Fuzzy BIon

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We construct a solution of the Banks-Fischler-Shenker-Susskind matrix theory, which is a counterpart of the BIon solution representing a fundamental string ending on a bound state of a D2-brane and D0-branes. We call this solution the ‘‘fuzzy BIon’’ and show that this configuration preserves 1/4 supersymmetry of type IIA superstring theory. We also construct an effective action for the fuzzy BIon by analyzing the non-Abelian Born-Infeld action for D0-branes. When we take the continuous limit, with some conditions, this action coincides with the effective action for the BIon configuration.

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I. INTRODUCTION

It is well recognized that a bound state of the D$(p+2)$-brane and Dp-branes has dual descriptions. From the D$(p+2)$-brane picture, the Dp-brane charge is represented by the magnetic flux on it, and from the Dp-branes viewpoint, the D$(p+2)$-brane is realized by some fuzzy configuration of Dp-branes. The latter description has the advantage of dealing with multibody systems. For example, it is easy to realize the situation where the Dp-branes move around the D$(p+2)$-branes.

In type IIA superstring theory there exists the BIon configuration which represents a fundamental string ending on a bound state of a D2-brane and D0-branes [1,2]. From the viewpoint of the world-volume theory on the D2-brane, which is labeled by the radial direction ρ and the angular direction ϕ, the fundamental string is expressed as a source of a Coulomb-like electric field and D0-branes are realized as a uniform magnetic flux on the (ρ,ϕ) plane. One of the transverse scalars, say z, on the D2-brane is also excited to be the classical solution of field equations. This configuration preserves 1/4 supersymmetry of type IIA superstring theory.

In this paper we give a dual description of the BIon solution, which is obtained as a classical solution of the Banks-Fischler-Shenker-Susskind (BFSS) matrix theory. In order to execute this, we employ the matrix representations for the general fuzzy surface with axial symmetry, and explicitly write down the equations of motion of the BFSS matrix theory in terms of their components $\rho_{m+1/2}$, z_m and a_m. These components contain information about the shape of a fuzzy surface. In fact, $\rho_{m+1/2}$ and z_m almost correspond to the radial direction ρ and the transverse scalar z, respectively, and nontrivial a_m give electric flux on the fuzzy surface. We uniquely determine values of $\rho_{m+1/2}$, z_m and a_m which equip the properties of the BIon solution. We will call this solution the ‘‘fuzzy BIon.’’

The organization of this paper is as follows. In Sec. II we briefly review the BIon configuration. In Sec. III we solve the equations of motion of BFSS matrix theory and obtain the solution which represents the fuzzy BIon. It will be confirmed that this solution satisfies the discrete version of the differential equation for the BIon solution and preserves 1/4 supersymmetry of type IIA superstring theory. In Sec. IV the effective action for the fuzzy BIon is constructed by analyzing the non-Abelian Born-Infeld action for D0-branes. When we take the continuous limit, with some condition, this action coincides with the effective action for the BIon configuration. Conclusions are given in Sec. V.

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II. REVIEW OF THE BION SOLUTION

In this section we briefly review the BIon solution which represents a fundamental string ending on a bound state of a D2-brane and D0-branes [1,2]. This brane configuration preserves 1/4 supersymmetry of type IIA superstring theory.

Let us choose the line element of the flat space-time as

$$ds^2=-dt^2+d\rho^2+\rho^2 d\phi^2+dz^2+\sum_{i=4}^{9}(dx^i)^2, \qquad (1)$$

and identify the world-volume coordinates on the D2-brane with (t,ρ,ϕ). The D2-brane is embedded into the target space like $x^i=0$ and $z=z(\rho)$. In order to obtain the BIon configuration, we also assume that gauge field strengths $F_{t\rho}$ and $F_{\rho\phi}$ are nontrivial. Then the Born-Infeld action for the D2-brane is given by [3]

$$S_{\rm D2}=-T_2\int dt d\rho d\phi \sqrt{\rho^2(1+z'^2)+\lambda^2F_{\rho\phi}^2-\rho^2\lambda^2F_{t\rho}^2}. \qquad (2)$$

Here we used the definitions $\lambda=2\pi\ell_s^2$ and $z'=dz/d\rho$. T_2 is the tension of the D2-brane.

First we consider the world-volume supersymmetry on the D2-brane. In this case the killing spinor equation $(1-\Gamma)\epsilon=0$ is transformed into the form [4,5]

$$\left[\lambda F_{t\rho}\rho\Gamma_{11}\Gamma_{\hat{\phi}}\left(1+\frac{z'}{\lambda F_{t\rho}}\Gamma_{\hat{t}\hat{z}}\Gamma_{11}\right)+\sqrt{X}\left(1-\frac{\rho\Gamma_{\hat{t}\hat{\rho}\hat{\phi}}+\lambda F_{\rho\phi}\Gamma_{\hat{t}}\Gamma_{11}}{\sqrt{X}}\right)\right]\epsilon=0, \qquad (3)$$

and we defined X as

$$X=\rho^2(1+z'^2)+\lambda^2F_{\rho\phi}^2-\rho^2\lambda^2F_{t\rho}^2. \tag{4}$$

The caret notation for the subscripts of the gamma matrices is used to declare the Lorentz indices. When we choose the gauge field strength as

$$F=\pm\frac{z'}{\lambda}dt\wedge d\rho\pm\frac{\rho}{b}d\rho\wedge d\phi, \tag{5}$$

the above Killing spinor equation is solved like

$$\epsilon=\frac{1}{2}(1\mp\Gamma_{\hat{t}\hat{z}}\Gamma_{11})\frac{1}{2}\left(1+\frac{b\Gamma_{\hat{t}\hat{\rho}\hat{\phi}}\pm\lambda\Gamma_{\hat{t}}\Gamma_{11}}{\sqrt{b^2+\lambda^2}}\right)\epsilon_0, \tag{6}$$

where ϵ_0 is an arbitrary constant Majorana spinor. Therefore the configuration (5) preserves 1/4 supersymmetry. The first term of Eq. (5) represents the existence of the constant electric flux along the z direction, and plus or minus sign corresponds to the orientation of the fundamental string. The second term of Eq. (5) insists that the magnetic flux projected on the (ρ,ϕ) plane is uniform, and plus or minus sign corresponds to the sign of the D0-brane charge. By using the flux quantization condition of the magnetic flux, we see that the area per a unit of magnetic flux projected on the (ρ,ϕ) plane is $2\pi b$.

Second, the Gauss law constraint which is obtained by varying the action with the gauge field a_t is written as

$$\frac{d}{d\rho}\left(\frac{T_2\lambda^2F_{t\rho}\rho^2}{\sqrt{X}}\right)=0. \tag{7}$$

The conditions (5) simplify the above constraint into the form

$$dz=\frac{L\,d\rho^2}{2\rho^2}, \tag{8}$$

and the solution is written as $z=z_0+L\ln\rho$. L and z_0 are integral constants. From this we see that the first term of Eq. (5) represents the Coulomb-like electric field on the (ρ,ϕ) plane. The BIon configuration is characterized by Eqs. (5) and (8).

III. THE FUZZY BION

In the previous section we reviewed the BIon configuration by analyzing the world-volume theory on the D2-brane. Here we construct a counterpart of the BIon from the viewpoint of D0-branes, that is, in the framework of the BFSS matrix theory. We call this solution the "fuzzy BIon."

Let us begin with the action of the BFSS matrix theory [6],

$$S_{\rm BFSS}=T_0\int dt\,{\rm Tr}\left(\frac{1}{2}(D_tX^i)^2+\frac{1}{4\lambda^2}[X^i,X^j]^2\right), \tag{9}$$

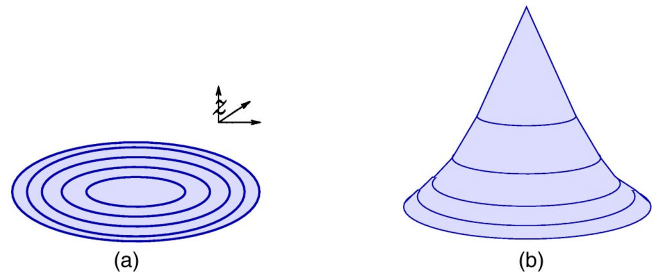

FIG. 1. (a) The fuzzy plane. (b) The fuzzy BIon. This configuration can be obtained by pulling the fuzzy plane into the z direction.

where i,j run $1,\ldots,9$. The covariant derivative is defined as $D_tX^i=\partial_tX^i+i[A_t,X^i]$, and T_0 is the mass of a D0-brane. The equations of motion for X^i and the Gauss law constraint on A_t are written as

$$-D_t(D_tX^i)+\frac{1}{\lambda^2}[X^j,[X^i,X^j]]=0,\quad [X^i,D_tX^i]=0. \tag{10}$$

Now we solve these equations to obtain the fuzzy BIon solution. In order to execute this, we set $X^4=\cdots=X^9=0$ and choose the matrices X^1, X^2, X^3 and A_t like

$$X^1_{mn}=\frac{1}{2}\rho_{m+1/2}\delta_{m+1,n}+\frac{1}{2}\rho_{m-1/2}\delta_{m,n+1},$$

$$X^2_{mn}=\frac{i}{2}\rho_{m+1/2}\delta_{m+1,n}-\frac{i}{2}\rho_{m-1/2}\delta_{m,n+1},$$

$$X^3_{mn}=z_m\delta_{m,n},\quad A_{tmn}=a_m\delta_{m,n}, \tag{11}$$

where $m,n\in\mathbb{N}$. These matrices represent the fuzzy surface with axial symmetry around the $x^3(=z)$ direction [7,8]. Since what we want is a static configuration, each component does not depend on the time t. After some calculations, the equations (10) are transformed into the forms

$$2\lambda^2(a_{m+1}-a_m)^2-2(z_{m+1}-z_m)^2+(\rho_{m+3/2}^2-2\rho_{m+1/2}^2+\rho_{m-1/2}^2)=0,$$

$$\rho_{m+1/2}^2(z_{m+1}-z_m)-\rho_{m-1/2}^2(z_m-z_{m-1})=0,$$

$$\rho_{m+1/2}^2(a_{m+1}-a_m)-\rho_{m-1/2}^2(a_m-a_{m-1})=0. \tag{12}$$

Here we should set $\rho_{1/2}=0$. Note that when z_m and a_m are all equal to zero, we obtain nontrivial solutions by choosing $\rho_{m+1/2}$ as

$$\rho_{m+1/2}^c=\sqrt{2bm}. \tag{13}$$

The superscript c is denoted to emphasize that this is a classical solution. These matrices are known to represent the fuzzy plane [6,8] [see Fig. 1(a)]. The commutation relation for X^1 and X^2 becomes $[X^1,X^2]=-ib$, and $2\pi b$ represents the fuzziness of a D0-brane.

The explicit values of $\rho_{m+1/2}$, z_m and a_m which represent the fuzzy BIon can be obtained by referring to Eq. (5) obtained in the previous section. The latter term of Eq. (5) insists that the magnetic flux of the BIon configuration is uniform on the projected (ρ,ϕ) plane and the area occupied per a unit of magnetic flux is $2\pi b$. If we neglect the electric flux, this means that the BIon configuration can be obtained by pulling the planar D2-brane with uniform magnetic flux on it to the z direction. Of course, the equations of motion determined the form of the function $z(\rho)$ and the preservation of 1/4 supersymmetry required the existence of the electric flux.

Now we trace the same procedure as the above. First we identify $\rho_{m+1/2}$ for the fuzzy BIon configuration with that of the fuzzy plane. Then by substituting $\rho_{m+1/2}=\sqrt{2bm}$ into Eq. (12), the equations for z_m and a_m are obtained like

$$z_{m+1}-z_m=\pm\lambda(a_{m+1}-a_m),$$

$$m(z_{m+1}-z_m)-(m-1)(z_m-z_{m-1})=0, \qquad (14)$$

and z_m and a_m are easily solved as

$$z_m^c=\pm\lambda a_m^c=z_1^c+\frac{L}{2}\sum_{i=1}^{m-1}\frac{1}{i}. \qquad (15)$$

Here $m\geqslant 2$ and L and $z_1^c=\pm\lambda a_1^c$ are some constant parameters. This solution insists the existence of fundamental string charge along the z direction because of the relation $A_t=\pm\frac{1}{\lambda}X^3$ [9–11]. The sign corresponds to the orientation of the fundamental string. We can confirm that this solution really represents the fuzzy BIon configuration, by noting the relation

$$z_{m+1}^c-z_m^c=\frac{L(\rho_{m+1}^{c2}-\rho_m^{c2})}{2\rho_{m+1/2}^{c2}}, \qquad (16)$$

where $\rho_{m+1}^{c2}\equiv(\rho_{m+3/2}^{c2}+\rho_{m+1/2}^{c2})/2$. This is the matrix version of Eq. (8).

The commutation relations which represents the fuzzy BIon are written as

$$[X^1,X^2]_{mn}=-ib\,\delta_{m,n},$$

$$[X^2,X^3]_{mn}=\frac{i}{2}bL\rho_{m+1/2}^{c-1}\delta_{m+1,n}+\frac{i}{2}bL\rho_{m-1/2}^{c-1}\delta_{m,n+1},$$

$$[X^3,X^1]_{mn}=-\frac{1}{2}bL\rho_{m+1/2}^{c-1}\delta_{m+1,n}+\frac{1}{2}bL\rho_{m-1/2}^{c-1}\delta_{m,n+1}. \qquad (17)$$

And the figure of the fuzzy BIon is drawn in Fig. 1(b).

Our final confirmation is to check the preservation of 1/4 supersymmetry. In this case the Killing spinor equation $\delta\Theta=0$ becomes

$$0=\left(D_tX^i\Gamma^{0i}+\frac{i}{2\lambda}[X^i,X^j]\Gamma^{ij}\right)P_+\epsilon+P_+\epsilon'$$

$$=\pm\frac{i}{\lambda}([X^3,X^1]\Gamma^{01}+[X^3,X^2]\Gamma^{02})P_+(1\mp\Gamma^{03}\Gamma_{11})\epsilon+\frac{i}{\lambda}[X^1,X^2]P_+\Gamma^{12}\epsilon+P_+\epsilon', \qquad (18)$$

where ϵ and ϵ' are constant Majorana spinors and $P_+=(1+\Gamma_{11})/2$. Thus we see that 1/4 supersymmetry is preserved when

$$\epsilon'=\left(\frac{\sqrt{b^2+\lambda^2}}{\lambda}\Gamma_0+\Gamma_{11}\right)\epsilon,$$

$$\epsilon=\frac{1}{2}(1\mp\Gamma_{03}\Gamma_{11})\frac{1}{2}\left(1+\frac{b\Gamma_{012}+\lambda\Gamma_0\Gamma_{11}}{\sqrt{b^2+\lambda^2}}\right)\epsilon_0, \qquad (19)$$

where ϵ_0 is an arbitrary constant Majorana spinor.

IV. THE EFFECTIVE ACTION FOR THE FUZZY BION

In the previous section we obtained the fuzzy BIon configuration as the classical solution of the BFSS matrix theory. We also checked that the fuzzy BIon preserves 1/4 supersymmetry of type IIA superstring theory. Now it is natural to ask whether the effective action for D0-branes which is described by the non-Abelian Born-Infeld action contains the same solution. In this section we do not prove it directly, but we construct the effective action for the fuzzy BIon by analyzing the non-Abelian Born-Infeld action for D0-branes. We show that, with some conditions, the effective action obtained in this way correctly reproduces that for the BIon configuration in the continuous limit.

The non-Abelian Born-Infeld action is given by Refs. [12–14]. In the background of the flat space-time, when we set $X^4=\cdots=X^9=0$, this action is transformed into the form [8]

$$S_{\rm D0}=-T_0\int dt\,{\rm Tr}\sqrt{1-(D_tX^i)^2-\frac{1}{2\lambda^2}[X^i,X^j]^2+\frac{1}{4\lambda^2}(\epsilon_{ijk}\{D_tX^i,[X^j,X^k]\})^2}, \qquad (20)$$

where $i,j,k=1,2,3$ and $\{A,B\}\equiv(AB+BA)/2$. Now we choose representations of three adjoint scalars X^1, X^2, X^3 and a gauge field A_t as

$$X^1_{mn}=\frac{1}{2}\rho e^{i\theta}\Big|_{m+1/2}\delta_{m+1,n}+\frac{1}{2}\rho e^{-i\theta}\Big|_{m-1/2}\delta_{m,n+1},$$

$$X^2_{mn}=\frac{i}{2}\rho e^{i\theta}\Big|_{m+1/2}\delta_{m+1,n}-\frac{i}{2}\rho e^{-i\theta}\Big|_{m-1/2}\delta_{m,n+1},$$

$$X^3_{mn}=z|_m\delta_{m,n},\quad A_{tmn}=a|_m\delta_{m,n}. \tag{21}$$

The notation $|_m$ is employed to show functions before this have the subscript m. Here the elements $\rho_{m+1/2}$, $\theta_{m+1/2}$, z_m and a_m are functions of the time t. Then by introducing the following definitions:

$$\delta z|_{m+1/2}=z|_{m+1}-z|_m,$$

$$\delta a|_{m+1/2}=a|_{m+1}-a|_m,$$

$$\rho^2\delta z^2|_m=\frac{1}{2}(\rho^2\delta z^2|_{m+1/2}+\rho^2\delta z^2|_{m-1/2}),$$

$$\delta\rho^2|_m=\rho^2|_{m+1/2}-\rho^2|_{m-1/2},$$

$$\dot\rho^2|_m=\frac{1}{2}(\dot\rho^2|_{m+1/2}+\dot\rho^2|_{m-1/2}),$$

$$\rho^2(\dot\theta-\delta a)^2|_m=\frac{1}{2}\{\rho^2(\dot\theta-\delta a)^2|_{m+1/2}+\rho^2(\dot\theta-\delta a)^2|_{m-1/2}\},$$

$$\rho\dot\rho\delta z|_m=\frac{1}{2}(\rho\dot\rho\delta z|_{m+1/2}+\rho\dot\rho\delta z|_{m-1/2}), \tag{22}$$

after some calculations, the effective action (20) is evaluated as

$$S_{\mathrm{D0}}=-T_0\int dt\sum_m\Bigg[1+\left\{-\dot z^2+\frac{1}{\lambda^2}\rho^2\delta z^2\right\}+\lambda^2\left\{-\frac{1}{\lambda^2}\rho^2(\dot\theta-\delta a)^2+\frac{1}{4\lambda^4}(\delta\rho^2)^2-\frac{1}{\lambda^2}\dot\rho^2\right\}-\lambda^2\left\{\frac{1}{2\lambda^2}\dot z\delta\rho^2-\frac{1}{\lambda^2}\rho\dot\rho\delta z\right\}^2\Bigg]^{1/2}\Bigg|_m. \tag{23}$$

Note that the interior of the square root is proportional to the identity matrix and the trace operation is simply replaced with the sum.

Let us return to the case of the fuzzy BIon configuration. Now we add a scalar fluctuation $\hat z|_m$ and gauge fluctuations $a_\rho|_{m+1/2}$, $a_\phi|_{m+1/2}$ and $a_t|_m$ around the fuzzy BIon configuration (13) and (15) like

$$X^1_{mn}=\frac{1}{2}(\rho^c+la_\phi)e^{ila_\rho}\Big|_{m+1/2}\delta_{m+1,n}+\frac{1}{2}(\rho^c+la_\phi)e^{-ila_\rho}\Big|_{m-1/2}\delta_{m,n+1},$$

$$X^2_{mn}=\frac{i}{2}(\rho^c+la_\phi)e^{ila_\rho}\Big|_{m+1/2}\delta_{m+1,n}-\frac{i}{2}(\rho^c+la_\phi)e^{-ila_\rho}\Big|_{m-1/2}\delta_{m,n+1},$$

$$X^3_{mn}=(z^c+\hat z)|_m\delta_{m,n},\quad A_{tmn}=(a^c+a_t)|_m\delta_{m,n}. \tag{24}$$

Here we defined $l|_{m+1/2}=b/\rho^c|_{m+1/2}$ which is interpreted as a separation between mth and $(m+1)$th segments. We also define $\rho^c|_m=(\rho^c|_{m+1/2}+\rho^c|_{m-1/2})/2$ and $l|_m=b/\rho^c|_m$ for later use. Now the definitions (22) are translated into the forms

$$\delta z|_{m+1/2}=\frac{b}{\rho^c}\left(\frac{L}{\rho^c}+\hat z'\right)\Bigg|_{m+1/2},$$

$$\hat z'|_{m+1/2}=\frac{\hat z|_{m+1}-\hat z|_m}{l|_{m+1/2}},$$

$$\delta a|_{m+1/2}=\frac{b}{\rho^c}\left(\pm\frac{L}{\lambda\rho^c}+a_t'\right)\Bigg|_{m+1/2},$$

$$a_t'|_{m+1/2}=\frac{a_t|_{m+1}-a_t|_m}{l|_{m+1/2}},$$

$$\delta\rho^2|_m\cong\frac{2b^2}{\rho^c}\left(\frac{\rho^c}{b}+a_\phi'\right)\Bigg|_m,$$

$$a_\phi'|_m=\frac{a_\phi|_{m+1/2}-a_\phi|_{m-1/2}}{l|_m},$$

$$\rho^2\delta z^2|_m\cong\frac{1}{2}\left\{b^2\left(\frac{L}{\rho^c}+\hat z'\right)^2\Bigg|_{m+1/2}+b^2\left(\frac{L}{\rho^c}+\hat z'\right)^2\Bigg|_{m-1/2}\right\}\equiv b^2\left(\frac{L}{\rho^c}+\hat z'\right)^2\Bigg|_m,$$

$$\dot\rho^2|_m=\frac{1}{2}\left(\frac{b^2}{\rho^{c2}}\dot a_\phi^2\Big|_{m+1/2}+\frac{b^2}{\rho^{c2}}\dot a_\phi^2\Big|_{m-1/2}\right)\equiv\frac{b^2}{\rho^{c2}}\dot a_\phi^2\Big|_m,$$

$$\rho^2(\dot{\theta}-\delta a)^2|_m \cong \frac{1}{2}\left\{ b^2\left(\mp\frac{L}{\lambda\rho^c}+f_{t\rho}\right)^2\Bigg|_{m+1/2} + b^2\left(\mp\frac{L}{\lambda\rho^c}+f_{t\rho}\right)^2\Bigg|_{m-1/2}\right\} \equiv b^2\left(\mp\frac{L}{\lambda\rho^c}+f_{t\rho}\right)^2\Bigg|_m ,$$

$$\rho\dot{\rho}\,\delta z|_m \cong \frac{1}{2}\left\{ \frac{b^2}{\rho^c}\left(\frac{L}{\rho^c}+\hat{z}'\right)\dot{a}_\phi\Bigg|_{m+1/2} + \frac{b^2}{\rho^c}\left(\frac{L}{\rho^c}+\hat{z}'\right)\dot{a}_\phi\Bigg|_{m-1/2}\right\} \equiv \frac{b^2}{\rho^c}\left(\frac{L}{\rho^c}+\hat{z}'\right)\dot{a}_\phi\Bigg|_m . \quad (25)$$

The symbol $\cong$ is used when we ignore higher order terms on $la_\phi/\rho^c|_m$. By substituting these values into Eq. (23), we obtain the effective action for the fuzzy BIon configuration,

$$S_{\mathrm{D0}} \cong -T_0\int dt\sum_m \frac{l\rho^c}{b}\left[1+\left\{ -\dot{\hat{z}}^2+\frac{b^2}{\lambda^2}\left(\frac{L}{\rho^c}+\hat{z}'\right)^2\right\} + \lambda^2\left\{ -\frac{b^2}{\lambda^2}\left(\mp\frac{L}{\lambda\rho^c}+f_{t\rho}\right)^2+\frac{b^4}{\lambda^4\rho^{c2}}\left(\frac{\rho^c}{b}+a'_\phi\right)^2 - \frac{b^2}{\lambda^2\rho^{c2}}\dot{a}_\phi^2\right\} - \lambda^2\left\{ \frac{b^2}{\lambda^2\rho^c}\dot{\hat{z}}\left(\frac{\rho^c}{b}+a'_\phi\right) - \frac{b^2}{\lambda^2\rho^c}\left(\frac{L}{\rho^c}+\hat{z}'\right)\dot{a}_\phi\right\}^2\right]^{1/2}\Bigg|_m . \quad (26)$$

Note that we used the relations $\rho^c l|_{m+1/2}=\rho^c l|_m=b$. Let us consider that l_m are sufficiently small and take the continuous limit. Then the above effective action for the fuzzy BIon reaches to

$$S_{\mathrm{D0}} = -\frac{T_0}{b}\int dt d\rho\rho\left[1+\left\{ -\dot{\hat{z}}^2+\frac{b^2}{\lambda^2}\left(\frac{L}{\rho}+\hat{z}'\right)^2\right\} + \lambda^2\left\{ -\frac{b^2}{\lambda^2}\left(\mp\frac{L}{\lambda\rho}+f_{t\rho}\right)^2+\frac{b^4}{\lambda^4\rho^2}\left(\frac{\rho}{b}+a'_\phi\right)^2 - \frac{b^2}{\lambda^2\rho^2}\dot{a}_\phi^2\right\} - \lambda^2\left\{ \frac{b^2}{\lambda^2\rho}\dot{\hat{z}}\left(\frac{\rho}{b}+a'_\phi\right) - \frac{b^2}{\lambda^2\rho}\left(\frac{L}{\rho}+\hat{z}'\right)\dot{a}_\phi\right\}^2\right]^{1/2} . \quad (27)$$

In order to justify this action, we should compare with the effective action for the BIon configuration.

The effective action for the D2-brane is described by

$$S_{\mathrm{D2}} = -T_2\int dt d\rho d\phi\rho \times\sqrt{1+\partial_\alpha z\partial^\alpha z+\frac{\lambda^2}{2}F_{\alpha\beta}F^{\alpha\beta}-\frac{\lambda^2}{4}(\epsilon^{\alpha\beta\gamma}\partial_\alpha zF_{\beta\gamma})^2}, \quad (28)$$

where indices are raised or lowered by the metric $ds^2=-dt^2+d\rho^2+\rho^2 d\phi^2$ and $\epsilon^{t\rho\phi}=1/\rho$. The fluctuations around the BIon solution is given by

$$z=z_0+L\ln\rho+\hat{z}(t,\rho), \quad F_{t\rho}=\pm\frac{L}{\lambda\rho}+f_{t\rho}(t,\rho),$$

$$F_{\rho\phi}=\frac{\rho}{b}+a'_\phi(t,\rho), \quad F_{t\phi}=\dot{a}_\phi(t,\rho). \quad (29)$$

We assumed that fluctuations $\hat{z}$, a_t, a_ρ and a_ϕ do not depend on the angular direction ϕ. By substituting these into the effective action, we obtain

$$S_{\mathrm{D2}} = -2\pi T_2\int dt d\rho\rho\left[1+\left\{ -\dot{\hat{z}}^2+\left(\frac{L}{\rho}+\hat{z}'\right)^2\right\} + \lambda^2\left\{ -\left(\mp\frac{L}{\lambda\rho}+f_{t\rho}\right)^2+\frac{1}{\rho^2}\left(\frac{\rho}{b}+a'_\phi\right)^2-\frac{1}{\rho^2}\dot{a}_\phi^2\right\} - \lambda^2\left\{ \frac{1}{\rho}\dot{\hat{z}}\left(\frac{\rho}{b}+a'_\phi\right)-\frac{1}{\rho}\left(\frac{L}{\rho}+\hat{z}'\right)\dot{a}_\phi\right\}^2\right]^{1/2} . \quad (30)$$

From these we see that the action (27) coincides with the action (30) in the case of $b=\lambda$. This result is the same as that in Ref. [8]. The condition $b=\lambda$ suggests that a D0-brane can transform into a D2-brane with the area $2\pi\lambda$.

V. CONCLUSION

In this paper we construct the fuzzy BIon configuration in the framework of BFSS matrix theory. This solution has the same properties as the BIon solution which represents a fundamental string ending on a bound state of a D2-brane and D0-branes. For instance, the differential equation (8) for the BIon corresponds to the discrete equations (16) for the fuzzy BIon. We also checked that the fuzzy BIon preserves 1/4 supersymmetry of type IIA superstring theory.

In Sec. IV we construct the effective action for the fuzzy BIon configuration by analyzing the non-Abelian Born-Infeld action for D0-branes. In the continuous limit, this action coincides with the effective action for the BIon configuration, in the case of $b=\lambda$. This means that a D0-brane can transform into a D2-brane with the area $2\pi\lambda$. This fact is also supported by the energy conservation that $T_0=2\pi\lambda T_2$. With these nontrivial confirmations, we conclude that the fuzzy BIon configuration is also a solution of nonabelian Born-Infeld action for D0-branes.

It is important to note that only a fundamental string exists around the origin of the fuzzy BIon. This would make us possible to compute the interaction between a fundamental string and D0-branes in the framework of BFSS matrix-theory. It is also interesting to generalize the fuzzy BIon configuration in the curved space background [15].

Note added. In Ref. [16] the same equations as Eq. (12) are obtained. I would like to thank Nakwoo Kim for pointing out this fact.

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[1] J.P. Gauntlett, R. Portugues, D. Tong, and P.K. Townsend, Phys. Rev. D **63**, 085002 (2001).

[2] D. Mateos and P.K. Townsend, Phys. Rev. Lett. **87**, 011602 (2001).

[3] R.G. Leigh, Mod. Phys. Lett. A **4**, 2767 (1989).

[4] M. Aganagic, C. Popescu, and J.H. Schwarz, Phys. Lett. B **393**, 311 (1997).

[5] E. Bergshoeff and P.K. Townsend, Nucl. Phys. **B490**, 145 (1997).

[6] T. Banks, W. Fischler, S. Shenker, and L. Susskind, Phys. Rev. D **55**, 5112 (1997).

[7] Y. Hyakutake, J. High Energy Phys. **05**, 013 (2001).

[8] Y. Hyakutake, hep-th/0302190.

[9] T. Banks, N. Seiberg, and S.H. Shenker, Nucl. Phys. **B490**, 91 (1997).

[10] D. Bak and K.-M. Lee, Phys. Lett. B **509**, 168 (2001).

[11] Y. Hyakutake, J. High Energy Phys. **01**, 021 (2002).

[12] A.A. Tseytlin, hep-th/9908105.

[13] A.A. Tseytlin, Nucl. Phys. **B501**, 41 (1997).

[14] R.C. Myers, J. High Energy Phys. **12**, 022 (1999).

[15] D.K. Park, S. Tamaryan, and H.J.W. Muller-Kirsten, Phys. Lett. B **563**, 224 (2003).

[16] N. Kim, Phys. Rev. D **59**, 067901 (1999).