# Fall of stringy de Sitter spacetime

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Kachru, Kallosh, Linde, and Trivedi recently constructed a four-dimensional de Sitter compactification of type IIB string theory, which they showed to be metastable in agreement with general arguments about de Sitter spacetimes in quantum gravity. In this paper, we describe how discrete flux choices lead to a closely spaced set of vacua and explore various decay channels. We find that in many situations NS5-brane mediated decays which exchange NSNS 3-form flux for D3-branes are comparatively very fast.

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# I. INSTABILITIES OF dS FLUX COMPACTIFICATIONS

de Sitter spacetime (dS) holds a special place in the study of quantum gravity. Constructing and exploring the maximally symmetric spacetime with a positive cosmological constant  $\Lambda$  has been the source of much recent interest despite (or perhaps because of) its stubborn opacity. While much progress has been made in the understanding of the other maximally symmetric solutions, Minkowski and anti-de Sitter (AdS) spaces, dS has until recently eluded string theoretic description because of some of its unique properties. The observer-dependent horizon of dS, like a black hole horizon, yields a thermal state with finite entropy. Not only are the S-matrix observables of string theory precluded in  $\Lambda > 0$  spaces [1], but, due to the inevitable Poincaré recurrences [2], all observables are ill-defined [3]. These issues would merely be of abstract theoretical importance were it not for recent observational evidence [4] indicating that not only was  $\Lambda > 0$  in the early Universe during inflation, but it seems to be so today.

It has become increasingly clear that dS cannot be a stable state in any theory of quantum gravity. The symmetries of dS are incommensurate with the discrete spectrum implied by finite entropy [5]. Rather than a stable vacuum, dS is instead a metastable resonance whose lifetime, on general entropic grounds, must be less than the recurrence time [5-7].

One would ask, then, what does string theory say about dS and its decay modes? String models of dS have been difficult to find partly because, as nonsupersymmetric vacua, they are isolated points in moduli space with all moduli stabilized. Notably, some dS compactifications of string theory were described in [8,9] and, in a well-controlled manner for critical strings, by Kachru, Kallosh, Linde, and Trivedi (KKLT) [6]. Generically, any string theoretic dS compactification can decay and decompactify [7,10] because the 10D Poincaré invariant string vacuum is supersymmetric and so has vanishing energy density. However, this is far from the only decay mode. For example, in any compactification in which RR fluxes contribute to the potential, D-brane instantons change the fluxes and the cosmological constant. This

has been an object of study in many papers, including [11-15].

We consider a slight twist on the brane instanton decays. In [6], the cosmological constant gets a positive contribution from D3-branes, rather than directly from the fluxes. This effect has been seen in the AdS conformal field theory (CFT) correspondence, where instantonic Neveu-Schwarz 5-branes (NS5-branes) provide a decay mode for the D3-branes [16]. These results apply to the similar dS compactifications of KKLT and are particularly of interest because they can end in a state of positive cosmological constant. Therefore, one might wonder whether this type of decay could occur quickly enough to affect the cosmological constant within the age of the Universe. In this paper, we generalize the results of Kachru, Pearson, and Verlinde (KPV) [16] to dS compactifications and compare the decay rate through the 5-brane channel to two other decays, one to decompactification and the other by D3-brane tunneling in the compactification manifold. We give explicit examples in which the 5-brane decays are much faster than the others.

In the next section, we review the dS vacuum construction that we will study. In Sec. III, we flesh out the discrete landscape of vacua that are available through tuning and among which our instantons will interpolate. We then review the AdS/CFT instantons of [16] and make the corrections necessary to compactify their backgrounds in Sec. IV. We apply our calculation to find decay times for specific sets of initial parameters in Sec. V and compare them to those of KKLT in Sec. VI. In addition, we comment on two other possible decay channels. We will generally keep factors of the gravitational coupling  $\kappa_4$  and the string length  $\alpha'$  explicit in formulas, but any numbers we cite should be taken in Planck or string units.

# II. BUILDING dS VACUA

Constructing a solution of string or M theory with a fourdimensional dS vacuum has been a longstanding challenge. Such a solution must be nonsupersymmetric and requires aspects of the theory beyond the low-energy SUGRA limit.

Recently, however, KKLT [6] presented a specific construction in critical string theory with no unfixed moduli. The model was based on the warped flux compactifications stud-

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ied by Giddings, Kachru, and Polchinski (GKP) [17].<sup>1</sup> Nonperturbative corrections fix the overall Kähler modulus of this tree-level no-scale model, resulting in a stable, supersymmetric AdS vacuum. KKLT then added  $\overline{D3}$ -branes to yield a metastable dS vacuum and showed, by considering decays to decompactification, the lifetime to be less than the Poincaré recurrence time.

The GKP compactification of type IIB string theory on a threefold M with 7-branes and O3-planes can be efficiently described as an F-theory compactification on a CY fourfold (CY) X. X is elliptically fibered over M such that the fiber's complex structure  $\tau = c_0 + ie^{-\phi}$  is the type IIB axion-dilaton (we take for simplicity  $\tau = i/g_s$ ). We will consider the orientifold limit of F theory in which M is an orientifolded CY threefold. Three-form fluxes and D3-branes are added subject to the global tadpole constraint, or the global conservation of RR 5-form  $F_5$  flux:

$$0 = N_{\rm D3} - N_{\rm \overline{D3}} + \frac{1}{2\kappa_{10}^2\mu_3} \int_M H_3 \wedge F_3 - \frac{\chi(X)}{24}.$$
 (1)

The Euler number of the CY fourfold  $\chi(X)$  gives the effective negative D3-brane charge in type IIB O3-planes and D7-branes wrapped on 4-cycles of *M*. For typical choices of *X*,  $\chi(X)$  can be up to  $O(10^5)$  [28]. This must be balanced by the charge from 4D space-filling D3-branes, D3-branes, and the wrapped NSNS and RR 3-form fluxes  $H_3$  and  $F_3$ , which also source  $F_5$ .

To construct their model, KKLT began by choosing *X* and a set of wrapped fluxes, while setting  $N_{D3} = N_{\overline{D3}} = 0$ . The CY threefold *M* has  $b_3 \ge 1$  three-cycles, and a particular choice of fluxes  $H_3$ ,  $F_3 \in H^3(M,Z)$  represents a point in a  $2b_3$ dimensional lattice. The fluxes combine into a single complex 3-form  $G_3 = F_3 - \tau H_3$ . For simplicity, KKLT chose  $h^{1,1}(X) = 2$ , so that *M* has a single Kähler modulus  $\rho$ . In addition to the moduli  $\tau$  and  $\rho$ , *M* has  $h^{2,1}(M)$  complex structure moduli  $z^{\alpha}$ .

In the presence of fluxes, the classical 4D effective  $\mathcal{N} = 1$  superpotential is [18]

$$W_0 = \frac{1}{\kappa_4^8} \int_M G_3 \wedge \Omega, \qquad (2)$$

where  $\Omega$  is the holomorphic (3,0) form on *M*. *W*<sub>0</sub> then is given by the (0,3) part of the *G*<sub>3</sub> flux which, because the fluxes are quantized, can only be tuned discretely. The tree-level Kähler potential (ignoring warping of the spacetime metric)

$$\mathcal{K} = -3 \log[-i(\rho - \bar{\rho})] - \log[-i(\tau - \bar{\tau})] - \log\left(-\frac{i}{\kappa_4^6} \int_M \Omega \wedge \bar{\Omega}\right)$$
(3)

along with  $W_0$  gives the no-scale potential

$$V = e^{\mathcal{K}} \sum_{i,\bar{J}} \mathcal{K}^{i\bar{J}} D_i W \bar{D}_{\bar{J}} \bar{W}$$
(4)

where i, j sum over all moduli but  $\rho$ ,  $\mathcal{K}_{i\overline{j}} = \partial_i \partial_{\overline{j}} \mathcal{K}$  is the Kähler metric, and  $D_i = \partial_i + \partial_i \mathcal{K}$  is the Kähler covariant derivative. Except for the volume modulus  $\rho$ , this potential generically fixes all other moduli such that  $G_3$  is imaginary self-dual.<sup>2</sup> The remaining condition for supersymmetry,  $D_\rho W = 0$  is satisfied only when  $W_0 = 0$ , which implies that in supersymmetric vacua  $G_3$  is a (2,1) form.

The geometry of M is, of course, very complicated but is accurately described near conifold points by the Klebanov-Strassler (KS) solution [30]. Wrapped fluxes warp and deform the conifold; at the tip y=0, the metric is

$$ds^{2} = h^{-1/2} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + bg_{s} M \alpha' (e^{2u} dy^{2} + d\Omega_{3}^{2} + e^{2u} y^{2} d\Omega_{2}^{2})$$
(5)

where  $b \sim 1$  is a numerical constant and  $e^u$  is the compactification length scale (here we use 10D string frame). Notice that the  $S^3$  at the tip has a fixed proper size depending only on the fluxes. Also, the  $S^2$  is nontrivially fibered over the  $S^3$ . Away from the tip, the throat has approximately a warped conifold metric

$$ds^{2} \approx h^{-1/2} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + h^{1/2} e^{2u} (dr^{2} + r^{2} ds^{2}_{T^{1,1}}) \qquad (6)$$

where  $ds_{T^{1,1}}^2$  is the metric on the base  $T^{1,1}$ . In this region, the warp factor is approximately

$$h = 1 + (L^4/r^4)\log(r/r_s)$$
(7)

with the length scale  $L^4 = \frac{81}{8}e^{-4u}g_sM\alpha'$ . Here, the radial coordinates *r* and *y* are complicated functions of each other, and the tip is at  $y=0,r=\tilde{r}$ . For the undeformed conifold, the singular tip is located at  $r=r_s=\tilde{r}e^{-1/4}$ . Splitting the conifold into the tip and throat in this manner is described in [31] and references therein.

The radial modulus Im  $\rho = e^{4u}/g_s \equiv \sigma$  is defined so that, at large radius, the total unwarped volume of the compactification is  $\int_M d^6x \sqrt{h^{-1/2}g} \approx e^{6u} \alpha'^3$ . The fluxes through any 3-cycle of *M* are quantized, and for a given conifold throat

$$M = \frac{1}{4\pi^{2}\alpha'} \int_{A} F_{3}, \quad K = -\frac{1}{4\pi^{2}\alpha'} \int_{B} H_{3}$$
(8)

where the A cycle is the  $S^3$  which stays finite at the tip and the B cycle is the six-dimensional dual of A. GKP found that

<sup>2</sup>It is natural to wonder if corrections to the Kähler potential due to warping could fix the radial modulus. While the precise form of  $\mathcal{K}$  is difficult to compute for  $\rho$ , because the 10D solution exists at tree level for all compactification scales, the final potential must be no-scale [29]. We will look at warping in the complex structure Kähler potential below.

<sup>&</sup>lt;sup>1</sup>The GKP type of compactification was studied earlier in simpler cases and in M theory by [18–21]. The supersymmetry conditions and equations of motion were considered in [22–24]. Explicit constructions on tori and K3 are in [25–27].

the warp factor at the tip of the conifold is related to the deformation parameter z of the tip, which is determined by the flux superpotential (2), by

$$h(y=0) \approx \frac{(g_s M)^2}{|z|^{4/3}}, \quad z = \exp\left[-\frac{2\pi K}{g_s M}\right].$$
 (9)

It is this particular form of the warp factor that gives the *A* cycle a fixed proper size at the tip. Note that this is not the  $r \rightarrow \tilde{r}$  limit of Eq. (7) because the conifold is deformed.

To generate a nontrivial potential for  $\rho$ , as suggested in [17], KKLT considered nonperturbative corrections to the superpotential (2). Both wrapped Euclidean D3-branes and gluino condensation on the worldvolume of non-Abelian D7-branes generate additional terms of the form

$$\delta W = A e^{ia\rho} \tag{10}$$

where the constants  $A \sim O(1)$  and  $a \sim O(10^{-1})$ . For simplicity, KKLT took  $\rho$  to be purely imaginary,  $\rho = i\sigma$ , and A, a,  $W_0$  to be real. The potential now becomes

$$V = \frac{aAe^{-a\sigma}}{2\sigma^2} \left\{ Ae^{-a\sigma} \left( 1 + \frac{a\sigma}{3} \right) + W_0 \right\}, \tag{11}$$

and for suitable  $W_0 < 0$  there is a supersymmetric vacuum with  $V_0 < 0$ , implying the noncompact directions are AdS. For  $|W_0| \ll 1$ , the AdS minimum lies at  $\sigma_{cr} \gg 1$  where the SUGRA can be trusted and  $\alpha'$  corrections are small.

The final step in the KKLT construction is to add enough  $\overline{\text{D3}}$ -branes so that  $V_0 > 0$  and the vacuum is dS. The global  $F_5$  charge must still be conserved via Eq. (1), and the addition of  $p \overline{\text{D3}}$ -branes gives  $N_{D3} = -p$ . By adjusting the fluxes, a corresponding increase in  $\int_M H_3 \wedge F_3$  balances this reduction. The  $\overline{\text{D3}}$ -branes break supersymmetry and add some extra energy [16],

$$\delta V = \frac{Dp}{\sigma^3}; \quad D = 2\mu_3 h^{-1}(r)$$
 (12)

where  $\mu_3$  is the brane charge. To minimize their energy, the  $\overline{D3}$ -branes migrate to a conifold tip, so the energy density per  $\overline{D3}$ -brane depends, through Eq. (9), on the fluxes. For sufficiently fine-tuned parameters, this additional term in the potential lifts the AdS global minimum to a dS local minimum.

Unlike the AdS vacuum, the dS minimum is only metastable. KKLT investigated one possible decay mode, tunneling to large  $\sigma$ . The potential becomes arbitrarily close to zero at large radius, so it is possible to tunnel to a runaway, decompactifying solution.

Coleman and De Luccia (CDL) [32] described such an instanton including gravitational back-reaction. In terms of a canonical scalar field  $\varphi = (\sqrt{3/2} \log \sigma)/\kappa_4$ , the Euclidean action is

$$S_E[\varphi] = \int d^4x \sqrt{g} \left( -\frac{1}{2\kappa_4^2} R + \frac{1}{2} (\partial \varphi)^2 + V(\varphi) \right)$$
$$= -\int d^4x \sqrt{g} V(\varphi)$$
(13)

using Einstein's equations to get the second line. The instanton  $\varphi_{CDL}$  is an O(4)-symmetric interpolation between the dS vacuum at  $\varphi_{cr}$  and the supersymmetric vacuum at  $\varphi = \infty$ . When Wick rotated back, this gives the usual expanding bubble of true vacuum inside the false dS vacuum. The action of the static dS vacuum is simply computed to give

$$S_0 = -\frac{24\pi^2}{\kappa_4^4 V_0} = -S_0 \tag{14}$$

where  $S_0$  is the entropy of the dS vacuum. The tunneling probability per unit volume is given by the difference between the action of the instanton solution and the static dS vacuum:

$$P_{decay}^{CDL} \sim e^{-S[\varphi_{CDL}] + S_0}.$$
(15)

From Eq. (13),  $S[\varphi] < 0$  for  $V(\varphi) > 0$ , and the resulting lifetime is exponentially less than the Poincaré recurrence time  $t_r \sim e^{S_0}$ :

$$t_{decay}^{CDL} \sim e^{S_0 - |S[\varphi]|} < t_r \tag{16}$$

which is in line with the general arguments of [5,7,33].

In addition to the CDL instanton, KKLT considered decompactification decay via the stochastic Hawking-Moss (HM) instanton [34]. Considering decays of general dS string compactifications, [5] and [7] also discussed thermal fluctuations using the HM reasoning. Whereas the CDL instanton tunnels through the potential barrier, the HM instanton relies on thermal fluctuations to carry  $\varphi$  to the top of the potential, where it can then roll down the other side to the true vacuum. While the original HM process is homogeneous, KKLT argued it should be interpreted as a horizon-sized fluctuation. If the potential has a broad, flat maximum at  $\varphi_1$ , the state there is approximately dS with energy  $V(\varphi_1) > V_0$  and entropy  $S_1$ . The probability per unit volume for a thermal fluctuation is given by the difference in entropies between the fluctuation and equilibrium:

$$P_{decav}^{HM} \sim e^{S_1 - S_0}. \tag{17}$$

The decay time  $t_{decay}^{HM} = (P_{decay}^{HM})^{-1}$  is again less than the recurrence time  $t_r$  and is also less than the CDL decay time (16) when the potential barrier is short and wide and thus the thin-wall approximation is invalid.

#### **III. FINDING dS PARAMETERS**

As described in Sec. II, obtaining the vacua constructed in [6] requires fine-tuning subject to several constraints. First, one must adjust the bulk fluxes so that  $|W_0| \ll 1$ . Moreover, a dS minimum requires fine-tuning of the fluxes, *K* and *M*, in



FIG. 1. The possible dS vacua with  $V_0$  for given *M* illustrate the density of states consistent with a discretuum.

the KS throat. A given value of  $W_0$  tightly constrains one's choice for Dp [cf. Eq. (12)]. For example, KKLT presented a model with  $W_0 = -10^{-4}$  and an AdS minimum of  $V_0 = -2.00 \times 10^{-15}$ ; by adding one D3-brane with  $D = 3 \times 10^{-9}$ , they achieved a dS minimum of  $V_0 = 1.77 \times 10^{-17}$ . This is a very special choice of fluxes indeed. For  $Dp \leq 3 \times 10^{-9}$  the minimum is at  $V_0 < 0$  and is AdS, and for  $Dp \geq 7.5 \times 10^{-9}$  a local minimum no longer exists. There are additional constraints as well. In [16] it is shown that there exists a classical instability if  $p/M \geq 0.08$ . Furthermore, results from Sec. III of [17] rely on approximations valid when  $K/(g_sM) \geq 1/2$ .

With such fine-tuning and taking into account that the tuning parameters K and M are discrete, one might question if it is possible to build such a model at all. Such tuning would require the existence of a "discretuum."<sup>3</sup> We have done numerical searches in order to map out the discrete landscape of dS vacua. Figure 1 shows the existence of the discretuum. Here we have plotted the possible values  $V_0$  that have a dS minimum and can be achieved with integer fluxes for the parameters used in KKLT,  $W_0 = -10^{-4}$ , a = .1, A = 1,  $g_s = 0.1$ ,  $\kappa_4 = \alpha' = 1$ .<sup>4</sup> It is clear that for a desired value of  $V_0$  there exists a configuration of fluxes with  $\tilde{V}_0 = V_0$  $+\epsilon$ , where  $\epsilon$  is very small; i.e. a discretuum does exist. For each of the models studied  $K/g_{s}M > 1/2$ . Here we have allowed M to range from 75 to 1000. The lower bound avoids the classical instability (for  $p \leq 6$ ). As one goes to higher and higher values of M, one must also increase the amount of induced D3-brane charge on the D7-branes in order to satisfy Eq. (1). This might require adding more D7-branes and, thus, more degrees of freedom, which, though massive, could cause problems when considering loop corrections.

The smallest possible value of  $V_0$ , for the parameters used in KKLT, is  $\mathcal{O}(10^{-20})$ , a far cry from the desired  $\mathcal{O}(10^{-120})$ . In order to obtain a more realistic vacuum energy, one must attempt to construct a background with  $|W_0| \sim \mathcal{O}(10^{-55})$ . While such a fine-tuning seems improbable, with  $b_3$  sufficiently large, it is at least possible,<sup>5</sup> if not particularly natural [13].

We have so far considered only a single KS throat, as in KKLT. However, a general CY has many of them. By considering backgrounds with multiple KS throats the discretuum density is increased dramatically. One finds that Eq. (12) becomes,

$$\delta V = \sum_{i} \frac{D_{i} p_{i}}{\sigma^{3}}; \quad D_{i} = 2 \mu_{3} h^{-1}(\tilde{r}_{i}), \qquad (18)$$

where *i* labels the different throats. Clearly, by adjusting the fluxes in each individual throat, one may tune  $\delta V$  with greater accuracy. For a single KS throat we found  $\mathcal{O}(10^3)$  configurations with a dS minimum. Analogously, for 2 KS throats  $(75 \le M_1 \le M_2, 75 \le M_2 \le 300)$  we find  $\mathcal{O}(10^5)$  dS minima. It is easy to find configurations with  $\mathcal{O}(10)$  KS throats,<sup>6</sup> leading to an amazingly dense set of vacua. The inclusion of a second throat also lowers our minimum value of  $V_0$  by an order of magnitude. Though this is nice, it does little good in helping build a model with a realistic cosmological constant. We suspect that even with the addition of 10 or more throats the lofty goal of  $V_0 \sim 10^{-120}$  would still be far out of reach.

The following sections describe various decays, analogous to those studied in [16], in which one unit of  $H_3$  flux is exchanged for M D3-branes. For geometries with single KS throats, after one decay the final state has a negative cosmological constant and a big crunch in its future. It has been argued that these decays should not be allowed in a quantum theory of gravity and also that instantons mediating these decays may not be possible to construct [36]. We will not worry about these subtleties (other than the well-known effects on the instanton action [32]) since our main focus is on instanton decays ending in dS. The configuration with multiple KS throats is more interesting. As with the single throat, these may decay directly into states with negative cosmological constant. However, there can now be decays from one dS vacuum to another with smaller  $\Lambda$  (modulo some classical evolution we will discuss later). This process is of particular interest, since it allows for a rather generic set of fluxes on several KS throats to undergo a series of decays to dS vacua with smaller and smaller cosmological constant; this situation is similar to that envisioned by [37] and expanded upon by [11,12,14].

<sup>&</sup>lt;sup>3</sup>The authors of [13] coined this term to refer to situations in which a discrete spectrum is sufficiently dense to allow for an (almost) arbitrarily fine-tuning. Our discretuum is not as finely spaced as those in [13].

<sup>&</sup>lt;sup>4</sup>In addition to tuning  $V_0$  by varying the fluxes M and K, one could, in principle, vary  $W_0$  by adjusting the bulk fluxes. While this would certainly increase the discretuum density, we leave  $W_0$  constant as explicit calculation of  $W_0$  in terms of bulk fluxes is prohibitively complicated.

<sup>&</sup>lt;sup>5</sup>One can estimate the smallest  $|W_0|$  to have  $\log(|W_0|) \sim -2b_3$ . We thank S. Kachru for discussion on this point.

<sup>&</sup>lt;sup>6</sup>For example, in [35] a family of quintics are constructed with 16 conifold singularities.



FIG. 2. Top: In the KPV process,  $p \ \overline{D3}$ -branes polarize into an NS5-brane wrapping an  $S^2$  on the *A* cycle. The NS5-brane then slides to the opposite pole, becoming M - p D3-branes. Bottom: In the thin-wall limit, the NS5-brane instead wraps the *A* cycle at a particular Euclidean radius.

#### IV. DECAYS IN THE MANNER OF KPV

#### A. Review of NS5-brane instantons

The KS geometry found at conifold points of GKP compactifications was first studied in the usual decoupling limit of string-gauge theory dualities [30]. The relevant gauge theory dual is a duality cascade with an energy dependent effective number of D3-branes; in the IR, most of the D3branes have been transformed into 3-form fluxes. The BPS domain wall that transforms the D3-branes to fluxes was described by KPV; it is a polarized NS5-brane that carries D3brane charges and bends over the *A* cycle at the deformed conifold tip [16]. As the NS5-brane moves over the *A* cycle, the D3-branes are absorbed into the background RR flux, and the background NSNS flux jumps by a unit due to the NS5brane charge.

KPV also described nonsupersymmetric gauge theories with p D3-branes at the tip of the conifold, as in KKLT. Due to the 3-form flux background, the D3-branes suffer a classical instability to brane polarization (first discovered in [38]) as an NS5-brane wrapping an  $S^2$  in the A cycle. However, for  $p \ge M/12$ , the NS5-brane itself is unstable to collapse around the A cycle, reducing the NSNS flux and turning the D3-branes into supersymmetric D3-branes. For smaller p, the decay of the NS5-brane proceeds by tunneling; in Euclidean spacetime, the NS5-brane is slightly polarized with D3 charge at infinity and bends around the A cycle to leave D3-branes at the origin [16]. This process is illustrated in the top line of Fig. 2.

For *p* small enough, KPV showed numerically that the thin-wall approximation is very reasonable. In that limit, the instanton appears to be an NS5-brane wrapping the full  $S^3$  of the *A* cycle at a fixed radius, as shown in the bottom line of Fig. 2. The wrapped  $F_3$  flux induces *M* units of charge in the NS5 worldvolume gauge theory which is canceled by the charge carried by the ends of *M* D3-branes. The *p* D3-branes end on the outside of the NS5-brane, and M-p D3-branes end on the inside. The bubble tension in the effective theory is just the NS5-brane tension times the volume of the *A* cycle. These instantons are clearly related to the BPS domain walls KPV found.

In the rest of this paper, we will focus on the thin-wall limit to estimate the instanton bubble tension. As has been argued strenuously [36], the thin-wall limit certainly does not describe the full picture of the decays, but the other contributions to the Euclidean path integral (such as Hawking-Moss instantons at the other extreme) should only enhance the decay rate. Therefore, we take the point of view that the thin-wall limit estimates an upper limit for the decay time. As a consequence of the thin-wall limit, we may, as in KKLT, ignore the polarization of the D3-branes in the initial metastable state. Before we turn to the modifications necessary for including KPV instantons in KKLT compactifications, let us also note that our instantons are cousins of the supersymmetry-changing domain wall bubbles found in [39], just as the AdS/CFT instantons of KPV are related to the BPS domain walls.

#### **B.** Corrections for compactifications

There are several modifications that we have to make to the KPV instanton decay formula due to the fact that we have a compact GKP geometry rather than a noncompact conifold.

The first and most obvious correction is that gravity is no longer decoupled, so we should include the effects of gravitation on the decay time. These effects are well known [11,12,32]; in Appendix B, we work out the specific formula we need. The decay time, including gravity (but ignoring the large number of massive fields in the compactification), is  $t_{decay} \sim \exp[-\Delta S_E]$ , where  $\Delta S_E$  is the difference of the Euclidean actions for the instanton and the initial background state as given in Eq. (B10). It depends only on the bubble tension, the initial vacuum energy density, and the change in energy density. Given two dS states from Sec. III, we just need to calculate the bubble tension and plug into Eq. (B10).

There are also modifications to the tension of the bubble. The easiest to calculate is an effect of working in the 4D Einstein frame. Let us emphasize that we need to work in the 4D Einstein frame to use the superpotential formalism of Sec. II, and this is also the frame in which the potential has been calculated. The Einstein frame is also the frame used in calculating the instanton decay time. It is easiest to get this by going to the NS5-brane action

$$S_E = \frac{\mu_5}{g_s^2} \int d^4x \sqrt{\det g_{\mu\nu}} \delta \int d^3x \sqrt{g_{S^3}},$$
 (19)

where  $g_{\mu\nu}$  is the 4D pullback of the 10D metric,  $\delta$  is the delta function at the radius of the bubble (with the determinant of the metric included), and  $g_{S^3}$  is the determinant of the metric on the *A* cycle. The 10D string frame and 4D Einstein frame are related by  $h^{-1/2}g_{\mu\nu}^E = g_s^{-2}e^{6u}g_{\mu\nu}$ , so the NS5 action becomes

$$S_{E} = 2 \pi^{2} r^{3} \tau_{5},$$
  

$$\tau_{5} \equiv \mu_{5} g_{s} e^{-9u} \left( \frac{z^{2/3}}{g_{s} M} \right)^{3/2} (2 \pi^{2}) (b g_{s} M \alpha')^{3/2}$$
  

$$= \frac{b^{3/2} z}{16 \pi^{3} \alpha'^{3/2} g_{s}^{5/4} \sigma^{9/4}}.$$
(20)

(Henceforth  $\tau$  is the instanton bubble tension.) In the first equality for the tension  $\tau_5$ , we have separated the contribution from the conversion to Einstein frame, the warp factor, and the volume of the *A* cycle. We have ignored the contribution to the action from the NSNS 6-form potential, which KPV showed is negligible in the thin-wall limit. Heuristically this is because the 6-form potential only has two legs on the *A* cycle and the 5-brane fills the entire cycle, as shown in Fig. 2. However, the RR field strength  $F_3$  gives a worldvolume anomaly that requires *M* D3-branes to attach to the 5-brane. Here, there are p D3-branes on the outside and M-p D3-branes on the inside.

The other correction we should make is due to the action for the moduli. Since the moduli are fixed by the flux superpotential (2), after the NS5-brane bubble changes the flux, the VEVs of the moduli will change. Therefore, we need to take into account the rolling of the moduli to the new vacuum. We will focus on the deformation modulus z of the conifold for the following reasons. First, it clearly changes significantly when K changes [see Eq. (9)]. Also, for a noncompact conifold, K does not affect the dilaton or other moduli, so we would expect that they would be only minimally affected by a change of K in the compact case (the other moduli are typically fixed by fluxes on other cycles). Also, KKLT have shown that the VEV of  $\sigma$  does not change much due to the presence of D3-branes. Therefore, since we expect  $g_s$  and  $\sigma$  to keep roughly the same values before and after the decay, we expect that they will not roll much, and we will treat them as constants. There is actually a significant tree-level potential for  $g_s$  and  $\sigma$  when z is not at its VEV, and we will consider its effects in the next subsection. Nevertheless, we expect our estimate of the contribution from z not to be affected significantly by other moduli. To be conservative, one could multiply the contribution from *z* by a fudge factor, but we note that we are only making an estimate to begin with, so we are not quite that careful.

To estimate the tension due to the rolling of z, we will assume that just inside the NS5-brane z is in its original vacuum value outside of the NS5-brane and rolls quickly to the new VEV inside. This is probably not the exact classical solution, but we will use it and the thin-wall approximation as an upper limit. At tree level (where we are working), we can write the action as

$$S_{E}(z) = \frac{1}{\kappa_{4}^{2}} \int d^{4}x \sqrt{g_{4}} [\mathcal{K}_{z\overline{z}}\partial_{\mu}z \partial^{\mu}\overline{z} + \kappa_{4}^{4}e^{\mathcal{K}}\mathcal{K}^{z\overline{z}}D_{z}W\overline{D}_{\overline{z}}\overline{W}]$$

$$= \frac{2\pi^{2}}{\kappa_{4}^{2}} \int d\xi r^{3} [\mathcal{K}^{z\overline{z}}(K_{z\overline{z}}\partial_{\xi}z - \kappa_{4}^{2}e^{\mathcal{K}/2 - i\omega}\overline{D}_{\overline{z}}\overline{W})$$

$$\times (K_{z\overline{z}}\partial_{\xi}\overline{z} - \kappa_{4}^{2}e^{\mathcal{K}/2 + i\omega}D_{z}W)$$

$$+ \kappa_{4}^{2}e^{\mathcal{K}/2 + i\omega}\partial_{\xi}zD_{z}W + \kappa_{4}^{2}e^{\mathcal{K}/2 - i\omega}\partial_{\xi}\overline{z}\overline{D}_{\overline{z}}\overline{W}], \quad (21)$$

where  $\omega$  is some phase (physically, we have to take it so that the Euclidean action comes out positive because it started positive definite). As above, *r* is the radius of curvature of the bubble, while  $\xi$  is the radial coordinate corresponding to proper distance. This is clearly minimized when only the last two terms contribute. Taking the average Kähler potential in the exponential, we get (up to numerical factors of order unity)

$$\tau_z \approx 2 \pi^2 e^{\langle \mathcal{K} \rangle/2} (|\Delta W| + |\Delta \mathcal{K}|| \langle W \rangle|).$$
(22)

(This comes from the definition of the covariant derivative and the chain rule.) This derivation is very similar to that of BPS domain walls and is also used in [40]. Actually, it is easy to generalize this estimate to include other moduli, but we will only consider z in the superpotential and Kähler potential. We should note that  $\Delta W$  and  $\Delta K$  are calculated from the inside of the NS5-brane (where z is not in a vacuum state) to the new vacuum on the interior of the instanton and not from the original vacuum to the new vacuum. Since we are just making an estimate,  $\langle \cdots \rangle$  will be an average value over the region of variation of z.

The change in the superpotential is given entirely by the superpotential of the conifold just inside the NS5-brane minus  $W_0$ . This is because in the vacuum states, the *K* and *M* fluxes are (2,1) forms and so do not contribute to the superpotential (see, for example, [31]). Using the notation and conventions of [17,29], we get

$$\Delta W = -W(z) = -\frac{(2\pi)^2 \alpha'^{5/2}}{\kappa_4^8} \left( M \mathcal{G}(z) - i \frac{K}{g_s} z \right)$$
  
$$\approx -\frac{(2\pi)^2 \alpha'^{5/2}}{\kappa_4^8} z \left( \frac{M}{2\pi i} \ln z - i \frac{K}{g_s} \right)$$
  
$$\approx -i \frac{(2\pi)^2 \alpha'^{5/2}}{g_s \kappa_4^8} z, \qquad (23)$$

where K is the NSNS flux on the inside of the bubble and z is evaluated outside the bubble. This follows from the definitions

$$\int_{A} \Omega = \alpha'^{3/2} z, \qquad \int_{B} \Omega = \alpha'^{3/2} \mathcal{G}(z),$$
$$\mathcal{G} = \frac{1}{2\pi i} z \log z + \text{holomorphic} \qquad (24)$$

and the relation from Eq. (9) that  $z(\text{outside}) \approx \exp(-2\pi/g_s M) z(\text{inside})$ . To overestimate  $\langle W \rangle$ , we will take

$$|\langle W \rangle| \approx |W_0| + |\Delta W|. \tag{25}$$

The Kähler potential is significantly more complicated, and, because we are concerned with a modulus that lives at the bottom of a throat, we need to take the warp factor into account. Including warping and bunching the Kähler potential for all other complex moduli together into  $\mathcal{K}_c$  (that is, integrals over other cycles), Eq. (3) becomes [29]

$$\mathcal{K}(\text{complex}) = -\log \left[ e^{-\mathcal{K}_c} - \frac{i}{\kappa_4^6} \left( \int_A \Omega \int_B \bar{\Omega} h - \int_A \bar{\Omega} \int_B \Omega h \right) \right].$$
(26)

To compute  $\int_B \Omega h$ , we use the trick that the cycles have a monodromy  $B \rightarrow B + A$  around z = 0 (in the same way it was used to find the leading term in  $\mathcal{G}$ ) and Eq. (9) (which is valid at points both inside and outside the bubble) to expand out

$$\mathcal{K}(\text{complex}) \approx \mathcal{K}_c - e^{\mathcal{K}_c} \frac{\alpha'^3 (g_s M)^2}{2\pi\kappa_4^6} |z|^{2/3} \log|z|^2.$$
(27)

(This is, to our knowledge, the first calculation of part of a Kähler potential with warping included.) Actually, there will be other terms in the *B* cycle integral, but it is reasonable to believe that, as in the unwarped case, this is the leading term that depends on *z*. Then, using Eq. (3) and assuming the complex structure gives small contributions to  $\mathcal{K}_c$ , we get roughly

$$e^{\langle \mathcal{K} \rangle} \approx \frac{g_s}{16\sigma^3}, \quad \Delta \mathcal{K} \approx -\frac{\alpha'^3 (g_s M)^2}{2\pi\kappa_4^6} |z|^{2/3} \left[ \frac{4\pi}{g_s M} e^{4\pi/3g_s M} + \log|z|^2 (e^{4\pi/3g_s M} - 1) \right].$$
(28)

The total bubble tension is therefore

$$\tau = \tau_{5} + \tau_{z} \approx \frac{b^{3/2}z}{16\pi^{3}\alpha'^{3/2}g_{s}^{5/4}\sigma^{9/4}} + \frac{2\pi^{2}g_{s}^{1/2}}{4\sigma^{3/2}} \left\{ \frac{(2\pi)^{2}\alpha'^{5/2}}{g_{s}\kappa_{4}^{8}} z + \left( W_{0} + \frac{(2\pi)^{2}\alpha'^{5/2}}{g_{s}\kappa_{4}^{8}} z \right) \frac{\alpha'^{3}(g_{s}M)^{2}}{2\pi\kappa_{4}^{6}} |z|^{2/3} \times \left| \frac{4\pi}{g_{s}M}e^{4\pi/3g_{s}M} + \log|z|^{2}(e^{4\pi/3g_{s}M} - 1) \right| \right\}.$$
(29)

With the bubble tension in hand we are now in a position to calculate decay rates. However, before moving on we would like to take a closer look at subtle issues ignored in the above calculation. The anxious reader, fretting over the fate of his or her universe, may skip ahead to Sec. V, and leave the following subsection for a more careful reading.

#### C. Other considerations

## 1. D3-brane migration

The KPV instanton bubble not only reduces the NSNS flux *K* and annihilates  $\overline{D3}$ -branes, but it also leaves behind D3-branes. If there are  $\overline{D3}$ -branes in other throats, the D3-branes will feel an attraction and roll through the bulk<sup>7</sup> and into the throat with the  $\overline{D3}$ -branes. Eventually, they will annihilate with the  $\overline{D3}$ -branes via tachyon condensation. If this migration is part of the instanton, then, in many cases, all the  $\overline{D3}$ -branes will be annihilated, leaving a Big Crunch spacetime with negative energy density. If there are more  $\overline{D3}$ -branes to start, the final state could still be dS.

However, we argue that we should not consider the migration of the D3-branes to be part of the instanton, but rather as a classical process that occurs after the bubble nucleates. Our logic is something like the discussion of the bounce instanton of quantum mechanics in [42]; the instanton should only tunnel through the barrier to some energy slightly lower than the initial state, and classical evolution should take over. Typically, in the thin-wall limit, we just assume that the inside of the instanton is just the final state. In our case, though, we expect that the D3 migration would not be well approximated at all by a thin-wall instanton because they are very far from the D3-branes, so the potential is very flat. (Contrast this to the case for the *z* modulus, where the gradient of the potential is Planck scale.) This logic is consistent with the discussion of Hawking-Moss and related instantons in [6,36].

We expect the migration time to be similar to the bubble thickness for the motion of the D3-branes in the Euclidean description of the instanton. The migration times are larger than the bubble radius for the rest of the instanton, so we will treat the D3-brane migration as a classical process. In fact, the migration times are larger than the initial dS radius itself for the models we consider, which is the maximum bubble radius.

In Appendix A, we estimate the classical migration time for a single D3-brane migrating from one tip to another. For the particular model we examine,  $\Delta t_M \sim O(10^{15})$  (in string units). As discussed below, decay times for the instantons we are considering are much larger,  $O(\exp[10^9])$ . Thus, in spite of the fact that total migration will vary a great deal from model to model, the total decay time  $\Delta t_{TOT} = \Delta t_{decay}$  $+ \Delta t_M \approx \Delta t_{decay}$  is relatively unaffected.

We should note that the classical D3-brane migration followed by  $D3/\overline{D3}$  annihilation could leave a state with nega-

<sup>&</sup>lt;sup>7</sup>We assume forces due to objects in the bulk, such as D7-branes with gluino condensation [41], can be ignored. We thank S. Kachru and L. McAllister for discussion on this point.

Model	<i>p</i> <sub>1</sub> , <i>p</i> <sub>2</sub>	$K_1$	$M_1$	$K_2$	$M_2$	$z_1 \times 10^{17}$	$z_2 \times 10^5$	$\Delta\Lambda  imes 10^{31}$	$\Lambda  imes 10^{17}$
1	1,1	9	15	3	19	4.2	4.9	3.9	69
2	1,1	9	15	4	26	4.2	6.3	3.9	2.7
3	1,1	9	15	9	69	4.2	28	3.9	4.5
4	1,5	9	15	8	51	4.2	5.2	3.9	4.5
5	1,5	9	15	13	91	4.2	13	3.9	7.6

TABLE I. Models and cosmological constants.

tive cosmological constant. In cosmology, if the spatial slices have non-negative curvature, the FRW constraint equation means that the universe cannot actually transition to a negative cosmological constant. Instead, there is a Big Crunch singularity [43-45]. Though it is preferable to end in a dS state after the full decay, this is not necessary as long as the initial instanton has a lifetime much longer than the age of the universe. Note that the instanton, however, ends in a state of positive cosmological constant, so we avoid the concerns raised by [36].

### 2. Rolling radius

Now we should go back and examine the classical potential for  $\sigma$  that arises because z is away from its VEV. The behavior of the radial modulus in flux-generated potentials has been studied in an attempt to find inflationary behavior in [46]; we are in a different regime here because we do not take z to be slowly rolling. One point to address is that we cannot actually calculate the Kähler potential with warping for z excited because it is not clear if Eq. (9) would still hold as z changes. However, we will assume that it is valid since the starting and ending points of our evolution are vacuum states for some values of the flux K. The key point is that for instantons that go from dS to dS, the boundary conditions on  $\sigma$  mean it should not roll much, so the following discussion does not apply. What we are doing here is comparing instantons with different boundary conditions, one with  $\sigma$  unchanged in the final state and one with  $\sigma \rightarrow \infty$  in the final state.

We make the comparison as follows. The classical potential for  $\sigma$  and z naturally pushes  $\sigma$  to large radius as long as z is not in its vacuum state (note that this potential is extremely large compared to the KKLT potential (11), so we can ignore the KKLT potential here). We will make a very rough estimate of the change in  $\sigma$  while z rolls to its vacuum. If we believe that  $\sigma$  changes enough to get over the barrier of the KKLT potential before z reaches its vacuum and the classical potential vanishes, then we expect dS to Minkowski decays-mediated by NS5-branes-will dominate over dS to dS decays. This is because the classical evolution should have a lower action. Otherwise, the dS to dS decays will dominate, at least in the NS5-brane channel. We will not say anything else about these dS to Minkowski decays since they are less computationally tractable and are somewhat redundant with other decays to large radius.

Now we can roughly estimate the potential for  $\sigma$  and z. As in GKP [17], we work assuming small z, which implies that  $\partial_z W > (\partial_z \mathcal{K})W$ , so we will consider only the derivative of the superpotential. As before, the  $D_{\rho}W$  terms cancel with  $-3|W|^2$ . As a final approximation, we take only the leading terms of the Kähler metric for *z* small. Thus, we approximate the potential as

$$V = g_s^4 e^{-12u} (2\pi)^5 \frac{\alpha'^2}{\kappa_4^8 (g_s M)^2} \frac{|z|^{4/3}}{|\log|z|^2|} \\ \times \left| \frac{M}{2\pi} \log z + \frac{K}{g_s} \right|^2.$$
(30)

We have used

$$\mathcal{K}_{z\bar{z}} = -\frac{(g_s M)^2 \alpha'^3}{18\pi\kappa_4^6} |z|^{-4/3} \log|z|^2 \tag{31}$$

as the Kähler metric for z. This is singular at z=0, but our evolution never takes  $z \rightarrow 0$ .

To get a very rough estimate of the change in radial modulus u (remember that  $\sigma = e^{4u}/g_s$ ) while z changes, we approximate that the proper distance in the u direction of moduli space is proportional to the proper distance moved in the z direction of moduli space. The proportionality constant is given by the directional derivative (in the moduli space orthonormal frame) of the potential. Using the Kähler metric (31) to get the orthonormal frame, we find that

$$\Delta u \approx \frac{\nabla_{u}^{2} V}{\nabla_{z}^{2} V} \frac{\sqrt{\mathcal{K}_{zz}} \Delta z}{\sqrt{12}}$$
(32)

$$\Delta u \approx \frac{(g_s M)^2 \alpha'^3}{18\pi \kappa_4^6} |z|^{2/3} |\log |z|^2 |(e^{2\pi/g_s M} - 1)$$
(33)

up to factors of order unity. We have used  $\sqrt{\mathcal{K}_{zz}}\Delta z$  for the proper distance in the *z* direction. The factor of  $\sqrt{12}$  in Eq. (32) comes from the normalization of *u*.

Using the potential graphed in KKLT as a guide, we expect that  $\Delta u$  only needs to be  $\geq 0.1$  for the Minkowski decay to predominate, which is achieved by  $z \geq 10^{-3}$ . As it turns out, we will mainly be interested in cases with smaller *z*, so we will not consider the 5-brane mediated dS to Minkowski decays any further.

## 3. Thermal enhancements

Due to the fact that dS has a temperature, we might expect that the 5-branes that make up our instantons should have some nonzero entropy. Since the exponential of the entropy gives a density of states, the decay time should be reduced

Model	KPV: $(\tau/\tau_c)$	$\ln(t_{decay}^{KPV}) \times 10^{-9}$	KKLT: $(\tau/\tau_c)$	$\ln(t_{decay}^{KKLT}) \times 10^{-18}$	T2T: $(\tau/\tau_c)$	$\ln(t_{decay}^{T2T}) \times 10^{-18}$
1	0.163	0.66	1.8	0.32	24970	0.35
2	0.164	86	7.7	8.9	24257	8.9
3	0.164	40	5.9	5.2	16512	5.2
4	0.163	3.7	2.9	1.1	34054	1.1
5	0.164	18	4.6	3.1	33517	3.1

TABLE II. Tensions and decay times.

by a factor  $\exp[-S(NS5)]$ . This argument was first given in [14]. There it was argued that the brane instantons probably are out of thermal equilibrium with any matter or radiation in the cosmology, so they should have a temperature corresponding to the dS temperature. However, whether the temperature should be the initial dS temperature, final dS temperature, or the geometric mean was undetermined. It is now clear [47] that the brane has a well-defined temperature because it corresponds to accelerating observers in the two dS spacetimes.8

We will, however, neglect this effect. The bubble temperature is just the inverse radius,  $T = 1/(2\pi r)$  [47]. Therefore, the temperature is not high enough to excite the "Kaluza-Klein" modes of the bubble much, and the entropy would access only the zero-mode quantum mechanics. We expect that the enhancement factor would be relatively weak, therefore.

### V. CALCULATION OF DECAY TIMES

Throughout this paper, we have mainly discussed the KPV instantons as CDL thin-wall instantons. However, they contain an NS5-brane, which makes them also of the membrane class of instantons studied by [11,12]. In Appendix B, we demonstrate the equivalence of these two formalisms by showing that they give the same decay rate given initial and final cosmological constants and instanton tension.

Using the results of Sec. IV and Appendix B, we are able to calculate decay rates. For illustrative purposes let us first consider a model with a single KS throat. In particular, for 3 D3-branes sitting at the tip of a throat with K = 12, M = 87, one finds that the probability per unit volume for NS5-brane mediated decay is  $P \sim \exp(-10^{19})$ . Decays to decompactification are much faster,  $P \sim \exp(-10^{17})$ . We expect this to generally be the case for single throat models. Moreover, as discussed in [36], since all single throat decays will have  $\Lambda$ <0 in the final state, the instantons mediating these decays might not exist. It is for this reason we have chosen to focus on models with 2 KS throats, which, after the initial decay, have  $\Lambda > 0$ .

What follows is a discussion of the decay rates for several different two throat models. Table I shows the fluxes and number of D3-branes,  $p_i$ , in each throat. In each model the initial KPV instanton occurs in throat 1. This decay is driven by the notably small value of  $z_1$ , which makes the tension very small. Note that we have specifically chosen models where this is the case. The change in and resulting value of the cosmological constant ( $\Delta\Lambda$  and  $\Lambda_{-}$  respectively), due to KPV decay, are also given in Table I. The small value of  $z_1$ corresponds to small  $|\Delta\Lambda|$ , which would increase the decay time, but this effect is compensated by the small bubble tension. How the decay rate depends on these values is given explicitly by Eq. (B10). We list the tensions and decay times<sup>9</sup> for the KPV instantons in Table II, along with tensions and decay times for two other decay modes discussed in Sec. VI below. Note that the lifetimes for these models are  $\sim \exp(10^9)$ , where as the age of the universe (times the horizon volume) is  $\sim \exp(10^3)$ , so even the most anxious reader can now relax and enjoy the rest of the paper.

For each of the models discussed above, although the initial instanton decay yields a spacetime with positive cosmological constant, the ensuing D3-brane migration results in a negative cosmological constant, a situation which, as discussed in [43–45], ultimately leads to a Big Crunch singularity. Note, as previously mentioned, this is a classical process and thus avoids arguments given against instanton decays to negative  $\Lambda$  [36]. The total migration time, as shown in Appendix A, is negligible compared to the decay time<sup>10</sup> and will thus be ignored. We should also note that, although it seems difficult to find two throat models with a positive cosmological constant after D3/D3 annihilation, it should be possible to construct multiple (>2) throat models that end in dS.

## VI. COMPARISON TO OTHER DECAY MODES

The KPV instanton is just one of several avenues by which these dS vacua can decay. One particular mode, thoroughly studied in [6,7] and reviewed at the end of Sec. II, is tunneling to decompactification (in the CDL formalism). In these decays, or for any decay in which  $\Lambda_{-}=0$ ,  $\Delta S_{E}$  takes a particularly simple form,

$$\Delta S_E = -\frac{S_0}{(1+\tau_c^2/\tau^2)^2}.$$
(34)

<sup>&</sup>lt;sup>8</sup>We thank the authors of [47] for sharing their results with us prior to publication.

<sup>&</sup>lt;sup>9</sup>Note that these are the decay times for a unit volume, i.e.  $t_{decay} = P^{-1}$ . <sup>10</sup>Note, however, that it is long compared to the string scale,

 $O(10^{15}).$ 

For comparison purposes, we have calculated the CDL tensions and decay times  $(t_{decay}^{KKLT})$  for five models discussed above. These are also listed in Table II. Note that in each model the tensions are super-critical,  $\tau/\tau_c > 1$ . This will in fact always be true for decays to decompactification since,

$$\frac{\tau}{\tau_c} = \frac{1}{\sqrt{4V(\phi_+)/3}} \int_{\phi_+}^{\infty} d\phi \sqrt{2V(\phi)} \ge 1, \qquad (35)$$

for any  $V(\phi)$  whose barrier width (in string or Planck units for our normalization) is greater that  $\sqrt{2/3}$ . Noting that  $S_0$ <0, it is clear from Eq. (34) that the lifetime,  $t_{decay}$  $\sim \exp(-\Delta S_E)$ , increases with  $\tau/\tau_c$ . Though the story is more complicated when comparing to decays with  $\Lambda_{-} \neq 0$ , this will generally still be the case, and it is this fact which drives  $t_{decay}^{KKLT}$  to be much greater than  $t_{decay}^{KPV}$ . Take careful note that Table II lists the logs of the decay times. For these models  $t_{decay}^{KKLT}/t_{decay}^{KPV} \sim \exp(10^8)$ . These KPV instantons are, in technical terms, much much faster. It is possible to find super-critical KPV instantons in which  $t_{decay}^{KKLT} < t_{decay}^{KPV}$ . However, these require larger z in the decaying throat, leading to larger initial cosmological constant and slower decay times.

Another particularly simple decay mode occurs in models with multiple KS throats. The potential energy of a  $\overline{D3}$ -brane is proportional to  $h^{-1}(r)$ , the inverse warp factor given by Eq. (9), which is locally minimized at the tip of each throat. However, the energy is lower still at the tip of other throats with smaller z.  $\overline{D3}$ -branes can therefore tunnel from one throat to another. On the other hand,  $h \sim 1$  in the bulk, presenting a substantial potential barrier through which to tunnel. These instantons are similar to the glueball decays considered in [48,49].

As in previous examples, we consider models with two KS throats. The  $\overline{D3}$ -brane portion of the total potential is initially [cf. Eq. (18)]

$$\delta V = \frac{2\mu_3}{\sigma^3} h^{-1}(\tilde{r}_1) p_1 + \frac{2\mu_3}{\sigma^3} h^{-1}(\tilde{r}_2) p_2.$$
(36)

After the tunneling occurs, the form of  $\delta V$  is unchanged except for  $p_1 \rightarrow p_1 + 1$  and  $p_2 \rightarrow p_2 - 1$ . These decays have little effect on  $\sigma$ , and thus  $\sigma$  will be treated as a constant throughout this calculation.

To find the decay rate, we compute the instanton tension from the Euclidean brane action in the thin-wall limit using [32]:

$$\tau = (2 \pi \sqrt{\sigma})^{3/2} g_s^{1/4} \alpha' \int_{\tilde{r}_1}^{\tilde{r}_2} dr \ 2 \sqrt{\frac{\mu_3}{\sigma^3} [h^{-1}(r) - h^{-1}(\tilde{r}_1)]}.$$
(37)

The prefactor is from the conversion between *r* and a canonically normalized scalar in the 4D Einstein frame. Note that here we are using rescaled coordinates so  $e^{2u}$  does not appear in the metric (6). We then plug  $\tau$  and  $\Lambda_{\pm} = \kappa_4^2 (V + \delta V_{\pm})$  into Eq. (B10) to obtain the "throat-to-throat" decay

time  $(t_{decay}^{T2T})$ . Once again, these decay times and tensions are listed in Table II. Note, however, that these instantons tunnel to negative cosmological constant; while they would be ruled out by [36], they are not forbidden by the original calculation of [32]. As with the KKLT decays, the tensions are uniformly supercritical, and  $t_{decay}^{T2T}$  is remarkably similar to  $t_{decay}^{KKLT}$ . Indeed, we expect these decays to be super-critical because the conifold throats are long in string units, giving a wide potential barrier. Moreover, taking the limit  $\kappa_4^2 \tau \gg \Lambda_+$  in Eq. (B10), it is easy to see that,

$$(\Delta S_E)_{T2T} \approx \frac{24\pi^2}{\Lambda_+},\tag{38}$$

and thus from Eqs. (14) and (34), one can see that  $t_{decay}^{KKLT} \sim t_{decay}^{T2T}$ .

The reader should remember that we are working only in the thin-wall limit and that Hawking-Moss instantons can also give significant contributions to the decay rate. However, in the models we have described, the dS to dS decays are subcritical, so the Hawking-Moss contributions seem unlikely to change our qualitative results; KKLT found that Hawking-Moss instantons begin to dominate over thin-wall instantons only when  $\tau \sim \sqrt{V(\varphi_1)} > \tau_c$ .

The KPV instanton deals only with changes to fluxes and branes in Eq. (1). One might speculate about processes which could involve changes to  $\chi(X)$ , or the induced D3 charge on wrapped (p,q) 7-branes in the type IIB. From the F-theory viewpoint this would obviously involve topology change. While one could consider nontrivial D7-brane worldvolume gauge fields undergoing a small instanton transition and emitting D3-branes into the bulk, we know of no analog for nontrivial curvature on a four-cycle. A possible  $\chi$ changing instanton would involve D7-branes unwrapping a particular 4-cycle and wrapping a different one; however, these 4-cycles would be homologous unless the D7-brane can tear, so the induced D3-brane charge would remain the same. However, it may be interesting to explore whether  $\chi$ -changing mechanisms are possible.

### VII. CONCLUSIONS

The decays considered in this paper in a very real sense would represent the end of the universe for anyone unfortunate enough to experience it. Note, however, unlike the decays in CDL, even when  $D3/\overline{D3}$  annihilation following a KPV decay results in a Big Crunch, lifeforms might be capable of knowing joy for  $10^{-28}$  s while the D3-branes migrate across the compact manifold. We can all take comfort in the fact that even the fastest decays we consider have decay times incredibly greater than the age of our universe. Assuming that our calculations hold even approximately for a compactification with a realistic cosmological constant, we will have to worry about the death of the Sun long before the death of the universe.

Of interest, however, is the fact that we constructed decay modes other than the straightforward decay to decompactification discussed in [6,7,33]. In fact, we found it easy to construct NS5-brane mediated decays that occur much more



FIG. 3. Potential in throat 1 (model 1).

rapidly than the decompactification decays. We reiterate that the NS5-brane decays can have a subcritical tension. It is also noteworthy that the final state of many decays is not 10D Minkowski spacetime but is instead dS or a space with negative cosmological constant which ends in a Big Crunch. In fact, depending on the region of parameter space, we found that decays mediated by NS5-branes can end in dS, 10D Minkowski, or with negative cosmological constant, without considering other decay channels. The lesson is that, even in the KKLT models, there are many different metastable vacua and many different possible decay modes.

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#### **APPENDIX A: MIGRATION OF D3-BRANES**

The KPV decay leaves D3-branes at the tip of the KS throat where it occurs. In this appendix we analyze their subsequent classical motion in configurations with multiple KS throats. The D3-branes, produced by a decay in one throat (throat 1), are attracted by  $\overline{D3}$ -branes in another throat (throat 2), migrate across the compact manifold *M*, and eventually annihilate the  $\overline{D3}$ -branes. Here we work in the SUGRA limit to approximate the total migration time, in a two-throat geometry. As discussed in Sec. V, the KPV/CDL decay times are so large that the migration times have little effect. This appendix, therefore serves largely to show that one may, in fact, ignore the migration time and to illuminate how the migration itself proceeds.

It will be assumed that the back-reaction of the migrating D3-branes is negligible, the proper velocity of the branes remains small, and that the majority of the travel time comes

from the two throats (i.e. the time through the bulk of the CY may be ignored).

We turn our attention once more to the metric (6) (with the overall scale of the manifold scaled back in). In [30] it was shown that the  $F_3$  flux wrapped on the A-cycle smoothly deforms the tip of the conifold. Though we will find that the majority of the travel time comes from the tip of throat 1, we may ignore most of the details coming from the deformation of the conifold since the motion is assumed to be radial. We will use the undeformed metric and, when working near the tip, multiply the warp factor *h* by an overall constant ~0.4 to account for the deformation.<sup>11</sup> This has little effect on the final result, however it was such a trivial correction it seemed silly not to include it. The warp factor, away from the tip, is

$$h = \frac{L^4}{r^4} \ln(r/r_s) \tag{A1}$$

$$r_s = r_0 \exp\left(-\frac{2\pi(N+p)}{3g_s M^2} - \frac{1}{4}\right)$$
 (A2)

$$p \equiv \text{no. of } \overline{\text{D3}} - \text{branes}; \quad r_0^2 = 3/2^{5/3}.$$
 (A3)

Due to the deformation of the conifold discussed above we will only be interested in the region,  $\tilde{r} = r_s \exp(1/4) \le r \le r_0$ . Note that this avoids the naked singularity at  $r = r_s$ .

The action for the D3-branes is

$$S_{3} = -\frac{\mu_{3}}{g_{s}} \int d^{4}\xi \sqrt{-\tilde{G}} + \mu_{3} \int_{\Sigma(D3)} \tilde{C}_{4}.$$
 (A4)

The Ramond-Ramond potential  $\tilde{C}_4$  depends only on the radial distance *r*:

$$\tilde{C}_4 = \frac{f(r)}{g_s} dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \tag{A5}$$

where  $t \equiv x^0$ . Working in the gauge  $\xi^0 = \tau(t)$  and  $\xi^i = x^i$ , the Lagrangian becomes,

$$\mu_3^{-1}\mathcal{L} = -\frac{h^{-1}}{g_s}(\sqrt{\dot{t}^2 - h\dot{r}^2}) + f(r)\dot{t}.$$
 (A6)

Assuming that the proper velocity is small,

$$\mu_3^{-1} \mathcal{L} \simeq \frac{1}{2} \frac{\dot{r}^2 \dot{t}^{-1}}{g_s} - \frac{[h^{-1} - g_s f(r)]}{g_s} \dot{t}.$$
 (A7)

It is easy to check that this a valid approximation. In particular, one only needs to consider throat 2, since this is where the D3-branes are moving fastest. One can show that  $p \ll (g_s M^2)/8$  will insure that Eq. (A7) is valid.

<sup>&</sup>lt;sup>11</sup>One finds this correction by comparing the "near tip" warp factor found in [30] to the naive limit of the undeformed Klebanov-Tseytlin geometry.

Since  $t(\tau)$  is a cyclic variable, we know that  $\partial \mathcal{L}/\partial \dot{t} = -E/g_s$  is constant, leaving us with

$$E = \frac{1}{2} \left( \frac{\partial r}{\partial t} \right)^2 + V(r); \quad V(r) \equiv [h^{-1} - f(r)], \quad (A8)$$

and the travel time through a single throat is therefore,

$$\Delta t = \pm \int_{r_0}^{\tilde{r}} \frac{dr}{\sqrt{2[E - V(r)]}}.$$
 (A9)

The +/- corresponds to branes traveling into/out of the throat.

Note that the time here is a coordinate time, but we will see that it is so small compared to decay times that we do not need to worry about conversion to proper time in the 4D Einstein frame.

## 1. Throat 1

The D3-branes, produced at rest in throat 1, feel a slight gravitational attraction to the  $\overline{\text{D3}}$ -branes at the bottom of throat 2. Since the gravitational attraction is weakest while in throat 1, one suspects the majority of the migration time it comes from throat 1. We show below that, in fact, throat 2 can be ignored completely. This also justifies not including the travel time through the bulk of the CY.

The Ramond-Ramond potential,  $\tilde{C}_4 = h_1^{-1}/g_s$ , is not affected by charge contained in throat 2. Physically, this is due to the charge being screened; mathematically, we are working on a compact manifold and may consider just the charge enclosed in throat 1. However, the geometry does, albeit slightly, know about what is happening in throat 2.

In order to get an estimate of the migration time we will make the (perhaps bold) assumption that, as with most multipole solutions in gravity, the warp factor is changed by an additive factor,

$$h_1(r) \rightarrow h_1(r) + \delta(r);$$
  
$$\delta(r) \equiv -\frac{27\pi\alpha'}{(2r_0 - r)^4} (N_2 + p + 3g_s M^2 / 8\pi).$$
  
(A10)

Here, the subscripts indicate the throat in which the fluxes are contained, p is the number of D3-branes in throat 2 (there are no D3-branes in throat 1). Expanding to first order in  $\delta$ , we see that

$$V(r) = -\frac{27\pi\alpha'}{(2r_0 - r)^4} (N_2 + p + 3g_s M^2 / 8\pi) h_1^{-2}(r).$$
(A11)

Figure 3 shows the potential. Note that the majority of the time will be spent at the tip of this throat. We have been unable to evaluate the integral (A9) explicitly. Numerical methods also proved difficult, due to the singular behavior of the integrand as  $r \rightarrow \tilde{r}$ . This results from the fact that the



FIG. 4. Potential in throat 2 (model 1).

D3-branes are produced at rest. However, one may gain control of the situation by linearizing the potential near  $r = \tilde{r}$  and integrating away from the problematic singular point. It is in this limit that we multiply  $h_1$  by the numerical factor ~0.4 discussed above. Once a safe distance away from  $r = \tilde{r}$ , which for technical reasons coming from the linearization of Eq. (A11) we take to be  $r \sim \tilde{r} + zr_0$ , one may evaluate the rest of the integral (A9) numerically. For model 1, one finds that

$$\Delta t_1 = 1.5 \times 10^{16}.$$
 (A12)

#### 2. Throat 2

For completeness, we will determine time spent in throat 2, subsequently showing that it is of no importance. In order to deduce  $\tilde{C}_4$ , recall that  $\int \star d\tilde{C}_4 \sim (N_{eff} - p)$ . Plugging in the appropriate constants, this leaves us with

$$r^{5}h^{2}\partial_{r}f = (27\pi\alpha'^{2})(N_{eff}-p).$$
 (A13)

Using what we know from the KS geometry, we define  $f = h^{-1} + V$ , where the potential, V(r), satisfies

$$\partial_r V = \frac{(27\pi\alpha'^2 g_s)p}{L^5} \frac{r^3/L^3}{[\ln(r/r_{s+})]}.$$
 (A14)

This can be integrated and gives a solution depending on exponential-integral functions. The potential is shown in Fig. 4. Numerically integrating Eq. (A9) for throat 2 gives

$$\Delta t_2 \approx 9.9, \tag{A15}$$

which is clearly negligible compared to Eq. (A12).

# APPENDIX B: INCLUSION OF GRAVITY IN BUBBLE NUCLEATION

In recent literature, there has been some confusion concerning the relation of two formalisms for studying thin-wall instantons. The method of Coleman and De Luccia (CDL) [32,36] describes smooth instantons in the limit of large radius of curvature; this formalism was used by [6] to argue that bubbles will always nucleate in a dS background before the recurrence time. Alternately, however, we could imagine that the bubble wall is truly an infinitesimally thin membrane, such as a D-brane, with a delta function stress tensor. This type of configuration was studied by Brown and Teitelboim (BT) [11,12]. In a description of dS solutions in noncritical string theory [8], Ref. [15] uses the BT formalism to argue that there is a critical tension above which the bubble occupies more than half the original de Sitter sphere and above which the decay time changes behavior as a function of the bubble tension. Additionally, [15] claims that the decay time is of order the recurrence time at the critical tension.

In this appendix, we show that the CDL and BT formalisms actually agree; this is reassuring, since even D-branes should be described as smooth objects in a complete version of string theory. Our results show that the decay time is always less than the recurrence time, as in [5-7,33]. Additionally, we confirm that more than half the original de Sitter sphere decays above the critical tension, but we show that (due to some technical considerations) the decay time is actually a smooth function of the bubble tension. We work in a 4D effective theory throughout.

We begin by describing the two formalisms. In both CDL and BT, the nucleation/decay time is given by exponentiating the difference of the (Euclidean) bubble and background actions. Therefore, the exterior of the bubble, which is approximated by the background, contributes nothing in both formalisms.

At that point, CDL note that, since both bubble and background have the same behavior at infinity, they can integrate some terms in the Ricci tensor by parts. After determining the bubble tension  $\tau$  as a functional of the potential, they find that the action is

$$\Delta S_{E} = 2 \pi^{2} r^{3} \tau + \frac{12 \pi^{2}}{\kappa_{4}^{2}} \left\{ \frac{1}{\Lambda_{-}} \left[ \left( 1 - \frac{\Lambda_{-}}{3} r^{2} \right)^{3/2} - 1 \right] - \frac{1}{\Lambda_{+}} \left[ \left( 1 - \frac{\Lambda_{+}}{3} r^{2} \right)^{3/2} - 1 \right] \right\}$$
(B1)

where  $\Lambda_{\pm} = \kappa_4^2 V_{\pm}$  are the potential outside and inside the bubble respectively [this is a combination of Eqs. (3.11) and (3.13) from [32]]. Minimizing this action with respect to the bubble curvature radius *r* gives the decay rate.

On the other hand, BT cannot use the same integration by parts because the infinite stress of the bubble wall separates the interior and exterior regions. Instead, the bubble action must include extrinsic curvature terms; it is these terms that will explain the apparent contradiction between CDL and BT formalisms. The extrinsic curvatures of the interior and exterior regions are

$$K_{\pm} = -3 \sigma_{\pm} \left( \frac{1}{r^2} - \frac{\Lambda_{\pm}}{3} \right)^{1/2}$$
 (B2)

where  $\sigma_{\pm} = 1$  if the radius of curvature of the outside/inside region of the bubble is increasing toward the exterior of the

bubble and is -1 if the radius is decreasing. Since the Ricci scalar in the bubble is given by the cosmological constant, the action just becomes

$$\Delta S_E = 2 \pi^2 \tau r^3 + \frac{2 \pi^2}{\kappa_4^2} r^3 (K_- - K_+) - \frac{1}{\kappa_4^2} (\Lambda_- \mathcal{V}_- - \Lambda_+ \mathcal{V}_+).$$
(B3)

The interior volumes for the bubble and background are given by (for either sign of the cosmological constant)

$$\mathcal{V}_{\pm} = 2 \,\pi^2 \left(\frac{3}{\Lambda_{\pm}}\right)^2 \left\{ \frac{1}{3} \left[ \sigma_{\pm} \left( 1 - \frac{\Lambda_{\pm}}{3} r^2 \right)^{3/2} - 1 \right] - \left[ \sigma_{\pm} \left( 1 - \frac{\Lambda_{\pm}}{3} r^2 \right)^{1/2} - 1 \right] \right\}.$$
(B4)

It is algebraically simple to see that the extrinsic curvature terms combine with the square root terms from the volume to give exactly the CDL action (B1) up to the signs  $\sigma_{\pm}$ . The reason [15] found a different action is that they omitted the extrinsic curvature terms.

Now we should see why the CDL result should actually have the signs  $\sigma_{\pm}$ . For a de Sitter background, the terms in square brackets of Eq. (B1) come from integrals

$$\int_{0}^{\xi(r)} d\xi r \left( 1 - \frac{\Lambda_{\pm}}{3} r^2 \right), \tag{B5}$$

which CDL evaluate by replacing  $d\xi = dr(1 - r^2 \Lambda_{\pm}/3)^{-1/2}$ . However, as they note,  $r = \sqrt{3/\Lambda_{\pm}} \sin[\sqrt{\Lambda_{\pm}/3}\xi]$ , so  $\xi(r)$  is double-valued. In fact, the correct integral is

$$\frac{3}{\Lambda_{\pm}} \int_{\sigma_{\pm}(1-r^2\Lambda_{\pm}/3)^{1/2}}^{1} dy y^2$$
(B6)

which just introduces a factor of  $\sigma_{\pm}$  in the  $(\cdots)^{3/2}$  terms. This precisely agrees with the BT results. In this paper, we will be concerned only with the case  $\sigma_{-}=1$ , and  $\sigma_{+}=-1$  only for  $\Lambda_{+}>0$  and tension above critical.

To find the radius of the bubble given the two cosmological constants and the tension, we could minimize the action with respect to *r*. However, it is easier to use the Israel matching condition across the bubble wall, which has trace  $K_+ - K_- = (3/2) \kappa_4^2 \tau$ . The answer is given by [15] and can be written as

$$\frac{1}{r^2} = \left(\frac{\kappa_4^2 \tau}{4}\right)^2 + \frac{\overline{\Lambda}}{3} + \left(\frac{\Delta \Lambda}{3\kappa_4^2 \tau}\right)^2,$$
$$\overline{\Lambda} = \frac{\Lambda_+ + \Lambda_-}{2}, \ \Delta \Lambda = \Lambda_- - \Lambda_+.$$
(B7)

It is tedious but straightforward to check that this matches the result from minimizing the action. The maximum radius occurs at critical tension

$$\kappa_4^2 \tau_c = \left(\frac{4}{3} |\Delta\Lambda|\right)^{1/2} \tag{B8}$$

and is  $1/r^2 = \Lambda_+/3 = 1/R_{dS}^2$  for positive initial cosmological constant. Note that for vanishing initial vacuum energy, gravity stabilizes the false vacuum for tension bigger than critical, as in [32]. Also, for negative initial cosmological constant, the radius becomes infinite for tensions lower than critical. We will concern ourselves only with initial de Sitter spacetimes, so we do not face some of the concerns raised by [36] about decays of Minkowski and AdS spacetimes.

We will finally write down the action for the bubbles:

$$\Delta S_E = 2 \pi^2 r^3 \left( \tau + \frac{6}{\kappa_4^2 \Lambda_+ \Lambda_-} \left\{ \frac{\Delta \Lambda}{r^3} + \Lambda_+ \left[ \left( \frac{\kappa_4^2 \tau}{4} \right)^2 - \frac{\Delta \Lambda}{6} + \left( \frac{\Delta \Lambda}{3 \kappa_4^2 \tau} \right)^2 \right]^{3/2} - \sigma_+ \Lambda_- \left[ \left( \frac{\kappa_4^2 \tau}{4} \right)^2 + \frac{\Delta \Lambda}{6} + \left( \frac{\Delta \Lambda}{3 \kappa_4^2 \tau} \right)^2 \right]^{3/2} \right\} \right)$$
(B9)

$$= \frac{2\pi^{2}}{\left[\left(\frac{\kappa_{4}^{2}\tau}{4}\right)^{2} + \frac{\Lambda_{-} - \Delta\Lambda/2}{3} + \left(\frac{\Delta\Lambda}{3\kappa_{4}^{2}\tau}\right)^{2}\right]^{3/2}} \times \left(\frac{\tau + \frac{6}{\kappa_{4}^{2}\left[\left(\Lambda_{-} - \frac{\Delta\Lambda}{2}\right)^{2} - \frac{\Delta\Lambda^{2}}{4}\right]}}{\kappa_{4}^{2}\left[\left(\Lambda_{-} - \frac{\Delta\Lambda}{2}\right)^{2} - \frac{\Delta\Lambda^{2}}{4}\right]}\right] \left\{\Delta\Lambda \left[\left(\frac{\kappa_{4}^{2}\tau}{4}\right)^{2} + \frac{\Lambda_{-} - \frac{\Delta\Lambda}{2}}{3} + \left(\frac{\Delta\Lambda}{3\kappa_{4}^{2}\tau}\right)^{2}\right]^{3/2} + (\Lambda_{-}\Delta\Lambda) \times \left(\frac{\kappa_{4}^{2}\tau}{4} - \frac{\Delta\Lambda}{3\kappa_{4}^{2}\tau}\right)^{3} + \Lambda_{-}\left(\frac{\kappa_{4}^{2}\tau}{4} + \frac{\Delta\Lambda}{3\kappa_{4}^{2}\tau}\right)^{3}\right\}\right).$$
(B10)

While this is a mess, the reader should note that the sign of the last term is independent of the tension. That is because, for  $\tau < \tau_c$ ,  $\sigma_+ = 1$  but the quantity in the square brackets in the last term of Eq. (B9) is the square of a negative number, so the square root introduces a sign. For supercritical tension, that quantity is the square of a positive number, but then  $\sigma_+ = -1$ . We have chosen to write the variables in this form in order to illuminate the dependence on the level spacing. Please see Fig. 5 for the qualitative features of the action  $\Delta S$ as a function of  $\Lambda_-$ ,  $\Delta \Lambda$ . While in some ways the physics depends more directly on the initial cosmological constant  $\Lambda_+$ , in this paper we typically work with a fixed final  $\Lambda_-$ , and  $\Delta \Lambda$  depends on the same moduli that control the bubble tension.

We should note that this action reduces to the known formulas in special cases. In particular, the result of CDL as quoted in KKLT,

$$\Delta S_E = -\frac{S_0}{(1 + \tau_c^2 / \tau^2)^2}$$
(B11)

is valid for all tensions when the final vacuum energy vanishes. A related result is that, for any  $\Lambda_{\pm} \ge 0$ , as the bubble tension goes to infinity, the decay time goes to the recurrence time of the original dS.

As final comments, let us reemphasize, following [32,36], that the final states are not maximally symmetric spacetimes but rather cosmological ones. In particular, decays with a negative final cosmological constant lead not to AdS but to a Big Crunch singularity within the bubble. Additionally, as mentioned in [36], these instantons are technically different from instanton decays of inflationary spacetimes. It seems reasonable that for sufficiently small decay rates treating the initial spacetime as dS is a good approximation, but it remains an interesting problem to study decays of possibly more cosmologically relevant spacetimes.



FIG. 5. The bubble minus background action as a function of the cosmological constants. The variables are  $S = \kappa_4^4 \tau^2 \Delta S$ ,  $L = \Lambda_- / \kappa_4^4 \tau^2$ , and  $dL = \Delta \Lambda / \kappa_4^4 \tau^2$ .

### FALL OF STRINGY de SITTER SPACETIME

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