Determining γ using $B^\pm \to DK^\pm$ with multibody D decays

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We propose a method for determining γ using $B^\pm \to DK^\pm$ decays followed by a multibody D decay, such as $D\to K_S\pi^-\pi^+$, $D\to K_S K^- K^+$, and $D\to K_S\pi^-\pi^+\pi^0$. The main advantages of the method are that it uses only Cabibbo allowed D decays, and that large strong phases are expected due to the presence of resonances. Since no knowledge about the resonance structure is needed, γ can be extracted without any hadronic uncertainty.

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I. INTRODUCTION

Theoretically, the cleanest way of determining the angle

$$\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*) \tag{1}$$

is to utilize the interference between the $b\to c\bar{u}s$ and $b\to u\bar{c}s$ decay amplitudes [1–12]. Because these transitions involve only distinct quark flavors, there are no penguin contributions to these decays. In the original idea of Gronau and Wyler (GW) [3] the $B^\pm \to D_{CP}K^\pm$ decay modes are used, where D_{CP} represents a D meson which decays into a CP eigenstate. The dependence on γ arises from the interference between the $B^\pm\to D^0K^\pm$ and $B^\pm\to\bar{D}^0K^\pm$ decay amplitudes. The main advantage of the GW method is that, in principle, the hadronic parameters can be cleanly extracted from data, by measuring the $B^\pm\to D^0K^\pm$ and $B^\pm\to\bar{D}^0K^\pm$ decay rates.

In practice, however, measuring γ in this way is not an easy task. Because of the values of the Cabibbo-Kobayashi-Maskawa (CKM) coefficients and color suppression, the ratio between the two interfering amplitudes r_B [see Eq. (4)] is expected to be small, of order 10%–20%. This reduces the sensitivity to γ, which is roughly proportional to the magnitude of the smaller amplitude. In addition, if the strong phases vanish, measuring γ makes use of terms of order r_B^2. In contrast, if a large strong phase is involved in the interference, there is a sensitivity to γ at order r_B with most methods. Thus, in general, having large interfering amplitudes with large relative strong phases is a favorable situation.

Since the hadronic parameters are not yet known, it is still not clear which of the proposed methods is more sensitive. In addition, all the methods are expected to be statistically limited. It is therefore important to make use of all modes and methods, as well as to try to find new methods. Any new method that is based on "unused" decay channels increases the total statistics. Moreover, many of the analyses are sensitive to common hadronic parameters, for example, r_B. Combining them will increase the sensitivity of the measurement by more than just the increase in statistics.

Here we study the possibility to use $B^\pm\to DK^\pm$, followed by a multibody D decay, in order to cleanly determine γ. While this idea was already discussed in [5], most of our results and applications are new. For the sake of concreteness, we concentrate on the $D\to K_S\pi^-\pi^+$ decay mode. The advantage of using such decay chains is threefold. First, one expects large strong phases due to the presence of resonances. Second, only Cabibbo allowed D decay modes are needed. Third, the final state involves only charged particles, which have a higher reconstruction efficiency and lower background than neutrals. The price one has to pay is that a Dalitz plot analysis of the data is needed. We describe how to do the Dalitz plot analysis in a model independent way, and explore the advantages gained by introducing verifiable model dependence. The final balance between the advantages and disadvantages depends on yet-to-be-determined hadronic parameters and experimental considerations.

II. MODEL INDEPENDENT DETERMINATION OF γ

As we shall show in this section, to perform a model independent determination of the angle γ one needs to measure the two CP-conjugate decay modes $B^\pm\to DK^\pm\to(K_S\pi^-\pi^+)_DK^\pm$ and to perform a Dalitz plot analysis of the $K_S\pi^-\pi^+$ final state originating from the intermediate D meson. (In the following discussion we neglect D^0-$\bar{D}^0$ mixing, which is a good approximation in the context of the standard model. See Appendix A for details.)

Let us first focus on the cascade decay

$$B^-\to DK^-\to(K_S\pi^-\pi^+)_DK^- \tag{2}$$

and define the amplitudes

$$A(B^-\to D^0K^-)\equiv A_B, \tag{3}$$

$$A(B^-\to\bar{D}^0K^-)\equiv A_B r_B e^{i(\delta_B-\gamma)}. \tag{4}$$

The same definitions apply to the amplitudes for the CP conjugate cascade $B^+\to DK^+\to(K_S\pi^+\pi^-)_DK^+$, with the change of weak phase sign $\gamma\to-\gamma$ in Eq. (4). Since we have set the strong phase of A_B to zero by convention, δ_B is the difference of strong phases between the two amplitudes. For the CKM elements, the usual convention of the weak phases has been used. (The deviation of the weak phase from $-\gamma$ has been neglected, as it is suppressed by the factor $\lambda^4\sim 2\times10^{-3}$, with λ being the sine of the Cabibbo angle.) The value of $|A_B|$ is known from the measurement of the B^-

$\to D^0 K^-$ decay width using flavor specific decays of D^0 and the precision of its determination is expected to further improve [13]. The amplitude $A(B^- \to \bar{D}^0 K^-)$ is color suppressed and cannot be determined from experiment in this way [4]. The color suppression together with the experimental values of the ratio of the relevant CKM elements leads to the theoretical expectation $r_B \sim 0.1$–0.2 (see the recent discussion in [11]).

For the three-body D meson decay we define

$$\begin{aligned} A_D(s_{12},s_{13}) &\equiv A_{12,13} e^{i\delta_{12,13}} \\ &\equiv A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) \\ &= A(\bar{D}^0 \to K_S(p_1)\pi^+(p_2)\pi^-(p_3)), \end{aligned} \tag{5}$$

where $s_{ij}=(p_i+p_j)^2$, and p_1,p_2,p_3 are the momenta of the K_S,π^-,π^+, respectively. We also set the magnitude $A_{12,13} \geqslant 0$, such that $\delta_{12,13}$ can vary between 0 and 2π. In the last equality the *CP* symmetry of the strong interaction together with the fact that the final state is a spin zero state has been used. With the above definitions, the amplitude for the cascade decay is

$$\begin{aligned} &A(B^- \to (K_S\pi^-\pi^+)_D K^-) \\ &\quad = A_B \mathcal{P}_D (A_D(s_{12},s_{13}) + r_B e^{i(\delta_B-\gamma)} A_D(s_{13},s_{12})), \end{aligned} \tag{6}$$

where $\mathcal{P}_D$ is the D meson propagator. Next, we write down the expression for the reduced partial decay width:

$$\begin{aligned} d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) &= \{A_{12,13}^2 + r_B^2 A_{13,12}^2 \\ &+ 2 r_B \,\mathrm{Re}[A_D(s_{12},s_{13}) A_D^*(s_{13},s_{12}) e^{-i(\delta_B-\gamma)}]\} dp, \end{aligned} \tag{7}$$

where dp denotes the phase space variables, and we used the extremely accurate narrow width approximation for the D meson propagator.

In general, there is no symmetry between the two arguments of A_D in Eq. (6), and thus in the rates over the Dalitz plot. A symmetry would be present if, for instance, the three-body D decay proceeded only through ρ-like resonances. We emphasize, however, that the product $A_D(s_{12},s_{13})A_D^*(s_{13},s_{12})$ in the interference term in Eq. (7) is symmetric under the exchange $s_{12} \leftrightarrow s_{13}$ followed by complex conjugation. This fact is used to simplify the analysis.

The moduli of the D decay amplitude $A_{12,13}$ can be measured from the Dalitz plot of the $D^0 \to K_S\pi^-\pi^+$ decay. To perform this measurement the flavor of the decaying neutral D meson has to be tagged. This can be best achieved by using the charge of the soft pion in the decay $D^{*+} \to D^0\pi^+$. However, the phase $\delta_{12,13}$ of the D meson decay amplitude is not measurable without further model dependent assumptions. The cosine of the relevant phase difference may be measured at a charm factory (see Sec. III). If the three-body decay $D^0 \to K_S\pi^-\pi^+$ is assumed to be resonance dominated, the Dalitz plot can be fitted to a sum of Breit-Wigner functions, determining also the relative phases of the resonant amplitudes. This is further discussed in Sec. IV. Here we assume that no charm factory data are available and develop the formalism without any model dependent assumptions.

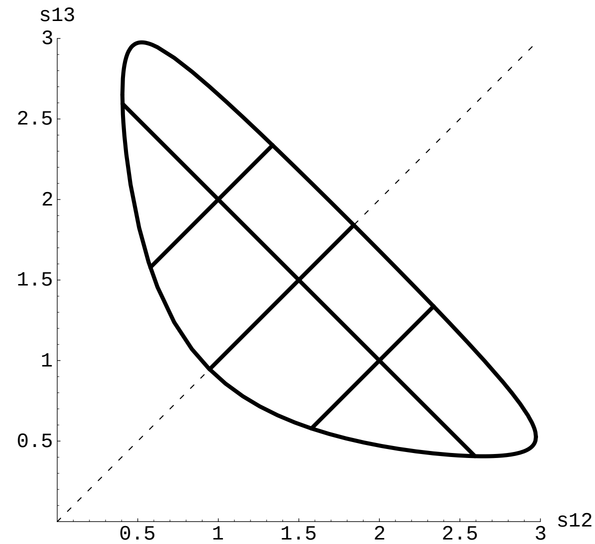

FIG. 1. The partitions of the Dalitz plot as discussed in the text. The symmetry axis is the dashed line. On the axes we have $s_{12} = m^2_{K_s\pi^-}$ and $s_{13} = m^2_{K_s\pi^+}$ in GeV^2.

Using the trigonometric relation $\cos(a+b) = \cos a \cos b - \sin a \sin b$, the last term of Eq. (7) can be written as

$$\begin{aligned} &\mathrm{Re}[A_D(s_{12},s_{13}) A_D^*(s_{13},s_{12}) e^{-i(\delta_B-\gamma)}] \\ &\quad = A_{12,13}A_{13,12}[\cos(\delta_{12,13}-\delta_{13,12})\cos(\delta_B-\gamma) \\ &\qquad + \sin(\delta_{12,13}-\delta_{13,12})\sin(\delta_B-\gamma)]. \end{aligned} \tag{8}$$

Obviously, to compare with the data, an integration over at least some part of the Dalitz plot has to be performed. We therefore partition the Dalitz plot into n bins and define

$$c_i \equiv \int_i dp A_{12,13} A_{13,12} \cos(\delta_{12,13}-\delta_{13,12}), \tag{9a}$$

$$s_i \equiv \int_i dp A_{12,13} A_{13,12} \sin(\delta_{12,13}-\delta_{13,12}), \tag{9b}$$

$$T_i \equiv \int_i dp A_{12,13}^2, \tag{9c}$$

where the integrals are done over the phase space of the ith bin. The variables c_i and s_i contain differences of strong phases and are therefore unknowns in the analysis. The variables T_i, on the other hand, can be measured from the flavor tagged D decays as discussed above, and are assumed to be known inputs into the analysis.

Due to the symmetry of the interference term, it is convenient to use pairs of bins that are placed symmetrically about the $12 \leftrightarrow 13$ line, as shown in Fig. 1. Consider an even, $n = 2k$, number of bins. The k bins lying below the symmetry axis are denoted by the index i, while the remaining bins are indexed with $\bar{i}$. The $\bar{i}$th bin is obtained by mirroring the ith bin over the axis of symmetry. The variables c_i, s_i of the ith bin are related to the variables of the $\bar{i}$th bin by

$$c_{\bar{i}} = c_i, \quad s_{\bar{i}} = -s_i, \tag{10}$$

while there is no relation between T_i and $T_{\bar{i}}$. Note that had one used $12\leftrightarrow 13$ symmetric bins centered on the symmetry axis, one would have had $s_i=0$.

Together with the information available from the B^+ decay, we arrive at a set of $4k$ equations:

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^-\to(K_S\pi^-\pi^+)_D K^-) = T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B-\gamma)c_i + \sin(\delta_B-\gamma)s_i], \tag{11a}$$

$$\hat{\Gamma}_{\bar{i}}^- \equiv \int_{\bar{i}} d\hat{\Gamma}(B^-\to(K_S\pi^-\pi^+)_D K^-) = T_{\bar{i}} + r_B^2 T_i + 2r_B[\cos(\delta_B-\gamma)c_i - \sin(\delta_B-\gamma)s_i], \tag{11b}$$

$$\hat{\Gamma}_i^+ \equiv \int_i d\hat{\Gamma}(B^+\to(K_S\pi^-\pi^+)_D K^+) = T_{\bar{i}} + r_B^2 T_i + 2r_B[\cos(\delta_B+\gamma)c_i - \sin(\delta_B+\gamma)s_i], \tag{11c}$$

$$\hat{\Gamma}_{\bar{i}}^+ \equiv \int_{\bar{i}} d\hat{\Gamma}(B^+\to(K_S\pi^-\pi^+)_D K^+) = T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B+\gamma)c_i + \sin(\delta_B+\gamma)s_i]. \tag{11d}$$

These equations are related to each other through $12\leftrightarrow 13$ and/or $\gamma\leftrightarrow -\gamma$ exchanges. All in all, there are $2k+3$ unknowns in Eq. (11),

$$c_i,\ s_i,\ r_B,\ \delta_B,\ \gamma, \tag{12}$$

so that the $4k$ relations (11) are solvable for $k\geqslant 2$. In other words, a partition of the D meson Dalitz plot into four or more bins allows for the determination of γ without hadronic uncertainties. This is our main result.

Alternatively to this binning, one can use a partition of the Dalitz plot into k bins which are symmetric under $12\leftrightarrow 13$. For that case, $s_i=0$ and the set of $4k$ equations (11) reduces to $2k$ relations [the first two and the last two equations in (11) are the same in this case]. Then, there are just $k+3$ unknowns to be solved for, which is possible for $k\geqslant 3$. While such binning may be needed due to low statistics, it has several disadvantages, which are further discussed below.

When $c_i=0$ or $s_i=0$ for all i, some equations become degenerate and γ cannot be extracted. However, due to resonances, we do not expect this to be the case. Degeneracy also occurs if $\delta_B=0$. In this case, γ can still be extracted if some of the c_i and/or s_i are independently measured, as discussed in the following section.

The optimal partition of the Dalitz plot as well as the number of bins is to be determined once the analysis is done. Some of the considerations that enter this choice are as follows. First, one would like to have as many small bins as possible, in order that c_i and s_i do not average out to small numbers. Second, the bins have to be large enough that there are significantly more events than bins. Otherwise there will be more unknowns than observables. There are also experimental considerations, such as optimal parametrization of backgrounds and reconstruction efficiency.

III. IMPROVED MEASUREMENT OF c_i AND s_i

So far, we have used the B decay sample to obtain all the unknowns, including c_i and s_i, which are parameters of the charm system. We now discuss ways to make use of high-statistics charm decays to improve the measurement of these parameters, or obtain them independently. Doing so will reduce the number of unknowns that need to be determined from the relatively low-statistics B sample, thereby reducing the error in the measurement of γ.

The first improvement in the measurement is obtained by making use of the large sample of tagged D decays, identified in the decay $D^{*+}\to D^0\pi^+$, at the B factories. So far we have assumed only that we use these data to determine T_i. In fact, they can also be used to bound the unknowns c_i and s_i defined in Eq. (9):

$$|s_i|,|c_i| \leqslant \int_i dp A_{12,13}A_{13,12} \leqslant \sqrt{T_i T_{\bar{i}}}. \tag{13}$$

This bound will help decrease the error in the determination of γ, with an especially significant effect when, due to low statistics in each bin, c_i and s_i are determined with large errors.

Next, we show that the c_i can be independently measured at a charm factory [14–16]. This is done by running the machine at the $\psi(3770)$ resonance, which decays into a $D\bar{D}$ pair. If one D meson is detected in a CP eigenstate decay mode, it tags the other D as an eigenstate of the opposite CP eigenvalue. The amplitude and partial decay width for this state to decay into the final state of interest are

$$A(D^0_{\pm}\to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) = \frac{1}{\sqrt{2}}[A_D(s_{12},s_{13}) \pm A_D(s_{13},s_{12})],$$

$$d\Gamma(D^0_{\pm}\to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) = \frac{1}{2}(A^2_{12,13}+A^2_{13,12}) \pm A_{12,13}A_{13,12} \times\cos(\delta_{12,13}-\delta_{13,12})dp, \tag{14}$$

where we defined $D^0_\pm \equiv (D^0 \pm \overline{D}^0)/\sqrt{2}$. With these relations, one readily obtains

$$c_i = \frac{1}{2}\left[\int_i d\Gamma(D^0_+ \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) - \int_i d\Gamma(D^0_- \to K_S(p_1)\pi^-(p_2)\pi^+(p_3))\right]. \quad (15)$$

As stated above, obtaining this independent measurement reduces the error in the measurement of γ by removing k of the $2k+3$ unknowns.

In addition, if one of the D mesons decays into a non-*CP* eigenstate, we are sensitive to the s_i variables as well. Consider, for instance, a $\psi(3770)$ decaying into a $D,\overline{D}$ pair, of which one decays into $K_S\pi^+\pi^-$ and the other decays into some general state g. The partial decay width corresponding to the ith bin of the $K_S\pi^+\pi^-$ Dalitz plot and the jth bin of the g final state's phase space is

$$\Gamma_{i,j} \propto T_i T^g_{\bar{j}} + T_{\bar{i}} T^g_j - 2(c_i c^g_j + s_i s^g_j), \quad (16)$$

where T^g_j, c^g_j, s^g_j are defined as in Eq. (9). In particular, if one chooses $g = K_S\pi^+\pi^-$ and $j = i$ (or $j = \bar{i}$) one measures s_i^2. If, on the other hand, g is a *CP* even (odd) eigenstate, $s^g_j = 0$, $T^g_j = T^g_{\bar{j}} = \pm c^g_j$, and Eq. (16) reduces to Eq. (15).

We can further improve the measurement by incorporating more relations between c_i and s_i. To do this, one takes each bin i and further divides it into n_i sub-bins, such that the quantities $A_{12,13}$, $\cos(\delta_{12,13}-\delta_{13,12})$, and $\sin(\delta_{12,13}-\delta_{13,12})$ do not change significantly within each sub-bin i'. Naively, this statement appears to introduce model dependence. In practice, however, the high statistics in the tagged D sample and the charm factory $\psi(3770)$ sample allow its verification up to a statistical error, which can be measured and propagated to the final measurement of γ.

Given this condition, Eq. (9a) may be written as

$$c_i = \sum_{i'} c_{i'} = \sum_{i'} A_{i'} A_{\bar{i}'} \cos(\delta_{i'} - \delta_{\bar{i}'}) \Delta p_{i'} = \sum_{i'} \sqrt{T_{i'} T_{\bar{i}'}} \cos(\delta_{i'} - \delta_{\bar{i}'}), \quad (17)$$

where the $\bar{i}'$th sub-bin is the $12 \leftrightarrow 13$ mirror image of the i'th sub-bin, $A_{i'}$ and $\delta_{i'}$ are the values of $A_{12,13}$ and $\delta_{12,13}$ on sub-bin i', taken to be constant throughout the sub-bin, and $\Delta p_{i'}$ is the area of sub-bin i'. Analogously to Eq. (9c), we have defined the quantities $T_{i'} = A^2_{12,13}\Delta p_{i'}$, which are measured using the tagged D sample. The $c_{i'}$'s are assumed to be measured at the charm factory, applying Eq. (15) to the sub-bin i'. Similarly, Eq. (9b) becomes

$$s_i = \sum_{i'} \sqrt{T_{i'} T_{\bar{i}'}} \sin(\delta_{i'} - \delta_{\bar{i}'}) = \sum_{i'} \pm\sqrt{T_{i'} T_{\bar{i}'} - c^2_{i'}}. \quad (18)$$

Equation (18) removes the k unknowns s_i, and replaces them with the twofold ambiguity associated with the sign of the square root. Thus, the best approach is to have the signs of s_i determined by the fit, while constraining their absolute values to satisfy Eq. (18). Doing so will reduce the "strain" on the B decay sample, reducing the error on γ.

Another option for removing the dependence on s_i is to use bins centered symmetrically about the $12 \leftrightarrow 13$ line, making s_i vanish, as discussed after Eq. (10). In this case, both the number of unknowns and the number of observables (bins) is reduced by k. By contrast, using Eqs. (16) and (18) introduces new information from the independent tagged D sample, and is therefore preferred. Doing so also preserves the $\sin(\delta_B-\gamma)$ terms in Eq. (11), which help resolve discrete ambiguities (see [7] and Sec. V).

IV. ASSUMING BREIT-WIGNER DEPENDENCE

If the functional dependence of both the moduli and the phases of the D^0 meson decay amplitudes $A_D(s_{12},s_{13})$ were known, then the analysis would be simplified. There would be only three variables, r_B, δ_B, and γ, that need to be fitted to the reduced partial decay widths in Eq. (7). A plausible assumption about their forms, which is also supported by experimental data [17–19], is that a significant part of the three-body $D^0 \to K_S\pi^-\pi^+$ decay proceeds via resonances. These include decay transitions of the form $D^0 \to K_S\rho^0 \to K_S\pi^-\pi^+$ or $D^0 \to K^{*-}(892)\pi^+ \to K_S\pi^-\pi^+$, as well as decays through higher resonances, e.g., $f_0(980)$, $f_2(1270)$, or $f_0(1370)$, inducing ρ-like transitions, or $K^*_0(1430)$, which induces a $K^*(892)$-like transition.

It is important to stress that these assumptions can be tested. By making use of the high statistics tagged D sample, one can test that the assumed shapes of the resonances are consistent with the data. While the error introduced by using the Breit-Wigner shapes is theoretical, it is expected to be much smaller than the statistical error in the measurement of γ. It will become a problem only when the B sample is large enough to provide a precision measurement of γ. By then the tagged D sample will have increased as well, allowing even more precise tests of these assumptions, as well as improving the precision of the methods presented in Sec. III.

The decay amplitude can then be fitted to a sum of Breit-Wigner functions and a constant term. Following the notations of Ref. [20] we write

$$A_D(s_{12},s_{13}) = A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12},s_{13}), \quad (19)$$

where the first term corresponds to the nonresonant term and the second to the resonant contributions. The Breit-Wigner function is defined as

$$\mathcal{A}_r(s_{12},s_{13}) = {}^J\mathcal{M}_r \times F^r_{BW}, \quad (20)$$

where r represent a specific resonance in either the $K_S(p_1)\pi^-(p_2)$, $K_S(p_1)\pi^+(p_3)$, or $\pi^-(p_2)\pi^+(p_3)$, channel. ${}^{J}\mathcal{M}_r$ is the term that accounts for the angular dependence. It depends on the spin J of the resonance. For example, ${}^{0}\mathcal{M}_r=1$ and ${}^{1}\mathcal{M}_r=-2\vec{k}_1\cdot\vec{k}_3$. Here $\vec{k}_1,\vec{k}_3$ are, respectively, the three-momenta of one of the particles originating from the resonance and of the remaining particle, as measured in the rest frame of the two resonating particles [20]. F^r_{BW} corresponds to the relativistic Breit-Wigner function and is given by

$$F^r_{BW}(s)=\frac{1}{s-M_r^2+iM_r\Gamma_r(\sqrt{s})}, \tag{21}$$

where M_r is the mass of the rth resonance and $\Gamma_r(\sqrt{s})$ denotes the mass dependent width. The argument of F^r_{BW} is s_{12} $[s_{13},s_{23}]$ for a $K_S(p_1)\pi^-(p_2)$ $[K_S(p_1)\pi^+(p_3)$, $\pi^-(p_2)\pi^+(p_3)]$ resonance. One can find detailed expressions for all the functions mentioned above in Ref. [20].

One of the strong phases δ_i in the ansatz (19) can be put to zero, while others are fitted to the experimental data together with the amplitudes a_i. The best option is to fit the Dalitz plot of tagged D decays, as was done a decade ago by the ARGUS and E687 Collaborations [17,18] and recently by the CLEO Collaboration [19]. The obtained functional form of $A_D(s_{12},s_{13})$ can then be fed into Eq. (7), which is then fitted to the Dalitz plot of the $B^{\pm}\to(K_S\pi^-\pi^+)_DK^{\pm}$ decay with r_B, δ_B, and γ left as free parameters. In Appendix B we provide a formula for the latter case, where only three resonances are included in the analysis.

V. DISCUSSION

The observables $\hat{\Gamma}_i^{\pm}$ defined in Eq. (11) can be used to look experimentally for direct CP violation. Explicitly,

$$a^i_{CP}\equiv\hat{\Gamma}_i^- -\hat{\Gamma}_{\bar{i}}^+ =4r_B\sin\gamma[c_i\sin\delta_B-s_i\cos\delta_B],$$

$$a^{\bar{i}}_{CP}\equiv\hat{\Gamma}_{\bar{i}}^- -\hat{\Gamma}_i^+ =4r_B\sin\gamma[c_i\sin\delta_B+s_i\cos\delta_B]. \tag{22}$$

It is manifest that finite a_{CP} requires nonvanishing strong and weak phases. The first terms in the brackets, in Eq. (22) depend on $\sin\delta_B$. This is the same dependence as for two-body D decays into CP eigenstates. In the second terms, which depend on $\cos\delta_B$, the required strong phase arises from the D decay amplitudes. Due to the resonances, we expect this strong phase to be large. Therefore, it may be that direct CP violation can be established in this mode even before the full analysis to measure γ is conducted. With more data, γ can be extracted assuming Breit-Wigner resonances (cf. Sec. IV). Eventually, a model independent extraction of γ can be done (cf. Secs. II and III).

The method proposed above for the model independent measurement of γ involves a fourfold ambiguity in the extracted value. The set of equations (11) are invariant under each of the two discrete transformations:

$$P_\pi\equiv\{\delta_B\to\delta_B+\pi,\gamma\to\gamma+\pi\},$$

$$P_-\equiv\{\delta_B\to-\delta_B,\gamma\to-\gamma,s_i\to-s_i\}. \tag{23}$$

We note that if all the bins used are symmetric under $12\leftrightarrow13$, the absence of the $\sin(\delta_B-\gamma)$ terms in Eq. (11) introduces a new ambiguity transformation, $P_{\rm ex}\equiv\gamma\to\delta_B,\delta_B\to\gamma$. The discrete transformation P_π is a symmetry of the amplitude (6) and is thus an irreducible uncertainty of the method. It can be lifted if the sign of either $\cos\delta_B$ or $\sin\delta_B$ is known. The ambiguity due to P_- can be resolved if the sign of $\sin\delta_B$ is known or if the sign of s_i can be determined in at least some part of the Dalitz plot. The latter can be done by fitting a part of the Dalitz plot to Breit-Wigner functions. We emphasize that only the sign of the phase of the resonance amplitude is required, and thus we can safely use a Breit-Wigner form for this purpose.

The r_B suppression present in the scheme outlined above can be somewhat lifted if the cascade decay $B^-\to DX_s^-\to(K_S\pi^-\pi^+)_DX_s^-$ is used [6,11]. Here X_s^- is a multibody hadronic state with an odd number of kaons (examples of such modes are $K^-\pi^-\pi^+$, $K^-\pi^0$, and $K_S\pi^-\pi^0$). Unlike the $B^-\to\overline{D}^0K^-$ decay, these modes have color-allowed contributions. This lifts the color suppression in r_B, while the mild suppression due to the CKM matrix elements remains. The major difference compared with the case of the two-body B^- decay is that now r_B and δ_B are functions of the $B\to DX_s^-$ decay phase space. Therefore, the experimental analysis has to deal with two Dalitz plots, one describing $B\to DX_s^-$ and the other describing the $D\to K_S\pi^-\pi^+$ decay. In Appendix C the necessary formalism that applies to this case is outlined. Note that the above mentioned treatment for multibody B decays also applies to quasi two-body B decays involving a resonance, such as $B\to DK^*$.

In addition to using different B modes, statistics may be increased by employing various D decay modes as well. An interesting possibility is the Cabibbo allowed $D\to K_S\pi^-\pi^+\pi^0$ decay. It comes with an even larger branching ratio than the $D\to K_S\pi^-\pi^+$ decay. In addition, it has many intermediate resonances contributing to the greatly varying decay amplitude, which is what is needed for the extraction of γ. The disadvantages of this mode are the low reconstruction efficiency of the π^0, as well as the binning difficulties introduced by the higher dimensionality of the four-body phase space. The formalism of Sec. II applies to this mode as well, but now the partition of the four-body phase space is meant in Eq. (11). In the equivalent of Eq. (5), this mode has an extra minus sign, since we have introduced a new CP-odd state, the π^0. The final set of equations is then obtained from Eq. (11) by replacing $r_B\to-r_B$. The Cabibbo allowed mode $D\to K^-K^+K_S$ may also be used for the extraction of γ, as can the Cabibbo suppressed decays to $K^-K^+\pi^0$, $\pi^-\pi^+\pi^0$, and $K_SK^+\pi^-$.

We note that use of our formalism is needed in order to measure γ with (almost) flavor eigenstate multibody decays, such as $D\to K^-\pi^+\pi^0$ and $D\to K^-\pi^+\pi^-\pi^+$, using the method of [4], if one does not wish to make assumptions about specific resonances in the decay, as was done in [5]. Here, the important interference is between the Cabibbo allowed D decay and the doubly Cabibbo suppressed $\bar{D}$ decay. Due to a strong phase different between these two amplitudes, every multibody D decay mode (or bin in the phase space of the mode) of this type introduces two unknowns c_i and s_i. As a result, these modes are not sufficient to measure γ without assuming resonances in the decays. However, they can be used in combination with additional D decay modes, such as the ones proposed here or *CP* eigenstate modes. In that case, one has enough observables to determine all the unknowns, and the flavor eigenstate modes contribute to the total measurement of γ.

While we concentrated on charged B decays, the Dalitz plot analysis presented here can also be applied to self-tagging decays of neutral B mesons [8]. It is also straightforward to apply it to cases where time dependent *CP* asymmetries are measured [2].

The sensitivity to γ is roughly proportional to the smaller of the two interfering amplitudes. Assuming that the only two small parameters are r_B and λ, our method is sensitive to γ at $O(r_B)$. However, the method is sensitive to γ only in parts of the Dalitz plot. The highest sensitivity is in regions with two or more overlapping resonances. The sensitivity of the proposed method is therefore of order $O(r_B\xi)$, where ξ^2 is the fraction of events that are in the interesting region of the Dalitz plot. This is to be compared with the sensitivity of the GW method, which is $O(r_B\lambda)$ [10]. One can see that the sensitivity of our method is comparable to that of the GW method even if ξ^2 is as small as 0.05. While a precise measurement of ξ has not been conducted yet, a rough estimate gives $\xi^2\sim 0.1$. We also note that the Cabibbo allowed branching fraction will result in a relatively easier experimental analysis, due to the large signal-to-background ratio.

A crucial point of our method is that it uses interference between two Cabibbo allowed D decay amplitudes. This is against the common intuition, which suggests that we must have a λ^2 suppression for such interference to take place, as we need a final state that is common to both D and $\bar{D}$. Specifically, one typically requires one Cabibbo allowed decay and another that is doubly Cabibbo suppressed, or two decays that are singly Cabibbo suppressed. To overcome this preconception, our method makes use of K^0-$\bar{K}^0$ mixing (which is also the case for the two-body $D\to K_S\pi^0$ decay), plus the existence of overlapping resonances, which are obtained by Cabibbo allowed D^0 and $\bar{D}^0$ decays. In addition, it is important that the hadronic three-body D meson decays have a widely changing amplitude over the Dalitz plot, which is ensured by the presence of resonances in this energy region. If the strong phases $\delta_{12,13}$ and the moduli $A_{12,13}$ in Eq. (9) were (almost) constant across the available phase space, the extraction of γ from Eqs. (11) would not be possible.

Before concluding, we mention that quasi two-body D decays where one of the particles is a resonance, such as $D\to K^{*+}\pi^-$ and $D\to K^+\rho^-$ [4], were proposed for use in measuring γ. But, in fact, using such decays requires a Dalitz plot analysis (see, e.g., [10,12]). What we showed here is that one can actually use the whole Dalitz plot to carry out the analysis and does not need to single out contributions of one particular resonance. Moreover, we showed that the assumption about the shapes of the resonances can be avoided, essentially with currently available data sets.

In conclusion, we have shown that the angle γ can be determined from the cascade decays $B^\pm\to K^\pm(K_S\pi^-\pi^+)_D$. The reason for the applicability of the proposed method lies in the presence of resonances in the three-body D meson decays which provide a necessary variation of both the phase and the magnitude of the decay amplitude across the phase space. The fact that no Cabibbo suppressed D decay amplitudes are used in the analysis is another advantage of the method. However, it does involve a Dalitz plot analysis with possibly only parts of the Dalitz plot being practically useful for the extraction of γ. In reality, many methods have to be combined in order to achieve the required statistics for a precise determination of γ [7].

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APPENDIX A: THE EFFECT OF D-$\bar{D}$ MIXING

In this section we focus on the contributions introduced by the fact that the flavor states $|D^0\rangle,|\bar{D}^0\rangle$ and the mass eigenstates $|D_{H,L}\rangle=p_D|D^0\rangle\pm q_D|\bar{D}^0\rangle$ do not coincide. This effect was studied in the general case in Ref. [21]. Here we apply their formalism to our case.

Following Ref. [21] we introduce the rephasing-invariant parameter χ_1,

$$\chi_1=\frac{\lambda_{D\to f}+\xi_{B^-\to D}}{1+\lambda_{D\to f}\xi_{B^-\to D}}, \tag{A1}$$

where

$$\lambda_{D\to f}=\frac{q_D}{p_D}\frac{A_{\bar{D}^0\to f}}{A_{D^0\to f}},$$

$$\xi_{B^-\to D}=\frac{A_{B^-\to\bar{D}^0K^-}}{A_{B^-\to D^0K^-}}\frac{p_D}{q_D}=r_Be^{-i(2\theta_D-\delta_B+\gamma)}, \tag{A2}$$

and we use the definitions of Eqs. (3) and (4) and allow for new physics effects in $q_D/p_D=e^{i2\theta_D}$. (In the phase convention where the D decay amplitudes are real, the phase θ_D is negligible in the standard model.) In our case, the final state f equals $K_S\pi^-\pi^+$, which leads to

$$\lambda_{D\to K_S(p_1)\pi^-(p_2)\pi^+(p_3)}=e^{i2\theta_D}\frac{A_D(s_{13},s_{12})}{A_D(s_{12},s_{13})}$$
$$=R_D(s_{12,},s_{13})e^{i(2\theta_D+\delta_{13,12}-\delta_{12,13})}. \tag{A3}$$

Once D-$\bar{D}$ mixing is taken into account in the analysis, the expression for the partial decay width (7) is multiplied by the correction term [21]

$$1-\mathrm{Re}(\chi_1)y_D+\mathrm{Im}(\chi_1)x_D, \tag{A4}$$

where we have expanded the correction term to first order in the small parameters

$$x_D=\frac{\Delta m}{\Gamma},\quad y_D=\frac{\Delta\Gamma}{2\Gamma}, \tag{A5}$$

where Δm and $\Delta\Gamma$ are the mass and decay width differences in the D-$\bar{D}$ system, and Γ is the D^0 decay width. The values of x_D and y_D are constrained by present measurements to be in the percent range, $y_D=(1.0\pm0.7)\%$ [22] and $|x|<2.8\%$ [23] (assuming small strong phases).

The ratio of magnitudes $R_D(s_{12,},s_{13})$ depends on the position in the Dalitz plot and can vary widely. Our method is useful for the model independent extraction of γ only in the region where R_D is of order 1. We therefore distinguish three limiting cases.

(1) $R_D\gg1\gg r_B$, for which $\mathrm{Re}(\chi_1),\mathrm{Im}(\chi_1)\sim O(1/r_B)$ and therefore the corrections in Eq. (A4) can be of order 10%. However, this is the region of the Dalitz plot where our method is mostly not sensitive to γ and therefore the induced corrections due to D-$\bar{D}$ mixing do not translate into an error on the extracted γ.

(2) $R_D\sim1\gg r_B$, for which $\mathrm{Re}(\chi_1),\mathrm{Im}(\chi_1)\sim O(1)$ and therefore the corrections in Eq. (A4) are at the percent level. This is the value of R_D for which our method is most sensitive to γ.

(3) $1\gg r_B\sim R_D$, for which $\mathrm{Re}(\chi_1),\mathrm{Im}(\chi_1)\sim O(r_B,R_D)$ and therefore the corrections in Eq. (A4) are very small.

In conclusion, we expect errors of at most a few percent due to neglecting D-$\bar{D}$ mixing in our method. In principle, even these errors can be taken into account [16,21,24].

APPENDIX B: A FIT TO BREIT-WIGNER FUNCTIONS: AN ILLUSTRATION FOR THREE RESONANCES

In this appendix we provide the formulas for the fit of the D meson decay amplitude to a sum of three Breit-Wigner functions describing $K^{*\pm}(892)$ and ρ^0 resonances. We write Eq. (19) explicitly as

$$A_D(s_{12},s_{13})=A(D^0\to K_S(p_1)\pi^-(p_2)\pi^+(p_3))$$
$$=a_\rho\mathcal{A}_{\rho^0}(s_{23})+a_{K^*}e^{i\delta_F}\mathcal{A}_{K^*}(s_{12})$$
$$+a_{K^*}r_De^{i\delta_D}\mathcal{A}_{K^*}(s_{13}), \tag{B1}$$

where δ_F (δ_D) is the strong phase of the Cabibbo favored (doubly Cabibbo suppressed) $D^0\to K^{*-}\pi^+$ ($D^0\to K^{*+}\pi^-$) decay with respect to the decay $D^0\to K_S\rho^0$. We further introduced

$$a_\rho\propto A(D^0\to\rho^0K_S)=A(\bar{D}^0\to\rho^0K_S),$$

$$a_{K^*}e^{i\delta_F}\propto A(D^0\to K^{*-}\pi^+)=A(\bar{D}^0\to K^{*+}\pi^-),$$

$$a_{K^*}r_De^{i\delta_D}\propto A(D^0\to K^{*+}\pi^-)=A(\bar{D}^0\to K^{*-}\pi^+). \tag{B2}$$

The Breit-Wigner functions $\mathcal{A}_r$ are defined in Eq. (20), where we write in Eq. (B1) only the s_{ab} dependence of the F^r_{BW} part, given in Eq. (21). The first index of s_{ab} is understood to denote also the particle appearing in the expression for ${}^1\mathcal{M}_r$ [Eq. (20)]. Exchanging $a\leftrightarrow b$ corresponds to ${}^1\mathcal{M}_r\leftrightarrow-{}^1\mathcal{M}_r$, in particular, $\mathcal{A}_{\rho^0}(s_{23})=-\mathcal{A}_{\rho^0}(s_{32})$. In the above we assumed that there is no CP violation in the D decay amplitudes. Note that there are two small parameters

$$r_B\sim0.1-0.2,\quad r_D\sim\lambda^2\sim0.05. \tag{B3}$$

We then obtain [cf. Eq. (6)]

$$A(B^-\to(K_S(p_1)\pi^-(p_2)\pi^+(p_3))_DK^-)$$
$$=A_B\mathcal{P}_D\times((a_\rho\mathcal{A}_{\rho^0}(s_{23})+a_{K^*}[e^{i\delta_F}\mathcal{A}_{K^*}(s_{12})$$
$$+r_De^{i\delta_D}\mathcal{A}_{K^*}(s_{13})])+r_Be^{i(\delta_B-\gamma)}\{a_\rho\mathcal{A}_{\rho^0}(s_{32})$$
$$+a_{K^*}[e^{i\delta_F}\mathcal{A}_{K^*}(s_{13})+r_De^{i\delta_D}\mathcal{A}_{K^*}(s_{12})]\}). \tag{B4}$$

The corresponding expressions for B^+ decays are obtained by changing $\gamma\to-\gamma$ and $\pi^-(p_2)\pi^+(p_3)\to\pi^+(p_2)\pi^-(p_3)$.

We further define

$$\delta_-=\arg[\mathcal{A}_{K^*}(s_{12})],\quad \delta_+=\arg[\mathcal{A}_{K^*}(s_{13})],$$
$$\delta_0=\arg[\mathcal{A}_{\rho^0}(s_{23})], \tag{B5}$$

where the dependence of $\delta_{\pm,0}$ on the position in the Dalitz plot is implicitly assumed. The reduced differential decay rate is then

$$d\hat{\Gamma}(B^-\to(K_S\pi^-\pi^+)_DK^-)\propto a_\rho^2|\mathcal{A}_{\rho^0}(s_{23})|^2[1-2r_B\cos(\delta_B-\gamma)+r_B^2]+a_{K*}^2|\mathcal{A}_{K*}(s_{12})|^2 \times[1+2r_Br_D\cos(\delta_{BD}^F-\gamma)+(r_Br_D)^2]+a_{K*}^2|\mathcal{A}_{K*}(s_{13})|^2[r_D^2+2r_Br_D\cos(\delta_{BF}^D-\gamma)+r_B^2] +2a_\rho a_{K*}|\mathcal{A}_{\rho^0}(s_{23})\mathcal{A}_{K*}(s_{13})|\{r_D\cos\delta_0^{D+}-r_B^2\cos\delta_0^{F+}-r_Br_D\cos(\delta_{B0}^{D+}-\gamma) +r_B\cos(\delta_0^{BF+}+\gamma)\}+2a_\rho a_{K*}|\mathcal{A}_{\rho^0}(s_{23})\mathcal{A}_{K*}(s_{12})|\{\cos\delta_0^{F-}-r_B\cos(\delta_{B0}^{F-}-\gamma)+r_Br_D\cos(\delta_0^{BD-} +\gamma)-r_B^2r_D\cos\delta_0^{D-}\}+2a_{K*}^2|\mathcal{A}_{K*}(s_{12})\mathcal{A}_{K*}(s_{13})|\{r_D\cos\delta_{F-}^{D+} +r_B\cos(\delta_-^{B+}+\gamma)+r_Br_D^2\cos(\delta_{B-}^{+}-\gamma)+r_B^2r_D\cos\delta_{D-}^{F+}\}, \tag{B6}$$

where the notation of the strong phases is such that the lower (upper) indices indicate phases appearing with a plus (minus) sign. For example,

$$\delta_{D-}^{F+}=\delta_D+\delta_--\delta_F-\delta_+. \tag{B7}$$

a_ρ, a_{K*}, and r_D are assumed to be known and thus there are five unknowns to fit, namely,

$$r_B,\ \ \delta_D,\ \ \delta_F,\ \ \delta_B,\ \ \gamma. \tag{B8}$$

Using both B^- and B^+ decays, there is enough information to determine them all. This is true even if one neglects terms that scale as r_B^2 and even if $r_D=0$. This indicates that the method does not rely on doubly Cabibbo suppressed decays of the D, and that it is sensitive to γ in terms of order r_B, rather than r_B^2. (See the discussion in [10].) Moreover, even if some or all of the strong phases that arise from two-body decays, namely, δ_B, δ_D, and δ_F, vanish, there is still enough information to determine γ.

APPENDIX C: MULTIBODY *B* DECAY

We consider the cascade decay $B^-\to DX_s^-\to(K_S\pi^-\pi^+)_DX_s^-$. Let us assume that the phase space of the first decay, $B^-\to DX_s^-$, is partitioned into m bins that we label by the index j, and the phase space of the D meson decay is partitioned into $n=2k$ bins labeled by i and $\bar{i}$ as in Sec. II. Instead of Eqs. (11) we now have the set of $4k\times m$ equations

$$\hat{\Gamma}_{i,j}^-\equiv\int_{i,j}d\Gamma(B^-\to(K_S\pi^-\pi^+)_DX_s^-) =T_i+R_j^BT_{\bar{i}}+\cos\gamma(c_ic_j^B+s_is_j^B) +\sin\gamma(c_is_j^B-s_ic_j^B), \tag{C1a}$$

$$\hat{\Gamma}_{\bar{i},j}^-\equiv\int_{\bar{i},j}d\Gamma(B^-\to(K_S\pi^-\pi^+)_DX_s^-) =T_{\bar{i}}+R_j^BT_i+\cos\gamma(c_ic_j^B-s_is_j^B) +\sin\gamma(c_is_j^B+s_ic_j^B), \tag{C1b}$$

$$\hat{\Gamma}_{i,j}^+\equiv\int_{i,j}d\Gamma(B^+\to(K_S\pi^-\pi^+)_DX_s^+) =T_{\bar{i}}+R_j^BT_i+\cos\gamma(c_ic_j^B-s_is_j^B) -\sin\gamma(c_is_j^B+s_ic_j^B), \tag{C1c}$$

$$\hat{\Gamma}_{\bar{i},j}^+\equiv\int_{\bar{i},j}d\Gamma(B^+\to(K_S\pi^-\pi^+)_DX_s^+) =T_i+R_j^BT_{\bar{i}}+\cos\gamma(c_ic_j^B+s_is_j^B) -\sin\gamma(c_is_j^B-s_ic_j^B), \tag{C1d}$$

where the integration is over the phase space of the jth bin in the B decay and the phase space of the ith bin in the D decay. The jth bin of the B^+ decay phase space is obtained from the jth bin of the B^- decay by CP conjugation. We also used

$$s_j^B=\int_j 2r_B\sin\delta_B,$$

$$c_j^B=\int_j 2r_B\cos\delta_B,$$

$$R_j^B=\int_j r_B^2, \tag{C2}$$

where r_B and δ_B are functions of the position in the B decay phase space. From the set of $4k\times m$ equations (C1), one has to determine $2k+3m+1$ unknowns c_i, s_i, c_j^B, s_j^B, R_j^B, and γ. With a partition of the D decay phase space into $2k\geqslant4$ bins and with a partition of the B decay phase space into $m\geqslant1$ bins, one has enough relations to determine all the unknowns, including the angle γ. This is true even for constant δ_B and r_B, in which case the above equations fall into $4k$ sets of m equivalent relations, i.e., the set of $4k\times m$ equations is reduced to the set of $4k$ independent relations (11).

Finally, we note that the above equations can be used to determine γ also for two-body D decays [6].

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