Quark spin coupling in baryons reexamined

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A direct connection can be made between mixing angles in negative parity baryons and the spin coupling of constituent quarks. The mixing angles do not depend on spectral data. These angles are recalculated for gluon exchange and pion exchange between quarks. The mixing determined for pion exchange is in disagreement with a calculation in the literature [Phys. Rep. **268**, 263 (1996)], but consistent with the results of another calculation [Phys. Rev. C **65**, 045209 (2002)] and with experiment. The experimental data on mixing are very similar to those derived from gluon exchange but substantially different from the values obtained for pion exchange. A preliminary estimate of spin orbit forces is found to give only small changes in these angles.

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I. INTRODUCTION

The spin-spin coupling between two fermions has two terms: a ''tensor'' term and a ''contact'' term. In atoms, electron spins interact with the nuclear spin and this explains the hyperfine structure: the tensor term is the ordinary magnetic dipole-dipole interaction, and the contact term is part of the same interaction when the dipoles are at the same point. In nuclei, nucleon spins interact through pion exchange and there is a similar coupling with a different weight for the contact term. For constituent quarks in baryons there is a controversy in the literature between ''one-gluon exchange'' (OGE), which mimics the magnetic coupling, and ''one-pion exchange'' (OPE). In the early days of constituent quarks OGE was applied to ground state [1] and excited baryons [2–4] with some success. Pion exchange was also tried, particularly in the context of bag models [5]. More recently, it has been argued that the entire spin dependent coupling between constituent quarks is due to Goldstone boson exchange [6], a generalization of OPE. This proposal was criticized [7,8].

We deal here with a single issue: the mixing of states in the lowest mass negative parity nucleons. These negative parity nucleons have internal orbital angular momentum $L=1$, which couples with an overall quark spin of $S=1/2$ or 3/2 to give the total angular momentum J. The physical states of $J=1/2$ (or 3/2) are mixtures of doublet and quartet spin states, and this mixing can be determined from decay data. This issue has been discussed already [7,8]. However, we find that the discussion was flawed since [7] used the estimates of [6] and the estimates of [6] are based on fitting the experimental mass spectrum. However, as we show below, these mixing angles are independent of the mass spectrum and depend only on the coupling and wave functions. We reevaluate these angles and find significant changes from those appearing in [6,7]. The differences between OPE and OGE become larger and the data more clearly favors OGE, the same coupling as between electrons and nuclei (i.e., magnetic dipole type hyperfine interactions). All this will be discussed in detail below, as well as some items in the literature.

II. QUARK-QUARK HYPERFINE INTERACTIONS

Our discussion here follows [3], although for the sake of clarity we repeat some of the material.

A. One-gluon exchange

This effective hyperfine interaction between two quarks in a baryon has the form of the magnetic dipole-dipole interaction in electrodynamics (with dipoles produced by current loops; see [9]):

$$H_{\mathrm{OGE}}=A\{(8\pi/3)\vec{S}_1\cdot\vec{S}_2\delta^3(\vec{\rho})+(3\vec{S}_1\cdot\hat{\rho}\vec{S}_2\cdot\hat{\rho}-\vec{S}_1\cdot\vec{S}_2)\rho^{-3}\}. \tag{1}$$

Here S_1, S_2 are the spins of the two quarks, $\sqrt{2}\vec{\rho}=\vec{r}_1-\vec{r}_2$ is a vector joining them, $\hat{\rho}=\vec{\rho}/|\vec{\rho}|$ is a unit vector, and A is an overall constant which determines the strength of the interaction. We do not need the value of A in what follows, since we do not engage in fitting spectra (but $A>0$). Nor does it matter if the value of A is too large to be interpreted as single gluon exchange. The first term is called the (Fermi) contact term and the second is the ''tensor'' term, but these names obscure the origin of the second term, which is the ordinary dipole-dipole interaction for two separated dipoles of spin one-half. Recall that the contact term contributes only when the two dipoles are in an orbital s-wave state ($l_{12}=0$), while the tensor term contributes only when the two dipoles are in an orbital state with l_{12} different from zero (*unity* here). It is also important to note that these two terms are parts of the same physical interaction. In Eq. (1) one assumes that the quarks are pointlike.

B. One-pion exchange

Here we assume that the two quarks interact by exchanging a massless pseudoscalar, the ''pion,'' and the coupling takes the form [6]

$$H_{\mathrm{OPE}}=B\{(-4\pi/3)\vec{S}_1\cdot\vec{S}_2\delta^3(\vec{\rho}) +(3\vec{S}_1\cdot\hat{\rho}\vec{S}_2\cdot\hat{\rho}-\vec{S}_1\cdot\vec{S}_2)\rho^{-3}\}\vec{\lambda}_1^f\cdot\vec{\lambda}_2^f, \tag{2}$$

where B is another constant and $\vec{\lambda}_{1,2}^f$ are the eight 3×3 Gell-Mann SU(3) flavor matrices for quarks number 1 and number 2. If we consider strictly pion exchange we should replace these 3×3 matrices by isospin matrices $\vec{\tau}_{1,2}$. For a pair of nonstrange quarks the difference between the two is small (see below). As noted already in the previous section,

if one is interested only in the mixing angles (and not in fitting mass spectra) the value of the constant B is immaterial, as we shall see. The main difference between Eq. (1) and Eq. (2) is the extra factor of $\vec{\lambda}_1 \cdot \vec{\lambda}_2$ in $H_{\rm OPE}$ and the coefficient of the contact term relative to the tensor term ($8\pi/3$ versus $-4\pi/3$, a factor of minus one-half in going from OGE to OPE). It is interesting to note that the coefficient of $(-4\pi/3)$ in the pion exchange case is the same as for the interaction of two electric dipoles [9]. For finite mass pions there are corrections to Eq. (2) that will be discussed elsewhere [15].

C. Negative parity eigenstates

The low mass negative parity baryons are assigned to a 70-plet of SU(6), which means that the spatial wave functions have mixed permutational symmetry (see [3,4,10]). In the notation we use [3], the spatial wave function ψ has two components ψ^λ and ψ^ρ which transform under permutations of the three quarks as a two-dimensional irreducible representation. The notation is explained, for example, in [10]. The total wave function Ψ is the sum of products of the spatial ψ, spin χ, and flavor ϕ wave functions. For spin 3/2, the spin wave function is totally symmetric under permutations while for spin 1/2 there are again two states of mixed symmetry χ^λ and χ^ρ. The flavor wave functions for $I=1/2$ also have mixed symmetry. Ignoring the color wave function (which is antisymmetric), the total wave function is totally symmetric under all permutations, and has the following forms:

$$S=3/2: \quad \Psi(^4P) = \frac{1}{\sqrt{2}} \chi^s \{\psi^\lambda \phi^\lambda + \psi^\rho \phi^\rho\}, \tag{3a}$$

$$S=1/2: \quad \Psi(^2P) = \frac{1}{2}\{\chi^\lambda \psi^\rho \phi^\rho + \chi^\rho \psi^\lambda \phi^\rho + \chi^\rho \psi^\rho \phi^\lambda - \chi^\lambda \psi^\lambda \phi^\lambda\}. \tag{3b}$$

III. COMPUTATIONS

The spin angular momentum $S=1/2,3/2$ has to be coupled with the orbital angular momentum $L=1$ to give the total angular momentum $J=L+S$. As a result there are two states each at $J=1/2$ and $J=3/2$, namely, spin doublet and spin quartet states: $^2P_{1/2}$, $^4P_{1/2}$ and $^2P_{3/2}$, $^4P_{3/2}$. The physical eigenstates are linear combinations of these two states, and can be obtained by diagonalizing the Hamiltonian ($H_{\rm OGE}$ or $H_{\rm OPE}$) in this space of states. For example, the $J^P=3/2^-$ states are eigenstates of the matrix

$$\begin{pmatrix} \langle ^4P_{3/2}|H|^4P_{3/2}\rangle & \langle ^4P_{3/2}|H|^2P_{3/2}\rangle \\ \langle ^2P_{3/2}|H|^4P_{3/2}\rangle & \langle ^2P_{3/2}|H|^2P_{3/2}\rangle \end{pmatrix}, \tag{4}$$

where H is either $H_{\rm OGE}$ or $H_{\rm OPE}$, which are given in Eq. (1) or Eq. (2) for a single pair of quarks. The total Hamiltonian sums over all three quark pairs, and since the wave functions in Eq. (3) are symmetric under all permutations, we can pick a single pair of quarks $H^{(12)}$ and multiply the result by 3. The computation of these matrix elements is simple but a little tedious. We illustrate the computation for the case of $J^P=3/2^-$ given above, successively for both gluon and pion exchange. Before we start, we comment that the doublet-doublet matrix elements receive contributions only from the contact terms, while the doublet-quartet matrix elements come only from the tensor terms. The quartet-quartet matrix element receives contributions from both tensor and contact terms. Thus the relative size of contact and tensor terms come into play. We find

$$\langle ^4P_{3/2}|H_{\rm OPE}|^4P_{3/2}\rangle = \left(\frac{3}{2}\right)\{\langle \chi^s\psi^\lambda|H^{12}|\chi^s\psi^\lambda\rangle\langle \phi^\lambda|\vec{\lambda}_1\cdot\vec{\lambda}_2|\phi^\lambda\rangle + \langle \chi^s\psi^\rho|H^{12}|\chi^s\psi^\rho\rangle\langle \phi^\rho|\vec{\lambda}_1\cdot\vec{\lambda}_2|\phi^\rho\rangle\}, \tag{5}$$

where the leading factor of 3/2 comes from the number of pairs and the normalization in Eq. (3). We further take note that

$$\langle \phi^\lambda|\vec{\lambda}_1\cdot\vec{\lambda}_2|\phi^\lambda\rangle = 4/3 \quad \text{and} \quad \langle \phi^\rho|\vec{\lambda}_1\cdot\vec{\lambda}_2|\phi^\rho\rangle = -8/3. \tag{6}$$

It is amusing that these flavor matrix elements have coefficients similar to the contact interaction in pion or gluon exchange, but this is a simple numerical coincidence. We also assume harmonic oscillator spatial wave functions $\psi^{\rho,\lambda}_{1M}$ in common with [3,6]. Then there is a further simplification: the contact term that contains the delta function $\delta^3(\vec{\rho})$ vanishes in the state ψ^ρ and receives a contribution only in the state ψ^λ. The tensor term survives only in ψ^ρ, which has unit orbital angular momentum $l_\rho=1$. As a result we can write

$$H_{\rm OGE}(^4P) = (3/2)\{\langle \chi^s\psi^\lambda|H^{12}_{\rm contact}|\chi^s\psi^\lambda\rangle + \langle \chi^s\psi^\rho|H^{12}_{\rm tensor}|\chi^s\psi^\rho\rangle\}, \tag{7}$$

where for one-gluon exchange

$$\langle \chi^s\psi^\lambda|H^{12}_{\rm contact}|\chi^s\psi^\lambda\rangle = A(8\pi/3)\langle \chi^s|\vec{S}_1\cdot\vec{S}_2|\chi^s\rangle \times \langle \psi^\lambda|\,\delta^3(\vec{\rho})\,|\psi^\lambda\rangle = (2/3)A\alpha^3\pi^{-1/2}. \tag{8}$$

Here α is an oscillator parameter; the corresponding tensor term is

$$\langle \chi^s\psi^\rho|H^{12}_{\rm tensor}|\chi^s\psi^\rho\rangle = (8/15)A\alpha^3\pi^{-1/2}. \tag{9}$$

Inserting Eqs. (8), (9) into Eq. (7) we obtain (in agreement with [3])

$$\langle ^4P_{3/2}|H_{\rm OGE}|^4P_{3/2}\rangle = (3/2)\{(2/3)+(8/15)\} = (9/5) \quad (\text{in units of } A\alpha^3\pi^{-1/2}). \tag{10}$$

Similarly, for pion exchange one obtains

$$\langle {}^4P_{3/2}|H_{\rm OPE}|{}^4P_{3/2}\rangle = (3/2)\{(-1/2)(2/3)(4/3)+(1)(8/15) \times(-8/3)\} = (-14/5) \quad (\text{in units of } B\alpha^3\pi^{-1/2}), \tag{11}$$

where the factor of $(-1/2)$ is the change in the contact term from Eq. (1) to Eq. (2). There are only two more matrix elements (for each of the pion and gluon exchanges), and they are

$$\langle {}^2P_{3/2}|H_{\rm OGE}|{}^4P_{3/2}\rangle = (10)^{-1/2}A\alpha^3\pi^{-1/2}, \tag{12a}$$

$$\langle {}^2P_{3/2}|H_{\rm OGE}|{}^2P_{3/2}\rangle = -A\alpha^3\pi^{-1/2}, \tag{12b}$$

$$\langle {}^2P_{3/2}|H_{\rm OPE}|{}^4P_{3/2}\rangle = (-8/3)(10)^{-1/2}B\alpha^3\pi^{-1/2}, \tag{12c}$$

$$\langle {}^2P_{3/2}|H_{\rm OPE}|{}^2P_{3/2}\rangle = (-7/3)B\alpha^3\pi^{-1/2}. \tag{12d}$$

With these matrix elements we find for OGE the Hamiltonian for $J=3/2$ to have the form

$$\begin{pmatrix} 9/5 & 10^{-1/2} \\ 10^{-1/2} & -1 \end{pmatrix} \begin{pmatrix} {}^4P_{3/2} \\ {}^2P_{3/2} \end{pmatrix}, \tag{13}$$

where we have omitted the common units $A\alpha^3\pi^{-1/2}$. We now find the mixing $\sin\theta_d \simeq (10^{-1/2})/(14/5) = 0.11$ corresponding to a mixing angle of $\theta_d = 6.3°$, in agreement with [3,4]. The definition we follow has the lowest energy state ("$E_{\rm low} = -1.035$") with composition $|E_{\rm low}\rangle = -\sin\theta_d|{}^4P_{3/2}\rangle + \cos\theta_d|{}^2P_{3/2}\rangle$. This means that the lowest eigenstate of the matrix above is $|J^P=3/2^-;{\rm OGE}\rangle = -0.110|{}^4P_{3/2}\rangle + 0.994|{}^2P_{3/2}\rangle$. We emphasize that this mixing is the same for all possible values of the constant $A\alpha^3\pi^{-1/2}$, whether they fit the masses or not. Similarly for $H_{\rm OPE}$ we have to diagonalize the matrix

$$\begin{pmatrix} -14/5 & -(8/3)10^{-1/2} \\ -(8/3)10^{-1/2} & -7/3 \end{pmatrix} \begin{pmatrix} {}^4P_{3/2} \\ {}^2P_{3/2} \end{pmatrix}. \tag{14}$$

With this matrix, the mixing angle θ_d is found to be $\theta_d = -52.7°$. This means that the lowest eigenstate of $H_{\rm OPE}$ has the composition $|J^P=3/2^-;{\rm OPE}\rangle = 0.796|{}^4P_{3/2}\rangle + 0.606|{}^2P_{3/2}\rangle$. This is very different from the composition of the state with OGE coupling, given above. Whereas with OPE coupling the lowest $3/2^-$ state is about 63% spin quartet, with OGE it is about 1% spin quartet. The decay data favor a 1% contamination [11]. Furthermore, [6,7] quote a mixing angle $\theta_d = \pm 8°$ for OPE which differs substantially from $-53°$. Note that had we used the coupling $\vec{\tau}_1\cdot\vec{\tau}_2$ instead of $\vec{\lambda}_1\cdot\vec{\lambda}_2$ the OPE composition would change slightly to $0.78|{}^4P_{3/2}\rangle + 0.63|{}^2P_{3/2}\rangle$.

We give now briefly the corresponding numbers in the $J^P=1/2^-$ sector, referring to the lowest energy states in units of $A\alpha^3\pi^{-1/2}$ (for OGE) and $B\alpha^3\pi^{-1/2}$ (for OPE):

$$\text{OGE:} \quad |E=-1.62\rangle = 0.526|{}^4P_{1/2}\rangle + 0.85|{}^2P_{1/2}\rangle, \qquad \theta_s = -32°, \tag{15}$$

$$\text{OPE} \quad |E=-3.60\rangle = -0.43|{}^4P_{1/2}\rangle + 0.903|{}^2P_{1/2}\rangle, \qquad \theta_s = +25.5°. \tag{16}$$

References [6] and [7] quote a mixing angle $\theta_s = \pm 13°$. The data [11] support a composition close to OGE and a mixing angle of $-32°$. For reference we quote the OPE Hamiltonian in the $1/2^-$ sector in matrix form (in units of $B\alpha^3\pi^{-1/2}$):

$$\begin{pmatrix} 2 & 8/3 \\ 8/3 & -7/3 \end{pmatrix} \begin{pmatrix} {}^4P_{1/2} \\ {}^2P_{1/2} \end{pmatrix}. \tag{17}$$

Note that for OPE coupling the lowest lying state is predominantly (81%) spin doublet, while for OGE coupling the ground state is 72% spin doublet.

IV. SUMMARY AND DISCUSSION

In addition to the spin-spin couplings of Eqs. (1) or (2) discussed above, spin in the negative parity baryons also couples to the orbital angular momentum. This is the spin orbit coupling $(\vec{L}\cdot\vec{S})$. It is an empirical observation that this coupling is rather weak in negative parity nucleons, and as a result it has been neglected in some of the literature [3,4,6]. There is a great deal of discussion about the physical origin of this effect [7,8]. If some spin orbit coupling is included it will contribute to the diagonal matrix elements of the 2×2 matrices that we diagonalize and will shift the mixing angles. The inclusion of this effect, however, will negate a parameter-free determination of the mixing angles. That is, one must use the spectroscopic mass data in order to find the relative strengths of the hyperfine interaction [Eqs. (1), (2)] and the spin orbit interaction. This we have done in a preliminary manner, and we find the changes to the mixing angles to be small—less than the experimental error of 10°. The spectroscopic data utilized were the nucleon–delta (P_{33} resonance) and the D_{13} (low)–D_{15} mass splitting. With these splittings, we find that the OGE mixing angle changes from $-32°$ to $-36°$, while for OPE the angle changes from 25.5° to 27.5° (both for the $J^P=1/2^-$ sector, with similar results holding for $J^P=3/2^-$). For more complete results, one should attempt a reasonable fit to all states in the multiplet and this has not yet been done.

We summarize in Table I the results quoted in Sec. III. We emphasize again that these results are independent of spectral fits to the masses of these states. The results depend only on the couplings and the wave functions assumed. The strength of the coupling (either A or B here) factorizes from the mixing matrices and the same mixing is obtained regardless of this coupling strength. The wave functions were assumed to be of harmonic oscillator type—appropriate for these states since they are the ground states in the negative parity sector. Moreover, this assumption is common to [3,4] and [6]. We further assumed in the couplings for OPE that the pion mass is zero; results for nonzero pion masses will be given elsewhere. They do not change significantly the numbers given in Table I.

Table I shows a substantial change from the values for OPE in the literature [6,7]; this change is relevant since the

TABLE I. Summary of results.

	Coupling	Reference	Mixing angle	% ^4P_j
$J^P=3/2^-$	OPE	[6,7]	$\pm 8°$	2%
	OPE	This paper	$-52.7°$	63%
	OGE	[3,4] and this paper	$+6°$	1%
	Expt.	[11]	$+10°$	3%
$J^P=1/2^-$	OPE	[6,7]	$\pm 13°$	5%
	OPE	This paper	$+25.5°$	19%
	OGE	[3,4] and this paper	$-32°$	28%
	Expt.	[11]	$-32°$	28%

error of the ''experimental'' value is of the order of 10° [11], and the preference for the OGE solution is now unambiguous. It has been argued [8] that the addition of vector meson exchange to pseudoscalar exchange will remedy this problem. That may indeed be the case, but one should recall that the primary controversy is whether the quark coupling in baryons is by OGE or OPE, and the data answer this question unequivocally. One may just as well argue that the atomic hyperfine interaction—which has the same form as Eq. (1)—is really due to the superposition of a pseudoscalar and massive vector field, rather than a massless gauge field. Similar mixings for OPE have also been obtained elsewhere [12]; however, the emphasis on the independence of spectral data is new.

There have been comments in the literature on the issue of color versus flavor exchange. In particular, [13] fits the mass spectrum in the $L=1$ sector in a rather ingenious way, using only permutation symmetry and SU(6), with a number of free parameters, essentially reduced matrix elements. But by treating the matrix elements corresponding to the contact and tensor terms as independent parameters, one sidesteps the controversy between vector exchange [1,3,4] and pseudoscalar exchange [6]. In addition, as noted, if one has well defined Hamiltonians and wave functions these mixings are independent of mass fits. Although not in these precise words, similar conclusions are stated by the authors of [13].

Finally, one may argue [14] that both Hamiltonians (1) and (2) contain delta functions and therefore would be illegal in a larger space of functions when one might wish to compute all possible states of the system—since the delta function leads to a collapse. This criticism is irrelevant, as we do not compute anything other than the effect of these Hamiltonians in the subspace of first excited P-wave states. This is very similar to the idea of the Fermi pseudopotential in condensed matter physics, which is not meant to be a fundamental interaction, but should be considered only in lowest order. All we are interested in here is the mixing in this multiplet, and we find that experiment prefers the so-called OGE solution. A similar point of view is taken in atomic hyperfine interactions.

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