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Phenomenology of Lorentz-conserving noncommutative QED

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Recently a version of Lorentz-conserving noncommutative field theory (NCFT) has been suggested. The underlying Lie algebra of the theory is the same as that of Doplicher, Fredenhagen, and Roberts. In Lorentz-conserving NCFT the matrix parameter $\theta^{\mu\nu}$ which characterizes the canonical NCFT's is promoted to an operator $\hat{\theta}^{\mu\nu}$ that transforms as a Lorentz tensor. In this paper, we calculate the phenomenological consequences of the QED version of this theory by looking at various collider processes. In particular we calculate modifications to Mϕller scattering, Bhabha scattering, $e^+e^-\to\mu^+\mu^-$ and $e^+e^-\to\gamma\gamma$. We obtain bounds on the noncommutativity scale from the existing experiments at CERN LEP and make predictions for what may be seen in future collider experiments.

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I. INTRODUCTION

It is interesting to consider the possibility that the structure of spacetime is nontrivial. In one of the most popular scenarios position four-vectors are promoted to operators that do not commute at short distance scales [1–25]. There has been a lot of work on field theories with an underlying noncommutative spacetime structure. Jurčo *et al.* [6] have presented a formalism on how to construct non-Abelian gauge theories in noncommutative spaces from a consistency relation. Using a similar approach Carlson, Carone and Zobin (CCZ) [22] have formulated noncommutative Lorentz-conserving QED based on a contracted Snyder [25] algebra, thus offering a general prescription as how to formulate noncommutative Lorentz-conserving gauge theories. In this algebra the self-adjoint spacetime coordinate operators satisfy the following commutation relation:

$$[\hat{x}^\mu,\hat{x}^\nu]=i\hat{\theta}^{\mu\nu}. \tag{1.1}$$

Here $\hat{\theta}^{\mu\nu}=-\hat{\theta}^{\nu\mu}$ transforms as a Lorentz tensor and is in the same algebra with $\hat{x}^\mu$. This algebra is Lorentz covariant.

The Lie algebra considered by CCZ is the same as the Lie algebra of Doplicher, Fredenhagen, and Roberts (DFR) [24]. Interestingly enough DFR came to the formulation of their algebra by considering modifications of spacetime structure in theories that are designed to quantize gravity. The DFR algebra places limitations on the precision of localization in spacetime. As noted in [24], quantum spacetime can be regarded as a novel underlying geometry for a quantum field theory of gravity.

Interest in noncommutative spacetime originated with the work of Connes and collaborators [26] and has gained more attention due to developments in string theory [27], where noncommutative spacetime has been shown to arise in a low energy limit. In string theories $\theta^{\mu\nu}$ is just an antisymmetric c-number. Theories involving noncommutative spacetime structure based on algebras with c-number $\theta^{\mu\nu}$ suffer from Lorentz-violating effects. Such effects are severely constrained [9–17] by a variety of low energy experiments [28]. Lorentz-violating effects appear in field theories as a consequence of θ^{0i} and $\epsilon^{ijk}\theta^{ij}$ defining preferred direction in a given Lorentz frame. In contrast with this the noncommutative QED (NCQED) formulated by CCZ based on Eq. (1.1) is free from Lorentz-violating effects.

Carlson, Carone and Zobin have connected the DFR Lie algebra Eq. (1.1), and the antisymmetric tensor $\hat{\theta}^{\mu\nu}$ to experimental observables, by showing how to formulate a quantum field theory on this noncommutative spacetime. Similar issues have been discussed by Morita *et al.* [23]. These theories make it possible to study phenomenological consequences of Lorentz-conserving noncommutative spacetime. As a beginning, CCZ have studied light-by-light elastic scattering and obtained contributions that can be significant with respect to the standard model background.

In this paper we calculate other phenomenological consequences of Lorentz-conserving NCQED formulated by CCZ. We consider various collider processes such as Bhabha and Mϕller scattering, $e^+e^-\to\mu^+\mu^-$ and $e^+e^-\to\gamma\gamma$. The experiments at planned colliders will provide a means of testing the properties and the structure of spacetime at smaller distance scales. We note that any property prescribed to spacetime, if confirmed experimentally, must affect all interactions.

In the following section we discuss the underlying formalism of noncommutative Lorentz-conserving gauge theories, with emphasis on NCQED. In Sec. III we study the Lorentz-conserving NCQED by considering various collider processes. In Sec. IV we obtain bounds on the noncommutativity scale from Bhabha scattering, $e^+e^-\to\mu^+\mu^-$ and $e^+e^-\to\gamma\gamma$ experiments. We summarize our discussion in Sec. V with some concluding remarks.

II. ALGEBRA AND QED FORMULATION

The simplest construction of a Lorentz-conserving noncommutative theory involves promoting the position four-vector to an operator which satisfies the DFR Lie algebra

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$$[\hat{x}^\mu,\hat{x}^\nu]=i\,\hat{\theta}^{\mu\nu},$$

$$[\hat{\theta}^{\mu\nu},\hat{x}^\lambda]=0,$$

$$[\hat{\theta}^{\mu\nu},\hat{\theta}^{\alpha\beta}]=0, \tag{2.1}$$

where $\theta^{\mu\nu}$ is antisymmetric and transforms as a Lorentz tensor.

On the other hand, CCZ took as the starting point Snyder's algebra,

$$[\hat{x}^\mu,\hat{x}^\nu]=ia^2\hat{M}^{\mu\nu},$$

$$[\hat{M}^{\mu\nu},\hat{x}^\lambda]=i(\hat{x}^\mu g^{\nu\lambda}-\hat{x}^\nu g^{\mu\lambda}),$$

$$[\hat{M}^{\mu\nu},\hat{M}^{\alpha\beta}]=i(\hat{M}^{\mu\beta}g^{\nu\alpha}+\hat{M}^{\nu\alpha}g^{\mu\beta}-\hat{M}^{\mu\alpha}g^{\nu\beta}-\hat{M}^{\nu\beta}g^{\mu\alpha}). \tag{2.2}$$

Snyder's algebra [which is the same as the algebra of SO(4,1)] describes a Lorentz-invariant noncommutative discrete spacetime characterized by a fundamental length scale a. By constructing an explicit representation for $\hat{x}$ and $\hat{M}$ in terms of differential operators, the Lorentz invariance of Eq. (2.2) was demonstrated [25]. CCZ then extracted the DFR Lie algebra by performing a particular contraction on Eq. (2.2). Specifically, by rescaling $M^{\mu\nu}=\hat{\theta}^{\mu\nu}/b$ and holding the ratio $a^2/b=1$ fixed, the limit $b\to 0$, $a\to 0$ yields the DFR Lie algebra. Thus, the Lorentz covariance of Snyder's Lie algebra implies the Lorentz covariance of Eq. (2.1) [22]. The commutator of $\hat{\theta}^{\mu\nu}$ and $\hat{M}^{\mu\nu}$ is

$$[\hat{M}^{\mu\nu},\hat{\theta}^{\alpha\beta}]=i(\hat{\theta}^{\mu\beta}g^{\nu\alpha}+\hat{\theta}^{\nu\alpha}g^{\mu\beta}-\hat{\theta}^{\mu\alpha}g^{\nu\beta}-\hat{\theta}^{\nu\beta}g^{\mu\alpha}), \tag{2.3}$$

as one would expect if $\hat{\theta}^{\mu\nu}$ is a Lorentz tensor. Note that the contraction also implies that the eigenvalues of the position operator of the DFR algebra are continuous.

To develop a field theory on a noncommutative spacetime, one defines a one-to-one mapping which associates functions of the noncommuting coordinates with functions of the typical c-number coordinates. In the canonical noncommutative theory this is achieved via a Fourier transform

$$\hat{f}(\hat{x})=\frac{1}{2\pi^n}\int d^nke^{-ik\hat{x}}\int d^nxe^{ikx}f(x). \tag{2.4}$$

In the Lorentz-conserving case the presence of the operator $\hat{\theta}^{\mu\nu}$ requires that the mapping involve a new c-number coordinate $\theta^{\mu\nu}$ (no hat). Functions of the noncommuting coordinates are then related to functions of c-number coordinates by

$$\hat{f}(\hat{x},\hat{\theta})=\int \frac{d^4\alpha}{(2\pi)^4}\frac{d^6B}{(2\pi)^6}\times e^{-i[\alpha_\mu\hat{x}^\mu+(B_{\mu\nu}\hat{\theta}^{\mu\nu}/2)]}\tilde{f}(\alpha,B), \tag{2.5}$$

where

$$\tilde{f}(\alpha,B)=\int d^4xd^6\theta e^{i[\alpha_\mu x^\mu+(B_{\mu\nu}\theta^{\mu\nu}/2)]}f(x,\theta). \tag{2.6}$$

Lorentz invariance requires that B transform as a two index Lorentz tensor.

To ensure that operator multiplication be preserved, $\hat{f}\hat{g}=\widehat{f\star g}$, one finds that the rule for ordinary multiplication must be modified:

$$(f\star g)(x,\theta)=f(x,\theta)\exp\left[\frac{i}{2}\overleftarrow{\partial}_\mu\theta^{\mu\nu}\overrightarrow{\partial}_\nu\right]g(x,\theta). \tag{2.7}$$

The θ dependence of the functions distinguishes this result from the $\star$-product of the canonical noncommutative theory. Equations (2.5) and (2.6) allow one to work solely with functions of classical coordinates x and θ, provided that all multiplication be promoted to a $\star$-product.

The introduction of a Lorentz invariant weighting function $W(\theta)$ allows for the following generalization of the operator trace:

$$\mathrm{Tr}\,\hat{f}=\int d^4xd^6\theta W(\theta)f(x,\theta). \tag{2.8}$$

In [22] CCZ took the normalization to be

$$\int d^6\theta W(\theta)=1. \tag{2.9}$$

It is straightforward to demonstrate the cyclic property of Eq. (2.8), i.e. $\mathrm{Tr}\,\hat{f}\hat{g}=\mathrm{Tr}\,\hat{g}\hat{f}$. One requires that for large $|\theta^{\mu\nu}|$, $W(\theta)$ dies off sufficiently fast in order that all integrals be well defined [22]. Lorentz-invariance requires that W be an even function of θ, which yields

$$\int d^6\theta W(\theta)\theta^{\mu\nu}=0. \tag{2.10}$$

As will be seen, this restriction has interesting consequences on possible collider signatures of the theory.

Field theory interactions are extracted by performing the $d^6\theta$ integral, resulting in the action

$$\mathcal{S}=\mathrm{Tr}\,\hat{\mathcal{L}}=\int d^4xd^6\theta W(\theta)\mathcal{L}(\phi,\partial\phi)_\star, \tag{2.11}$$

where the notation in $\mathcal{L}(\phi,\partial\phi)_\star$ indicates $\star$-product multiplication.

As was mentioned, in the Lorentz-conserving noncommutative theory the initial ''fields'' are generally functions of x and θ, and must be related to ordinary quantum fields which are only functions of x. CCZ showed how this can be done for NCQED using a nonlinear field redefinition and an expansion in θ. Since the phenomenology of NCQED is the topic of this paper, all developments will be directed toward a U(1) gauge theory. For completeness the formalism presented in [22] is reviewed.

In Lorentz-conserving NCQED, one has a matter field ψ and gauge field A. For a U(1) gauge transformation characterized by a parameter $\Lambda(x,\theta)$, the fields transform as

$$\psi(x,\theta)\rightarrow U\star\psi(x,\theta), \tag{2.12}$$

and

$$A_\mu(x,\theta)\rightarrow U\star A_\mu(x,\theta)\star U^{-1}+\frac{i}{e}U\star\partial_\mu U^{-1}, \tag{2.13}$$

where

$$U=(e^{i\Lambda})_\star =1+i\Lambda(x,\theta)+\frac{1}{2!}i\Lambda(x,\theta)\star i\Lambda(x,\theta)+\cdots. \tag{2.14}$$

A U(1) gauge invariant Lagrangian is

$$\mathcal{L}=\int d^6\theta W(\theta)\left[-\frac{1}{4}F_{\mu\nu}\star F^{\mu\nu}+\bar{\psi}\star(i\not{D}-m)\star\psi\right], \tag{2.15}$$

where

$$D_\mu=\partial_\mu-ieA_\mu, \tag{2.16}$$

and the field strength is

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-ie[A_\mu \overset{\star}{,} A_\nu]. \tag{2.17}$$

In demonstrating the gauge invariance of Eq. (2.15) and the cyclic property of Eq. (2.8), the following identity is useful:

$$\int d^4x f\star g=\int d^4x fg. \tag{2.18}$$

Equations (2.15), (2.16), and (2.17) are similar in form to those obtained in the canonical NCQED case, the difference again being the θ dependence of the fields $\psi(x,\theta)$ and $A(x,\theta)$ in Eq. (2.15). One must have a way of relating ψ and A to ordinary quantum fields which are only functions of x. This is accomplished by utilizing the behavior of the weighting function Eq. (2.8), which allows an expansion of the fields and gauge parameter in powers of θ. A similar technique involving field expansions was first used in constructing a noncommutative SU(N) gauge theory in [6]. The coefficients of the power series are thus only functions of x and correspond to ordinary quantum fields. From requirements of gauge invariance and noncommutativity, these coefficients can be determined order by order in θ.

The gauge parameter, gauge field, and matter field of NCQED are expanded as

$$\Lambda_\alpha(x,\theta)=\alpha(x)+\theta^{\mu\nu}\Lambda^{(1)}_{\mu\nu}(x;\alpha)+\theta^{\mu\nu}\theta^{\eta\sigma}\Lambda^{(2)}_{\mu\nu\eta\sigma}(x;\alpha)+\cdots, \tag{2.19}$$

$$A_\rho(x,\theta)=A_\rho(x)+\theta^{\mu\nu}A^{(1)}_{\mu\nu\rho}(x)+\theta^{\mu\nu}\theta^{\eta\sigma}A^{(2)}_{\mu\nu\eta\sigma\rho}(x)+\cdots, \tag{2.20}$$

$$\psi(x,\theta)=\psi(x)+\theta^{\mu\nu}\psi^{(1)}_{\mu\nu}+\theta^{\mu\nu}\theta^{\eta\sigma}\psi^{(2)}_{\mu\nu\eta\sigma}(x)+\cdots. \tag{2.21}$$

The lowest order term in each expansion corresponds to the ordinary QED term. Thus, ordinary QED can be extracted by taking the commutative limit, $\theta^{\mu\nu}\rightarrow 0$.

Consider an infinitesimal transformation of a matter field $\psi(x)$ in an ordinary U(1) gauge theory:

$$\delta_\alpha\psi(x)=i\alpha(x)\psi(x). \tag{2.22}$$

For a Lorentz-conserving noncommutative theory, this is generalized to

$$\delta_\alpha\psi(x,\theta)=i\Lambda_\alpha(x,\theta)\star\psi(x,\theta). \tag{2.23}$$

In an Abelian gauge theory two successive gauge transformations must then satisfy the relation

$$(\delta_\alpha\delta_\beta-\delta_\beta\delta_\alpha)\psi(x,\theta)=0. \tag{2.24}$$

For Eq. (2.24) to hold, Λ must satisfy

$$i\delta_\alpha\Lambda_\beta-i\delta_\beta\Lambda_\alpha+[\Lambda_\alpha \overset{\star}{,} \Lambda_\beta]=0. \tag{2.25}$$

The parameter Λ can then be determined at each order in θ. Specifically, it can be shown that

$$\Lambda^{(1)}_{\mu\nu}(x;\alpha)=\frac{e}{2}\partial_\mu\alpha(x)A_\nu(x) \tag{2.26}$$

and

$$\Lambda^{(2)}_{\mu\nu\eta\sigma}(x;\alpha)=-\frac{e^2}{2}\partial_\mu\alpha(x)A_\eta(x)\partial_\sigma A_\nu(x) \tag{2.27}$$

satisfy the condition of Eq. (2.25). The gauge and matter fields are treated in a similar manner.

The restriction of a gauge field transforming infinitesimally as

$$\delta_\alpha A_\sigma=\partial_\sigma\Lambda_\alpha+i[\Lambda_\alpha \overset{\star}{,} A_\sigma], \tag{2.28}$$

is satisfied by the following expressions for $A^{(1)}$ and $A^{(2)}$:

$$A^{(1)}_{\mu\nu\rho}(x)=-\frac{e}{2}A_\mu(\partial_\nu A_\rho+F^0_{\nu\rho}), \tag{2.29}$$

$$A^{(2)}_{\mu\nu\eta\sigma\rho}(x)=\frac{e^2}{2}(A_\mu A_\eta\partial_\sigma F^0_{\nu\rho}-\partial_\nu A_\rho\partial_\eta A_\mu A_\sigma+A_\mu F^0_{\nu\eta}F^0_{\sigma\rho}), \tag{2.30}$$

where

$$F^0_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \tag{2.31}$$

is the ordinary QED field strength tensor.

Likewise, one can show that for a matter field transforming infinitesimally as Eq. (2.23), the appropriate forms of $\psi^{(1)}$ and $\psi^{(2)}$ are

$$\psi^{(1)}_{\mu\nu}(x) = -\frac{e}{2}A_\mu\partial_\nu\psi \tag{2.32}$$

and

$$\psi^{(2)}_{\mu\nu\eta\sigma}(x) = \frac{e}{8}\Big(-i\partial_\mu A_\eta\partial_\nu\partial_\sigma\psi + eA_\mu A_\eta\partial_\nu\partial_\sigma\psi + 2eA_\mu\partial_\nu A_\eta\partial_\sigma\psi + eA_\mu F^0_{\nu\eta}\partial_\sigma\psi - \frac{e}{2}\partial_\mu A_\eta\partial_\nu A_\sigma\psi + ie^2 A_\mu A_\sigma\partial_\eta A_\nu\psi\Big). \tag{2.33}$$

Interactions are extracted by substituting Eqs. (2.26), (2.27), (2.29), (2.30), (2.32), (2.33) into the Lagrangian Eq. (2.15). We expand the Lagrangian through θ^2 and evaluate the $d^6\theta$ integral using the weighted average

$$\int d^6\theta W(\theta)\theta^{\mu\nu}\theta^{\eta\rho} = \frac{\langle\theta^2\rangle}{12}(g^{\mu\eta}g^{\nu\rho} - g^{\mu\rho}g^{\eta\nu}), \tag{2.34}$$

where the expectation value is defined as

$$\langle\theta^2\rangle \equiv \int d^6\theta W(\theta)\theta_{\mu\nu}\theta^{\mu\nu}. \tag{2.35}$$

It is natural to define $\Lambda_{NC} = (12/\langle\theta^2\rangle)^{1/4}$ which characterizes the energy scale where noncommutative effects become relevant. The restriction on W from Eq. (2.10) demands that only terms containing even powers of θ will result in interaction vertices. Thus, for example, the three-photon vertex of canonical NCQED is not present. The next section focuses on the phenomenology of a U(1) theory whose spacetime coordinate operators obey the DFR Lie algebra. Possible collider signatures are considered and bounds on the energy scale Λ_{NC} are obtained.

III. COLLIDER SIGNATURES

The Lagrangian for QED with Lorentz-invariant noncommutative spacetime Eq. (2.15) can be written as an expansion in θ order by order using the nonlinear field redefinition described above. The zeroth order in θ will give the ordinary QED Lagrangian. The first order is zero due to the evenness of the weighting function $W(\theta)$. The first nontrivial contributions come from the second order; they include the following.

(i) The 4-photon vertex, which has been discussed extensively in [22].

(ii) The correction to 2-fermion-1-photon vertex (ordinary QED vertex).

(iii) The 2-fermion-2-photon vertex.

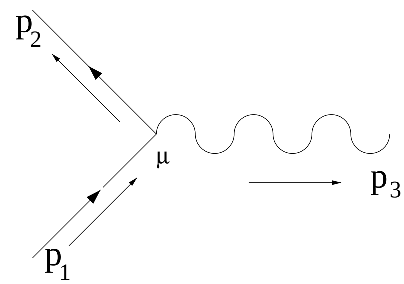

FIG. 1. 2-fermions-1-photon vertex.

The lowest order correction to the ordinary QED vertex comes from the following terms in Lagrangian density:

$$\overline{\psi}^{(2)}(i\partial\!\!\!/ - m)\psi^{(0)} + \overline{\psi}^{(0)}(i\partial\!\!\!/ - m)\psi^{(2)} + \frac{e}{2}\{(\overline{\psi}^{(0)}\star A\!\!\!/^{(0)})\psi^{(0)} + \overline{\psi}^{(0)}(A\!\!\!/^{(0)}\star\psi^{(0)})\}, \tag{3.1}$$

where we retain only the second order term in contributions to the $\star$-product shown in the last two terms. The first two terms will go to zero if both fermion fields are on shell. And the 2-fermion-2-photon vertex comes from

$$\overline{\psi}^{(2)}(i\partial\!\!\!/ - m)\psi^{(0)} + \overline{\psi}^{(0)}(i\partial\!\!\!/ - m)\psi^{(2)} + \overline{\psi}^{(1)}(i\partial\!\!\!/ - m)\psi^{(1)} + e\{\overline{\psi}^{(2)}A\!\!\!/^{(0)}\psi^{(0)} + \overline{\psi}^{(0)}A\!\!\!/^{(0)}\psi^{(2)}\} + e\{(\overline{\psi}^{(0)}\star A\!\!\!/^{(0)})\psi^{(1)} + \overline{\psi}^{(1)}(A\!\!\!/^{(0)}\star\psi^{(0)}) + (\overline{\psi}^{(0)}\star A\!\!\!/^{(1)})\psi^{(0)}\}, \tag{3.2}$$

where this time we retain only the first order in the $\star$-product shown.

A. Dilepton production, $e^+e^-\to l^+l^-$

First we consider processes in which all fermions are on shell, i.e. dilepton production $e^+e^-\to l^+l^-$. For processes up to tree level Feynman diagram, only

$$\frac{e}{2}\{(\overline{\psi}^{(0)}\star A\!\!\!/^{(0)})\psi^{(0)} + \overline{\psi}^{(0)}(A\!\!\!/^{(0)}\star\psi^{(0)})\}$$

will contribute to the vertex correction since all the fermions are on shell. This Lagrangian term reduces to

$$\frac{e}{2}\frac{\langle\theta^2\rangle}{96}\{\overline{\psi}(\partial_\mu\partial_\nu A\!\!\!/)(\partial^\mu\partial^\nu\psi) + (\partial^\mu\partial^\nu\overline{\psi})(\partial_\mu\partial_\nu A\!\!\!/)\psi\}. \tag{3.3}$$

From this we obtain the following Feynman rule for the 2-fermion-1-photon vertex with all fermions on shell and with momenta labeled as in Fig. 1:

$$ie\left\{1 + \frac{\langle\theta^2\rangle}{384}(p_3)^4\right\}\gamma^\mu, \tag{3.4}$$

where we have not made the assumption that the fermions are massless (although we do set $m=0$ in the cross section formula).

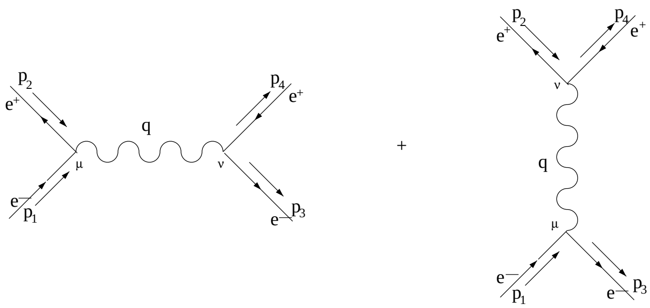

FIG. 2. Bhabha scattering.

We will consider the following processes which are affected by this vertex correction: Bhabha scattering, $e^+e^-\rightarrow\mu^+\mu^-$ and Møller scattering. The matrix element with vertex correction for Bhabha scattering (Fig. 2) is

$$\begin{aligned} i\mathcal{M}=&\overline{u(}p_3)(ie\gamma^\nu)\left(1+\frac{\langle\theta^2\rangle}{384}q^4\right)v(p_4)\frac{-ig_{\mu\nu}}{q^2+i\epsilon}\\ &\times\overline{v(}p_2)(ie\gamma^\mu)\left(1+\frac{\langle\theta^2\rangle}{384}q^4\right)u(p_1)\\ &-\overline{v(}p_2)(ie\gamma^\nu)\left(1+\frac{\langle\theta^2\rangle}{384}q'^4\right)v(p_4)\frac{-ig_{\mu\nu}}{q'^2+i\epsilon}\\ &\times\overline{u(}p_3)(ie\gamma^\mu)\left(1+\frac{\langle\theta^2\rangle}{384}q'^4\right)u(p_1). \end{aligned}\tag{3.5}$$

Squaring the matrix element and summing (averaging) over the final (initial) fermion spin states will give

$$\overline{|\mathcal{M}|^2}=2e^4\left\{F_s^2\left(\frac{t^2+u^2}{s^2}\right)+2F_sF_t\frac{u^2}{st}+F_t^2\left(\frac{u^2+s^2}{t^2}\right)\right\},\tag{3.6}$$

where we define $F_s=\{1+(\langle\theta^2\rangle/96)(s^2/4)\}^2$ where s, t and u are the Mandelstam variables. To first order in $\langle\theta^2\rangle/12$ this will give us the center of mass (CM) differential cross section:

$$\frac{d\sigma}{d\cos\theta}=\left(\frac{d\sigma}{d\cos\theta}\right)_{QED}+\frac{\pi\alpha^2}{s}\frac{\langle\theta^2\rangle}{96}\left\{s^2+t^2+2u^2+u^2\left(\frac{t}{s}+\frac{s}{t}\right)\right\},\tag{3.7}$$

where θ is the CM scattering angle.

The same results for $e^+e^-\rightarrow\mu^+\mu^-$ can be obtained easily by just throwing away the t channel in the Bhabha scattering calculation, assuming the muons are massless. The spin average square matrix element is

$$\overline{|\mathcal{M}|^2}=2e^4F_s^2\left(\frac{t^2+u^2}{s^2}\right).\tag{3.8}$$

And to first order in $\langle\theta^2\rangle/12$ this will give us

$$\frac{d\sigma}{d\cos\theta}=\left(\frac{d\sigma}{d\cos\theta}\right)_{QED}\left(1+\frac{\langle\theta^2\rangle}{96}s^2\right).\tag{3.9}$$

B. Møller scattering

For Møller scattering, the spin average square matrix element is obtained by using crossing symmetry from Bhabha scattering,

$$\overline{|\mathcal{M}|^2}=2e^4\left\{F_t^2\left(\frac{u^2+s^2}{t^2}\right)+2F_tF_u\frac{s^2}{tu}+F_u^2\left(\frac{s^2+t^2}{u^2}\right)\right\}.\tag{3.10}$$

To first order in $\langle\theta^2\rangle/12$ this gives us the CM differential cross section:

$$\frac{d\sigma}{d\cos\theta}=\left(\frac{d\sigma}{d\cos\theta}\right)_{QED}+\frac{\pi\alpha^2}{s}\frac{\langle\theta^2\rangle}{96}\left\{t^2+u^2+2s^2+s^2\left(\frac{u}{t}+\frac{t}{u}\right)\right\}.\tag{3.11}$$

C. Diphoton production, $e^+e^-\rightarrow\gamma\gamma$

In order to calculate the cross section for $e^+e^-\rightarrow\gamma\gamma$, we first need to calculate the full correction to ordinary QED vertex, not just the case when all fermions are on shell. This requirement comes from the fact that in diphoton production we have fermion propagators in the Feynman diagrams. By using the nonlinear field redefinition for $\psi^{(2)}$, the Lagrangian for the full correction can be written as

$$\begin{aligned} ie\frac{\langle\theta^2\rangle}{96}\Big[&(\partial_\mu A^\mu)((\partial^2\overline{\psi})(i\partial\!\!\!/-m)\psi+\{(i\partial_\alpha+m)\overline{\psi}\}\gamma^\alpha(\partial^2\psi))\\ &-(\partial_\mu A_\nu)((\partial^\mu\partial^\nu\overline{\psi})(i\partial\!\!\!/-m)\psi+\{(i\partial_\alpha+m)\overline{\psi}\}\gamma^\alpha(\partial^\mu\partial^\nu\psi))\\ &-\frac{i}{2}\{\overline{\psi}(\partial_\mu\partial_\nu A\!\!\!/)(\partial^\mu\partial^\nu\psi)+(\partial^\mu\partial^\nu\overline{\psi})(\partial_\mu\partial_\nu A\!\!\!/)\psi\}\Big]. \end{aligned}\tag{3.12}$$

Then the Feynman rule for the 2-fermion-1-photon vertex with all fermions and photons possibly off-shell is (Fig. 1)

$$\begin{aligned} ie\Big\{\gamma^\mu+\frac{\langle\theta^2\rangle}{96}\Big[&(p\!\!\!/_1-m)p_2^2p_3^\mu-(p\!\!\!/_2-m)p_1^2p_3^\mu+(p\!\!\!/_2-m)\\ &\times(p_1.p_3)p_1^\mu-(p\!\!\!/_1-m)(p_2.p_3)p_2^\mu+\frac{1}{2}\{(p_1.p_3)^2\\ &+(p_2.p_3)^2\}\gamma^\mu\Big]\Big\}. \end{aligned}\tag{3.13}$$

Next we need to calculate the contribution from the new vertex, i.e., 2-fermion-2-photon vertex. The Lagrangian for this vertex is

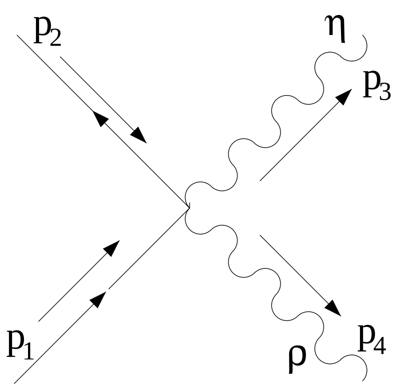

FIG. 3. Two fermions–two photon vertex.

$$ie^2\frac{\langle\theta^2\rangle}{96}[A_\mu(\partial_\alpha A_\nu)\{(\partial^\mu\bar{\psi})\gamma^\alpha(\partial^\nu\psi)-(\partial^\nu\bar{\psi})\gamma^\alpha(\partial^\mu\psi)\}$$
$$-(\partial_\mu A_\nu)\{(\partial^\mu\partial^\nu\bar{\psi})\not{A}\psi-\bar{\psi}\not{A}(\partial^\mu\partial^\nu\psi)\}$$
$$+2A_\mu F_{\nu\alpha}\{(\partial^\mu\bar{\psi})\gamma^\alpha(\partial^\nu\psi)-(\partial^\nu\bar{\psi})\gamma^\alpha(\partial^\mu\psi)\}],\tag{3.14}$$

and we put all the fermions and photons on shell to simplify the calculation. This simplification is possible since in the calculation for diphoton production up to second order in θ for the 2-fermion-2-photon vertex all fermions and photons are on shell. Labeling momenta as in Fig. 3, we obtain the Feynman rule for the 2-fermion-2-photon vertex with all fermions and photons on shell:

$$ie^2\frac{\langle\theta^2\rangle}{96}[(p_1.p_3)\{p_2^\rho\gamma^\eta-p_1^\eta\gamma^\rho\}+(p_1.p_4)\{p_2^\eta\gamma^\rho-p_1^\rho\gamma^\eta\}$$
$$+(\not{p}_3-\not{p}_4)\{p_1^\rho p_2^\eta-p_1^\eta p_2^\rho\}].\tag{3.15}$$

Putting all these rules together, the cross section up to first order in $\langle\theta^2\rangle/12$ for diphoton production can be calculated (Fig. 4). The matrix element for diphoton production can be written as the sum of the three diagrams: $i\mathcal{M}=i\mathcal{M}_1+i\mathcal{M}_2+i\mathcal{M}_3$, with each matrix element defined below:

$$i\mathcal{M}_1=-ie^2\epsilon_\mu^*(p_3)\epsilon_\nu^*(p_4)\overline{v}(p_2)\left[\frac{\gamma^\nu\not{q}\gamma^\mu}{t}+\frac{\langle\theta^2\rangle}{96}\frac{t}{2}\{\gamma^\nu\not{q}\gamma^\mu\right.$$
$$\left.+p_2^\nu\gamma^\mu-p_1^\mu\gamma^\nu\}\right]u(p_1),\tag{3.16}$$

$$i\mathcal{M}_2=-ie^2\epsilon_\mu^*(p_3)\epsilon_\nu^*(p_4)\overline{v}(p_2)\left[\frac{\gamma^\mu\not{q}_1\gamma^\nu}{u}\right.$$
$$\left.+\frac{\langle\theta^2\rangle}{96}\frac{u}{2}\{\gamma^\mu\not{q}_1\gamma^\nu+p_2^\mu\gamma^\nu-p_1^\nu\gamma^\mu\}\right]u(p_1),\tag{3.17}$$

$$i\mathcal{M}_3=ie^2\epsilon_\mu^*(p_3)\epsilon_\nu^*(p_4)\frac{\langle\theta^2\rangle}{192}\overline{v}(p_2)[t\{p_1^\mu\gamma^\nu-p_2^\nu\gamma^\mu\}$$
$$+u\{p_1^\nu\gamma^\mu-p_2^\mu\gamma^\nu\}+2(\not{p}_3-\not{p}_4)(p_1^\nu p_2^\mu$$
$$-p_1^\mu p_2^\nu)]u(p_1).\tag{3.18}$$

It is easy to show that if either one of the polarization vectors is replaced with its momentum, the matrix element will be zero as we expect from gauge invariance. Next it is straightforward to show that the spin average square matrix element is

$$\overline{|\mathcal{M}|^2}=2e^4\left[\frac{t}{u}+\frac{u}{t}-\frac{\langle\theta^2\rangle}{96}(t^2+u^2)\right].\tag{3.19}$$

To first order in $\langle\theta^2\rangle/12$ this gives the following CM differential cross section:

$$\frac{d\sigma}{d\cos\theta}=\left(\frac{d\sigma}{d\cos\theta}\right)_{QED}\left[1-\frac{\langle\theta^2\rangle}{192}\frac{s^2}{2}\sin^2\theta\right].\tag{3.20}$$

IV. BOUNDS ON Λ_{NC} FROM COLLIDERS

Møller scattering experiments do not provide data at high enough energy to set a bound comparable to the one obtained from Bhabha scattering. For Bhabha scattering the bound can be extracted from a series of LEP experiments [29]. The total cross section integrated between θ_0 and $180°-\theta_0$ predicted by our calculation can be written as

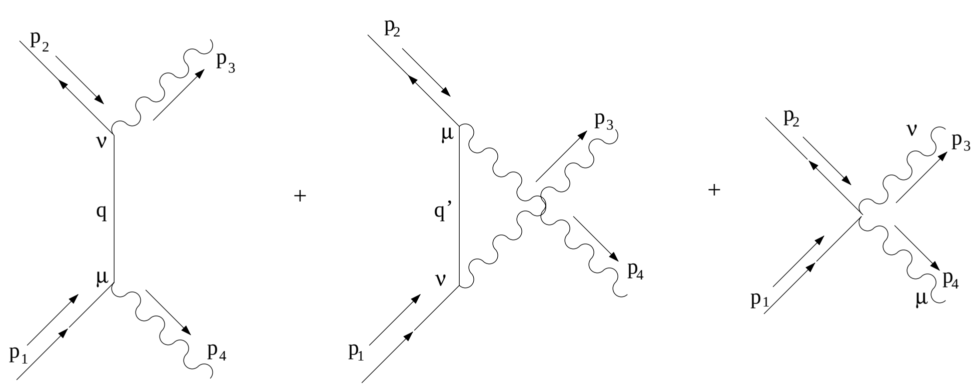

FIG. 4. Feynman diagrams for $e^+e^-\to\gamma\gamma$.

TABLE I. Bhabha scattering: Data from L3 experiment at LEP and SM prediction [29].

$\sqrt{s}$ (GeV)	$\sigma_{exp}\pm\Delta_{stat}\pm\Delta_{sys}$ (pb)	σ_{SM} (pb)
130.10	51.10 ± 2.90 ± 0.20	56.50
136.10	49.30 ± 2.90 ± 0.20	50.90
161.30	34.00 ± 1.90 ± 1.00	35.10
172.30	30.80 ± 1.90 ± 0.90	30.30
182.70	27.60 ± 0.70 ± 0.20	26.70
188.70	25.10 ± 0.40 ± 0.10	24.90

$$\sigma=\sigma_{SM}+\frac{\pi\alpha^2 s}{8\Lambda_{NC}^4}\left\{\frac{25}{4}a+\frac{7}{12}a^3+2\ln\frac{1-a}{1+a}\right\}, \qquad (4.1)$$

with $a=\cos\theta_0$. This matches the cut introduced by the L3 experiment where $\theta_0=44°$ is the angle relevant to the L3 detector. Here we use σ_{SM} instead of σ_{QED} to take into account the weak interaction and radiative corrections. We have neglected the noncommutative correction to higher order QED and weak interactions. We use the numerical values of the data above (Table I) [29], and for the theoretical prediction we add the correction due to noncommutativity obtained in the previous section to the listed SM cross section. The χ^2 function is defined as follows:

$$\chi^2=\sum_i\left(\frac{\sigma_{exp}^i-\sigma_{theor}^i}{\Delta_{exp}^i}\right)^2 \qquad (4.2)$$

with $\Delta_{exp}^2=\Delta_{stat}^2+\Delta_{sys}^2$ and i sums over the energy range. Performing the χ^2 analysis over the energy range shown in Table I, we obtain the bound $\Lambda_{NC}\geqslant 137$ GeV (95% C.L.).

A similar analysis can be performed on $e^+e^-\to\mu^+\mu^-$ using the data from the same experiment at LEP [29]. The total cross section integrated between θ_0 and $180°-\theta_0$ is

$$\sigma=\sigma_{SM}+\frac{\pi\alpha^2 s}{8\Lambda_{NC}^4}\frac{a^3}{3}, \qquad (4.3)$$

with a defined above and $\theta_0=44°$. Fitting our theoretical prediction to LEP data (Table II) [29] using χ^2 fit will set the bound for $\Lambda_{NC}\geqslant 86$ GeV (95% C.L.).

TABLE II. $e^+e^-\to\mu^+\mu^-$: Data from L3 experiment and SM prediction [29].

$\sqrt{s}$ (GeV)	$\sigma_{exp}\pm\Delta_{stat}\pm\Delta_{sys}$ (pb)	σ_{SM} (pb)
130.10	21.00 ± 2.30 ± 1.00	20.90
136.10	17.50 ± 2.20 ± 0.90	17.80
161.30	12.50 ± 1.40 ± 0.50	10.90
172.30	9.20 ± 1.30 ± 0.40	9.20
182.70	7.34 ± 0.59 ± 0.27	7.90
188.70	7.28 ± 0.29 ± 0.19	7.29

For diphoton production, the bound can be extracted from a series of experiments at LEP [30]. The total cross section integrated between θ_0 and $180°-\theta_0$ predicted by our calculation can be written as

$$\sigma=\sigma_{SM}-\frac{\pi\alpha^2 s}{16\Lambda_{NC}^4}\left\{a+\frac{a^3}{3}\right\}, \qquad (4.4)$$

with $a=\cos\theta_0$. This time the bound is obtained from an analysis done by the experimenters themselves for the purpose of bounding a generic contribution for "new physics." The bound set from diphoton production experiments at LEP, as obtained by the DELPHI Collaboration and translated to our definition of noncommutativity scale is $\Lambda_{NC}\geqslant 160$ GeV [30]. A similar analysis by the L3 Collaboration yields a similar bound [30].

A next linear collider (NLC) with a luminosity 3.4×10^{34} cm^{-2} s^{-1} and center of mass energy 1.5 TeV will set a better bound for Λ_{NC}. We calculated the number of events predicted by ordinary QED at 1.5 TeV and took the statistical uncertainty from the square root of the number of events. By requiring the "new physics" effect to be significant only if it can produce an effect at least 2 standard deviations away from this predicted value, a prediction for the bound that could be set for the noncommutative scale can be obtained. Our calculation for Bhabha scattering predicts a reach for $\Lambda_{NC}\approx 2.0$ TeV, for $e^+e^-\to\mu^+\mu^-$ $\Lambda_{NC}\approx 1.7$ TeV, for Møller scattering $\Lambda_{NC}\approx 2.7$ TeV and for diphoton production $\Lambda_{NC}\approx 2.0$ TeV. From this we can conclude that the bound obtained from these experiments will be about ≈ 2 TeV and is comparable to the energy scales where the experiments are performed.

V. CONCLUSION

We have considered the phenomenology of a Lorentz-conserving version of noncommutative QED. In this theory, spacetime coordinates are promoted to operators satisfying the DFR Lie algebra. As opposed to the Lorentz-violating canonical noncommutative theory, field theory variables have an additional dependence on the operator θ which characterizes the noncommutativity. This is handled by expanding the fields in powers of θ, and using gauge invariance and noncommutativity restrictions to determine the fields order by order in θ. Lorentz-invariance restricts interaction vertices to contain only even powers of θ, which has distinct consequences on the phenomenology of the theory. We considered various e^+e^- and e^-e^- collider processes. The cross section was calculated to second order in θ for Bhabha, Møller, and $e^+e^-\to\mu^+\mu^-$ scattering, as well as $e^+e^-\to\gamma\gamma$. Results were then compared to LEP data, and bounds on the energy scale of noncommutativity, Λ_{NC}, were obtained. The tightest bound came from diphoton production which yielded $\Lambda_{NC}>160$ GeV at the 95% confidence level. We also determined that an NLC running at 1.5 TeV with a luminosity of 3.4×10^{34} cm^{-2} s^{-1} will be able to probe Λ_{NC} up to ~ 2 TeV.

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