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Gravity, *p*-branes, and a spacetime counterpart of the Higgs effect

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We point out that the worldvolume coordinate functions $\hat{x}^\mu(\xi)$ of a *p*-brane, treated as an independent object interacting with dynamical gravity, are Goldstone fields for spacetime diffeomorphisms gauge symmetry. The presence of this gauge invariance is exhibited by its associated Noether identity, which expresses that the source equations follow from the gravitational equations. We discuss the spacetime counterpart of the Higgs effect and show that a *p*-brane does not carry any local degrees of freedom, extending early known general relativity features. Our considerations are also relevant for brane world scenarios.

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I. INTRODUCTION

In the standard Higgs effect the degrees of freedom of the Goldstone bosons associated with the spontaneously broken *internal* gauge symmetry generators are incorporated by the gauge fields, which acquire mass as a result. Goldstone fields for local *spacetime* symmetries have been studied in [1] (fermionic), [2] (both bosonic and fermionic), and in [3].

The problem we address here, the Goldstone nature of the *p*-brane coordinate functions, and the spacetime counterpart of the Higgs effect in the presence of dynamical gravity, goes beyond previously considered cases. In it, the Goldstone fields are not spacetime (M^D) fields, but *worldvolume* ($\mathcal{W}^{p+1}\subset M^D$) fields; the gauge group, *spacetime* diffeomorphisms, is not internal, and the breaking of this invariance is the result of the location of the *p*-brane in spacetime i.e., of its mere existence. We shall argue, using the weak field approximation, that the removal of the *p*-brane Goldstone fields does not modify the number of polarizations of the graviton. This indicates that a *p*-brane, when coupled to dynamical gravity, does not carry any local degrees of freedom (although it provides the source in the Einstein equations).

We stress that our conclusions refer to a *p*-brane *object* described by an independent action S_{pD} which is added to the Einstein-Hilbert action S_{EHD} for dynamical gravity [see Eq. (11)], not to solitonic *p*-brane *solutions* of the Einstein field equations.

II. *p*-BRANE EQUATIONS FROM THE EINSTEIN FIELD EQUATIONS AND DIFFEOMORPHISM INVARIANCE

Interestingly enough, the Goldstone nature of the particle coordinate functions in gravity goes back to the classical papers [4] (see also [5]). It is well known that the Bianchi identity $\mathcal{G}^\mu_{\nu;\mu}\equiv 0$ for the Einstein tensor density $\mathcal{G}^{\mu\nu}$ in the gravitational field equations

$$\mathcal{G}_{\mu\nu}\equiv\sqrt{|g|}\left(\mathcal{R}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\mathcal{R}\right)=\kappa T_{\mu\nu}, \tag{1}$$

implies the covariant conservation of the energy-momentum tensor density $T_{\mu\nu}$,

$$T^\mu{}_{\nu;\mu}\equiv\partial_\mu(T^{\mu\rho}g_{\rho\nu})-\frac{1}{2}T^{\mu\rho}\partial_\nu g_{\rho\mu}=0. \tag{2}$$

For a particle $T_{\mu\nu}$ has support on the worldline $\mathcal{W}^1$,

$$T^{\mu\nu}=\frac{1}{2}\int d\tau l(\tau)\dot{\hat{x}}^\mu(\tau)\dot{\hat{x}}^\nu(\tau)\,\delta^4(x-\hat{x}(\tau)), \tag{3}$$

and then Eq. (2) is equivalent [4] to the particle (geodesic) equations $[\hat{x}^\mu\equiv\hat{x}^\mu(\tau)]$,

$$\partial_\tau[l(\tau)g_{\mu\nu}(\hat{x})\dot{\hat{x}}^\nu]-\frac{l(\tau)}{2}\dot{\hat{x}}^\nu\dot{\hat{x}}^\rho(\partial_\mu g_{\nu\rho})(\hat{x})=0 \tag{4}$$

or $d^2\hat{x}^\mu/ds^2+\Gamma^\mu_{\nu\rho}d\hat{x}^\nu/ds\,d\hat{x}^\rho/ds=0$ for $ds=d\tau/l(\tau)$.

This result exhibits a dependence among Eqs. (4) and (1) which, by the second Noether theorem, implies the existence of a gauge symmetry. This is the *diffeomorphism invariance* (the freedom of choosing a *local coordinate system*) or *passive* form of general coordinate invariance. For the gravity sector δ_{diff} is defined by

$$\delta x^\mu\equiv x^{\mu\prime}-x^\mu=b^\mu(x), \tag{5}$$

$$\begin{aligned}\delta' g_{\mu\nu}(x)&\equiv g'_{\mu\nu}(x)-g_{\mu\nu}(x)\\&=-(b_{\mu;\nu}+b_{\nu;\mu})\\&=-(\partial_\mu b^\rho g_{\nu\rho}+\partial_\nu b^\rho g_{\mu\rho}+b^\rho\partial_\rho g_{\mu\nu}),\end{aligned} \tag{6}$$

and for the particle sector by

$$\delta\hat{x}^\mu(\tau)\equiv\hat{x}^{\mu\prime}(\tau)-\hat{x}^\mu(\tau)=b^\mu(\hat{x}(\tau)). \tag{7}$$

Equations (5)–(7) indeed preserve the coupled action

$$S=S_{EH}+S_{0m},\quad S_{EH}=\frac{1}{2\kappa}\int d^4x\sqrt{|g|}\mathcal{R}, \tag{8}$$

$$S_{0m}=\frac{1}{2}\int d\tau[l(\tau)g_{\mu\nu}(\hat{x})\dot{\hat{x}}^{\mu}(\tau)\dot{\hat{x}}^{\nu}(\tau)+l^{-1}(\tau)m^2], \tag{9}$$

$\delta_{diff}S=0$. Indeed, the general variation δS (omitting for brevity the δx^{μ} and $\delta l(\tau)$ terms; the latter produces the algebraic equation $l(\tau)=m[g_{\mu\nu}(\hat{x})\dot{\hat{x}}^{\mu}\dot{\hat{x}}^{\nu}]^{-1/2}$) is

$$\delta S=-\frac{1}{2\kappa}\int d^4x\sqrt{|g|}\left(\mathcal{R}^{\mu\nu}-\frac{1}{2}g^{\mu\nu}\mathcal{R}\right)\delta' g_{\mu\nu}(x)+\frac{1}{2}\int d^4x\int d\tau l(\tau)\dot{\hat{x}}^{\mu}(\tau)\dot{\hat{x}}^{\nu}(\tau)\delta^4(x-\hat{x}(\tau))\delta' g_{\mu\nu}(x)$$
$$-\int d\tau\left(\partial_{\tau}[l(\tau)g_{\mu\nu}(\hat{x})\dot{\hat{x}}^{\nu}]-\frac{l(\tau)}{2}\dot{\hat{x}}^{\nu}\dot{\hat{x}}^{\rho}(\partial_{\mu}g_{\nu\rho})(\hat{x})\right)\delta\hat{x}^{\mu}(\tau). \tag{10}$$

For $\delta_{diff}S$ the first term vanishes since $\mathcal{G}^{\mu}{}_{\nu;\mu}\equiv 0$ [see the first form of Eq. (6)], and the second and third terms cancel [using the second form of Eq. (6)]. This cancellation just reflects the equivalence of Eq. (4) with the consequence (2) of the Einstein equation (1), i.e. the Noether identity for diffeomorphism symmetry [6].

Similarly, the action for D-dimensional gravity interacting with a string or p-brane $[\hat{x}^{\mu}\equiv\hat{x}^{\mu}(\xi)]$

$$S=S_{EHD}+S_{pD}=\frac{1}{2\kappa}\int d^Dx\sqrt{|g|}\mathcal{R}+\frac{T_p}{4}\int d^{p+1}\xi\sqrt{|\gamma|}[\gamma^{mn}(\xi)\partial_m\hat{x}^{\mu}\partial_n\hat{x}^{\nu}g_{\mu\nu}(\hat{x})+(p-1)] \tag{11}$$

is invariant under diffeomorphisms i.e., Eqs. (5),(6) plus

$$\hat{x}^{\mu\prime}(\xi)=\hat{x}^{\mu}(\xi)+\delta\hat{x}^{\mu}(\xi),\quad \delta\hat{x}^{\mu}(\xi)=b^{\mu}(\hat{x}(\xi)), \tag{12}$$

where $\xi^m=(\tau,\vec{\sigma})=(\tau,\sigma^1,\ldots,\sigma^p)$ are the local coordinates of the p-brane worldvolume $\mathcal{W}^{(p+1)}\subset M^D$, $\partial_m=\partial/\partial\xi^m$. The auxiliary worldvolume metric $\gamma_{mn}(\xi)$ is identified with the induced metric,

$$\gamma_{mn}(\xi)=\partial_m\hat{x}^{\mu}\partial_n\hat{x}^{\nu}g_{\mu\nu}(\hat{x}), \tag{13}$$

by the $\delta S/\delta\gamma^{mn}=0$ equation. In the language of the second Noether theorem, the diffeomorphism gauge invariance is reflected by the Noether identity [7–9] stating that the p-brane equations $\delta S/\delta\hat{x}^{\mu}(\xi)=0$,

$$\partial_m[\sqrt{|\gamma|}\gamma^{mn}g_{\mu\nu}(\hat{x})\partial_n\hat{x}^{\nu}(\xi)]$$
$$-\frac{1}{2}\sqrt{|\gamma|}\gamma^{mn}\partial_m\hat{x}^{\nu}\partial_n\hat{x}^{\rho}(\partial_{\mu}g_{\nu\rho})(\hat{x})=0 \tag{14}$$

[cf. Eq. (4)], also follow from the field equations $\delta S/\delta g_{\mu\nu}(x)=0$, Eq. (1) with $T^{\mu\nu}=\delta S_{pD}/\delta g_{\mu\nu}(x)$,

$$T^{\mu\nu}=\frac{T_p}{4}\int d^{p+1}\xi\sqrt{\gamma}\gamma^{mn}\partial_m\hat{x}^{\mu}\partial_n\hat{x}^{\nu}\delta^D(x-\hat{x}(\xi)) \tag{15}$$

(see [7,8] for the string and [9] for D-dimensional p-brane sources in M^D).

III. DIFFEOMORPHISM GAUGE SYMMETRY AND GOLDSTONE NATURE OF THE *p*-BRANE COORDINATE FUNCTIONS

The fact that diffeomorphism invariance [Eqs. (5), (6) and (7) or (12)] is *the gauge symmetry* of the dynamical system (8) or (11) allows one to conclude that the coordinate functions $\hat{x}(\xi)$ have a *pure gauge* nature. This is not surprising if we recall the situation for *flat* spacetime where they are known to be *Goldstone fields* [10–12] for spontaneously broken *global* translational symmetry. More precisely, the Goldstone fields correspond to the $(D-p-1)$ orthogonal directions, $\hat{x}^I(\xi)$, while the $\hat{x}^m(\xi)$ corresponding to tangential directions can be identified with the worldvolume coordinates, $\hat{x}^m=\xi^m$,

$$\hat{x}^{\mu}(\tau,\vec{\sigma})=(\xi^m,\hat{x}^I(\xi)),\quad I=(p+1),\ldots(D-1), \tag{16}$$

using ξ-reparametrizations i.e., *worldvolume* diffeomorphisms. These are the gauge symmetry of the p-brane action S_{pD}, Eq. (11), and are given by

$$\delta\xi^m\equiv\xi^{m\prime}-\xi^m=\beta^m(\xi),\quad \hat{x}^{\mu\prime}(\xi')=\hat{x}^{\mu}(\xi) \tag{17}$$

$$\Leftrightarrow\delta'\hat{x}^{\mu}(\xi)\equiv\hat{x}^{\mu\prime}(\xi)-\hat{x}^{\mu}(\xi)=-\beta^m(\xi)\partial_m\hat{x}^{\mu}(\xi). \tag{18}$$

Then, when the p-brane is in the curved spacetime determined by *dynamical* gravity, the rigid translation symmetry of the p-brane action in flat spacetime is replaced by the gauge diffeomorphism symmetry (5),(6),(12) of the coupled action (11) or (8), and the worldvolume fields $\hat{x}^{\mu}(\xi)$ become Goldstone fields for this *gauge* symmetry. Goldstone fields for a gauge symmetry always have a pure gauge nature, and their presence indicates spontaneous breaking of the gauge symmetry. Thus, a spacetime counterpart of the Higgs effect must occur in the interacting system of *dynamical* gravity and a p-brane described by the action (8) $(p=0)$ or (11).

In the standard Higgs effect one often uses the "unitary" gauge that sets the Goldstone fields equal to zero. Its counterpart in our gravity-brane interacting system,

$$\hat{x}^m(\xi)=\xi^m=(\tau,\sigma^1,\ldots,\sigma^p),\quad \hat{x}^I(\xi)=0, \tag{19}$$

can be called *static gauge* [although in the case of a brane in flat spacetime this name is also used for Eq. (16), we find it more proper for Eq. (19)]. Given a brane configuration, a gauge [i.e., a reference system $x^{\mu\prime}=x^{\mu\prime}(x)$] can be fixed in a tubular neighborhood of a point of $\mathcal{W}^{p+1}$ in such a way that Eq. (12) gives $\hat{x}^{I\prime}(\xi)=0$. In other words, the freedom of choosing any local coordinate system (i.e. the general relativity principle) allows one to put the particle or (a region of) the *p*-brane worldvolume in any convenient "position" with respect to the spacetime local coordinate system. The static gauge (19) breaks spacetime diffeomorphism invariance on the worldvolume $(b^\mu(\hat{x}(\xi)))$ down to the worldvolume diffeomorphisms (17),

$$b^\mu(\hat{x}(\xi))=\beta^m(\xi)\,\delta^\mu_m. \tag{20}$$

This actually reflects the spontaneous breaking of the diffeomorphism invariance due to the presence of the brane.

Let us stress that, although the brane action in a gravity *background* is also diffeomorphism invariant (i.e. it can be written in any coordinate system), when no gravity action is assumed, the spacetime diffeomorphisms cannot be treated as a *gauge* symmetry of the brane action since they transform the metric $g_{\mu\nu}(x)$ nontrivially, and now $g_{\mu\nu}(x)$ is a background and not a dynamical variable. Hence, in that case, diffeomorphism invariance cannot be used to fix the static gauge (19).

In the static gauge $\partial_m\hat{x}^\mu(\xi)=\delta^\mu_m$, $\gamma_{mn}(\xi)=g_{mn}(\xi,\vec{0})$, $\gamma^{mn}(\xi)=g^{[p+1]mn}(\xi,\vec{0})$, and the brane equations (14) become conditions for the gravitational field on $\mathcal{W}^{p+1}$,

$$\partial_m[|g^{[p+1]}|^{1/2}g^{[p+1]mn}g_{n\mu}(\xi,\vec{0})]$$
$$-1/2|g^{[p+1]}|^{1/2}g^{[p+1]mn}(\partial_\mu g_{mn})(\xi,\vec{0})=0. \tag{21}$$

The $\mu=m$ components of Eq. (21) are satisfied identically [this is the Noether identity for the worldvolume reparametrization gauge symmetry (17)], while those for $\mu=I$ can be recognized as the gauge fixing conditions often used to select the physical polarizations of the gravitational field, but on the worldvolume.

IV. SPACETIME COUNTERPART OF THE HIGGS EFFECT IN DYNAMICAL GRAVITY INTERACTING WITH A *p*-BRANE

The vielbein $e^a_\mu(x)$ or $g_{\mu\nu}(x)=e^a_\mu(x)e_{\nu a}(x)$ are the spacetime gauge fields. Since the Goldstone fields $\hat{x}^\mu(\xi)$ for the *spacetime* gauge symmetry (diffeomorphisms) are defined on the worldvolume $\mathcal{W}^{p+1}\subset M^D$, we should expect a modification of the gauge field equations (as in the usual Higgs effect), but here produced by (singular) terms with support on $\mathcal{W}^{p+1}$. These are just the *source terms* that account for the *p*-brane-gravity interaction i.e., $T^{\mu\nu}$ [Eq. (3) or (15)] in the Einstein equation (1). Clearly, $T^{\mu\nu}$ cannot be gauged away by a diffeomorphism (as the mass term in the standard Higgs phenomenon). The role of the vacuum expectation value of the Higgs field is here played by the brane tension T_p ($T_1=1/2\pi\alpha'$ for the string).

In particular, the source (15) is nonvanishing in the static gauge (19), although in the cases where this gauge can be fixed globally (as, e.g., in searching for an almost flat infinite *p*-brane solution of gravity equations) it simplifies the energy-momentum tensor (15) down to

$$T^{\mu\nu}=\frac{T_p}{4}\sqrt{|g^{[p+1]}|}g^{[p+1]mn}(\xi,\vec{0})\,\delta^\mu_m\delta^\nu_n\delta^{(D-p-1)}(x^I). \tag{22}$$

V. GRAVITON POLARIZATIONS, BRANE DEGREES OF FREEDOM AND THE MEANING OF SPACETIME POINTS IN GENERAL RELATIVITY

A question remains: do the Goldstone degrees of freedom reappear as additional polarizations of the gauge field $g_{\mu\nu}(x)$ on $\mathcal{W}^{p+1}$ so that on the brane the graviton behaves like a massive field? Can the singular energy-momentum tensor then be treated as a kind of mass term for the graviton? This is a subtle question, as the Einstein equation is essentially nonlinear. However, as far as the *perturbative* degrees of freedom are concerned, the answers to the above questions are negative.

To see this we use the weak field approximation where $g_{\mu\nu}(x)=\eta_{\mu\nu}+\kappa h_{\mu\nu}(x)$ and κT_p in the source is assumed to be small for consistency. The extraction of the Einstein coupling constant κ makes $h_{\mu\nu}(x)$ dimensionful, but allows us to present, formally, the weak field limit as the first order of an expansion in κ. With $\chi_{\mu\nu}:=h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}\eta^{\rho\sigma}h_{\rho\sigma}$ we find

$$\mathcal{G}^{\mu\nu}(h)=\kappa\mathcal{G}^{(0)\mu\nu}(h)+O(\kappa^2), \tag{23}$$

$$\mathcal{G}^{(0)\mu\nu}(h)=\frac{1}{4}(\Box\chi^{\mu\nu}+2\partial^{(\mu}\partial_\rho\chi^{\nu)\rho}-\eta^{\mu\nu}\partial_\rho\partial_\sigma\chi^{\sigma\rho}). \tag{24}$$

Similarly, e.g. in the static gauge when it can be fixed globally and Eq. (15) gives Eq. (22), we find

$$\kappa T^{\mu\nu}(h)=\kappa T^{(0)\mu\nu}+\kappa^2T^{(1)\mu\nu}(h)+O(\kappa^3), \tag{25}$$

$$T^{(0)\mu\nu}=1/4T_p\eta^{mn}\delta_m{}^\mu\delta_n{}^\nu\delta^{(D-p-1)}(x^I), \tag{26}$$

$$T^{(1)\mu\nu}(h)=\frac{1}{4}T_p\left(h^{mn}-\frac{1}{2}\eta^{mn}h_{kl}\eta^{kl}\right)\delta^\mu_m\delta^\nu_n\delta(x^I). \tag{27}$$

At first order in κ, Eq. (1) reduces to the linear inhomogeneous equation $\mathcal{G}^{(0)\mu\nu}(h)=T^{(0)\mu\nu}$, where $T^{(0)\mu\nu}$ is h-independent. Hence, the graviton degrees of freedom are determined by the solutions of $\mathcal{G}^{(0)\mu\nu}(h)=0$. Thus, the standard arguments that determine the $D(D-3)/2$ physical polarizations of the graviton field (see e.g. [13,14] for $D=4$) will also apply here *provided* there is full (linearized) diffeomorphism symmetry. This is an evident symmetry of the linearized Einstein tensor; however, in our case, there is a potential problem. The use of the gauge diffeomorphism symmetry to remove the Goldstone degrees of freedom on the *p*-brane worldvolume, e.g. the static gauge (19), partially breaks spacetime diffeomorphisms on the worldvolume down to reparametrization symmetry, Eq. (20). Hence $b^I(\xi,\vec{0})=0$ and one cannot use this parameter to obtain additional conditions on $h_{\mu\nu}$. Nevertheless, an analysis similar to the one presented in [9] for $p=0$ shows that the $\mu=I$ components of the *p*-brane equation in the static gauge (Eq. (21); for a weak field, $\partial_m[\eta^{mn}h_{nI}(\xi,\vec{0})] - \frac{1}{2}\partial_I(\eta^{mn}h_{nm})(\xi,\vec{0})=0$) replace the lost gauge fixing conditions so that, on the worldvolume, we still obtain the $D(D-3)/2$ polarizations of the massless graviton.

Thus in the static gauge, the removed brane degrees of freedom $[\hat{x}^I(\xi)=0]$ do *not* reappear as additional polarizations of the graviton on the worldvolume. This indicates that, in the interacting system of dynamical supergravity and a *p*-brane, the *p*-brane does not carry any local degrees of freedom.

This property of the spacetime counterpart of the Higgs effect is related to an old conceptual discussion, the lack of physical meaning of spacetime points in general relativity [15] (see also [16]). The passive form of the general coordinate invariance (diffeomorphism symmetry) is quite natural and provides a realization of the general relativity principle. However, the fact that the Einstein-Hilbert action and the Einstein equations are invariant as well under the *active* form of the general coordinate transformations (i.e. under a change of ‘‘physical’’ spacetime points, see [6]), “*takes away from space and time the last remnant of physical objectivity*” [15,16].

In our case, this corresponds to the absence of local degrees of freedom for a *p*-brane when the *p*-brane interacts with *dynamical* gravity. Indeed, the local brane degrees of freedom could have a meaning by specifying its position in spacetime M^D, i.e. by locating $\mathcal{W}^{p+1}$ in M^D. However, in a general coordinate invariant theory the spacetime point concept becomes ‘‘unphysical’’: it is not invariant and thus cannot be treated as an observable in so far as observables are identified with gauge invariant entities. The only physical information is the existence of the brane worldvolume (as reflected by the source term in the Einstein field equation), not where $\mathcal{W}^{p+1}$ is in M^D, and, if there are several branes, also the possible intersections of their worldvolumes (for a particle see [15] and [16]). This implies, and it is implied by, the pure gauge nature of the (local) degrees of freedom of a brane interacting with dynamical gravity.

This conclusion also holds for a system of several branes interacting with dynamical gravity. In contrast, global properties, like whether the *p*-brane is open or closed, or whether two branes intersect or not, do contain physical information. For topologically trivial branes the diffeomorphism gauge symmetry allows one to fix globally the static gauge, i.e. to choose the local coordinate system in which all the branes are parametrized as infinite planes, possibly intersecting ones. Let us stress that the questions about distances between nonintersecting (e.g. parallel) brane worldvolumes or about the angles between intersecting branes have to be addressed *after* the specific spacetime metric has been determined by solving the Einstein equations with the sources produced by these branes, as it enters into the definition of the invariant interval, $ds^2=dx^\mu dx^\nu g_{\mu\nu}(x)$.

VI. *p*-BRANE OBJECT VS *p*-BRANE SOLITONIC SOLUTIONS

Our statement about the absence of the brane degrees of freedom refers to a *p*-brane object, as described by its action added to the Einstein-Hilbert gravity action. This situation is not to be confused with the moduli space of solitonic *solutions* of (super)gravity equations, as considered, e.g., in [17]. Such a moduli space is spanned by deformations $h_{\mu\nu}(\xi^m,x^I)$ of a particular *metric* solution $g^{(1)}_{\mu\nu}(\xi^m,x^I)$ of the Einstein equation (1) with the source (22), such that $g^{(1)}_{\mu\nu}$ and the deformed metric $g_{\mu\nu}(\xi^m,x^I)=g^{(1)}_{\mu\nu}(\xi^m,x^I)+h_{\mu\nu}(\xi^m,x^I)$ are solutions of the same equation. Thus, this moduli space is associated with the gravity degrees of freedom rather than with those of the *p*-brane object.[1]

When discussing these solitonic solutions, in particular their zero modes[2] [i.e., metric deformations that are independent of x^I, $\hat{h}_{\mu\nu}(\xi)\equiv h_{\mu\nu}(\xi,0)$], topological considerations are important. In contrast, in our situation neither boundary conditions on W^{p+1} nor asymptotic properties are assumed, and the metric is a dynamical field variable and not a specific solution. Our statement is about the absence of *local* degrees of freedom of the brane object and, as such, it refers to ‘‘small’’ diffeomorphisms.[3]

VII. OUTLOOK

First, we note that there is some similarity between our results and the idea of holography. Their common basic statement is that a theory invariant under diffeomorphisms (and thus general coordinate transformations) cannot have observables (gauge invariant variables) in the bulk. The usual holography approach [19] concludes from this that the physical observables may be defined on a boundary of spacetime (e.g., on the conformal boundary of the AdS space which is the Minkowski space). Our statement about the pure gauge nature of the brane degrees of freedom in the dynamical gravity-brane interacting system is different, but similar in spirit: the variables describing the spacetime location of

[1]As it follows from the linearized situation when the *p*-brane tension T_p is assumed to be weak, these small deformations $h_{\mu\nu}(\xi,x^I)$ satisfy the free linearized homogeneous Einstein equation $\mathcal{G}^{(0)}_{\mu\nu}(h)=0$ [see Eqs. (23)–(27) and below] and, hence, just describe the graviton polarizations (i.e. the gravity degrees of freedom).

[2]These zero modes $h_{\mu\nu}(\xi^m,0)$ for a solitonic solution $g^{(1)}_{\mu\nu}(\xi^m,x^I)$ of the Einstein field equations with a *p*-brane source (22) located at $\hat{x}^I=0$ simulate the *p*-brane dynamics. Namely, $h_{\mu\nu}(\xi^m,0)$ appears to be expressed through $(D-p-1)$ independent field parameters $\phi^I(\xi)$ that satisfy equations that coincide with the equations of motion for the coordinate functions $\hat{x}^I(\xi)$ of a *p*-brane in a spacetime with metric $g^{(1)}_{\mu\nu}(\xi^m,x^I)$.

[3]Thus, neither the idea of ‘‘large gauge transformations’’ as defined, e.g., in [17], nor the Goldstone analysis of the breaking of global symmetries by a particular metric ansatz [12,18] apply here.

the brane are unphysical; the only physical information is the existence of one or several branes and the possible intersections of their worldvolumes.

Secondly, we mention that our conclusions also apply to a brane carrying worldvolume fields (such as D-branes in string or M theory). In that case, by fixing the static gauge (19), one arrives at a model similar to the ones considered in brane world scenarios [20,21] [an additional Einstein term in the brane action, $\int d^{p+1}\xi |\gamma|^{1/2}\mathcal{R}^{[p+1]}(\gamma)$, could be looked at as induced by quantum corrections [21]]. This observation indicates that in such scenarios the brane universe is not forced to be a ''frozen'' *fixed* hypersurface in a higher-dimensional spacetime, but could be rather considered as a brane described by a diffeomorphism invariant action interacting with dynamical gravity.

The fact that the *D*-dimensional graviton does not acquire any additional perturbative degrees of freedom on the *p*-brane worldvolume is also natural for a brane world scenario. Indeed, if it were not massless on, say, a four-dimensional worldvolume, this would produce a difficulty in treating the three-brane as a model for a universe with physically acceptable ($\sim 1/r^2$) long range gravitational forces.

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