Twisted moduli stabilization in type I string models

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We consider a model with dilaton and twisted moduli fields, which is inspired by type I string models. The stabilization of their vacuum expectation values is studied. We find that the stabilization of the twisted moduli field has different aspects from dilaton stabilization.

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I. INTRODUCTION

Superstring theory is a promising candidate for a unified theory including gravity. It has no free parameter, and the gauge couplings, Yukawa couplings, and other couplings of low energy effective field theory are determined by vacuum expectation values (VEVs) of dilaton or moduli fields. Thus, it is important to stabilize these VEVs. Indeed, several stabilization mechanisms have been proposed.

The dilaton or moduli fields have no potential perturbatively. Only nonperturbative effects lift their potential. Gaugino condensations are nonperturbative effects that could are plausibly generate a nonperturbative superpotential of dilaton or moduli fields. However, one cannot stabilize the VEV of the dilaton field to a fine value in the model with a superpotential generated by a single gaugino condensation and a tree-level Kähler potential. One of the simple extensions is the model with double gaugino condensation and a tree-level Kähler potential, i.e., the so-called racetrack model [1–6], while a nonperturbative Kähler potential has also been considered [7–9]. In fact, one can stabilize the VEV of the dilaton field to a finite value depending on the beta function coefficients of the gauge couplings relevant to gaugino condensation.

Twisted moduli fields appear in orbifold or orientifold models. These are localized at fixed points. In type I models, the twisted moduli fields are gauge singlets, while they are charged in heterotic models. The gauge kinetic functions depend on twisted moduli in type I models [10,11]. They play a role in 4D Green-Schwarz anomaly cancellation, e.g., for anomalous $U(1)$ [12,13], while the dilaton field plays the same role in heterotic models [14].[1] Thus, their VEVs determine the magnitude of the Fayet-Iliopoulos terms. The prediction of the gauge couplings depends on the VEVs of the twisted moduli fields. The mirage unification of gauge couplings is one possibility to explain the experimental values of gauge couplings with a lower string scale [16]. Hence, the magnitude of twisted moduli field VEVs is phenomenologically important.

In this paper, we consider a model with dilaton and twisted moduli fields that is inspired by type I string models, and study the stabilization of dilaton and twisted moduli fields. For a similar purpose, models with twisted moduli fields were studied in Refs. [17,18]. The Kähler potential of the twisted moduli fields is not clear. Here we will use the assumption of the canonical form, which was studied in Ref. [19],[2] and show that this form is important to stabilize the VEV of twisted moduli. As another example, we will assume the logarithmic form of the Kähler potential the twisted moduli fields like the dilaton and other moduli fields. That is an example of a Kähler potential that has a different behavior from the canonical form. However, we will show that even in the case with the logarithmic Kähler potential the positive exponent in the nonperturbative superpotential is useful for the stabilization of twisted moduli fields.

This paper is organized as follows. In the next section, we briefly review the stabilization of the dilaton VEV in the racetrack model. In Sec. III, we study the model with dilaton and twisted moduli fields. In Sec. III A, we briefly discuss the twisted moduli fields. In Sec. III B we consider the single gaugino condensation model and show how different the stabilization of twisted moduli fields is from the dilaton stabilization. In Sec. III C we consider a specific double gaugino condensation model in order to study the simultaneous stabilization of the dilaton and twisted moduli fields. In Sec. III D we give a comment on the effects of twisted moduli fields on the dilaton VEV. Section IV is devoted to a conclusion and discussion.

II. THE RACETRACK MODEL

The tree-level Kähler potential of the dilaton field is obtained as

$$K = -\ln(S+\bar{S}). \tag{1}$$

The gauge kinetic function of heterotic models is obtained as

$$f = S \tag{2}$$

up to the Kac-Moody level, and the gauge coupling g is obtained as $\mathrm{Re}(S) = 1/g^2$. This is the same for the gauge multiplets originating from D9-branes in type I models. Perturbatively, the dilaton field has a flat potential. Single gaugino condensation induces the nonperturbative superpotential

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[1]See also Ref. [15] for anomalous $U(1)$ in heterotic models.

[2]See also Ref. [20].

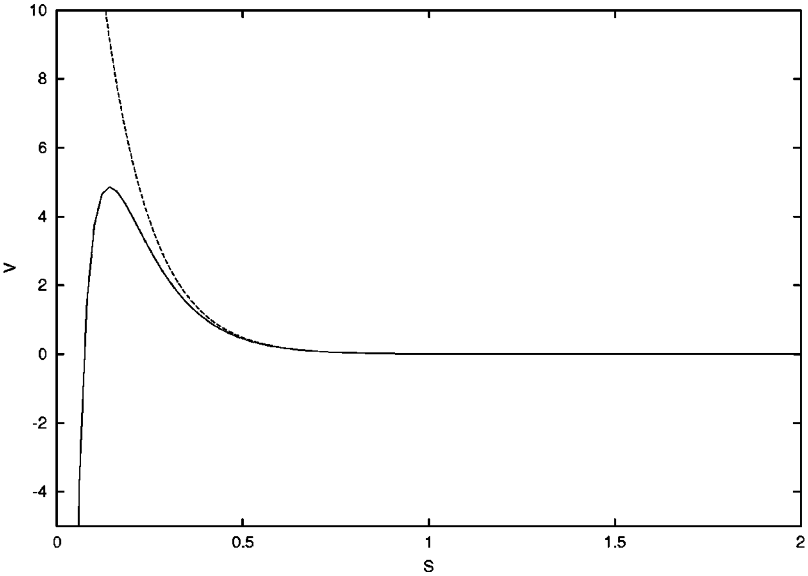

FIG. 1. The lower line corresponds to the scalar potential for $\Delta=10$ and $d=1$ without $\hat{W}(T)$. The upper line corresponds to $(T+\bar{T})^3 V/|\hat{W}(T)|^2$ for $\Delta=10$, $d=1$, and $g(T,\bar{T})=-0.5$.

$$W=de^{-\Delta S}, \tag{3}$$

where d is a constant, $\Delta=-24\pi^2/b$, and b is the one-loop beta function coefficient, e.g., $b=-3N_c$ for pure $N=1$ $SU(N_c)$ Yang-Mills theory. With the above Kähler potential, the scalar potential V is written as

$$V=\frac{1}{S+\bar{S}}[|(S+\bar{S})W_S-W|^2-3|W|^2], \tag{4}$$

where W_S denotes the first derivative of W with respect to S, i.e., $W_S=\partial W/\partial S$. Here we have not taken into account the D terms, although S has a D-term potential in heterotic models if the model has anomalous $U(1)$.[3] We have the following solutions for $\partial V/\partial S=0$:

$$(S+\bar{S})W_S-W=0, \tag{5}$$

or

$$(S+\bar{S})^2 W_{SS}=2\bar{W}\frac{(S+\bar{S})W_S-W}{(S+\bar{S})\bar{W}_S-\bar{W}}. \tag{6}$$

With the single gaugino condensation superpotential (3), the solution (5) leads to $S+\bar{S}=-1/\Delta$, which is not a realistic VEV for S in the asymptotically free case. The solution (6) leads to $\Delta(S+\bar{S})=\sqrt{2}$, but this corresponds to the maximum point of V. See Fig. 1, where the lower line shows the scalar potential against $s\equiv S+\bar{S}$ in the case with $\Delta=10$ and $d=1$.

In heterotic models, the requirement of $SL(2,Z)$ duality invariance of the overall moduli field T leads to the following superpotential [21–23]:

$$W=de^{-\Delta S}\hat{W}(T). \tag{7}$$

The corresponding scalar potential is written as

$$V=\frac{|e^{-\Delta S}|^2|\hat{W}(T)|^2}{(S+\bar{S})(T+\bar{T})^3}\{[(S+\bar{S})\Delta+1]^2+g(T,\bar{T}]\}, \tag{8}$$

with

$$g(T,\bar{T})\equiv\frac{1}{3}\left|(T+\bar{T})\frac{W_T}{W}-3\right|^2-3. \tag{9}$$

Here we have used the Kähler potential of T as

$$-3\ln(T+\bar{T}). \tag{10}$$

However, the inclusion of $\hat{W}(T)$ does not help the stabilization of S. If $g(T,\hat{T})<-1$, the situation is the same as in the case without $\hat{W}(T)$. If $g(T,\hat{T})>-1$, the scalar potential monotonically decreases with s. The upper line in Fig. 1 shows $(T+\bar{T})^3 V/|\hat{W}(T)|^2$ for $\Delta=10$, $d=1$, and $g(T,\bar{T})=-0.5$.

One mechanism to stabilize the VEV of S is to consider the superpotential with double gaugino condensations,

$$W=d_1 e^{-\Delta_1 S}+d_2 e^{-\Delta_2 S}. \tag{11}$$

With this superpotential, the solution (5) of $\partial V/\partial S=0$ is given as

$$\mathrm{Im}(S)=\frac{\pi}{\Delta_1-\Delta_2}(2n+1), \tag{12}$$

$$\mathrm{Re}(S)=\frac{1}{\Delta_1-\Delta_2}\ln\frac{[1+2\Delta_1\mathrm{Re}(S)]d_1}{[1+2\Delta_2\mathrm{Re}(S)]d_2}. \tag{13}$$

If $\Delta_a\mathrm{Re}(S)\gg 1$, the latter equation becomes the simple equation

[3]In this case, the dilaton field S is relevant to Green-Schwarz anomaly cancellation.

$$\mathrm{Re}(S)=\frac{1}{\Delta_1-\Delta_2}\ln\frac{\Delta_1 d_1}{\Delta_2 d_2}. \tag{14}$$

In the case with $\ln(\Delta_1 d_1/\Delta_2 d_2)=O(1)$, the stabilized value of $\mathrm{Re}(S)$ is determined by $1/(\Delta_1-\Delta_2)$. Thus, the natural order of S is $O(1/\Delta)$. If Δ_1 and Δ_2 are close to each other, the VEV of S is enhanced. For example, one can obtain $O(1)$ of S in two cases (i) the case with large beta-function coefficients[4] $b_i=O(100)$ and (ii) the case with fine-tuning values (Δ_1,Δ_2). For the latter case, we have $\mathrm{Re}(S)=1$, e.g., for $(\Delta_2-\Delta_1)/\Delta_1=0.04$ and $b_1=10$.

III. STABILIZATION IN MODEL WITH TWISTED MODULI

A. Twisted moduli

Twisted moduli fields M are localized at orbifold fixed points and these moduli fields are important from several phenomenological viewpoints in 4D models obtained from type I and type II orientifold models. For example, the gauge kinetic functions corresponding to gauge groups originating from D9-branes are written as

$$f_a=S+\sigma_a M, \tag{15}$$

where σ_a is a model-dependent constant [12,25]. Concerning σ_a, here we take the purely phenomenological standpoint, that is, we treat σ_a as free parameters. Similarly, for the gauge groups originating from, e.g., D5-branes, which are wrapped on the ith torus $(i=1,2,3)$, the corresponding gauge kinetic functions are written as

$$f_{5a}=T_i+\sigma_{5a}M, \tag{16}$$

where T_i is the moduli field corresponding to the ith torus and its Kähler potential is obtained as

$$K(T_i,\bar{T}_i)=-\ln(T_i+\bar{T}_i), \tag{17}$$

that is, its form is exactly the same as the Kähler potential of the dilaton field (1). Thus, we can discuss the stabilization of T_i due to gaugino condensation from D5-originating gauge groups in the same way as the stabilization of S due to condensation from D9 gaugino fields. Here, we concentrate on the S stabilization.

One of the important aspects is that the twisted moduli field M plays a role in the 4D Green-Schwarz anomaly cancellation mechanism. For example, under anomalous $U(1)$ symmetry, the twisted moduli fields are assumed to transform at the one-loop level:

$$M\rightarrow M+i\delta_{GS}\Lambda, \tag{18}$$

with the transformation parameter Λ. The Fayet-Iliopoulos term is written by the first derivative of the Kähler potential $\partial K(M,\bar{M})/\partial M$, where $K(M,\bar{M})$ is the Kähler potential of the twisted moduli field. Thus, the magnitude of the Fayet-Iliopoulos term is determined by the VEV of M.

Unfortunately, the Kähler potential of M, $K(M,\bar{M})$, is still unclear. In the limit $M\rightarrow 0$, the Kähler metric has no singularity. Hence, the Kähler potential $K(M,\bar{M})$ could be expanded as

$$K(M,\bar{M})=\frac{1}{2}(M+\bar{M})^2+\cdots. \tag{19}$$

Actually, this form has been studied in Ref. [19]. Thus, we use the assumption of the Kähler potential as $K(M,\bar{M})=\frac{1}{2}(M+\bar{M})^2$ in one-half of our analyses. However, since its reliability for $M=O(1)$ may be unclear, we assume[5]

$$K(M,\bar{M})=-\ln(M+\bar{M}) \tag{20}$$

as a trial form of the Kähler potential for $M\geqslant O(1)$.

We also give comments on the gauge coupling unification. Within the framework of the minimal supersymmetric standard model (MSSM), three gauge couplings of $SU(3)\times SU(2)\times U(1)_Y$ meet around $M_X=2\times 10^{16}$ GeV. Suppose that the three gauge groups originate from different sets of D9-branes. If one can stabilize $\mathrm{Re}(S)\gg\sigma_a\mathrm{Re}(M)$, the gauge couplings are universal at the string scale M_s. That implies $M_s\approx M_X$. Otherwise, if $\sigma_a\mathrm{Re}(M)$ is sizable, the gauge couplings are, in general, nonuniversal at M_s. However, one of the interesting possibilities to explain the experimental values of the gauge couplings is the so-called mirage unification [16]. The MSSM gauge coupling at μ is obtained as

$$\frac{1}{g_a^2(\mu)}=S+\sigma_a^{MSSM}M+\frac{b_a^{MSSM}}{16\pi^2}\ln\frac{M_s^2}{\mu^2}, \tag{21}$$

where b_a^{MSSM} are the one-loop beta-function coefficients for the MSSM. Let us consider a specific model where the constants σ_a^{MSSM} are proportional to b_a^{MSSM}. In this scenario, the gauge couplings are nonuniversal at M_s, but its prediction is the same as the universal gauge coupling around M_X. The string scale M_s can be low depending on $\sigma_a^{MSSM}M$. Note that even a small value of $\mathrm{Re}(M)$ such as $\sigma_a^{MSSM}M=O(0.01)$ is important. If the ratio of M_s to M_X satisfies

$$\log_{10}\frac{M_s}{M_X}\sim\frac{\sigma_a^{MSSM}\mathrm{Re}(M)}{0.03}, \tag{22}$$

that leads to MSSM gauge couplings consistent with the experimental values.

Thus, it is important to study the stabilization of the twisted moduli field M. That is the issue we will study in the following sections. We will also discuss how the twisted moduli field M affects the stabilization of the dilaton field S.

[4]In Ref. [24] large beta-function coefficients are studied from the viewpoint of F theory.

[5]We would like to thank Kiwoon Choi for suggesting this point.

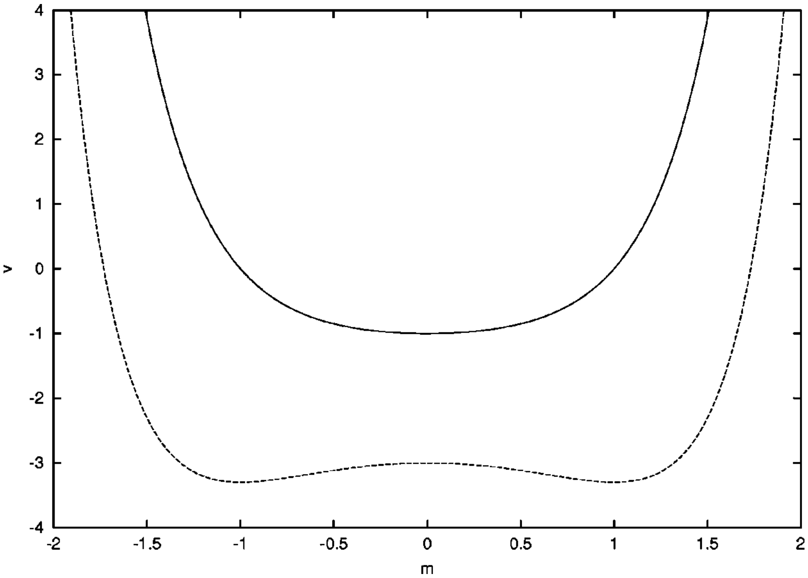

FIG. 2. The upper and lower lines show $v \equiv e^{m^2/2}[m^2+g(S+\overline{S})]$ for $g(S+\overline{S})=-1$ and -3, respectively.

B. Single gaugino condensation

Here we study the case with single gaugino condensation, although one cannot stabilize the dilaton field with single gaugino condensation as seen in Sec. II. That will be useful for later discussions. The Kähler potential is written as

$$K=-\ln(S+\overline{S})+K(M,\overline{M}), \tag{23}$$

and the superpotential due to the gaugino condensation is obtained as

$$W=de^{-\Delta(S+\sigma M)}. \tag{24}$$

Using the Kähler potential and the superpotential, we can write the scalar potential as

$$V=\frac{e^{K(M,\overline{M})}}{S+\overline{S}}\left[(K^{-1})^{M}_{\overline{M}}\left|\frac{\partial K(M,\overline{M})}{\partial M}W-W_M\right|^2 + |(S+\overline{S})W_S-W|^2-3|W|^2\right], \tag{25}$$

where $(K^{-1})^{M}_{\overline{M}}$ denotes the inverse of the Kähler metric for M and $\overline{M}$. Again, we do not take into account D terms. Inclusion of D terms will be studied elsewhere. For this scalar potential, one of the solutions to the stationary condition $\partial V/\partial M=0$ is

$$\left(\frac{\partial K(M,\overline{M})}{\partial M}-\Delta\sigma\right)W=0, \tag{26}$$

that is, $\partial K(M,\overline{M})/\partial M=\Delta\sigma$ is one solution.

1. The case with $K=\frac{1}{2}(M+\overline{M})^2$

To be concrete, we use the assumption of the Kähler potential $K=\frac{1}{2}(M+\overline{M})^2$. In this case, it is convenient to define m as

$$m\equiv M+\overline{M}-\Delta\sigma. \tag{27}$$

Then the scalar potential is written as

$$V=\frac{e^{-\Delta(S+\overline{S})-\sigma^2\Delta^2/2}}{S+\overline{S}}e^{m^2/2}[m^2+g(S+\overline{S})], \tag{28}$$

where

$$g(S+\overline{S})\equiv\left|(S+\overline{S})\frac{W_S}{W}-1\right|^2-3. \tag{29}$$

For single gaugino condensation, we have

$$g(S+\overline{S})=(\Delta(S+\overline{S})+1)^2-3. \tag{30}$$

The solutions of the stationary condition $\partial V/\partial m=0$ are obtained as follows:

$$m=0, \quad m=\pm\sqrt{-2-g(S+\overline{S})}. \tag{31}$$

The former solution corresponds to Eq. (26). The latter solutions are allowed only if

$$2+g(S+\overline{S})<0. \tag{32}$$

By the definition (30), this inequality is never satisfied for $(S+\overline{S})>0$. We have $\partial^2 V/\partial m^2>0$ for the former solution $m=0$ if

$$2+g(S+\overline{S})>0. \tag{33}$$

By the definition (30), this inequality is always satisfied for $(S+\overline{S})>0$. In addition, for the latter solution, we always have $\partial^2 V/\partial m^2>0$ if the solution is realized, i.e., $2+g(S+\overline{S})<0$. In Fig. 2 the upper and lower lines show $v\equiv e^{m^2/2}[m^2+g(S+\overline{S})]$ for $g(S+\overline{S})=-1$ and -3, respectively.

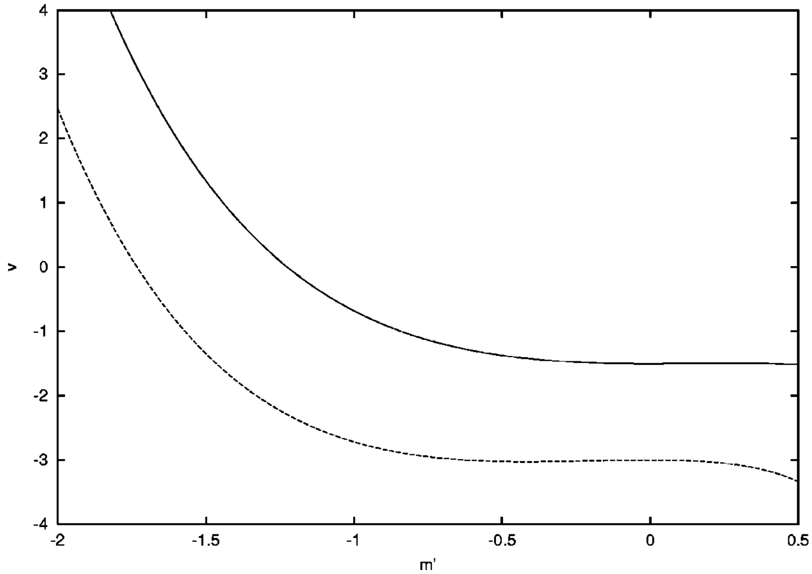

FIG. 3. The upper and lower lines show $v = -[e^{-m'}/(m'-1)][m'^2 + g(S+\bar{S})]$ for $g(S+\bar{S}) = -1.5$ and -3, respectively.

More explicitly, these solutions lead to the following values of $\mathrm{Re}(M)$:

$$2\,\mathrm{Re}(M) = \Delta\sigma, \quad 2\,\mathrm{Re}(M) = \Delta\sigma \pm \sqrt{-2 - g(S+\bar{S})}. \tag{34}$$

In the case that the Kähler potential $K(M,\bar{M}) = \frac{1}{2}(M+\bar{M})^2$ is reliable, in particular $\mathrm{Re}(M) < O(1)$, these results are valid. Similar analyses can be done for the polynomial Kähler potential. However, it is not clear that the expansion of the Kähler potential $K(M,\bar{M}) = \frac{1}{2}(M+\bar{M}^2) + \cdots$ is reliable for $\mathrm{Re}(M) \geqslant O(1)$. Thus, in the next subsection we will perform the same analysis by assuming $K = -\ln(M+\bar{M})$ as a trial. That is an example of a Kähler potentials that has behavior opposite to the canonical form at large M.

2. The case with $K = -\ln(M+\bar{M})$

Here the same analysis as in Sec. III B 1 will be done with the assumption $K = -\ln(M+\bar{M})$. In this case, it is convenient to define

$$m' \equiv (M+\bar{M})\sigma\Delta + 1. \tag{35}$$

Using this variable, we can write the scalar potential (25) as

$$V = \frac{\sigma\Delta}{(S+\bar{S})(m'-1)} e^{-\Delta(S+\bar{S})+2} e^{-m'} [m'^2 + g(S+\bar{S})]. \tag{36}$$

The solutions of the stationary condition $\partial V/\partial m' = 0$ are obtained as

$$m' = 0, \quad m' = 1 \pm \sqrt{-g(S+\bar{S}) - 1}. \tag{37}$$

The latter solution is allowed only if

$$g(S+\bar{S}) < -1. \tag{38}$$

For $\sigma < 0$, the region with $\mathrm{Re}(M) > 0$ corresponds to $m' < 1$. In this case, the second derivative of the scalar potential, $\partial^2 V/\partial m'^2$, is positive at $m' = 0$ if

$$g(S+\bar{S}) > -2. \tag{39}$$

This is always satisfied by the definition (30) if $(S+\bar{S}) > 0$. At $m' = 1 - \sqrt{-g(S+\bar{S}) - 1}$, we have $\partial^2 V/\partial m'^2 > 0$ if

$$g(S+\bar{S}) < -2. \tag{40}$$

This is never satisfied by the definition (30) if $(S+\bar{S}) > 0$. Figure 3 shows $v = -[e^{-m'}/(m'-1)][m'^2 + g(S+\bar{S})]$ for $g(S+\bar{S}) = -1.5$ and -3, respectively. The scalar potential has a singularity at $m' = 1$, which comes from the singularity of the Kähler potential at $M = 0$. However, in the vicinity of $M = 0$ the Kähler potential $K(M,\bar{M}) = \frac{1}{2}(M+\bar{M})^2$ as studied in the previous subsection is more reliable than the Kähler potential $-\ln(M+\bar{M})$.

For $\sigma > 0$, the region with $\mathrm{Re}(M) > 0$ corresponds to $m' > 1$. However, the second derivative of the scalar potential, $\partial^2 V/\partial m'^2$, is always negative at $m' = 1 + \sqrt{-g(S+\bar{S}) - 1}$. This situation is the same as the problem of dilaton stabilization by single gaugino condensation as seen in Sec. II.

Thus, the model with $\sigma < 0$ is interesting for $K(M,\bar{M}) = -\ln(M+\bar{M})$, that is, the positive exponent of M in the superpotential is useful. The solutions $m' = 0$ and $1 - \sqrt{-g(S+\bar{S}) - 1}$ correspond to

$$2\mathrm{Re}(M) = \frac{-1}{\sigma\Delta}, \quad \frac{-\sqrt{-g(S+\bar{S}) - 1}}{\sigma\Delta}, \tag{41}$$

respectively.

Assuming the Kähler potentials $K(M,\bar{M}) = \frac{1}{2}(M+\bar{M})^2$ and $-\ln(M+\bar{M})$, we have shown that the VEV of $\mathrm{Re}(M)$ can be stabilized with the VEV of S fixed. The former case implies that the canonical Kähler potential is important for stabilization of the twisted moduli. This analysis can be extended to the case with a polynomial Kähler potential $K(M,\bar{M})$. On the other hand, the latter case with $K(M,\bar{M}) = -\ln(M+\bar{M})$ shows that, even with the logarithmic Kähler potential, the positive exponent in the nonperturbative superpotential is useful to stabilize the VEV of twisted moduli fields. It is speculative whether really $K(M,\bar{M}) = -\ln(M+\bar{M})$ for large M, but that is an example of a Kähler potential that has behavior opposite to the canonical form at large M. For other forms of the Kähler potential, the analysis can be extended. The key point in the stabilization of twisted moduli is that the polynomial form of the Kähler potential is useful and the positive exponent of M in the superpotential is helpful. These aspects differ from the dilaton stabilization. The positive exponent of the dilaton field in the nonperturbative superpotential corresponds to the asymptotically nonfree case.[6]

Of course, the VEV of S is not stabilized in the case with single gaugino condensation which was discussed in this subsection. In order to study the stabilization of S and M at the same time, we will consider double gaugino condensations in the following subsections.

C. Mirage model

Here we consider the superpotential generated from double gaugino condensations, i.e., the racetrack model,

$$W = d_1 e^{-\Delta_1(S+\sigma_1 M)} + d_2 e^{-\Delta_2(S+\sigma_2 M)}. \tag{42}$$

Mirage unification can occur in the case that σ_a^{MSSM} for the MSSM are proportional to the one-loop beta-function coefficients b_a^{MSSM}. Here we consider a specific type of gaugino condensation model where σ_a for double gaugino condensations are proportional to their one-loop beta-function coefficients, that is, we can write

$$\Delta_a \sigma_a = C, \tag{43}$$

where C is common for the double gaugino condensations for $a=1,2$. Then the superpotential can be written as

$$W = e^{-CM}\tilde{W}, \quad \tilde{W} = (d_1 e^{-\Delta_1 S} + d_2 e^{-\Delta_2 S}). \tag{44}$$

The corresponding scalar potential is written as

$$V = \frac{e^{K(M,\bar{M}) - C(M+\bar{M})}}{S+\bar{S}} \Bigg[(K^{-1})^{M}_{\bar{M}} \times \left| \frac{\partial K(M,\bar{M})}{\partial M} - C \right|^2 |\tilde{W}|^2 + |(S+\bar{S})\tilde{W}_S - \tilde{W}|^2 - 3|\tilde{W}|^2 \Bigg]. \tag{45}$$

The racetrack solution (13),(14) corresponding to $(S+\bar{S})\tilde{W}_S - \tilde{W} = 0$ is still a solution of $\partial V/\partial S = 0$ for the present scalar potential. Here we restrict ourselves to this solution and the VEV of S itself is obtained by Eqs. (13), (14). The analysis of the scalar potential for the twisted moduli is almost the same as what was done in Sec. II B. The present case corresponds to the case with $g(S+\bar{S}) = -3$ and $\Delta\sigma = C$.

To be concrete, we again use the assumption of $K(M,\bar{M})$ as the canonical form and logarithmic form. First, in the case with $K(M,\bar{M}) = \frac{1}{2}(M+\bar{M})^2$ the solutions of $\partial V/\partial m = 0$ are obtained as

$$\mathrm{Re}(M) = \frac{C}{2}, \quad \frac{C \pm 1}{2}. \tag{46}$$

For the former solution, we have $\partial^2 V/\partial m^2 < 0$ because of $g(S+\bar{S}) = -3$. If there are additional contributions increasing the value of $g(S+\bar{S})$, this solution could be a local minimum. On the other hand, for the latter solution $2\,\mathrm{Re}(M) = C \pm 1$, we have $\partial^2 V/\partial m^2 > 0$ as well as $\partial^2 V/\partial m \partial S > 0$. At this point, the F component of M is obtained as

$$|F_M| = \frac{1}{\sqrt{S+\bar{S}}} e^{(1-C^2)/4} |\tilde{W}|. \tag{47}$$

Similarly, we can analyze the potential minima for the assumed Kähler potential $K(M,\bar{M}) = -\ln(M+\bar{M})$. We are interested in the case with $C<0$. The solutions of $\partial V/\partial m' = 0$ are obtained as

$$\mathrm{Re}(M) = -\frac{1}{2C}, \quad \pm\frac{1}{\sqrt{2}C}. \tag{48}$$

For the former solution, $\mathrm{Re}(M) = -1/2C$, we have $\partial^2 V/\partial m'^2 < 0$ because of $g(S+\bar{S}) = -3$. Additional contributions increasing $g(S+\bar{S})$ might make this point a local minimum. For the solution $\mathrm{Re}(M) = -1/\sqrt{2}C$, we have $\partial^2 V/\partial m'^2 > 0$ as well as $\partial^2 V/\partial m' \partial S > 0$. At this point, the F component of M does not vanish. Furthermore, stabilized values must satisfy the constraint $\mathrm{Re}(f_a) = \mathrm{Re}(S) + \sigma_a \mathrm{Re}(M) > 0$. For the above solution $\mathrm{Re}(M) = -1/\sqrt{2}C$, we can write

$$\mathrm{Re}(f_a) = \mathrm{Re}(S) - \frac{1}{\sqrt{2}\Delta_a}. \tag{49}$$

Thus, the stabilized value of S [Eq. (14)] must satisfy $\mathrm{Re}(S) > 1/\sqrt{2}\Delta_a$. Recall that the natural order of S is of $O(1/\Delta)$ unless Δ_a are close each other or $\ln(\Delta_1 d_1/\Delta_2 d_2)$ is large.

D. Generic racetrack model

In the previous section, we considered the specific racetrack model, i.e., $\sigma_1\Delta_1 = \sigma_2\Delta_2$. For the generic case $\sigma_1\Delta_1 \neq \sigma_2\Delta_2$, the analyses become complicated. Here we give a comment on this generic case.

[6] See Ref. [26] for dilaton stabilization in the asymptotically nonfree case.

As solutions of $\partial V/\partial S=0$, we again concentrate on the solution (5). That leads to the following equations:

$$\ln\frac{[2\Delta_1\mathrm{Re}(S)+1]d_1}{[2\Delta_2\mathrm{Re}(S)+1]d_2}=(\Delta_1-\Delta_2)\mathrm{Re}(S)+(\Delta_1\sigma_1-\Delta_2\sigma_2)\mathrm{Re}(M), \quad (50)$$

$$(\Delta_1-\Delta_2)\mathrm{Im}(S)+(\Delta_1\sigma_1-\Delta_2\sigma_2)\mathrm{Im}(M)=(2n+1)\pi. \quad (51)$$

Furthermore, if $\Delta\mathrm{Re}(S)\gg 1$, we obtain

$$\mathrm{Re}(S)=\frac{(\Delta_1\sigma_1-\Delta_2\sigma_2)}{\Delta_2-\Delta_1}\mathrm{Re}(M)+\frac{1}{\Delta_1-\Delta_2}\ln\frac{\Delta_1 d_1}{\Delta_2 d_2}. \quad (52)$$

The second term in the right hand side is the same as Eq. (14). The first term is a new contribution from M. When $\Delta_1\sigma_1=\Delta_2\sigma_2$, the first term vanishes and that is consistent with Sec. III C. However, if $\Delta_1\sigma_1\neq\Delta_2\sigma_2$, the VEV of $\mathrm{Re}(M)$ corresponds effectively to a large difference of d_a in Eq. (14) as can be seen by replacing $d_a\to d_a e^{-\Delta_a\sigma_a\mathrm{Re}(M)}$. Thus, the value of $\mathrm{Re}(M)$ is important for the stabilized value of $\mathrm{Re}(S)$.

Suppose that the VEV of $\mathrm{Re}(M)$ is also stabilized by the following equation similar to Eq. (5):

$$\frac{\partial K(M,\bar{M})}{\partial M}W+W_M=0. \quad (53)$$

Combined with Eq. (50), for $\Delta\mathrm{Re}(S)\gg 1$, we obtain

$$\frac{\partial K}{\partial M}=\frac{\Delta_1\Delta_2(\sigma_1-\sigma_2)}{(\Delta_2-\Delta_1)}. \quad (54)$$

For example, that leads to

$$\mathrm{Re}(M)=\frac{\Delta_1\Delta_2(\sigma_1-\sigma_2)}{2(\Delta_2-\Delta_1)} \quad (55)$$

for $K(M,\bar{M})=\frac{1}{2}(M+\bar{M})^2$, and

$$\mathrm{Re}(M)=\frac{\Delta_1-\Delta_2}{2\Delta_1\Delta_2(\sigma_1-\sigma_2)} \quad (56)$$

for $K(M,\bar{M})=-\ln(M+\bar{M})$. In the former (latter) case, the value of $\mathrm{Re}(M)$ is enhanced (suppressed) for fine-tuning $\Delta_1\approx\Delta_2$, while it is suppressed (enhanced) for fine-tuning $\sigma_1\approx\sigma_2$. Equation (52) becomes

$$\mathrm{Re}(S)=\left(-\sigma_1+\frac{2}{\Delta_1}\mathrm{Re}(M)\right)\mathrm{Re}(M)+\frac{1}{\Delta_1-\Delta_2}\ln\frac{\Delta_1 d_1}{\Delta_2 d_2} \quad (57)$$

for $K(M,\bar{M})=\frac{1}{2}(M+\bar{M})^2$, and

$$\mathrm{Re}(S)=-\sigma_1\mathrm{Re}(M)+\frac{1}{2\Delta_1}+\frac{1}{\Delta_1-\Delta_2}\ln\frac{\Delta_1 d_1}{\Delta_2 d_2} \quad (58)$$

for $K(M,\bar{M})=-\ln(M+\bar{M})$. For the latter case, the first term in the right-hand side would be important when $\mathrm{Re}(M)$ is enhanced by the fine-tuning $\sigma_1\approx\sigma_2$. Thus, the value of $\mathrm{Re}(M)$ has an interesting effect on the stabilized value of $\mathrm{Re}(S)$.

IV. CONCLUSION

We have studied stabilization of the dilaton and twisted moduli by assuming canonical and logarithmic forms for the Kähler potential of the twisted moduli field. The canonical Kähler potential plays a role in the stabilization of the twisted moduli. This analysis can be extended to the case with a polynomial Kähler potential. On the other hand, even with the logarithmic Kähler potential, the positive exponent of the twisted moduli field in the nonperturbative superpotential is significant. The logarithmic form was used as an example of a Kähler potential that has different behavior from the canonical form. That suggests that even for this case the positive exponent of the twisted moduli fields in the superpotential would be helpful. These aspects are different from the dilaton stabilization.

Similarly, in models where the gauge kinetic functions depend linearly on two or more moduli fields, the positive exponent of these fields in the superpotential might be helpful for moduli stabilization.

We have also considered the specific racetrack model with $\sigma_1\Delta_1=\sigma_2\Delta_2$ in order to discuss stabilization of the dilaton and twisted moduli at the same time. In the generic case, the VEV of M affects the stabilized value of the dilaton VEV. This point is also important in the stabilization of the twisted moduli fields.

Knowledge of the Kähler potential of the twisted moduli field is necessary to investigate numerically reliable results. We have not taken into account D terms. Inclusion of D terms will be studied elsewhere.

The models that have been studied lead to a negative cosmological constant. That is a common problem in dilaton stabilization. A vanishing cosmological constant could be realized by models with more gaugino condensation [5], a nonperturbative Kähler potential [9], or R symmetry [27].

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