Expectation values of four-quark operators in the nucleon

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We calculate the expectation values of QCD operators consisting of the products of the four operators of the light quarks $\bar{q}\Gamma^X q\bar{q}\Gamma^Y q$, with $\Gamma^{X,Y}$ corresponding to the scalar, pseudoscalar, vector, pseudovector (axial), and tensor Lorentz structures, in the nucleon. All combinations of the light flavors are considered. For the evaluation we use elements of the perturbative chiral quark model (PCQM), approximating the contribution of the valence quarks by the contribution of the PCQM constituent quarks. The contribution of the sea quarks is treated by averaging over the QCD pions with the distribution of the pion field being determined by the PCQM. For quarks with the same flavor the expectation values are dominated by the contribution of the sea quarks. In the scalar case the contribution of the sea quarks is dominated by the ‘‘disconnected terms’’ where one of the pairs of the $\bar{q}q$ operators acts on the vacuum while the other one acts on the quarks of the pion. The role of the interference terms with one of the $\bar{q}q$ pairs acting on the sea quarks and the other one acting on the valence quarks increases for quarks with different flavors. The result for the scalar condensate is compared to that obtained earlier in the framework of the Nambu–Jona-Lasinio model.

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I. INTRODUCTION

It is known that the expectation values of the two-quark QCD operators $\bar{q}q$ give the total number of quarks and antiquarks in a hadron under certain reasonable assumptions [1,2]. The motivation for studies of the expectation values of the four-quark operators in hadrons is that they carry information about the correlation of $\bar{q}q$ pairs. These expectation values in nucleons determine the coefficients of the next-to-leading order operators of deep inelastic scattering [3]. Another more reason is the manifestation of such operators in QCD sum rules in nuclear matter [4,5]. The lack of data on the expectation values of these operators became one of the main obstacles for further development of this approach.

Unfortunately, the calculations of these expectation values require some model assumptions on the quark structure of the nucleon. As it stands now, the only calculation of the four-quark condensates in the nucleon is that carried out by Celenza *et al.* [6] for the scalar case in the framework of the Nambu–Jona-Lasinio (NJL) model [7] under certain additional assumptions. Also, some constraints on the values were obtained by Johnson and Kisslinger [8] by analyzing QCD sum rules for nucleons and isobars.

In this paper we calculate the expectation values of the four-quark condensates in nucleons by using elements of the perturbative chiral quark model (PCQM). The chiral quark model, originally suggested in [9], was fully set up in [10–12]. In the PCQM the nucleon is treated as a system of relativistic valence quarks moving in an effective static field. In addition, the valence quarks are supplemented by a perturbative cloud of pseudoscalar mesons as dictated by chiral symmetry requirements. In this paper we restrict to the simplest SU(2) version of the PCQM, which includes only pions. To facilitate the evaluation of the four-quark operators we resort to previously derived results [11], such as nucleon wave function renormalization and self-energy contributions, which are used as an input in the present derivation. Although we do not apply the full perturbative machinery of the PCQM, as laid out in [10], the present evaluation serves as a first indication for the values of the four-quark condensates.

We calculate the expectation values of the four-quark operators as the matrix elements of the PCQM nucleon. In general the QCD operators q act on the valence and the sea quarks. We approximate the averaging over the valence quarks by averaging over the PCQM constituent quarks and also approximate the averaging over the sea quarks by averaging over the pions. In the original formulation of the PCQM the pions were treated as separate degrees of freedom, without taking into account their quark structure. Actually, in [11] the sigma-term was calculated in the framework of PCQM, with the pions determining the sea-quark contribution. We shall employ an additional extension of the PCQM, considering the pions as physical particles with their quark contents being determined by QCD. In this approach we therefore obtain the excess of the four-quark operator expectation value over that of the vacuum in the nucleon volume.

In the following we calculate the expectation values

$$U^{XY,f_1f_2}=\langle N|T^{XY,f_1f_2}|N\rangle \quad (1)$$

of the four-quark operators of light quarks

$$T^{XY,f_1f_2}=(:\overline{q}^{f_1a}\Gamma^X q^{f_1a'}\overline{q}^{f_2b}\Gamma^Y q^{f_2b'}:)(\delta_{aa'}\delta_{bb'}-\delta_{ab'}\delta_{ba'}). \quad (2)$$

Here Γ^X, Γ^Y are the matrices acting on the Lorentz indices, q^f denote QCD quark operators with f standing for the flavor.

The dots denote the normal ordering of the operators, a,a',b,b' represent the color indices. We calculate the condensates for the basic 4×4 matrices

$$\Gamma^S=I,\quad \Gamma^{Ps}=\gamma_5,\quad \Gamma^V_\mu=\gamma_\mu,\quad \Gamma^A_\mu=\gamma_\mu\gamma_5,$$

$$\Gamma^T_{\mu\nu}=\frac{i}{2}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu) \tag{3}$$

with γ_μ being the Dirac matrices, that is we consider the scalar, pseudoscalar, vector, pseudovector (axial), and tensor cases. We also obtain the results for the mixed condensate $U^{SV,du}$ which is important in applications.

These are three types of contributions to U^{XY,f_1f_2} in our approach. All four operators q can act on the constituent quarks, providing the term C^{XY,f_1f_2}. Also, four operators can act on the pions providing the term P^{XY,f_1f_2}. There is also a possibility that two of the operators act on the constituent quarks while the other two act on the pions. Denoting the last term as J^{XY,f_1f_2}, we present the expectation values as

$$U^{XY,f_1f_2}=C^{XY,f_1f_2}+P^{XY,f_1f_2}+J^{XY,f_1f_2}. \tag{4}$$

In Appendix A we show how these contributions manifest themselves in the PCQM formalism.

To simplify the notations we introduce

$$U^{X,f_1f_2}=U^{XX,f_1f_2} \tag{5}$$

with a similar convention for the other functions (C,P,J,T) involved.

While the four-quark condensates are Lorentz scalars in the scalar and pseudoscalar channels, they have a more complicated structure in the case of the vector and axial channels:

$$U^{V(A)}_{\mu\nu}=a^{V(A)}g_{\mu\nu}+b^{V(A)}\frac{p_\mu p_\nu}{m^2}. \tag{6}$$

Here p_μ is the momentum of the nucleon, while m denotes the nucleon mass. Also, in the tensor channel we have

$$U^T_{\mu\nu,\alpha\beta}=a^T s_{\mu\nu,\alpha\beta}+b^T t_{\mu\nu,\alpha\beta} \tag{7}$$

with

$$s_{\mu\nu,\alpha\beta}=g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha}, \tag{8}$$

$$t_{\mu\nu,\alpha\beta}=\frac{1}{m^2}(p_\mu p_\alpha g_{\nu\beta}+p_\nu p_\beta g_{\mu\alpha}-p_\mu p_\beta g_{\nu\alpha}-p_\nu p_\alpha g_{\mu\beta}).$$

We shall denote the values of $a^{V,A,T}$ and $b^{V,A,T}$ corresponding to the contributions C, P, and J by the lower indices, i.e. $a^V_{C,P,J}$, etc.

We approximate the averaging of the operator $\bar{q}\Gamma^X q\bar{q}\Gamma^Y q$ over the valence quarks by the expectation values of the products of the constituent quark operators Q averaged over the renormalized constituent quark PCQM states. Due to the normal ordering of the operators they should act on two different quarks. The expectation value is proportional to the probability to find the two constituent quarks at the same space point

$$\int |\psi(r)|^4 d^3r \sim \frac{1}{4\pi R^3}$$

with R standing for the size of the system of the three constituent quarks (quark core radius), while ψ is the constituent quark wave function. The factor $1/4\pi$ comes from the four angular wave functions $1/(4\pi)^{1/2}$ integrated over the solid angle. Since the PCQM deals with a quark wave function $\psi(r)$ provided in explicit form, the contributions C^{XY,f_1f_2} are also evaluated explicitly. The effect of the wave function renormalization induced by the interaction of the constituent quarks with the pion provides noticeable corrections in the case of scalar and vector structures only.

The averaging of the operators $\bar{q}\Gamma^X q\bar{q}\Gamma^Y q$ over the sea quarks is treated as the expectation value of these operators in pions. The distribution of the pion field is assumed to be that determined by the PCQM.

The pion expectation values where expressed in paper [13] by using the current algebra technique through the four-quark expectation values in vacuum. We obtain the expressions for P^{XY,f_1f_2} through these expectation values. However, to obtain the specific numbers, we use the factorization approximation for the vacuum expectation values, suggested first by Shifman *et al.* [14]. In the factorization approximation it is assumed that the vacuum states dominate in the sum over the intermediate states. While there are indications that this approximation may be violated in some of the channels [3], the factorization was advocated recently in [15]. Under this approximation the expectation values P^{XY,f_1f_2} are expressed by the $\bar{q}q$ vacuum expectation values which are known to be [16]

$$\langle 0|\bar{q}q|0\rangle=-\frac{M_\pi^2F_\pi^2}{m_u+m_d} \tag{9}$$

for each of the light flavors with M_π and F_π being the mass and the decay constant of the pion, while $m_{u(d)}$ are the current quark masses. Here we adopt the notations accepted in chiral perturbation theory [17]. In Eq. (9) q stands for the u or d quark field, and isotopic invariance of the vacuum is assumed. The value $\langle 0|\bar{q}q|0\rangle$ gives the characteristic size of the contribution of the pion sea. Thus the contribution of the constituent quarks is expected to be smaller than that of the sea quarks since $(1/4\pi)|\langle 0|\bar{q}q|0\rangle R^3|\simeq\frac{1}{5}$.

In the "interference term" one of the operators $\bar{q}\Gamma^X q$ acts on the valence quarks while the other $\bar{q}\Gamma^Y q$ acts on the sea quarks. Following our strategy, we approximate the corresponding matrix elements by those averaged over the constituent quarks and over the PCQM pion field. There are several possibilities to insert this four-quark operator. The operator can connect the pion with any of the constituent quarks of the nucleon. This contribution (the "contact interference") is proportional to $\langle\pi|\bar{q}\Gamma^Y q|\pi\rangle$ which does not vanish for the scalar case only. Thus, only the contributions $J^{SV}=J^{VS}$ and J^{SS} have nonzero values which are proportional to the expectation value [17]

$$\langle \pi|\bar{q}q|\pi\rangle = \frac{M_\pi^2}{m_u+m_d} = -\frac{\langle 0|\bar{q}q|0\rangle}{F_\pi^2}. \tag{10}$$

However, there is an additional small factor besides the characteristic parameter $\langle 0|\bar{q}q|0\rangle$. This factor reflects the small probability for the pion and the constituent quark to overlap at the same space point for a nucleon Fock state described by the valence quarks and a pion. The interference process can determine a $QQ\pi$ vertex as well, since the pseudovector and pseudoscalar currents connect the pion and vacuum states. Thus, one can consider the self-energy diagram with one of the PCQM vertices being replaced by the four-quark operator. That would be the first-order diagram in the PCQM πQ interaction. This "vertex interference" leads to a numerically larger contribution, except for the case of the scalar-vector condensate. The neutral pions provide the contribution to the terms $J^{A,ff}$ and $J^{Ps,ff}$ of the quarks with the same flavor. The charged pions contribute to the condensates $\bar{u}\Gamma^A d\bar{d}\Gamma^A u$ and $\bar{u}\Gamma^{Ps} d\bar{d}\Gamma^{Ps} u$. Thus, they provide the contributions to all the structures $J^{X,ud}$ with coefficients defined by the Fierz transform.

Following the general strategy of the PCQM we include only the lowest-order contributions in the πQ interactions. We also assume that only the ground states of the constituent quarks are included as intermediate states in the self-energy diagrams. This means that the nucleon and delta isobars only are included as intermediate states of the nucleon self-energy. This is a standard assumption of PCQM calculations [10,11].

It was shown in [13] that in the scalar case in the four-quark pion expectation value a "disconnected term," in which one of $\bar{q}q$ pairs acts on vacuum, can be singled out in a natural way. This is strongly pronounced in the case of the color-singlet four-quark operator $\bar{q}^a q^a \bar{q}^b q^b$ for which

$$\langle \pi|\bar{q}q\bar{q}q|\pi\rangle = 2\langle 0|\bar{q}q|0\rangle\langle \pi|\bar{q}q|\pi\rangle + \langle \pi|(\bar{q}q\bar{q}q)_{int}|\pi\rangle. \tag{11}$$

The "internal" contribution presented by the second term on the right hand side (rhs) of Eq. (11) appeared to be about an order of magnitude smaller than the "disconnected" one, presented by the first term. For the color asymmetric operator determined by Eq. (2) the disconnected terms still provide about $\frac{3}{4}$ of the total pion expectation value. This leads to the natural presentation.

$$P^{S,f_1f_2} = P^{S,f_1f_2}_{dis} + P^{S,f_1f_2}_{int} \tag{12}$$

with

$$P^{S,ud}_{dis} = \tfrac{2}{3}[2\langle 0|\bar{q}q|0\rangle\langle N|(\bar{q}q)_{sea}|N\rangle] \tag{13}$$

for the different flavors, where $|N\rangle$ represents the fully dressed nucleon state. The factor $\frac{2}{3}$ (it is $\frac{5}{6}$ for identical flavors) on the right hand side of Eq. (13) is the weight of the colorless combination $\bar{q}^a q^a$ in the color asymmetric expectation value defined by Eq. (2). In the case of the scalar condensate the sea quarks provide the main contribution. A large part of it is determined by the disconnected term, related by Eq. (13) to the sea-quark contribution to the πN sigma term. The internal contributions, coming mostly from the sea quarks, are several times smaller. The expectation values of the operators with the same flavor $(\bar{u}\Gamma u)^2$ in the other channels are also determined mostly by the sea quarks. For the mixed-flavor condensate $\bar{u}\Gamma u\bar{d}\Gamma d$ the sea-quark terms and the interference terms provide contributions of the same magnitude in most of the channels. The valence quarks provide a smaller correction. In the case of the scalar-vector condensate $P^{SV}=0$, and for the neutron, the valence quarks provide the main contribution, while in the proton the interference effects contribute to the same order.

In the calculations carried out below, we use the values $F_\pi = 92.4$ MeV for the pion decay constant and the value $m_u+m_d=11$ MeV for the sum of the light quark masses. Latter value, given in [18], leads to the conventional value $\langle 0|\bar{q}q|0\rangle = (-245\ \text{MeV})^3$ at the normalization scale of 1 GeV. This set of values was also used in paper [6]. Note that in papers [10,11] another value for the sum of the quark masses has been used, e.g. $m_u+m_d=14$ MeV. This value was also given in [18] as one of the possible values. Both values for m_u+m_d are consistent with nowadays experimental data [19].

We present the results for the condensates $(\bar{u}\Gamma^X u)^2$ both for protons and neutrons. The values of the $(\bar{d}\Gamma^X d)^2$ condensates are determined by the isotopic invariance relations

$$\langle p|(\bar{d}\Gamma^X d)^2|p\rangle = \langle n|(\bar{u}\Gamma^X u)^2|n\rangle,$$

$$\langle n|(\bar{d}\Gamma^X d)^2|n\rangle = \langle p|(\bar{u}\Gamma^X u)^2|p\rangle$$

while for the mixed-flavor condensates we have

$$\langle p|\bar{u}\Gamma^X u\bar{d}\Gamma^X d|p\rangle = \langle n|\bar{u}\Gamma^X u\bar{d}\Gamma^X d|n\rangle$$

except for the pseudoscalar case $\Gamma^X=\Gamma^Y=\gamma_5$. In the pseudoscalar case an explicit dependence of the interference terms on the current quark masses $m_{u,d}$ causes contributions which break the isotopic invariance. These terms are numerically small.

The results enable to obtain also values for the condensates $\bar{u}\Gamma^X d\bar{d}\Gamma^X u$. This can be done by using the Fierz transform.

We compare the value of the contribution $\langle N|(\bar{u}u+\bar{d}d)^2|N\rangle$ with the value obtained in [6] in the framework of the NJL model under certain additional assumptions. The values appear to differ by about 70%.

For the sake of simplicity we shall use the wording "scalar," "pseudoscalar," etc. condensates for the expectation values of the operators with the repeated Lorentz structures $\bar{q}\Gamma q\bar{q}\Gamma q$. Thus the scalar expectation values are rather scalar-scalar ones, etc.

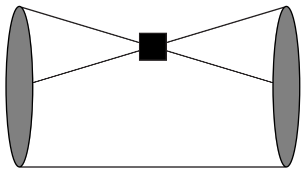

FIG. 1. The contribution of the valence quarks to the expectation values of the four-quark operators. Solid lines denote the valence quarks, the dark squares denote the four-quark operator.

II. CONTRIBUTION OF THE VALENCE QUARKS

In this section we calculate the contribution to the four-quark expectation values arising from averaging over the system of three valence quarks. Using the results of Appendix A we present this contribution by

$$C^{XY,f_1f_2}=\langle\phi_0|\int d^3x q^{f_1}(x)\Gamma^X q^{f_1}(x)\overline{q}^{f_2}(x)\Gamma^Y q^{f_2}(x)|\phi_0\rangle \tag{14}$$

with ϕ_0 denoting the nucleon as a bound state of three valence quarks. Our main assumption here is that the matrix element in the rhs of Eq. (14) is approximated by the matrix element of the renormalized constituent quark operators Q^r, i.e.,

$$C^{XY,f_1f_2}=\langle\phi_0|\int d^3x\overline{Q}^{r,f_1}(x)\Gamma^X Q^{r,f_1}(x)\overline{Q}^{r,f_2}(x)\Gamma^Y \times Q^{r,f_2}(x)|\phi_0\rangle. \tag{15}$$

The renormalization effects are expected to manifest themselves through small corrections only. As we shall see below, these corrections are of the order of several percent only, except for the scalar and vector structures. Thus, we start with the unrenormalized constituent quark operators Q in which the pion cloud is not included. The corresponding contribution

$$\mathring{C}^{XY,f_1f_2}=\langle\phi_0|\int d^3x\overline{Q}^{f_1}(x)\Gamma^X Q^{f_1}(x)\overline{Q}^{f_2}(x) \times\Gamma^Y Q^{f_2}(x)|\phi_0\rangle \tag{16}$$

is illustrated by Fig. 1. The constituent quark operators provide a nonzero value while acting on different quarks of the ϕ_0 system only. This is due to their normal ordering. Thus we find immediately

$$\mathring{C}_n^{XY,uu}=0 \tag{17}$$

for the neutron. Using Eq. (16) we obtain expressions for the contributions of the constituent quarks through the wave functions $\psi_i(x)$. Assuming that the constituent quarks U and D are described by the same wave functions $\psi_u(x)=\psi_d(x)=\psi(x)$, we present the general expressions for the proton as

$$\mathring{C}_p^{X,uu}=\int d^3x\mathcal{F}(x) \tag{18}$$

with

$$\mathcal{F}(x)=\hat{P}\,\overline{\psi}(x)\Gamma^X\psi(x)\overline{\psi}(x)\Gamma^X\psi(x). \tag{19}$$

Here $\hat{P}$ stands for the projection on the symmetric spin state of the two-quark system, while the total antisymmetrization is provided by the color variables. For the condensate $\overline{u}\Gamma^X u\overline{d}\Gamma^X d$ we find both for the proton and neutron

$$\mathring{C}^{X,ud}=2\int d^3x\mathcal{F}(x) \tag{20}$$

since there are two ud pairs.

The invariant coefficients of the rhs of Eqs. (6) and (7) can be obtained in a specific reference frame. Assuming i and j to be the three-dimensional indices, corresponding to the four-dimensional indices μ and ν, we find in the rest frame of the nucleon

$$a_c=-\tfrac{1}{3}C_{ij}^{XX,f_1f_1}\delta_{ij},\quad b_c=C_{00}^{XX,f_1f_1}-a_c \tag{21}$$

for the coefficients of Eq. (6), i.e., for the vector and pseudovector cases. Denoting the three-dimensional indices corresponding to the four-dimensional indices α and β as k and l, respectively, we obtain in the same frame

$$a_c^T=\tfrac{1}{6}C_{ijkl}^{T,uu}\delta_{ik}\delta_{jl},\quad b_c^T=-a_c^T-\tfrac{1}{3}\delta_{jl}C_{0j,0l}^{T,uu}. \tag{22}$$

Above equations are true for any constituent quark model. In the specific case of the PCQM the wave functions of both U and D constituent quarks are [11]

$$\psi(\vec{x})=Ne^{(-\vec{x}^2/2R^2)}\begin{pmatrix}\chi\\ i\beta\dfrac{(\vec{\sigma}\vec{x})}{R}\chi\end{pmatrix} \tag{23}$$

with the normalization constant

$$N=[\pi^{3/2}R^3(1+\tfrac{3}{2}\beta^2)]^{-1/2} \tag{24}$$

and χ being the two-component spinor.

The model parameters

$$\beta=0.39,\quad R=(0.6\pm0.05)\ \text{fm} \tag{25}$$

are fitted to reproduce the value of the axial coupling constant and of the proton charge radius. We will present the numerical values for the mean value of $R=0.6$ fm.

A straightforward calculation provides for the expectation values $\mathring{C}_p^{XY,uu}$ in the proton

$$\mathring{C}_p^{S,uu}=(1-\tfrac{3}{2}\beta^2+\tfrac{15}{16}\beta^4)\mathcal{N}^2 \tag{26}$$

with

$$\mathcal{N}^2=\frac{N^2}{2^{3/2}(1+\frac{3}{2}\beta^2)} \tag{27}$$

while the value is zero for neutron. For the $\bar{u}u\bar{d}d$ condensate we get

$$\mathring{C}^{S,ud}=2(1-\tfrac{3}{2}\beta^2+\tfrac{15}{16}\beta^4)\mathcal{N}^2. \quad (28)$$

The details of the calculations for the other structures are presented in Appendix B.

For the structures $(\bar{u}\Gamma^X u)^2$ in the proton we find for the pseudoscalar case

$$\mathring{C}_p^{Ps,uu}=-\beta^2\mathcal{N}^2, \quad (29)$$

while in the vector channel

$$\mathring{a}^V_{C,p}=-\frac{2}{3}\beta^2\mathcal{N}^2, \quad \mathring{b}^V_{C,p}=(1+\tfrac{13}{6}\beta^2+\tfrac{15}{16}\beta^4)\mathcal{N}^2, \quad (30)$$

and for the pseudovector case

$$\mathring{a}^A_{C,p}=-\tfrac{1}{3}(1-\tfrac{1}{2}\beta^2+\tfrac{15}{16}\beta^4)\mathcal{N}^2, \quad \mathring{b}^A_{C,p}=-\mathring{a}^A_{C,p}. \quad (31)$$

Note that $\mathring{b}^A_C=-\mathring{a}^A_C$ since the matrix element of the time component of the pseudovector operator turns to zero. This is true for the solution of the Dirac equation in any effective field. For the tensor case we get

$$\mathring{a}^T_C=\tfrac{1}{3}(1+\tfrac{1}{2}\beta^2+\tfrac{15}{16}\beta^4)\mathcal{N}^2, \quad \mathring{b}^T_C=-\tfrac{1}{3}(1+\tfrac{7}{2}\beta^2+\tfrac{15}{16}\beta^4)\mathcal{N}^2. \quad (32)$$

Following the previous analysis, these values turn to zero for the neutron.

Turning to the case of different flavors, we find the expectation values of the operators $\bar{u}\Gamma^X u\bar{d}\Gamma^X d$ in a nucleon to be twice as large as the values of $(\bar{u}\Gamma^X u)^2$ in the proton

$$\mathring{C}_p^{X,ud}=2\mathring{C}_p^{X,uu}. \quad (33)$$

We also present an example of the condensate for the mixed scalar-vector structure $T^{SV,du}_\mu=\bar{d}d\bar{u}\gamma_\mu u$, which is needed in applications. In the rest frame of the nucleon only the time component of the vector T_μ survives, providing

$$\mathring{C}^{SV,du}=2(1-\tfrac{15}{16}\beta^4)\mathcal{N}^2. \quad (34)$$

It is convenient to express the values in "units" of the value ε_0^3, defined as

$$\varepsilon_0^3=-\langle 0|\bar{q}q|0\rangle, \quad \varepsilon_0\simeq 245 \text{ MeV}. \quad (35)$$

To get a feeling for the relative size of the contributions, we present the numerical value

$$\mathcal{N}^2=1.50\times 10^{-3}\ \text{GeV}^3\simeq 0.10\varepsilon_0^3 \quad (36)$$

which is the result of the straightforward computation of the rhs of Eq. (27).

The πQ interactions provide the nonzero values of the condensates $\bar{u}\Gamma^X u\bar{u}\Gamma^Y u$ in the neutron. This happens since the four-quark operators connect the only valence U quark with the intermediate U quark of the $\pi^- U$ self-energy loop of the valence D quark. The value is

$$C_n^{X,uu}=-2\frac{\partial\Sigma^-}{\partial E}\,\mathring{C}_p^{X,uu} \quad (37)$$

with Σ^- standing for the contribution of π^- to the self-energy of the valence quark with the energy E. The direct calculation gives $\partial\Sigma^-/\partial E=-0.082$.

Now we take into account the change $\delta\psi$ of the shape of the single quark wave function $\psi(x)$, caused by renormalization [10]

$$\psi^r(x)=\psi(x)+\delta\psi(x), \quad \delta\psi(x)=\Lambda(h(x)+\gamma_0)\psi(x) \quad (38)$$

with

$$\Lambda=\frac{\delta m}{2}\frac{\beta R}{1+\frac{3}{2}\beta^2}, \quad h(x)=\frac{\frac{1}{2}+\frac{21}{4}\beta^2}{1+\frac{3}{2}\beta^2}-\frac{x^2}{R^2}.$$

Here $\delta m<0$ is the shift of the effective mass of the constituent quark caused by the pion cloud. The numerical values are $\Lambda=-2.5\times 10^{-2}$, $h(x)=1.06-x^2/R^2$.

The term containing the function $h(x)$ provides corrections which do not exceed 3%. This happens due to the strong cancellations of the two terms, composing $h(x)$. We shall neglect these corrections. The term containing the Dirac matrix γ_0 mixes the Lorentz structure of the condensates. It provides nonvanishing contributions to the scalar, vector, and scalar-vector expectation values.

Thus, we obtain for renormalized values defined by Eq. (15)

$$C_p^{S,uu}=\mathring{C}_p^{S,uu}+4\Lambda\mathring{C}_p^{SV,uu}\frac{1}{\gamma},$$

$$C^{S,ud}=\mathring{C}^{S,ud}+2\Lambda(\mathring{C}^{SV,du}+\mathring{C}^{SV,ud})\frac{1}{\gamma}, \quad (39)$$

$$C^{SV,du}\approx\mathring{C}^{SV,du}+2\Lambda\left(\mathring{C}^{S,ud}\gamma+\mathring{b}^V_C\frac{1}{\gamma}\right)$$

with $\gamma=(1-\frac{3}{2}\beta^2)/(1+\frac{3}{2}\beta^2)$. Thus, the scalar-scalar condensates are reduced by 16% due to the renormalization effects. The scalar-vector condensate is reduced by 11%. Also, in the vector case we have

$$a^V_C=\mathring{a}^V_C, \quad b^V_C=\mathring{b}^V_C(1+4\Lambda\gamma) \quad (40)$$

reducing the value of $\mathring{b}^V_C$ by about 6%. For the pseudoscalar, axial, and tensor structures the corrections are negligibly small and we put

$$C_p^{X,f_1f_2}=\mathring{C}_p^{X,f_1f_2} \quad (41)$$

in these cases.

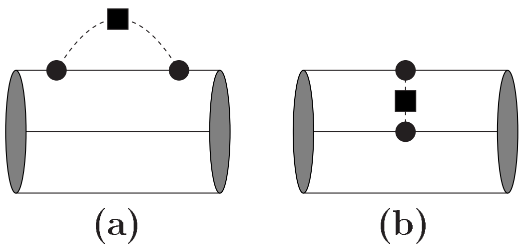

FIG. 2. The contribution of the sea quarks to the expectation values of the four-quark operators. The dashed lines represent the pions. Other notations are the same as in Fig. 1. The dark circles denote the vertices of the pion-quark interaction.

III. CONTRIBUTION OF THE SEA QUARKS

Now we calculate the contribution of the sea quarks. In the PCQM the excess of the sea quarks in nucleons over the QCD vacuum sea inside the nucleon volume is contained in the mesons, coupling to the constituent quarks. In the SU(2) version of the model, which we assume in this paper, only the pions are included. In the framework of the PCQM this contribution is contained in the next-to-leading order of the model. In other words, it is sufficient to include pion exchange in the one-loop approximation.

The distribution of the pion field $\pi^\alpha(x)$ is determined by the PCQM quark-pion interaction

$$H_I(x)=\bar{\Psi}(x)i\gamma_5\frac{S(x)\tau^\alpha\pi^\alpha(x)}{F_\pi}\Psi(x) \tag{42}$$

where $\Psi(x)$ represents the SU(2) doublet of light quarks, while $S(x)$ is the effective scalar field.

In the one-loop approximation of the PCQM the pions are contained in the constituent quark self-energy diagrams and in the diagram describing the pion exchange between the constituent quarks (Fig. 2). The contribution of the sea quarks [see Eq. (4)] can be presented as

$$P^{XY,f_1f_2}=\sum_\alpha \langle\pi^\alpha|T^{XY,f_1f_2}|\pi^\alpha\rangle\left(\frac{\partial\Sigma^\alpha}{\partial M_\pi^2}+\frac{\partial\Lambda^\alpha}{\partial M_\pi^2}\right) \tag{43}$$

with Σ^α and Λ^α standing for the self-energy and exchange contributions of the pions π^α $(\alpha=+,-,0)$. A similar presentation was actually used in [11] for the calculation of the sigma term. In that case T was the scalar quark operator $\bar{q}q$.

The rhs of Eq. (43) can be simplified by noticing that in the PCQM the relation for the total energy shifts caused by the self-energy and exchange diagrams [20]

$$\sum_\alpha \Lambda^\alpha=\frac{10}{9}\sum_\alpha \Sigma^\alpha \tag{44}$$

holds also for each pion α separately, when limiting single quark lines to the ground state.

Using Eq. (44) we present the total pion contribution to the nucleon mass as

$$\Sigma_t=(1+\tfrac{10}{9})\sum_\alpha \Sigma^\alpha.$$

This leads to

$$P^{XY,f_1f_2}=\frac{3\beta^0+4\beta^++2\beta^-}{9}\frac{\partial\Sigma_t}{\partial M_\pi^2} \tag{45}$$

for the proton with

$$\beta^\alpha=\langle\pi^\alpha|T^{XY,f_1f_2}|\pi^\alpha\rangle. \tag{46}$$

The coefficients multiplied by the expectation values β^α are determined by the number of the quarks which can emit the pion π^α and by the strength of the πQQ vertex. For example, $4\beta^+$ means that there are two quarks (these are U quarks) coupling to a π^+, and each of the π^+DU vertices contributes the factor $\sqrt{2}$, etc.

For the neutron we get

$$P^{XY,f_1f_2}=\frac{3\beta^0+2\beta^++4\beta^-}{9}\frac{\partial\Sigma_t}{\partial M_\pi^2}. \tag{47}$$

The value $\partial\Sigma_t/\partial M_\pi^2$ was evaluated earlier in the calculation of the sigma term [11], providing

$$\frac{\partial\Sigma_t}{\partial M_\pi^2}\approx 1.3\ \text{GeV}^{-1}. \tag{48}$$

The pion expectation values β^α can be expressed by the vacuum expectation values of the four-quark operators. This was done in [13] by using the reduction formula obtained by Lehmann, Symanzik, and Zimmermann [21]. Due to the partial conservation of the axial current (PCAC) the pion state vector can be expressed by the vacuum as (see, e.g., Ref. [22])

$$|\pi^\alpha\rangle=\frac{1}{\sqrt{2}F_\pi M_\pi^2}\partial_\mu\mathcal{A}^\alpha_{\mu5}(x)|0\rangle. \tag{49}$$

Here $\mathcal{A}^\alpha_{\mu5}(x)$ is the axial current of the light quarks,

$$\mathcal{A}^-_{\mu5}(x)=\sum_c \bar{d}^c(x)\gamma_\mu\gamma_5u^c(x) \tag{50}$$

with c being the color index. (We shall assume the summation over the colors in all the equations presented below.) This enables to present the expectation values defined in Eq. (46) by the vacuum matrix elements [13]

$$\beta^\alpha=\frac{1}{F_\pi^2}\langle 0|B^\alpha|0\rangle \tag{51}$$

with

$$B^\alpha=\frac{1}{2V}\int d^3x dy_0 dz_0\,\delta(x_0-y_0)\,\delta(z_0-x_0)\times[\bar{Q}_5^\alpha(z_0),[Q_5^\alpha(y_0),T^{XY,f_1f_2}(x)]]. \tag{52}$$

Here V is the normalization volume, and in the double commutator occur the axial charges Q_5^α, corresponding to the axial current $\mathcal{A}_{\mu 5}^{\alpha}(x)$. For example, in the scalar channel it was found

$$B^{\pm}=-(\bar{u}^a u^b \bar{u}^{a'} u^{b'}+\bar{u}^a u^b \bar{d}^{a'} d^{b'}+\bar{d}^a \gamma_5 u^b \bar{u}^{a'}\gamma_5 d^{b'})\times(\delta_{ab}\delta_{a'b'}-\delta_{ab'}\delta_{a'b}) \qquad (53)$$

for the operator $T^{S,uu}$, averaged over the charged pions $\pi^{\pm}$. The total contribution of the pion cloud to the $\bar{u}u\bar{u}u$ condensate in the SS channel—see Eqs. (45), (47)—takes the form

$$P^{S,uu}=-\frac{2}{3F_\pi^2}\langle 0|2\bar{u}^a u^b \bar{u}^{a'} u^{b'}+\bar{u}^a u^b \bar{d}^{a'} d^{b'}+\bar{u}^a \gamma_5 u^b \bar{u}^{b'}\gamma_5 u^{a'}+\bar{u}^a\gamma_5 d^b \bar{d}^{b'}\gamma_5 u^{a'})|0\rangle \times(\delta_{aa'}\delta_{bb'}-\delta_{ab'}\delta_{ba'})\frac{\partial\Sigma_t}{\partial M_\pi^2}. \qquad (54)$$

In [13] the pion expectation values were expressed by those of the vacuum for all channels. Thus similar equations can be presented for all structures. Since only the vacuum expectation values are involved, we find for the coefficients in the rhs of Eqs. (6), (7)

$$b_{p,n}^{V,A,T}=0. \qquad (55)$$

To avoid complicated formulas we shall present the final results in the framework of the factorization hypothesis for the vacuum expectation values.

It is convenient to present

$$\delta_{aa'}\delta_{bb'}-\delta_{ab'}\delta_{ba'}=\frac{2}{3}\delta_{aa'}\delta_{bb'}-\frac{1}{2}\sum_\rho \lambda_{aa'}^\rho\lambda_{bb'}^\rho \qquad (56)$$

with λ^ρ standing for the SU(3) Gell-Mann matrices normalized by the relation Sp $\lambda^\rho\lambda^\tau=2\delta^{\rho\tau}$.

In the factorization approximation we find for quarks of the same flavor

$$\langle 0|\bar{q}\Gamma_r q\bar{q}\Gamma_s q|0\rangle=\tfrac{1}{16}[\mathrm{Sp}\Gamma_r\mathrm{Sp}\Gamma_s-\tfrac{1}{3}\mathrm{Sp}(\Gamma_r\Gamma_s)](\langle 0|\bar{q}q|0\rangle)^2 \qquad (57)$$

for any 4×4 matrices $\Gamma_{s,r}$ acting on Lorentz indices. If the quarks have different flavors, we come to

$$\langle 0|\bar{q}_i\Gamma_r q_i\bar{q}_j\Gamma_s q_j|0\rangle=\tfrac{1}{16}\mathrm{Sp}\Gamma_r\,\mathrm{Sp}\Gamma_s\langle 0|\bar{q}_i q_i|0\rangle\langle 0|\bar{q}_j q_j|0\rangle \qquad (58)$$

and

$$\langle 0|\bar{q}_i\Gamma_r q_j\bar{q}_j\Gamma_s q_i|0\rangle=-\tfrac{1}{48}\mathrm{Sp}\Gamma_r\Gamma_s\langle 0|\bar{q}_i q_i|0\rangle\langle 0|\bar{q}_j q_j|0\rangle. \qquad (59)$$

For the matrices $\tilde{\Gamma}_{r,s}^\rho=\Gamma_{r,s}\lambda^\rho$ this approximation provides

$$\langle 0|\sum_\rho \bar{q}_i\tilde{\Gamma}_r^\rho q_j\bar{q}_j\tilde{\Gamma}_s^\rho q_i|0\rangle=-\frac{1}{9}\,\mathrm{Sp}(\Gamma_r\Gamma_s)\langle 0|\bar{q}_i q_i|0\rangle\times\langle 0|\bar{q}_j q_j|0\rangle \qquad (60)$$

which is true for $i=j$ and $i\neq j$, while

$$\langle 0|\sum_\rho \bar{q}_i\tilde{\Gamma}_r^\rho q_i\bar{q}_j\tilde{\Gamma}_s^\rho q_j|0\rangle=0 \qquad (61)$$

for $i\neq j$.

In the factorization approximation the contribution of the sea quarks contains the factor

$$\frac{(\langle 0|\bar{u}u|0\rangle)^2}{F_\pi^2}=-\frac{M_\pi^2}{m_u+m_d}\langle 0|\bar{u}u|0\rangle, \qquad (62)$$

see Eq. (9), and we can present

$$P_{p,n}^{X,f_1f_2}=\frac{M_\pi^2}{m_u+m_d}\frac{\partial\Sigma_t}{\partial M_\pi^2}\varepsilon_0^3 S_{p,n}^{X,f_1f_2}. \qquad (63)$$

Here we denoted $P_{p,n}^{XX,f_1f_2}=P_{p,n}^{X,f_1f_2}$ and the rhs of Eq. (46) turns to zero for $X\neq Y$. The subscript denotes proton or neutron.

Using the results of [13], we find for the same flavors

$$S_p^{S,uu}=S_n^{S,uu}=-\tfrac{16}{9},\quad S_p^{Ps,uu}=S_n^{Ps,uu}=-\tfrac{8}{9}, \qquad (64)$$

$$S_p^{V,uu}=-S_p^{A,uu}=-\tfrac{2}{9}g_{\mu\nu},\quad S_n^{V,uu}=-S_n^{A,uu}=-\tfrac{2}{9}g_{\mu\nu},$$

$$S_p^{T,uu}=S_n^{T,uu}=-\tfrac{4}{9}s_{\mu\nu,\alpha\beta}$$

with the tensor $s_{\mu\nu,\alpha\beta}$ defined by Eq. (8).

For the quarks of different flavors we obtain for the proton and neutron

$$S^{S,ud}=-\tfrac{14}{9},\quad S^{Ps,ud}=\tfrac{2}{9},\quad S^{V,ud}=-S^{A,ud}=\tfrac{2}{9}g_{\mu\nu},$$

$$S^{T,ud}=-\tfrac{2}{9}s_{\mu\nu,\alpha\beta}. \qquad (65)$$

We now show that in the scalar channel the disconnected terms are separated in a natural way. We have the result

$$S_{p,n}^{S,uu}=-\tfrac{5}{3}-\tfrac{1}{9},\quad S_{p,n}^{S,ud}=-\tfrac{4}{3}-\tfrac{2}{9}. \qquad (66)$$

As it was shown in [13] the expectation values of the scalar four-quark operators are dominated by the disconnected terms with one of the $\bar{q}q$ pairs coming from the vacuum. This corresponds to the approximation

$$S_{p,n}^{S,uu}=S_{dis}^{uu}=-\tfrac{5}{3},\quad S_{p,n}^{S,ud}=S_{dis}^{ud}=-\tfrac{4}{3}. \qquad (67)$$

On the other hand, the factor $M_\pi^2/(m_u+m_d)$ in the rhs of Eq. (63) is just the expectation value of the operator $\bar{q}q$ in the pion—see Eq. (10). Then we have

$$\frac{M_\pi^2}{m_u+m_d}\frac{\partial\Sigma_t}{\partial M_\pi^2}=\langle\pi|\bar{q}q|\pi\rangle\frac{\partial\Sigma_t}{\partial M_\pi^2}=\frac{1}{2}\langle N|\bar{u}u+\bar{d}d|N\rangle_{sea}. \tag{68}$$

Thus, in the scalar channel there is a contribution of disconnected terms

$$P^{S,uu}_{dis;p,n}=\tfrac{5}{6}(2\langle 0|\bar{u}u|0\rangle\langle N|\bar{u}u|N\rangle_{sea}) \tag{69}$$

corresponding to the approximation, expressed by Eq. (67). Of course, there is a similar expression for $P^{S,ud}_{dis;p,n}$

$$P^{S,ud}_{dis;p,n}=\tfrac{2}{3}(\langle 0|\bar{u}u|0\rangle\langle N|\bar{d}d|N\rangle_{sea}+\langle 0|\bar{d}d|0\rangle\langle N|\bar{u}u|N\rangle_{sea}). \tag{70}$$

IV. INTERFERENCE TERMS

We now turn to the situation when one of $\bar{q}\Gamma^X q$ operators acts on the constituent quark while the other one acts on the pion. In the one-loop approximation of the PCQM this contribution corresponds to the Feynman diagrams illustrated by Fig. 3.

A. Contact interference

The four-quark condensate can connect the pions of the nucleon self-energy loop with the quarks composing the nucleon. The contribution can be presented as

$$\langle N|\bar{q}\Gamma^X q\,\bar{q}\Gamma^Y q|N\rangle=\sum_{Q,n,n'}\int\frac{\langle\phi_0|H_I|\phi_n,\pi\rangle\langle\phi_n|\bar{q}\Gamma^X q|\phi_{n'}\rangle\langle\pi|\bar{q}\Gamma^Y q|\pi\rangle\langle\phi_{n'},\pi|H_I|\phi_0\rangle}{(E_0-k_{10}-E_n+i\varepsilon)(E_0-k_{20}-E_{n'}+i\varepsilon)}\Delta_\pi(k_1)\Delta_\pi(k_2)\frac{d^4k_1}{(2\pi)^4 i}\frac{d^4k_2}{(2\pi)^4 i} \tag{71}$$

where k_1,k_2 are the four-momenta of the pions, $\Delta_\pi(k)=1/(k^2-M_\pi^2+i\varepsilon)$ is the pion propagator. Recall that we include intermediate quark states with $n=n'=0$ only. The state vectors $|\phi_n\rangle$ compose the complete set of the quark states with the energy E_n, index 0 corresponds to the ground state and H_I denotes the quark-pion interaction (42). The summation is carried out over the quarks Q which compose the nucleon.

In the quark language this means that the four-quark condensate can connect the pions with the intermediate quark of the self-energy loop or with another quark. These contributions are shown in Figs. 3(a,b). The corresponding exchange diagrams are shown in Figs. 3(c,d). These expectation values contain the matrix elements $\langle\pi|\bar{q}\Gamma^X q|\pi\rangle$ and $\langle Q|\bar{q}\Gamma^Y q|Q\rangle$. The former has a nonvanishing value in the scalar case only. The latter matrix element survives in the scalar and vector cases only, for a unpolarized nucleon. Thus, only the expectation values J^{SS} and J^{SV} obtain nonzero values.

The connections of the pion π^α with the intermediate state quark I^{XY}_α [shown in Fig. 3(a)] and with another quark K^{XY}_α [shown in Fig. 3(b)] are tied by the relation

$$K^{XY}_\alpha=-2I^{XY}_\alpha \tag{72}$$

for a fixed quark flavor. This relation can be obtained by comparing the results of the integration over the pion energy in the loops of the diagrams shown in Figs. 3(a,b).

One can write

$$I^{XY}_\alpha=-\langle\pi^\alpha|\bar{q}\Gamma^X q|\pi^\alpha\rangle\int d^3z F^\alpha(z)\bar{\psi}(z)\Gamma^Y\psi(z)\bar{F}^\alpha(z) \tag{73}$$

with

$$F^\alpha(z)=\frac{1}{2F_\pi}\int d^3x\bar{\Psi}(x)i\gamma_5\tau^\alpha\Psi(x)S(x)D_\pi(x-z), \tag{74}$$

where Ψ is the SU(2) doublet of the light quarks. In Eq. (74)

$$D_\pi(x)=\frac{1}{4\pi}\frac{e^{-\mu x}}{x} \tag{75}$$

is the three-dimensional pion propagator with $\mu=M_\pi$, while

$$S(x)=M+cx^2 \tag{76}$$

is the scalar field with the parameters [11]

$$M=\frac{1-3\beta^2}{2\beta R},\quad c=\frac{\beta}{2R^3}. \tag{77}$$

Parameters β and R already occurred in the definition of the valence quark wave function of Eq. (23). Only terms with the scalar structure Γ^X provide a nonzero value. Also the integral in the rhs of Eq. (73) does not turn to zero for the scalar and vector structures Γ^Y only.

The total contribution of such interference terms to the expectation values of the operators $\bar{q}q\bar{q}\Gamma^Y q$ $(Y=S,V)$ can be expressed by the contribution I_0^{SY} of the $\pi_0 U$ loop to the self-energy diagram of the U quark—see Eq. (73),

$$J^{SY}=-\tfrac{2}{3}(1+\tfrac{10}{9})I_0^{SY}n \tag{78}$$

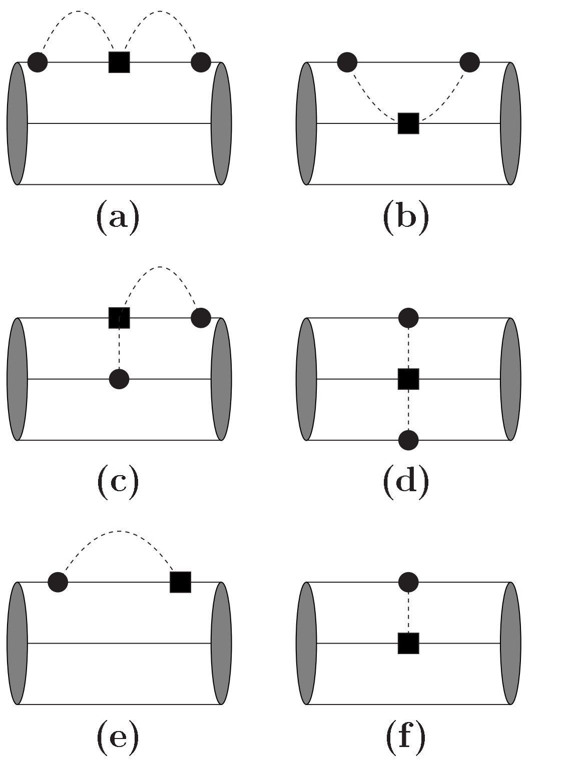

FIG. 3. The contribution of the interference term to the expectation values of the four-quark operators. The contact interference is illustrated by (a)–(d). The vertex interference is shown in (e)–(f). The permutated diagrams are not shown. The notations are the same as in Figs. 1 and 2.

with the factor $(1+\frac{10}{9})$ taking into account the exchange diagram shown in Fig. 3(c)—Eq. (44), while the coefficient $\frac{2}{3}$ is the weight of the color-asymmetric state. The factor n takes into account the charge dependence of the πQQ vertices and the number of the corresponding diagrams. We find $n=20$ for the $(\bar{u}u)^2$ condensate in the proton, turning to $n=7$ for the neutron. It is $n=27$ for the $\bar{u}u\bar{d}d$ condensate. For the scalar-vector condensate $\bar{d}d\bar{u}\gamma_\mu u$ we have $n=20$ for the proton and $n=7$ for the neutron.

Details of the calculation for the value I_0^{SY} are given in Appendix C. Here we present the result. By expressing the pion matrix element by the vacuum one—see Eq. (10), we obtain

$$I_0^{SY}=C_J A^Y\langle 0|\bar{q}q|0\rangle \tag{79}$$

with

$$A^Y=\int_o^\infty \frac{dt}{t^2} f^2(t)e^{-t^2}\varphi^Y(t), \tag{80}$$

while

$$f(t)=\frac{\sqrt{\pi}}{2}\mathrm{erf}(t)-te^{-t^2}-\frac{2\beta^2t^3}{2-\beta^2}e^{-t^2}. \tag{81}$$

Here we use the standard notation $\mathrm{erf}(x)=2/\sqrt{\pi}\int_o^x e^{-y^2}dy$ and

$$\varphi^S(t)=1-\beta^2t^2, \quad \varphi^V(t)=1+\beta^2t^2, \tag{82}$$

being caused by the matrix elements $\bar{\psi}\Gamma^S\psi=\bar{\psi}\psi$ and $\bar{\psi}\Gamma^V\psi=\bar{\psi}\gamma_0\psi$. For the coefficient C_J we get

$$C_J=\frac{\sqrt{\pi}}{(2\pi RF_\pi)^4}\frac{(1-\frac{1}{2}\beta^2)^2}{(1+\frac{3}{2}\beta^2)^3}\simeq 8.4\times 10^{-2}. \tag{83}$$

The interference terms provide for the scalar condensates

$$J_p^{S,uu}=6.2\times 10^{-2}\varepsilon_0^3, \quad J_n^{S,uu}=2.2\times 10^{-2}\varepsilon_0^3,$$

$$J_p^{S,ud}=J_n^{S,ud}=8.4\times 10^{-2}\varepsilon_0^3. \tag{84}$$

For the operator $\bar{d}d\bar{u}\gamma_\mu u$ we finally obtain

$$J_{p,\mu}^{SV,du}=9.6\times 10^{-2}\frac{p_\mu}{m}\varepsilon_0^3, \quad J_{n,\mu}^{SV,du}=3.4\times 10^{-2}\frac{p_\mu}{m}\varepsilon_0^3 \tag{85}$$

where the subscripts p,n represent the proton and neutron, $J_{p(n)\mu}^{SV}=J_{p(n)0}^{SV}\delta_{\mu 0}$ in the nucleon rest frame.

Note that the insertion of the four-quark operator can lead to a charge-exchange pion-quark interaction between the points of emission and absorption of the pion by the constituent quark. This mechanism is also described by the diagram of Figs. 3(a)–3(d) and provides a contribution to the expectation value $\bar{u}\Gamma^X d\bar{d}\Gamma^Y u$. The charge-exchange matrix element $\langle\pi^0|\bar{d}\gamma_\mu u|\pi^+\rangle$, which is related to the vector part of the weak decay amplitude $\pi^+\to\pi^0 e^+\nu_e$, has a nonzero value. However, the contribution is suppressed by an additional small factor m_q/M_π when compared to the expectation values $\langle\pi^\alpha|\bar{q}q|\pi^\alpha\rangle$. When calculated in the approach described in Sec. III the matrix elements $\langle\pi^0|\bar{d}\gamma_\mu u|\pi^+\rangle$ and $\langle\pi^0|\bar{d}u|\pi^+\rangle$ vanish. Nonvanishing values are provided by corrections of the relative order M_π to the PCAC relation expressed by Eq. (49).

B. Vertex interference

Another type of interference term, illustrated by Figs. 3(e,f), is due to the PCAC relation [22]

$$\langle 0|\bar{q}\gamma_\rho\gamma_5\tau^\alpha q|\pi^\alpha(k)\rangle=i\sqrt{2}F_\pi k_\rho, \tag{86}$$

where the pseudovector current connects the pion and vacuum states. The equations of motion lead to similar relations for the matrix elements of the pseudoscalar operator between the pion and vacuum states. In particular

$$\langle 0|\bar{u}\gamma_5 d|\pi^-\rangle=-\frac{i\sqrt{2}F_\pi q^2}{m_u+m_d}, \tag{87}$$

where q^2 denotes the square of the four-momentum of the pion. If one of the matrices acting on Lorentz indices, i.e., Γ^Y, has a pseudovector or a pseudoscalar structure, there is a nonvanishing matrix element

$$\langle \phi_0|\bar{q}\Gamma^Z\tau^\alpha q\bar{q}\Gamma^Y\tau^\alpha q|\phi_0,\pi^\alpha\rangle = \langle \phi_0|\bar{q}\Gamma^Z\tau^\alpha q|\phi_0\rangle\langle 0|\bar{q}\Gamma^Y\tau^\alpha q|\pi^\alpha\rangle \quad (88)$$

for any matrix Γ^Z and

$$\langle \phi_0|\bar{q}\Gamma^Z\tau^\alpha q|\phi_0\rangle = \int d^3x\bar{\Psi}(x)\Gamma^Z\tau^\alpha\Psi(x). \quad (89)$$

The sum over color is carried out in both matrix elements in the rhs of Eq. (88).

Replacing the PCQM vertex in the nucleon self-energy by the vertex defined by Eq. (88) we obtain [see Figs. 3(e,f)]

$$\langle N|\bar{q}\Gamma^X\tau^\alpha q\bar{q}\Gamma^Y\tau^\alpha q|N\rangle = \sum_{Q,n}\int\left(\frac{\langle \phi_0|H_I|\phi_n;\pi^\alpha\rangle\langle \phi_n|\bar{q}\Gamma^X\tau^\alpha q|\phi_0\rangle\langle \pi^\alpha|\bar{q}\Gamma^Y\tau^\alpha q|0\rangle}{E_0-k_0-E_n+i\varepsilon} + \frac{\langle 0|\bar{q}\Gamma^X\tau^\alpha q|\pi^\alpha\rangle\langle \phi_0|\bar{q}\Gamma^Y\tau^\alpha q|\phi_n\rangle\langle \phi_n;\pi^\alpha|H_I|\phi_0\rangle}{E_0-k_0-E_n+i\varepsilon}\right)\Delta_\pi(k)\frac{d^4k}{(2\pi)^4 i}. \quad (90)$$

The rhs of Eq. (90) does not turn to zero only when both Γ^X and Γ^Y are either pseudovector or pseudoscalar matrices. These cases must be treated separately. We shall use the standard PCQM approach [10,20] where the sum over n in Eq. (90) is restricted to the quark ground state.

1. Pseudovector case

We start with the case where both matrices Γ^X and Γ^Y in Eq. (90) are the pseudovector ones. The manifestation of the vertex interference in the self-energy diagrams is described by a certain tensor $I_{\rho\sigma}$ with vanishing time components—see Appendix B, Eq. (B11). Thus the integration over the energy k_0 in the rhs of Eq. (90) can be carried out in the same way as for the self-energy PCQM diagrams [20]. We present the contribution as

$$I_{\rho\sigma} = \sum_\alpha I^\alpha_{\rho\sigma}, \quad (91)$$

$$I^\alpha_{\rho\sigma} = \int d^3z[F^\alpha(z)\bar{\Psi}(z)\gamma_\sigma\gamma_5\tau^\alpha\Psi(z) \times\langle 0|\bar{q}(z)\gamma_\rho\gamma_5\tau^\alpha q(z)|\pi^\alpha\rangle + F^\alpha(z)\bar{\Psi}(z)\gamma_\sigma\gamma_5\tau^\alpha\Psi(z) \times\langle \pi^\alpha|\bar{q}(z)\gamma_\rho\gamma_5\tau^\alpha q(z)|0\rangle].$$

The two terms correspond to the manifestation of the mechanism in the two vertices of the one-loop self-energy diagram—see Fig. 3(e). The functions $F^\alpha(z)$ are determined by Eq. (74).

Since the operator $\bar{q}\tau^0 q$ does not change flavor, the neutral pions π^0 contribute to the pseudovector condensates $(\bar{u}\Gamma^A u)^2$ and $\bar{u}\Gamma^A u\bar{d}\Gamma^A d$ only. The charged pions $\pi^\pm$ provide contributions to the expectation values of the operators $\bar{u}\gamma_\rho\gamma_5 d\bar{d}\gamma_\sigma\gamma_5 u$. Thus the charged pions give contributions to all the structures $\bar{u}\Gamma^X u\bar{d}\Gamma^Y d$ with the weights being determined by the Fierz transform.

The tensor structure

$$I^\alpha_{\rho\sigma} = C^\alpha_I\left(g_{\rho\sigma} - \frac{p_\rho p_\sigma}{m^2}\right) \quad (92)$$

is determined by setting the time components to zero. The coefficients $C^\alpha_I = \frac{1}{3}I^\alpha_{\rho\sigma}g^{\rho\sigma}$ can be expressed through the contributions Σ^α_Q [Fig. 3(e)] of each quark to the total self-energy of the nucleon.

We start with the interference in the $\pi^+ UD$ vertex. Putting $\Gamma^X = \gamma_\sigma\gamma_5$, $\Gamma^Y = \gamma_\rho\gamma_5$ in Eq. (88) and projecting it on the quark states treated in momentum space, we obtain

$$\langle 0|\bar{d}\gamma_\rho\gamma_5 u|\pi^+\rangle\langle U|\bar{u}\gamma_\sigma\gamma_5 d|D\rangle g^{\rho\sigma} = \sqrt{2}F_\pi\int\frac{d^3k'}{(2\pi)^3}\bar{\psi}(\vec{k}')\gamma_\rho k^\rho i\gamma_5\psi(\vec{k}'-\vec{k}) = 2\sqrt{2}F_\pi\int\frac{d^3k'}{(2\pi)^3}\bar{\psi}(\vec{k}')i\gamma_5 S(k)\psi(\vec{k}'-\vec{k}) \quad (93)$$

with $S(k)$ standing for the scalar effective field, while $\vec{k}'$ denotes the momentum of the quark. The last equality is due to the PCQM equation of motion. The rhs of Eq. (93) is $2F^2_\pi$ times the PCQM quark-pion vertex—Eq. (42).

Being substituted to Eq. (91) for $I^{\pm 1}$, Eq. (93) provides the value $C^\pm_I = \frac{4}{3}F^2_\pi\Sigma^\pm_Q$ with $\Sigma^\pm_Q$ denoting the contributions of the single quarks in the self-energies $\Sigma^\pm$. One can also obtain that $C^0_I = \frac{4}{3}F^2_\pi\Sigma^0_Q$. To show this, note that

$$\bar{u}\Gamma u = \tfrac{1}{2}\bar{q}\Gamma(\tau^0 + I)q \quad (94)$$

(with I standing for the 2×2 unit matrix) for any 4×4 matrix Γ acting on Lorentz indices. Since $\langle 0|\bar{q}\Gamma Iq|\pi^0\rangle = 0$, one finds

$$\frac{1}{\sqrt{2}}\langle 0|\overline{u}\gamma_\rho\gamma_5 u|\pi^0\rangle=\frac{i\sqrt{2}F_\pi k_\rho}{2} \tag{95}$$

with the further procedure as in the charged case.

All the contributions can be expressed by the value

$$C_I^0=\frac{4}{3}F_\pi^2\Sigma^0, \tag{96}$$

which can be presented as

$$C_I^0=-\frac{\sqrt{2}}{6\pi^2}\frac{1}{(1+\frac{3}{2}\beta^2)^2}\frac{1}{R^3}\int_0^\infty dy\frac{y^4\left[1-\frac{\beta^2}{2}(1+y^2)\right]^2}{y^2+\frac{\mu^2R^2}{2}}e^{-y^2}. \tag{97}$$

In the chiral limit $\mu^2=0$ we obtain

$$C_I^0=-\frac{\sqrt{2}}{24\pi^{3/2}}\frac{1-\frac{5}{2}\beta^2+\frac{31}{16}\beta^4}{(1+\frac{3}{2}\beta^2)^2}\frac{1}{R^3}=-\frac{4.7\times10^{-3}}{R^3}$$
$$=-1.14\times10^{-2}\varepsilon_0^3 \tag{98}$$

while for $\mu=M_\pi$ we have

$$C_I^0=-1.0\times10^{-2}\varepsilon_0^3. \tag{99}$$

This provides

$$J_{\mu\nu}^{A,uu}=\tfrac{2}{3}n_u(1+\tfrac{10}{9})\delta_{\mu\nu}\delta_{\rho\sigma}I_{\rho\sigma}^0, \tag{100}$$

where the factor $(1+\frac{10}{9})$ also includes the exchange diagram shown in Fig. 3(f)—Eq. (44)—while the factor $\frac{2}{3}$ is the weight of the color asymmetric state. Thus we finally have

$$J_{\mu\nu}^{A,uu}=-1.4\times10^{-2}n_u\varepsilon_0^3\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{m^2}\right). \tag{101}$$

Turning to condensates of quarks with different flavors, we set for the contribution of the neutral pions to the pseudovector structure

$$J_{0\mu\nu}^{A,ud}=(-n_u-n_d)(1+\tfrac{10}{9})\tfrac{2}{3}\delta_{\mu\rho}\delta_{\nu\sigma}I_{\rho\sigma}^0$$
$$=4.2\times10^{-2}\varepsilon_0^3\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{m^2}\right). \tag{102}$$

The charged pions provide a direct contribution to the expectation values of the operators $\overline{u}\gamma_\rho\gamma_5 d\overline{d}\gamma_\sigma\gamma_5 u=\Sigma_{c,g}\overline{u^c}\gamma_\rho\gamma_5 d^c\overline{d^g}\gamma_\sigma\gamma_5 u^g$ with c and g standing for the color indices. Their contribution to the expectation values of the operators $\overline{u^a}\Gamma^X u^{a'}\overline{d^b}\Gamma^Y d^{b'}(\delta_{aa'}\delta_{bb'}-\delta_{ab'}\delta_{ba'})$, which we are looking for, is determined by the Fierz transform

$$u_\alpha^{a'}\overline{d_\beta^b}=-\frac{1}{12}\sum_A\Gamma_{\alpha\beta}^A\overline{d}\Gamma_A u\,\delta_{a'b}-\frac{1}{64}\sum_{A\kappa}\Gamma_{\alpha\beta}^A\lambda_{a'b}^\kappa\,\overline{d}\Gamma_A\lambda^\kappa u. \tag{103}$$

In our case the pseudovector term with the diagonal color structure contributes only to

$$\overline{u}\Gamma^X u\overline{d}\Gamma^Y d=-\tfrac{1}{4}\times\tfrac{2}{3}\overline{d}\gamma_\rho\gamma_5 u\overline{u}\Gamma^X\gamma^\rho\gamma_5\Gamma^Y d+\cdots. \tag{104}$$

Here the summation over colors is carried out, providing the factor $-\frac{2}{3}$. The dots denote the terms which do not contribute.

Following the previous analysis we must separate the pseudovector component $\gamma_\sigma\gamma_5$ in the operator $\Gamma^X\gamma^\rho\gamma_5\Gamma^Y$ in the rhs of Eq. (104). The interference terms can be expressed by the tensor

$$I_{\rho\sigma}^C=2(1+\tfrac{10}{9})(n_u+n_d)I_{\rho\sigma}^0. \tag{105}$$

The tensor $I_{\rho\sigma}^C$ is obtained by the summation of the rhs of Eq. (91) over the charged pion states and over the constituent quarks of the nucleon and by inclusion of the exchange terms. The coefficient C_I^0 is given by Eq. (96). The contributions are

$$J^{S,ud}=-\tfrac{1}{6}g_{\rho\sigma}I_{\rho\sigma}^C,\quad J^{Ps,ud}=-J^{S,ud}, \tag{106}$$
$$J_{\mu\nu}^{V,ud}=\tfrac{1}{3}\delta_{\mu\rho}\delta_{\nu\sigma}I_{\rho\sigma}^C-\tfrac{1}{6}g_{\mu\nu}g_{\rho\sigma}I_{\rho\sigma}^C,$$
$$J_{\mu\nu}^{A,ud}=J_{0\mu\nu}^{A,ud}+\tfrac{1}{3}\delta_{\mu\rho}\delta_{\nu\sigma}I_{\rho\sigma}^C-\tfrac{1}{6}g_{\mu\nu}g_{\rho\sigma}I_{\rho\sigma}^C,$$
$$J_{\mu\nu,\alpha\beta}^{T,ud}=-\tfrac{1}{24}\mathrm{Sp}(\sigma_{\mu\nu}\gamma_\rho\sigma_{\alpha\beta}\gamma_\sigma)I_{\rho\sigma}^C.$$

2. Pseudoscalar case

Now we consider the pseudoscalar case, i.e., $\Gamma^X=\Gamma^Y=\gamma_5$ in Eq. (90). If the charged pions are exchanged, the matrix elements of the quark operators between the vacuum and the pion states are given by Eq. (87) which respects the isospin symmetry. However, the contribution of the neutral pion exchange contains the matrix elements

$$\langle 0|\overline{u}\gamma_5 u|\pi^0\rangle=-\frac{iF_\pi q^2}{2m_u},\quad\langle 0|\overline{d}\gamma_5 d|\pi^0\rangle=\frac{iF_\pi q^2}{2m_d} \tag{107}$$

which depend on the quark masses $m_{u,d}$ separately. This breaks for $m_u\neq m_d$ the isospin symmetry explicitly.

After the integration over k_0 in the rhs of Eq. (90) (see Appendix D) we can present the contribution in a form similar to the pseudovector case—Eq. (91),

$$\tilde{I}=\sum_\alpha\tilde{I}^\alpha \tag{108}$$

$$\tilde{I}^\alpha=\int d^3z[F^\alpha(z)\overline{\psi}(z)\gamma_5\tau^\alpha\psi(z)\langle 0|\overline{q}(z)\gamma_5\tau^\alpha q(z)|\pi^\alpha\rangle$$
$$+F^\alpha(z)\overline{\psi}(z)\gamma_5\psi(z)\langle\pi^\alpha|\overline{q}(z)\gamma_5\tau^\alpha q(z)|0\rangle],$$

where we must put $q^2=m_\pi^2$ in the matrix elements determined by Eqs. (87), (108). The expectation values can be expressed by the term $\tilde{I}^+$ of Eq. (108) corresponding to the π^+ meson

$$\tilde{I}^+=\frac{2M_\pi^2}{m_u+m_d}\int d^3xd^3yS(x)\bar{\psi}(x)\gamma_5\psi(x)D_\pi(x-y)\times\bar{\psi}(y)\gamma_5\psi(y) \tag{109}$$

with D_π and S defined by Eqs. (75), (76). Numerically we get

$$\tilde{I}^+=7.0\times10^{-2}\varepsilon_0^3. \tag{110}$$

Proceeding in the same way as in the pseudovector case, we find for the contributions of the interference terms containing the neutral π^0 mesons to $(\bar{u}\gamma_5 u)^2$ and $\bar{u}\gamma_5 u\bar{d}\gamma_5 d$ condensates

$$J_0^{Ps,uu}=\tfrac{2}{3}(1+\tfrac{10}{9})n_u\tilde{I}_u \tag{111}$$

and

$$J_0^{Ps,ud}=-\tfrac{2}{3}(1+\tfrac{10}{9})(n_u\tilde{I}_d+n_d\tilde{I}_u) \tag{112}$$

with

$$\tilde{I}_{u,d}=\frac{\tilde{I}^+(m_u+m_d)}{4m_{u,d}}. \tag{113}$$

Using $m_u=4$ MeV, $m_d=7$ MeV we find

$$\tilde{I}_u=4.8\times10^{-2}\varepsilon_0^3,\quad \tilde{I}_d=2.8\times10^{-2}\varepsilon_0^3.$$

The charged $\pi^\pm$ mesons contribute to the expectation values of the operators $\bar{u}\gamma_5 d\bar{d}\gamma_5 u$ providing thus the contributions to all basic structures defined by Eq. (3). They contain the factor

$$\tilde{I}_C=(1+\tfrac{10}{9})(n_u+n_d)\tilde{I}^+=0.44\varepsilon_0^3, \tag{114}$$

for

$$J^{S,ud}=\tfrac{1}{6}\tilde{I}_C,\quad J^{Ps,ud}=J^{S,ud}+J_0^{Ps,ud}, \tag{115}$$

$$J_{\mu\nu}^{V,ud}=-\tfrac{1}{6}g_{\mu\nu}\tilde{I}_C,\quad J_{\mu\nu}^{A,ud}=\tfrac{1}{6}g_{\mu\nu}\tilde{I}_C,$$

$$J_{\mu\nu,\alpha\beta}^{T,ud}=\tfrac{2}{3}s_{\mu\nu,\alpha\beta}\tilde{I}_C.$$

In more sophisticated models of the pions [23] the quarks obtain large effective masses. Thus the terms I^α will become much smaller.

3. Mixed case

If one of the matrices in the rhs of Eq. (88) is a pseudoscalar (γ_5) while the other one describes the pseudovector ($\gamma_\rho\gamma_5$) we find contributions to the condensates with mixed Dirac structures $\bar{q}\Gamma^X q\bar{q}\Gamma^Y q$. If $\Gamma^X=\gamma_\rho\gamma_5,\Gamma^Y=\gamma_5$ the expectation value turns to zero when we neglect the possible intermediate state excitations of the constituent quarks—see Eq. (B11). The terms with $\Gamma^Y=\gamma_\rho\gamma_5,\Gamma^X=\gamma_5$ provide nonzero values. When we focus on the expectation value $\bar{d}d\bar{u}\gamma_\mu u$ (which is $\bar{d}d\bar{u}\gamma_0 u$ in the rest frame of the nucleon) among the mixed condensates, we must calculate the expectation value of the operators $\bar{u}\gamma_5 d\bar{d}\gamma_0\gamma_5 u$ and $\bar{u}\gamma_0\gamma_5 d\bar{d}\gamma_5 u$. Contrary to the pseudoscalar case $\Gamma^X=\Gamma^Y=\gamma_5$, such terms do not contain the large factor $M_\pi/(m_u+m_d)\approx12$—see Eq. (109), providing thus a minor contribution $\sim10^{-3}\varepsilon_0^3$.

C. Total contribution of the interference

Now we can present the total contribution of the interference terms. For the quarks of the same flavor they are presented in Eqs. (84), (101), (111), and

$$J_p^{S,uu}=0.06\varepsilon_0^3,\quad J_n^{S,uu}=0.02\varepsilon_0^3, \tag{116}$$

$$J^{Ps,uu}=0.07n_u\varepsilon_0^3,\quad J_{\mu\nu}^{A,uu}=-0.014n_u\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{m^2}\right)\varepsilon_0^3 \tag{117}$$

turning to zero for the other structures. Recall that n_u stands for a number of u valence quarks in a nucleon. For the quarks of different flavors Eqs. (84), (106), (115) provide for the proton

$$J^{S,ud}=0.22\varepsilon_0^3,\quad J^{Ps,ud}=-0.28\varepsilon_0^3, \tag{118}$$

$$J_{\mu\nu}^{V,ud}=\left(-0.05g_{\mu\nu}+0.04\frac{p_\mu p_\nu}{m^2}\right)\varepsilon_0^3,\quad J_{\mu\nu}^{A,ud}=0.14g_{\mu\nu}\varepsilon_0^3,$$

$$J_{\mu\nu,\alpha\beta}^{T,ud}=(0.25\,s_{\mu\nu,\alpha\beta}-0.08\,t_{\mu\nu,\alpha\beta})\varepsilon_0^3.$$

Of course, the values presented by Eq. (118) coincide for the proton and neutron except for the pseudoscalar case where

$$J_p^{Ps,ud}-J_n^{Ps,ud}=0.03\varepsilon_0^3. \tag{119}$$

Also the value for the $(\bar{u}\gamma_5 u)^2$ condensate for the proton differs from the value $(\bar{d}\gamma_5 d)^2$ for the neutron by

$$(J_0^{Ps,uu})_p-(J_0^{Ps,dd})_n=0.06\varepsilon_0^3. \tag{120}$$

These characteristics obtain nonzero values due to the explicit dependence on the current quark masses. As we noted earlier, these effects would be much smaller if more sophisticated models for the pions are used [23].

Finally, for the scalar-vector condensates we obtain by using Eq. (85)

$$J_{p,\mu}^{SV,du}=9.6\times10^{-2}\frac{p_\mu}{m}\varepsilon_0^3,\quad J_{n,\mu}^{SV,du}=3.4\times10^{-2}\frac{p_\mu}{m}\varepsilon_0^3.$$

V. THE VALUES OF THE FOUR-QUARK CONDENSATES

Now we sum the partial contributions obtained in the previous sections. We present the results in units of the characteristic scale $\varepsilon_0^3 = -\langle 0|\bar{q}q|0\rangle$ [see Eq. (35)]. The partial contributions to the expectation values defined by Eq. (1) are due to the constituent quarks (denoted by C and shown in Fig. 1), to the pion cloud (denoted by P and shown by Fig. 2), and to the interference terms (denoted by J and shown in Fig. 3). This is expressed by Eq. (4). We do not present the values of the parameters which are negligibly small in our scale.

A. Scalar channel

Recall that in the scalar case there are specific disconnected terms in which one of the products $\bar{q}q$ acts on the QCD vacuum. Such terms emerge from the contributions of the pion cloud. Thus in the scalar case the values of the four-quark condensates can be presented as the sum of the disconnected terms U_{dis} and the internal terms U_{int},

$$U_{p(n)} = U_{dis;p(n)} + U_{int;p(n)} \tag{121}$$

with the indices p,n denoting the proton or neutron. For the other structures the disconnected terms vanish.

We start by presenting the results for the disconnected terms. Using Eqs. (63) and (67) we obtain

$$U_{dis}^{S,uu} = P_{dis}^{S,uu} = -3.83\varepsilon_0^3 \tag{122}$$

for both proton and neutron.

Consider now the internal contributions. For the scalar case they are expressed by Eqs. (26), (37), (39), (63), (64), (116)

$$U_{int;p}^{S,uu} = C_p^{S,uu} + P_{int;p}^{S,uu} + J_p^{S,uu} = -0.11\varepsilon_0^3, \tag{123}$$

$$U_{int;n}^{S,uu} = C_n^{S,uu} + P_{int;n}^{S,uu} + J_n^{S,uu} = -0.23\varepsilon_0^3,$$

with the partial contribution

$$C_p^{S,uu} = 0.08\varepsilon_0^3, \quad C_n^{S,uu} = 0.01\varepsilon_0^3,$$

$$P_{int;p}^{S,uu} = P_{int;n}^{S,uu} = -0.25\varepsilon_0^3, \tag{124}$$

$$J_p^{S,uu} = 0.06\varepsilon_0^3, \quad J_n^{S,uu} = 0.02\varepsilon_0^3.$$

The total values of $U_{p,n}^{S,uu}$ (121) are the sums of Eq. (122) and (123)

$$U_p^{S,uu} = -3.94\varepsilon_0^3, \quad U_n^{S,uu} = -4.05\varepsilon_0^3. \tag{125}$$

In this case the sea quarks mainly contribute, the rest coming from the direct action of the four-quark operator on the constituent quarks and from the interference terms.

Considering the mixed-flavor condensates $\bar{u}u\bar{d}d$, we obtain for the disconnected terms presented by Eq. (70)

$$U_{dis}^{S,ud} = P_{dis}^{S,ud} = -3.06\varepsilon_0^3 \tag{126}$$

for both proton and neutron. The internal terms are expressed by Eqs. (28), (39), (63), (66) and Eq. (118):

$$C^{S,ud} = 0.16\varepsilon_0^3, \quad P_{int}^{S,ud} = -0.51\varepsilon_0^3, \quad J^{S,ud} = 0.22\varepsilon_0^3, \tag{127}$$

$$U_{int}^{S,ud} = C^{S,ud} + P_{int}^{S,ud} + J^{S,ud} = -0.13\varepsilon_0^3,$$

$$U_p^{S,ud} = U_n^{S,ud} = U_{dis}^{S,ud} + U_{int}^{S,ud} = -3.19\varepsilon_0^3. \tag{128}$$

There are no disconnected terms in the other channels, thus $U = U_{int}$.

B. Scalar-vector channel

For this case there is no contribution if all the four quarks belong to the sea. They contribute through the interference determined by Eq. (85) while the constituent quark contribution is given by Eqs. (34), (39)

$$C^{SV,du} = 0.18\varepsilon_0^3 \tag{129}$$

for both proton and neutron. The interference terms are, Eq. (85),

$$J_p^{SV,du} = 0.10\varepsilon_0^3, \quad J_n^{SV,du} = 0.03\varepsilon_0^3.$$

Thus Eqs. (34), (39), and (85) provide for the mixed scalar-vector condensate $\bar{d}d\bar{u}\gamma_0 u$

$$U_p^{SV,du} = C^{SV,du} + J_p^{SV,du} = 0.28\varepsilon_0^3, \tag{130}$$

$$U_n^{SV,du} = C^{SV,du} + J_n^{SV,du} = 0.21\varepsilon_0^3.$$

C. Pseudoscalar channel

For the pseudoscalar case we find by using Eqs. (29), (41), (63), (64), (117)

$$U_p^{Ps,uu} = C^{Ps,uu} + P^{Ps,ud} + J_p^{Ps,uu} = -1.91\varepsilon_0^3, \tag{131}$$

$$U_n^{Ps,uu} = C^{Ps,uu} + P^{Ps,ud} + J_n^{Ps,uu} = -1.96\varepsilon_0^3.$$

The partial values are

$$C_p^{Ps,uu} = -0.02\varepsilon_0^3, \quad P_p^{Ps,uu} = P_n^{Ps,uu} = -2.03\varepsilon_0^3,$$

$$J_p^{Ps,uu} = 0.14\varepsilon_0^3, \quad J_n^{Ps,uu} = 0.07\varepsilon_0^3.$$

The numerical values are determined mostly by the contribution $P^{Ps,uu}$ of the sea quarks. In the case of the condensate $\bar{u}\gamma_5 u\bar{d}\gamma_5 d$ we obtain from Eqs. (29), (33), (41), (63), (65), and (118)

$$C^{Ps,ud} = -0.03\varepsilon_0^3, \quad P^{Ps,ud} = 0.51\varepsilon_0^3, \tag{132}$$

$$J_p^{Ps,ud} = -0.28\varepsilon_0^3, \quad J_n^{Ps,ud} = -0.31\varepsilon_0^3$$

composing, following Eq. (4)

$$U_p^{Ps,ud} = C_p^{Ps,ud} + P_p^{Ps,ud} + J_p^{Ps,ud} = 0.20\varepsilon_0^3, \tag{133}$$

$$U_n^{Ps,ud} = C_n^{Ps,ud} + P_n^{Ps,ud} + J_n^{Ps,ud} = 0.17\varepsilon_0^3 .$$

The difference in the values $U_p^{Ps,ud}$ and $U_n^{Ps,ud}$ is caused by the explicit dependence on the current quark masses—see Eq. (119).

For the vector, axial, and tensor structures we present the values of the coefficients $a^{V(A,T)}$ and $b^{V(A,T)}$. Recall that we introduced a notation where the partial contributions of the valence and sea quarks and of the interference terms are denoted by the subscripts C,P, and J [see the text below Eq. (8)]. The second index labels the specific nucleon. Thus, $a^V_{P,p}$ denotes the contribution of the sea quarks to the parameter a^V of the proton, etc. The notations $a^X_{p(n)}$ and $b^X_{p(n)}$, labeling the vector V, axial A and tensor T cases, are kept for the total contributions to these parameters for the proton (neutron). We omit the subscript index if the values coincide for both nucleons. Note that in all the channels the sea quarks do not contribute to the parameter $b_{p,n}$, i.e.,

$$b^X_{P,p(n)} = 0 \tag{134}$$

for all composition of flavors. Recall also that the interference does not contribute to the expectation values of the operator of the same flavors in the vector and tensor cases—see Sec. IV.

D. Vector channel

By using Eqs. (30), (40), (63), (64) we obtain

$$a^{V,uu}_{C,p} = -0.01\varepsilon_0^3, \quad a^{V,uu}_{P,p} = a^{V,uu}_{P,n} = -0.51\varepsilon_0^3, \tag{135}$$

$$a^{V,uu}_p = a^{V,uu}_{C,p} + a^{V,uu}_{P,p} = -0.52\varepsilon_0^3, \quad a^{V,uu}_n = a^{V,uu}_{P,n} = -0.51\varepsilon_0^3$$

while

$$b^{V,uu}_p = b^{V,uu}_{C,p} = 0.13\varepsilon_0^3, \quad b^{V,uu}_n = b^{V,uu}_{C,n} = 0.02\varepsilon_0^3 . \tag{136}$$

For the mixed-flavor condensate we find Eqs. (30), (33), (40), (63), (65), (118),

$$a^{V,ud}_C = -0.02\varepsilon_0^3, \quad a^{V,ud}_P = 0.51\varepsilon_0^3, \quad a^{V,ud}_J = -0.05\varepsilon_0^3, \tag{137}$$

$$a^{V,ud} = a^{V,ud}_C + a^{V,ud}_P + a^{V,ud}_J = 0.44\varepsilon_0^3,$$

which are the same for the proton and neutron, as well as the parameters

$$b^{V,ud}_C = 0.25\varepsilon_0^3, \quad b^{V,ud}_J = 0.04\varepsilon_0^3, \tag{138}$$

$$b^{V,ud} = b^{V,ud}_C + b^{V,ud}_J = 0.29\varepsilon_0^3 .$$

E. Pseudovector channel

Here we find by using Eqs. (31), (41), (63), (64), (117)

$$a^{A,uu}_{C,p} = -0.03\varepsilon_0^3, \quad a^{A,uu}_{P,p} = a^{A,uu}_{P,n} = 0.51\varepsilon_0^3, \tag{139}$$

$$a^{A,uu}_{J,p} = -0.03\varepsilon_0^3, \quad a^{A,uu}_{J,n} = -0.01\varepsilon_0^3,$$

$$a^{A,uu}_p = a^{A,uu}_{C,p} + a^{A,uu}_{P,p} + a^{A,uu}_{J,p} = 0.45\varepsilon_0^3,$$

$$a^{A,uu}_n = a^{A,uu}_{C,n} + a^{A,uu}_{P,n} + a^{A,uu}_{J,n} = 0.50\varepsilon_0^3$$

and

$$b^{A,uu}_{C,p} = 0.03\varepsilon_0^3, \quad b^{A,uu}_{J,p} = 0.03\varepsilon_0^3, \quad b^{A,uu}_{J,n} = 0.01\varepsilon_0^3, \tag{140}$$

$$b^{A,uu}_p = b^{A,uu}_{C,p} + b^{A,uu}_{J,p} = 0.06\varepsilon_0^3,$$

$$b^{A,uu}_n = b^{A,uu}_{J,n} = 0.01\varepsilon_0^3 .$$

For the expectation value of the operator $\bar{u}\Gamma^A u \bar{d}\Gamma^A d$ we get with Eqs. (31), (33), (41), (63), (64), (118)

$$a^{A,ud}_C = -0.06\varepsilon_0^3, \quad a^{A,ud}_P = -0.51\varepsilon_0^3, \quad a^{A,ud}_J = 0.14\varepsilon_0^3, \tag{141}$$

$$a^{A,ud} = a^{A,ud}_C + a^{A,ud}_P + a^{A,ud}_J = -0.43\varepsilon_0^3$$

and

$$b^{A,ud} = b^{A,ud}_C = 0.06\varepsilon_0^3 . \tag{142}$$

F. Tensor channel

Using Eqs. (32), (41), (63), (64) we obtain

$$a^{T,uu}_{C,p} = 0.04\varepsilon_0^3, \quad a^{T,uu}_{P,p} = a^{T,uu}_{P,n} = -1.02\varepsilon_0^3, \tag{143}$$

$$a^{T,uu}_p = a^{T,uu}_{C,p} + a^{T,uu}_{P,p} = -0.98\varepsilon_0^3,$$

$$a^{T,uu}_n = a^{T,uu}_{C,n} + a^{T,uu}_{P,n} = -1.02\varepsilon_0^3$$

while

$$b^{T,uu}_p = b^{T,uu}_{C,p} = -0.05\varepsilon_0^3 . \tag{144}$$

For the mixed-flavor operator, Eqs. (32), (33), (41), (63), (65), (118)

$$a^{T,ud}_C = 0.07\varepsilon_0^3, \quad a^{T,ud}_P = -0.51\varepsilon_0^3, \quad a^{T,ud}_J = 0.25\varepsilon_0^3, \tag{145}$$

$$a^{T,ud} = a^{T,ud}_C + a^{T,ud}_P + a^{T,ud}_J = -0.19\varepsilon_0^3$$

while

$$b^{T,ud}_C = -0.10\varepsilon_0^3, \quad b^{T,ud}_J = -0.08\varepsilon_0^3, \tag{146}$$

$$b^{T,ud} = b^{T,ud}_C + b^{T,ud}_J = -0.18\varepsilon_0^3 .$$

The final results of this section are presented in a compact form in Tables I and II (keeping the values larger than 0.1 in modulus). The numbers are given in units of $\varepsilon_0^3 = 1.47\times 10^{-2}$ GeV^3, see Eq. (35). The values $U^{X,f_1 f_2}$ are

TABLE I. The nucleon expectation values of the four-quark operators in the scalar-scalar, scalar-vector, and pseudoscalar-pseudoscalar channels.

X	$U_p^{X,uu}$	$U_n^{X,uu}$	$U_p^{X,ud}$	$U_n^{X,ud}$
S	−3.9	−4.1	−3.2	−3.2
SV			0.3	0.3
Ps	−1.9	−2.0	0.2	0.2

$$U_N^{X,f_1f_2}=\langle N|:\overline{q}^{f_1a}\Gamma^X q^{f_1a'}\overline{q}^{f_2b}\Gamma^X q^{f_2b'}:|N\rangle \times(\delta_{aa'}\delta_{bb'}-\delta_{ab'}\delta_{a'b})$$

with $N=p,n$—see Eqs. (1),(2),(6),(7).

VI. SUMMARY

We calculated the expectation values of the four-quark QCD operators $\bar{q}\Gamma^X q\bar{q}\Gamma^Y q$ in nucleons for all basic Lorentz structures and for compositions of the light quark flavors.

We employed previously derived results of the perturbative chiral quark model (PCQM) which treats the nucleon as a system of three valence quarks surrounded by a pion cloud. We approximate the averaging of the product of operators over the valence quark by the matrix elements of the constituent quark operators over the PCQM constituent quarks. We present the expectation values of the operators acting on the sea quarks by the expectation values of QCD operators in pions. The intensity of the pion field is determined by the PCQM model result.

The expectation values of the scalar and pseudoscalar operators are Lorentz scalars. In the other channels they have a more complicated tensor structure being determined by the two parameters $a^{V(A,T)}$ and $b^{V(A,T)}$—Eqs. (6),(7). For the quark operators with the same flavor, e.g., $\bar{q}\Gamma^X q\bar{q}\Gamma^X q$, the scalar and pseudoscalar condensates, as well as the parameters $a^{V(A,T)}$ for the other structures are dominated by the contribution of the sea quarks. The averaging of four U-quark operators over the valence quarks in the neutron provide zero values in the lowest order of PCQM. This occurs because the operators $\bar{u}u$ should act on different quarks while there is only one U quark in the neutron. In the case of the proton both the constituent quark and the interference terms provide minor corrections of the order of several percent to the main contribution of the sea quarks. In the contrary, the sea quarks do not contribute to the coefficients $b^{V(A,T)}$. In the vector and tensor channels the values $b^{V,T}$ for the proton are determined by the contribution of the constituent quarks.

TABLE II. The values of the parameters for the four-quark expectation values in the vector-vector, axial-axial, and tensor-tensor channels.

X	$a_p^{X,uu}$	$b_p^{X,uu}$	$a_n^{X,uu}$	$b_n^{X,uu}$	$a_p^{X,ud}$	$b_p^{X,ud}$	$a_n^{X,ud}$	$b_n^{X,ud}$
V	−0.5	0.1	−0.5	0	0.4	0.3	0.4	0.3
A	0.5	0.1	0.5	0	−0.4	0.1	−0.4	0.1
T	−1.0	−0.1	−1.0	0	−0.2	−0.2	−0.2	−0.2

In the case of the mixed-flavor condensate $\bar{u}\Gamma^X u\bar{d}\Gamma^X d$ the role of the vertex interference increases due to the large combinatorial factor. These terms become as important as the sea-quark terms in most of the channels. The parameters $b^{V(A,T)}$ are determined mostly by the contributions of the constituent quarks.

The contributions of the sea quarks are expressed by the expectation values of the four-quark operators in pions. Latter values are in turn expressed by the expectation values of the four-quark operators in vacuum [13]. The specific numerical values are obtained by using the vacuum factorization approximations [14]. Thus the contribution of the sea quarks is expressed by the well known vacuum expectation value $\langle 0|\bar{q}q|0\rangle$.

In the case of the scalar-vector condensate $\bar{d}d\bar{u}\gamma_0 u$ there is no contribution coming from the pions only. Averaging over the neutron is dominated by the contribution of the constituent quarks. In the proton the interference and the constituent quark terms are of the same order of magnitude.

We can draw some conclusions on the chiral properties of the expectation values which we study in the present paper. The contribution of the sea quarks has the same explicit dependence on the pion mass M_π as the contribution of the sea-quarks to the expectation value $\langle N|\bar{q}q|N\rangle$. The latter expectation value, which is proportional to M_π^2 times the pion-nucleon σ term, is known to depend strongly on M_π. On the contrary, our interference terms exhibit only a weak dependence on M_π. The valence quark contribution does not contain an explicit dependence on M_π.

Due to the explicit dependence of the vertex interference terms on the quark current masses, we have the isotopic symmetry breaking effects in the pseudoscalar channel. The absolute magnitude of this effect is numerically small with several units of the value $10^{-2}\varepsilon_0^3$ for our scale ε_0. The effect is much smaller if the quarks composing pions are assumed to have the constituent (but not current) masses [23].

In the special case of the scalar condensate the expectation values are dominated by "disconnected terms" in which one of the quark operators acts "inside" the nucleon while the other one acts on the QCD vacuum. This contribution comes from the sea quarks, reflecting the pion structure [13].

Note that a nucleon expectation value is the excess of the density of the quark operator products over the vacuum density, integrated over the volume of the nucleon

$$\langle N|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|N\rangle=\langle N|\int d^3x[\bar{q}(x)\Gamma^X q(x)\bar{q}(x)\Gamma^X q(x) -\langle 0|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|0\rangle]|N\rangle. \tag{147}$$

Of course, the first term in the rhs of Eq. (147) is positive. However, the whole rhs of Eq. (147) can be negative. This is why some of the expectation values run negative.

In an earlier calculation [6] the scalar expectation value $\langle N|(\bar{u}u+\bar{d}d)^2|N\rangle$ was determined in the framework of the Nambu–Jona-Lasinio model [7] under certain additional assumptions. Actually, the expectation values of the color-singlet four-quark operators $\overline{q}^a q^a \overline{q}^b q^b$ have been obtained in [6]. Thus, to compare to the results of [6] we must extend our analysis to such operators as well.

In Ref. [6] the expectation value is presented as the composition of the contribution of the constituent quarks A_D and of σ and π mesons, A_σ and A_π. Our contribution of the constituent quarks appears to be several times smaller than the value of A_D. The large discrepancy is not surprising, since the conception of the constituent quarks is quite different in the two models. The meson contribution $A_\sigma + A_\pi$ of [6] could be compared with the internal sea-quark contribution of the present model, containing the expectation value, which is presented by the second term of the rhs of Eq. (11). The corresponding contribution $\hat{P}$ (with the "hat" sign labeling the color singlet operator) can be obtained by using the formula obtained in [13]. The result $\hat{P} = \frac{2}{3}(\partial\Sigma_t/\partial M_\pi^2)(\varepsilon_0^6/F_\pi^2)$ should be compared to the sum $A_\sigma + A_\pi$ of [6]. We find $\hat{P} = 1.53\varepsilon_0^3 = 2.3\times 10^{-2}$ GeV^3 while $A_\sigma + A_\pi = 3.6\times 10^{-2}$ GeV^3. The total values in the two models are $A_\sigma + A_\pi + A_D$ in [6] and the sum $\hat{U} = \hat{P} + \hat{C} + \hat{J}$ in our approach. We obtain $\hat{C} = \frac{1}{2}C$; $\hat{J} = \frac{3}{2}J$ where the additional term $\hat{J}$ dominates in the sum $\hat{C} + \hat{J}$. The NJL value is $A_\sigma + A_\pi + A_D = 6.4\times 10^{-2}$ GeV^3 while we obtain $\hat{U} = 2.5\varepsilon_0^3 = 3.7\times 10^{-2}$ GeV^3. The results provided by the two approaches differ by a factor of about 1.7. One of the possible reasons for the discrepancy is that some of the contributions have not been accounted for in both approaches.

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APPENDIX A

Here we show how the contributions to the four-quark expectation values, obtained in the paper, manifest themselves with the help of the PCQM formalism. As an example we consider the operator $\overline{u}u\overline{u}u$ averaged over the proton. In the framework of the PCQM the nucleon is a system of three constituent quarks, where the bare three-quark state is renormalized by πN interactions. Thus, the physical proton state $|N\rangle$ is expressed as

$$|N\rangle = T\exp\left(-i\!:\!\int_{-\infty}^{o} dt H_I^r(t)\!:\right)|\phi_0\rangle,$$

where $|\phi_0\rangle$ is the state of three valence quarks and H_I^r is the renormalized Hamiltonian of the interaction between the constituent quark and the pions, which includes the counterterms.

The expectation value $\langle N|(\overline{u}u)^2|N\rangle$ can then be written as

$$\begin{aligned}\langle N|(\overline{u}u)^2|N\rangle = {}& Z^2\langle\phi_0|(\overline{u}u)^2|\phi_0\rangle + 2\langle\phi_0|(\overline{u}u)^2|\phi_0\rangle \\ &\times\langle\phi_0|H_I^r|\phi_0,\pi\rangle\langle\phi_0,\pi|H_I^r|\phi_0\rangle \\ &+\langle\phi_0|H_I^r|\phi_0,\pi\rangle\langle\phi_0,\pi|(\overline{u}u)^2|\phi_0,\pi\rangle \\ &\times\langle\phi_0,\pi|H_I^r|\phi_0\rangle. \end{aligned} \tag{A1}$$

Here $Z = 1 + \partial\Sigma/\partial E$ is the renormalization factor, while Σ is the sum of the self-energies of the constituent quarks with energy E.

In the next step we present each pair of the operators $\overline{u}u$ as the sum of operators acting on the valence and the sea quarks

$$\overline{u}u = (\overline{u}u)_v + (\overline{u}u)_s. \tag{A2}$$

Thus

$$\langle\phi_0|(\overline{u}u)_s|\phi_0\rangle = 0, \quad \langle\pi|(\overline{u}u)_v|\pi\rangle = 0$$

and Eq. (A1) takes the form

$$\begin{aligned}\langle N|(\overline{u}u)^2|N\rangle = {}&\left(1 + 2\frac{\partial\Sigma}{\partial E}\right)\langle\phi_0|(\overline{u}u)_v^2|\phi_0\rangle + 2\langle\phi_0|(\overline{u}u)^2|\phi_0\rangle \\ &\times\langle\phi_0|H_I|\phi_0,\pi\rangle\langle\phi_0,\pi|H_I|\phi_0\rangle \\ &+\langle\phi_0|H_I|\phi_0,\pi\rangle\langle\phi_0|(\overline{u}u)_v^2|\phi_0\rangle \\ &\times\langle\phi_0,\pi|H_I|\phi_0\rangle + \langle\phi_0|H_I|\phi_0,\pi\rangle \\ &\times\langle\pi|(\overline{u}u)_s^2|\pi\rangle\langle\phi_0,\pi|H_I|\phi_0\rangle \\ &+2\langle\phi_0|H_I|\phi_0,\pi\rangle\langle\phi_0|(\overline{u}u)_v|\phi_0\rangle \\ &\times\langle\pi|(\overline{u}u)_s|\pi\rangle\langle\phi_0,\pi|H_I|\phi_0\rangle. \end{aligned} \tag{A3}$$

Since we include the quark-pion interactions to lowest order, the renormalization effects are taken into account in the first term of the rhs of Eq. (A3) only. We put $Z^2 = 1 + 2\partial\Sigma/\partial E$.

The rhs of Eq. (A3) can be simplified due to some cancellations. The second term in the rhs describes the self-energy insertions. These contributions are canceled by the counterterms of the PCQM Lagrangian [10]. Another cancellation occurs between the third term and the part of the first term

$$\begin{aligned}&2\frac{\partial\Sigma}{\partial E}\langle\phi_0|(\overline{u}u)_v^2|\phi_0\rangle + \langle\phi_0|H_I|\phi_0,\pi\rangle\langle\phi_0|(\overline{u}u)_v^2|\phi_0\rangle \\ &\quad\times\langle\phi_0,\pi|H_I|\phi_0\rangle = 0. \end{aligned} \tag{A4}$$

This can be obtained in a straightforward way. The last equation is a rather standard cancellation of the radiative correction by the renormalization factor. Note that the two terms in the rhs of Eq. (A4) do not cancel totally for the operators $(\overline{u}u)^2$ averaged over the neutron—see Sec. II.

Thus, Eq. (A1) takes the form

$$\langle N|(\bar{u}u)^2|N\rangle = \langle \phi_0|(\bar{u}u)_v^2|\phi_0\rangle + \langle \phi_0|H_I|\phi_0,\pi\rangle\langle \pi|(\bar{u}u)_s^2|\pi\rangle \times\langle \phi_0,\pi|H_I|\phi_0\rangle + 2\langle \phi_0|H_I|\phi_0,\pi\rangle \times\langle \phi_0|(\bar{u}u)_v|\phi_0\rangle\langle \pi|(\bar{u}u)_s|\pi\rangle \times\langle \phi_0,\pi|H_I|\phi_0\rangle. \tag{A5}$$

Now we can identify the terms in the rhs of Eq. (A5). The first term corresponds to the contributions of the constituent quarks. The second term describes the contribution of the sea quarks shown in Fig. 2. The third term presents the interference effects with one of the $\bar{u}u$ pairs coming from pions while another one comes from the constituent quark. The latter can be the same as that in the matrix element of the interaction H_I or the other one. These terms are shown in Figs. 3(a)–3(d). A cancellation similar to Eq. (A4) takes place for all the operators $(\bar{u}\Gamma^X u)^2$ in the proton, although in the general case the operator depends on the spin variables. However, the spin dependence manifests itself through the operator $(\vec{\sigma}^I\vec{\sigma}^{II})$ with I and II denoting the two quarks. Since the color wave function is asymmetric, the two quarks compose the spin-symmetric state being at the same space point. Thus, the two-quark spin wave function $|\chi^{I,II}\rangle$ is the eigenfunction of the operator $(\vec{\sigma}^I\vec{\sigma}^{II})$ with $(\vec{\sigma}^I\vec{\sigma}^{II})|\chi^{I,II}\rangle = |\chi^{I,II}\rangle$. Hence, the four-quark expectation value can be separated as a factor and the cancellation takes place as well as in the scalar case. Similar analysis can be carried out for the operators of the general form $\bar{q}\Gamma^X q\bar{q}\Gamma^Y q$.

In the special case of the axial and pseudoscalar operators, there can be the interference effects in the first order of the πQ interaction. This happens because the matrix elements $\langle 0|\bar{q}\Gamma^X q|\pi\rangle$ have nonzero values in these cases. Thus, the operators $\bar{q}\Gamma^X q\bar{q}\Gamma^X q$ determine a πQQ vertex $\langle Q|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|Q,\pi\rangle$. This causes the vertex interference contributions

$$\langle N|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|N\rangle_{intrf} = \langle \phi_0|H_I|\phi_0,\pi\rangle\langle \phi_0,\pi|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|\phi_0\rangle + \langle \phi_0|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|\phi_0,\pi\rangle\langle \phi_0,\pi|H_I|\phi_0\rangle \tag{A6}$$

with X labeling an axial or pseudoscalar. Such terms are shown in Figs. 3(e,f).

In the rhs of Eq. (A6) we have

$$\langle \phi_0,\pi|\bar{q}\Gamma^X q\bar{q}\Gamma^X q|\phi_0\rangle = \langle \pi|\bar{q}\Gamma^X q|0\rangle\langle \phi_0|\bar{q}\Gamma^X q|\phi_0\rangle \tag{A7}$$

$$\langle \phi_0|\bar{q}\Gamma^X q\,\bar{q}\Gamma^X q|\phi_0,\pi\rangle = \langle 0|\bar{q}\Gamma^X q|\pi\rangle\langle \phi_0|\bar{q}\Gamma^X q|\phi_0\rangle.$$

We assume that the matrix elements of the QCD operators $(\bar{q}q)_v^2$ over ϕ_0 are approximated by the matrix elements of the renormalized PCQM constituent quark operators, i.e.,

$$\langle \phi_0|(\bar{q}q)_v^2|\phi_0\rangle = \langle \phi_0|(\bar{Q}^r Q^r)^2|\phi_0\rangle \tag{A8}$$

in the first terms of the rhs of Eqs. (A3) and (A5). The renormalization [10] means that the shape of the constituent quark wave function is modified by the influence of the pion cloud. Also, we approximate the matrix element $\langle \phi_0|(\bar{q}q)_v|\phi_0\rangle = \langle \phi_0|\bar{Q}Q|\phi_0\rangle$ in the third term of the rhs of Eq. (A5).

APPENDIX B

Except for the scalar case, the matrix element between the two-quark states depends on the spin orientation, containing the factor $(\vec{\sigma}^I\vec{\sigma}^{II})$ with I and II denoting the two quarks. Since the color wave function is asymmetric, the two quarks compose the spin-symmetric state when being at the same space point. Thus we must put

$$\langle \chi^{I,II}|(\vec{\sigma}^I\vec{\sigma}^{II})|\chi^{I,II}\rangle = 1 \tag{B1}$$

for the value of the spin operator $(\vec{\sigma}^I\vec{\sigma}^{II})$ averaged over the spin two-quark wave function $\chi^{I,II}$ of the quarks I and II.

For the scalar case we find immediately

$$\mathcal{F}(x) = g(x)\left(1-\beta^2\frac{x^2}{R^2}\right)^2 \tag{B2}$$

with $\mathcal{F}(x)$ defined by Eq. (19), while $g(x) = e^{-2x^2/R^2}N^4$. This provides

$$C_{int}^{S,uu} = \mathcal{N}^2\left(1-\tfrac{3}{2}\beta^2+\tfrac{15}{16}\beta^4\right) \tag{B3}$$

for the proton, with $\mathcal{N}$ defined by Eq. (27). For the pseudoscalar case we get

$$\mathcal{F}(x) = -4\beta^2 g(x)\frac{(\vec{\sigma}^I\vec{x})(\vec{\sigma}^{II}\vec{x})}{R^2} \tag{B4}$$

leading to

$$C_{int}^{Ps,uu} = -\mathcal{N}^2\beta^2. \tag{B5}$$

For the vector and pseudovector structures we can find in the rest frame of the nucleon

$$a_C^{V(A)} = -\tfrac{1}{3}C_{ij}^{V(A)}\delta_{ij} \tag{B6}$$

with i and j being the space indices, corresponding to the four-dimensional indices μ and ν. A direct calculation provides for the vector case

$$\mathcal{F}_{ij} = 4\beta^2 g(x)\frac{x^2}{R^2}\frac{1}{3}\left(\delta_{ij}(\vec{\sigma}^I\vec{\sigma}^{II})-\sigma_i^I\sigma_j^{II}\right), \tag{B7}$$

leading to

$$a_C^V = \mathcal{N}^2\left(-\tfrac{2}{3}\beta^2\right). \tag{B8}$$

To determine the coefficient b_C^V, we calculate the time components

$$\mathcal{F}_{00}=g(x)\left(1+\beta^2\frac{x^2}{R^2}\right)^2 \tag{B9}$$

and, since $C_{00}=a_C^V+b_C^V$, we find

$$b_C^V=\mathcal{N}^2(1+\tfrac{13}{6}\beta^2+\tfrac{15}{16}\beta^4). \tag{B10}$$

For the pseudovector case notice that the time components turn to zero. We introduce the notation

$$\kappa=\begin{pmatrix}\chi\\ i\beta\dfrac{(\vec{\sigma}\vec{x})}{R}\chi\end{pmatrix}$$

for the bispinor entering the wave function—Eq. (23). We obtain

$$\bar{\kappa}\gamma_0\gamma_5\kappa=0. \tag{B11}$$

Thus

$$a_C^A+b_C^A=0. \tag{B12}$$

As to the value of a^A, it can be calculated by using Eq. (B6). In the pseudovector case we get

$$\bar{\kappa}\gamma_i\gamma_5\kappa=\sigma_i+\beta^2\frac{(\vec{\sigma}\vec{x})\sigma_i(\vec{\sigma}\vec{x})}{R^2}. \tag{B13}$$

By using the properties of the Pauli matrices one finds

$$(\vec{\sigma}\vec{x})\sigma_i(\vec{\sigma}\vec{x})=2x_i(\vec{\sigma}\vec{x})-x^2\sigma_i\,. \tag{B14}$$

Thus

$$\mathcal{F}_{ij}(x)=g(x)\left(\sigma_i^I+\beta^2\frac{2x_i(\vec{\sigma}^I\vec{x})-\sigma_i^Ix^2}{R^2}\right)\times\left(\sigma_j^{II}+\beta^2\frac{2x_j(\vec{\sigma}^{II}\vec{x})-\sigma_j^{II}x^2}{R^2}\right) \tag{B15}$$

leading to

$$a_C^A=\mathcal{N}^2(-\tfrac{1}{3}+\tfrac{1}{6}\beta^2-\tfrac{5}{16}\beta^4). \tag{B16}$$

Finally, in the tensor case we find for the space components

$$\bar{\kappa}\sigma_{ij}\kappa=\varepsilon_{ijk}\left(\sigma_k-\beta^2\frac{(\vec{\sigma}\vec{x})\sigma_k(\vec{\sigma}\vec{x})}{R^2}\right) \tag{B17}$$

and the function f can be obtained by using Eq. (B14). For the space-time components we have

$$\bar{\kappa}\sigma_{0j}\kappa=-2\beta\frac{x_j}{R} \tag{B18}$$

and

$$\mathcal{F}_{0j,0k}=\frac{4}{3}\beta^2g^2(x)\frac{x^2}{R^2}\delta_{jk} \tag{B19}$$

with the further procedure described in the main text.

APPENDIX C

In order to calculate the value I_0^{SY} introduced by Eq. (78), we must calculate the function $F^0(z)$—Eq. (74). For the u quark it takes the form

$$F^0(z)=-\frac{N^2}{F_\pi R}\beta\int d^3x\chi^*(\vec{\sigma}\vec{x})\chi S(x)D_\pi(x-z)\Phi^2(x), \tag{C1}$$

changing the sign for the d quark. We present the pion propagator (75) as

$$D_\pi(x-z)=\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\vec{k}(\vec{x}-\vec{z})}e^{-i\vec{x}\vec{a}}}{k^2+\mu^2},\quad (a=0). \tag{C2}$$

The factor $e^{-i(\vec{x}\vec{a})}(a=0)$ is introduced in order to simplify the calculations by expressing

$$\vec{x}D_\pi(x-z)=i\vec{\nabla}_aD_\pi(x-z). \tag{C3}$$

We obtain, by doing the integral over x,

$$F(z)=-\frac{\pi^{3/2}}{2}\frac{N^2\beta R^4}{F_\pi}\chi^*(\vec{\sigma}\vec{\nabla}_b)\chi\big(AT_1(z)+BT_2(z)\big)\quad (b=0) \tag{C4}$$

with $A=M+\frac{5}{2}cR^2$, $B=-\frac{1}{4}cR^4$, while c and R are determined by Eqs. (25), (77) and

$$T_1(z)=\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i\vec{k}(\vec{z}-\vec{b})-(1/4)k^2R^2}}{k^2+\mu^2}, \tag{C5}$$

$$T_2(z)=\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i\vec{k}(\vec{z}-\vec{b})-(1/4)k^2R^2}}{k^2+\mu^2}k^2. \tag{C6}$$

We can evaluate the rhs of Eqs. (C5), (C6) by presenting

$$\frac{1}{k^2+\mu^2}=\int_o^\infty d\alpha e^{-\alpha(k^2+\mu^2)}, \tag{C7}$$

leading to

$$T_1(z)=\int_o^\infty d\alpha\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}(\vec{z}-\vec{b})-(1/4)\,k^2R^2-\alpha(k^2+\mu^2)}, \tag{C8}$$

$$T_2(z)=\frac{1}{\pi^{3/2}}\frac{1}{R^3}e^{(\vec{z}-\vec{b})^2/R^2}-\mu^2T_1(z). \tag{C9}$$

Further calculations can be simplified by assuming the chiral limit $\mu^2=0$. The integral in the rhs of Eq. (C8) is dominated by the values of z^2 close to $\frac{2}{3}R^2$. Thus, the integral over k^2 is determined by $k^2 \sim \frac{3}{2} 1/R^2$, while the integral over α is dominated by $\alpha \sim \frac{2}{3}R^2$. Hence, the factor $\alpha\mu^2$ in the power of the exponent in the rhs of Eq. (C8) is about 0.12. Since I_0^{SY} provides a small correction only, this makes the calculation of this value in the chiral limit $\mu^2=0$ reasonable.

Calculation of the integrals over k and over α [by the substitution $t=(\frac{1}{4}R^2+\alpha)^{-1/2}$] leads to Eqs. (79)–(83) of the text.

APPENDIX D

The integrals over the time component k_0 in the rhs of Eq. (90) take the form

$$X=\int \frac{dk_0}{2\pi i}\frac{q^2}{q^2-M_\pi^2+i\varepsilon}\frac{1}{E_0-k_0-E_n+i\varepsilon} \tag{D1}$$

with $q^2=k_0^2-\vec{k}^2$. We can present $X=X_1+X_2$ with

$$X_1=M_\pi^2\int \frac{dk_0}{2\pi i}\frac{1}{q^2-M_\pi^2+i\varepsilon}\frac{1}{E_0-k_0-E_n+i\varepsilon}, \tag{D2}$$

$$X_2=\int \frac{dk_0}{2\pi i}\frac{1}{E_0-k_0-E_n+i\varepsilon}. \tag{D3}$$

The integral X_2 can be expressed through the contribution of the pole in the upper half-plane of the complex variable k_0. This corresponds to the negative-energy solutions of the Dirac equation. Such terms are neglected in the framework of the PCQM. Hence, we put $X=X_1$, leading to Eq. (109).

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