\overline{c}s}$	0.43	$(94.2 \pm 2.5)\%$
	$1^- \rightarrow 0^- + \pi^0$	48		$g_A\delta_{\eta\pi 0}$	0.0079	$(5.8 \pm 2.5)\%$
	total				0.44	
$(c\overline{s})$	$0^+ \rightarrow 1^- + \gamma$	212	2.794	$r_{\overline{c}s}$	1.74	
	$0^+ \rightarrow 0^- + \pi^0$	297		$G_A\delta_{\eta\pi 0}$	21.5	
	total				23.2	
$(c\overline{s})$	$1^+ \rightarrow 0^+ + \gamma$	138	0.992	$r'_{\overline{c}s}$	2.74	
	$1^+ \rightarrow 0^+ + \pi^0$	48		$g_A\delta_{\eta\pi 0}$	0.0079	
	$1^+ \rightarrow 1^- + \gamma$	323	2.638	$r_{\overline{c}s}$	4.66	
	$1^+ \rightarrow 0^- + \gamma$	442	2.437	$r_{\overline{c}s}$	5.08	
	$1^+ \rightarrow 1^- + \pi^0$	298		$G_A\delta_{\eta\pi 0}$	21.5	
	$1^+ \rightarrow 0^- + 2\pi$	221		$g_A\delta_{\sigma_1\sigma_3}$	4.2	
	total				38.2	
$(b\overline{u})$	$1^- \rightarrow 0^- + \gamma$	46	0.998	$r_{\overline{b}u}$	0.78	
	total				0.78	
$(b\overline{d})$	$1^- \rightarrow 0^- + \gamma$	46	0.998	$r_{\overline{b}d}$	0.24	
	total				0.24	
$(b\overline{s})$	$1^- \rightarrow 0^- + \gamma$	47	0.998	$r_{\overline{b}s}$	0.15	
	total				0.15	
$(b\overline{s})$	$0^+ \rightarrow 1^- + \gamma$	293	2.536	$r_{\overline{b}s}$	58.3	
	$0^+ \rightarrow 0^- + \pi^0$	297		$G_A\delta_{\eta\pi 0}$	21.5	
	total				79.8	
$(b\overline{s})$	$1^+ \rightarrow 0^+ + \gamma$	47	0.998	$r'_{\overline{b}s}$	0.061	
	$1^+ \rightarrow 1^- + \gamma$	335	2.483	$r_{\overline{b}s}$	56.9	
	$1^+ \rightarrow 0^- + \gamma$	381	2.423	$r_{\overline{b}s}$	39.1	
	$1^+ \rightarrow 1^- + \pi^0$	298		$G_A\delta_{\eta\pi 0}$	21.5	
	$1^+ \rightarrow 0^- + 2\pi$	125		$g_A\delta_{\sigma_1\sigma_3}$	0.12	
	total				117.7	

mesons, $[QQ]q \sim \overline{Q}'q$. The hyperfine mass splittings in these systems are suppressed.

The doubly heavy baryon ground states of the form $[QQ]_{J=1}q$ will consist of multiplets containing $(1/2^+, 3/2^+)$ heavy-spin fields. For example, the $[cc](u,d)$ ground state will contain one $I=\frac{1}{2}$ spin-1/2 baryon, and one $I=\frac{1}{2}$ spin-3/2 baryon (this is the analogue in $[ss](u,d)$ of a multiplet containing the (Ξ^0, Ξ^-) spin-1/2 baryon from the octet and the (Ξ^{*0}, Ξ^{*-}) spin-3/2 resonance from the decuplet). The hyperfine splitting mass differences within the multiplets, between the spin-3/2 and spin-1/2 members, have been estimated in [17]. Note that in the case of the $[cb]_{J=1}$ we have the normal $(1/2^+, 3/2^+)$ multiplet, while in the $[cb]_{J=0}$ system the spin-3/2 partner is absent.

The parity partner states are correspondingly p-wave resonances $(1/2^-, 3/2^-)$. The intramultiplet mass splitting will be approximately identical to the case of the ground state. The intramultiplet chiral mass gap for each system is therefore given by $\Delta M(\infty) \approx 349 \pm 35$ MeV, subject to modification by $SU(3)$ breaking effects.

The four systems with a strange quark will display narrow resonances in analogy to the $D_s(0^+, 1^+)$. These will have blocked kaonic decays to the ground state. They will decay mesonically in a manner identical to the HL meson system with the correspondence $(0^+,1^+)\leftrightarrow((1/2)^-,(3/2)^-)$. All of our computed intramultiplet widths will correspond identically, but will have significant modifications due to the hyperfine splittings affecting the phase space and kinematic factors, as well as explicit $SU(3)$ breaking effects.

The electromagnetic transitions will correspond with the meson case in a similar fashion. The intramultiplet transitions will be suppressed because of the reduced phase space. The widths in the $[bb]s$ system will have suppressed cancellations and will be predominantly governed by the light-quark terms alone.

VII. CONCLUSIONS

We have examined the chiral structure of the HL meson system in QCD. The ground state $(0^-,1^-)$ multiplet is paired with the $(0^+,1^+)$ multiplet through chiral symmetry. The physical significance of this statement is that the linear combinations of these states, $\mathcal{H}_L$ and $\mathcal{H}_R$, form definite representations under $SU(3)_L\times SU(3)_R$ of (3,1) and (1,3), respectively.

The spontaneous breaking of chiral symmetry in QCD leads to a mass term that elevates the $(0^+,1^+)$ above the $(0^-,1^-)$ by an amount ΔM. Chiral invariance alone implies that the true soft pion coupling constant for intermultiplet transitions obeys a modified Goldberger-Treiman relation $\tilde{g}_\pi = G_A\Delta M/f_\pi$. The analogue of ΔM for the nucleon system is the nucleon mass m_N, which satisfies the traditional Goldberger-Treiman relation $m_N = g_{NN\pi}f_\pi$. $\Delta M\approx m_N/3$ is close to its observed value in the BaBar data of 349 MeV. In a remarkable sense, the HL meson is displaying the chiral dynamical properties of a single light quark that is "tethered" to the heavy quark.

Our hypothesis fits the recent observation of the narrow resonance in $D_s\pi^0$ seen by the BaBar Collaboration. In general, D mesons form $SU(3)$ flavor triplets and approximate heavy-spin multiplets. The nonstrange $I=\frac{1}{2}(0^+,1^+)$ states can undergo $I=1$ transitions to $I=\frac{1}{2}$ $(0^-,1^-)$ by emission of a single pion. The coupling strength of this transition is governed by the GT relation, and these states are therefore broad, with total widths predicted to be 490 MeV.

The $D_s(0^+,1^+)$ resonance, on the other hand, is kinematically forbidden from undergoing its principal decay transition $D_s(0^+,1^+)\rightarrow D(0^-,1^-)+K$. The observed decay $D_s(0^+)\rightarrow D_s(0^-)+\pi^0$ violates isospin and therefore requires $SU(3)$ breaking effects. In the present paper we have addressed the main kinematically allowed effects $D_s(0^+)\rightarrow D_s(0^-)+\pi^0$ and $D_s(1^+)\rightarrow D_s(0^-)+2\pi$. The widths are small, consistent with the narrowness of the observed systems.

We have also tabulated the electromagnetic transitions. Due to the cancellations between light-quark and heavy-quark amplitudes, the intermultiplet $E1$ rates for the $\overline{c}s$ system are suppressed. This explains why the photonic decays are not seen in the BaBar data. In the analogous $\overline{b}s$ system the rates are not suppressed by a similar cancellation, and the electromagnetic transition widths are significantly larger.

It is important to realize that these heavy-quark and chiral symmetry arguments are quite general. What has been discovered by the BaBar experiment is a *phenomenon*. Analogous effects will be seen in the B meson system as well as with doubly heavy baryons.

In the B_s system we expect a splitting between the $(0^-,1^-)SU(3)$ triplet ground state mesons and the analogous $(0^+,1^+)$ resonance multiplet with a mass of 349 $\pm\mathcal{O}(\Lambda_{QCD}/m_{charm})$, or $\sim 349\pm 35$ MeV. Again, the channel $B_s(0^+,1^+)\rightarrow B_s(0^-,1^-)+K$ will be kinematically blocked, while the $B_{u,d}(0^+,1^+)$ states will have similar narrow meson transition widths. We have also described schematically how the doubly heavy baryons $[QQ]q$ will present an analogous situation.

Heavy-light meson states may be classified according to the total angular momentum carried by the light quark to leading order in the heavy-quark limit. In the present paper we have focused on the $j_\ell=1/2$ parity-doubled supermultiplet combining the *s*-wave $(0^-,1^-)$ mesons and the *p*-wave $(0^+,1^+)$ mesons. As mentioned in the Introduction, we expect all heavy-light states to be classified into parity-doubled supermultiplets where the mass splitting between parity partners is governed by the Goldberger-Treiman relation. For example, the $j_\ell=3/2$ supermultiplet consists of the *p*-wave $(1^+,2^+)$ mesons with $j_\ell=\ell+1/2$ and the *d*-wave $(1^-,2^-)$ mesons with $j_\ell=\ell-1/2$, and similarly for the higher angular momentum states. In QCD, string models, and linear potential models, the meson states are expected to be identified with linear Regge trajectories having a common slope [19,20]. A remarkable consequence of this observation is that the mass splitting between parity partners for the higher angular momentum supermultiplets will remain constant, i.e., the Yukawa coupling constant g_π governing the left-right transitions in Eq. (12) will be universal. The dynamical breaking of the light-quark chiral symmetries in QCD is apparently associated with the observed shifts in the opposite parity Regge trajectories by about half a unit of the Regge spacing from a completely parity-doubled picture of the Regge trajectories.

In a larger sense, parity doubling, which is required in any dynamics that can putatively restore spontaneously broken chiral symmetry of QCD without upsetting confinement, appears to play an important role in the real world and is controlling the chiral physics of HL systems.

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APPENDIX: NORMALIZATION CONVENTIONS

Consider a complex scalar field Φ with the Lagrangian

$$\partial_\mu\Phi^\dagger\partial^\mu\Phi-(M+\delta M)^2\Phi^\dagger\Phi. \tag{A1}$$

Define $\Phi' = \sqrt{2M}\exp(iMv\cdot x)\Phi$ (Φ' destroys incoming momentum $Mv_\mu + p_\mu$) and the Lagrangian becomes to order $1/M$

$$iv_\mu \Phi'^{\dagger}\partial^\mu \Phi' - \delta M \Phi'^{\dagger}\Phi'. \tag{A2}$$

Now let $\mathcal{H}_v = \frac{1}{2}(1-\not{v})i\gamma^5\Phi'$ and write in terms of traces (the field $\mathcal{H}_v$ with these conventions annihilates an incoming meson state $|B\rangle$)

$$-i\frac{1}{2}\mathrm{Tr}(\overline{\mathcal{H}}_v v\cdot\partial\mathcal{H}) + \delta M\frac{1}{2}\mathrm{Tr}(\overline{\mathcal{H}}\mathcal{H}_v). \tag{A3}$$

Thus, when the Lagrangian is written in terms of $\mathcal{H}$ and $\mathcal{H}'$, the normal sign conventions are those of the vector mesons, and opposite to those of scalars, i.e., the term in the Lagrangian $+\frac{1}{2}\delta M\,\mathrm{Tr}(\overline{\mathcal{H}}H)$ causes an *increase* in the $\mathcal{H}$ multiplet mass by an amount δM. A properly normalized kinetic term is $\frac{1}{2}\,\mathrm{Tr}(\overline{\mathcal{H}}i\not{\partial}\mathcal{H}) = -i\frac{1}{2}\,\mathrm{Tr}(\overline{\mathcal{H}}v\cdot\partial\mathcal{H})$.

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