# Conformal field theory couplings for intersecting D-branes on orientifolds

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We present a conformal field theory calculation of four-point and three-point correlation functions for the bosonic twist fields arising at the intersections of D-branes wrapping (supersymmetric) homology cycles of type II orientifold compactifications. Both the quantum contribution from local excitations at the intersections and the world-sheet disk instanton corrections are computed. As a consequence we obtain the complete expression for the Yukawa couplings of chiral fermions with the Higgs fields. The four-point correlation functions in turn lead to the determination of the four-point couplings in the effective theory, and may be of phenomenological interest.

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## I. INTRODUCTION

In recent years, the intersecting D-brane configurations have played an important role in several areas. The most prominent one is the construction of four-dimensional solutions [1-5] of type II string theory, compactified on orientifolds. In particular the appearance of the chiral matter [6,7] at the brane intersection provides a promising starting point to construct models with potential particle physics implications [6,7].

The model building with intersecting branes was developed in a series of papers. In particular, nonsupersymmetric [1-5] (and subsequently explored in [8-11]) and more recently, supersymmetric [12-18] constructions with quasirealistic features of the standardlike and grand-unified models have been given. One has tremendous freedom in the constructions of nonsupersymmetric models, since the Ramond-Ramond tadpole cancellation conditions can be satisfied for many brane configurations leading to the standard-model gauge group and three families of quarks and leptons. However, the fact that the theory is nonsupersymmetric introduces the Neveu-Schwarz-Neveu-Schwarz uncancelled tadpoles as well as the radiative corrections of the string scale. (For the constructions with intersecting D6-branes the string scale is necessarily of the order of the Planck scale. However, examples [19] with intersecting D5-branes have been given, where the string scale can be as low as the TeV scale.)

On the other hand, supersymmetric intersecting D-brane constructions are extremely constraining. Nevertheless such supersymmetric constructions with intersecting D6-branes, which have the standardlike [12,13,15,17] and grand-unified model spectra [13,18], have recently been constructed. In particular, these models have an additional quasi-hidden gauge sector that is typically confining which may have interesting implications for the supersymmetry breaking [20]. Note, however, that these models typically suffer from addi-

tional exotics [21]. Both supersymmetric and nonsupersymmetric constructions have adjoint matter associated with each brane configuration, since the toroidal cycles wrapped by the branes are *not* rigid. Interestingly, the embedding of supersymmetric four-dimensional models with intersecting D6-branes has a lift [13,14] into M theory that corresponds to the compactification of M theory on a singular  $G_2$  holonomy manifold [12,13,22–24].

While phenomenology of both nonsupersymmetric [6,11,25] and supersymmetric [20,21,26] models has been addressed, the actual string calculations of the couplings in this theory have been limited. While the tree-level gauge couplings are relatively easy to determine and their features have been studied, see, e.g., [21] and references therein. A calculation of gauge coupling threshold corrections [27] is also of interest, since it could be compared to the strong coupling limit of M-theory compactified on the corresponding  $G_2$  holonomy space [28].

An important set of tree level calculations involves the open-sector states that appear at the brane intersections. These states include the chiral matter. In the supersymmetric constructions the appearance of the full massless chiral supermultiplet is ensured there. The couplings of most interest are the three linear superpotential couplings, such as the coupling of quarks and leptons to the Higgs fields. On the other hand, the four-point couplings are also of interest, since they indicate the appearance of potentially other higher order terms in the effective Lagrangian.

The calculations of couplings of states (tachyons) appearing at the nonsupersymmetric intersections of branes also has interesting implications in the study of tachyon potential and the phenomenon of tachyon condensation. In particular, for specific *T*-dual models of [p-(p+2)] bound state configurations the four-point calculations have been addressed in [29–31].

The purpose of this paper is to perform explicit string calculations of the four-point and three-point correlation functions associated with the states appearing at the intersections of branes that wrap cycles of the internal tori. The

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FIG. 1. Target space: the intersection of two parallel branes separated by respective distances  $d_1$  and  $d_2$  and intersecting at angles  $\pi \nu$  [Fig. 1(a)]. World sheet: a disk diagram of the four twist fields located at  $x_{1,2,3,4}$  [Fig. 1(b)]. The calculation involves a map from the world sheet to target space.

nontrivial part of the calculation involves the evaluation of the correlation functions of four (three) bosonic twist fields, which signify the fact that the states at the intersection arise from the sector with twisted boundary conditions on the bosonic (and fermionic) string degrees of freedom. (For supersymmetric cycles the physical massless states at the intersection correspond to the chiral supermultiplets.) We employ the techniques of conformal field theory, which are related to the study of bosonic twist fields of the closed string theory on orbifolds [32]. Similar techniques were employed in the study of type II string theory for boundstates of p - (p+2) brane sectors [29–31].

Specifically, we focus on intersecting D-branes wrapping factorizable *N* cycles of  $T^{2N} = T^2 \times T^2 \cdots$ . Thus, in each  $T^2$  the D-branes wrap one-cycles, and the problem reduces to a calculation of correlation functions of bosonic twist fields associated with the twisted sectors at intersections of D-branes on a general  $T^2$ . Thus the final answer is a product of contributions from each correlation function on each  $T^2$ .

We provide a general result for

$$\langle \sigma_{\nu}(x_1)\sigma_{-\nu}(x_2)\sigma_{\nu}(x_3)\sigma_{-\nu}(x_4)\rangle, \qquad (1)$$

which corresponds to the bosonic twist field correlation function of states appearing at the intersection of two pairwise parallel branes with intersection angle  $\pi\nu$  (see Fig. 1).

In the case of the twist fields appearing at the same intersection, our result is interpreted in terms of the volume of the torus, the lengths of the one-cycles  $L_1$  and  $L_2$  that each set of branes wrap and the intersection numbers I.<sup>1</sup> We also address the case when the twist fields are associated with different intersections of the two branes. In particular we address in detail the summation over the instanton sectors for such general cases.



FIG. 2. Target space: the intersection of two branes intersecting, respectively, with the two parallel branes at angles  $\pi\nu$  and  $\pi\lambda$ , respectively [Fig. 2(a)]. World sheet: a disk diagram of the four twist fields located at  $x_{1,2,3,4}$  [Fig. 2(b)]. The calculation involves a map from the world sheet to target space, allowing for factorization on a three-point function.

The next calculation that we set out to do is that of the four-point correlation function:

$$\langle \sigma_{\nu}(x_1)\sigma_{-\nu}(x_2)\sigma_{-\lambda}(x_3)\sigma_{\lambda}(x_4)\rangle,$$
 (2)

which corresponds to the bosonic twist field correlation function of states appearing at the intersection of two branes intersecting at respective angles  $\pi\nu$  and  $\pi\lambda$  with the third set of parallel branes (see Fig. 2). This correlation function is specifically suited for taking the limit of  $x_2 \rightarrow x_3$  which factorizes to a three-point function associated with the intersection of three branes. This latter result is particularly interesting since it provides a key element in the calculation of the Yukawa coupling.

In this set of calculations we determine *both the classical* part and the quantum part of the amplitude and thus obtain the exact answer for the calculation. In particular, the calculation of the quantum part depends only on the angles (and is thus insensitive to the scales of the internal space). On the other hand, the classical part carries information on the actual separation among the branes and the overall volume of  $T^2$  as well.

In particular the full expression (both classical and quantum part) for the Yukawa couplings for branes wrapping factorizable cycles of  $T^6$  is written as

$$Y = \sqrt{2}g_0 2\pi \prod_{j=1}^{3} \sqrt{\frac{4\pi B(\nu_j, 1 - \nu_j)}{B(\nu_j, \lambda_j)B(\nu_j, 1 - \nu_j - \lambda_j)}}$$
$$\times \sum_m \exp\left(-\frac{A_j(m)}{2\pi\alpha'}\right), \tag{3}$$

where  $A_j(m)$  is the area of the triangle formed by the three intersecting branes on the *j*th torus and  $g_0 = e^{\Phi/2}$ , with  $\Phi$ corresponding to the type IIA dilaton. The coupling is between two fermion fields and a scalar field, i.e., the massless states appearing at the respective intersections, whose kinetic energies are taken to be canonically normalized.

While we were in a process of completing this work the paper [33] appeared where a comprehensive analysis of the classical part of the string contribution (disk instantons) to

<sup>&</sup>lt;sup>1</sup>This is a special case, when the branes wrap the canonical cycles of a torus with the complex structure  $\nu$ . A *T*-dual interpretation of this correlation function is that of the bosonic twist fields for D0-D2 brane with the magnetic flux  $B = \cot(\pi\nu)$ .

the Yukawa coupling in models with intersecting branes on Calabi-Yau manifolds was given, and extensive explicit calculations of the classical string contributions for models of intersecting branes on toroidal orientifolds were presented.

Our work has a certain overlap with that of [33]. In particular, our work focuses only on models with branes wrapping factorizable *N*-cycles of  $T^{2N} = T^2 \times T^2 \cdots$ . We evaluate the classical action contribution by explicitly solving for the classical solutions of the bosonic string with the boundary conditions governed by the locations of the D-branes. For the special case of the three-point function we therefore also derive the result of [33] that the classical string contribution to the three-point coupling involves a summation over the  $\exp(-A/2\pi\alpha')$ , where *A* corresponds to the area of the triangles associated with the intersections of the branes in each  $T^2$ . On the other hand, we have also determined the quantum part of the correlation functions, thus obtaining the full expression for the couplings.

The paper is organized as follows. In Sec. II we determine the correlation function (1). In Sec. III we calculate (2) and factorize it on a three-point function to determine the corresponding Yukawa coupling. Conclusions, which include comments on generalizations of these calculations as well as physical implications, are given in Sec. IV.

## II. FOUR-POINT FUNCTION WITH ONE INDEPENDENT ANGLE

In order to evaluate the path integral for the partition function of open strings stretching between D-branes intersecting at an angle  $\pi \nu$  we split the embedding fields  $X^i$  $=X_{cl}^{i}+X_{au}^{i}$  into a classical solution to the equation of motion, subject to the appropriate boundary conditions, and a quantum fluctuation. The mode expansion for the quantum fluctuation is not integer moded due to the boundary conditions. The vacuum of the  $X^i$  conformal field theory (CFT) is then created by primary fields  $\sigma_{\nu}$  acting on the  $SL(2,\mathbb{R})$ invariant vacuum. The partition function naturally factorizes into a classical contribution due to world sheet instanton sectors and a quantum amplitude due to quantum fluctuations. In contrast with the instanton contribution, the quantum amplitude contains no topological information about the world sheet embedding in target space, but it is still essential for the complete determination of Yukawa couplings in a general model with intersecting branes.

#### A. Evaluation of quantum amplitude

We shall employ the stress tensor method [29,30,32] to evaluate the quantum amplitude of four twist operators. For oriented theories the twist operators live on the boundary of the disk and change the boundary conditions as we move along the boundary. The boundary conditions are specified by the D-brane configuration in target space (see Fig. 1). We concentrate on a single  $T^2$  and D1-branes wrapping onecycles. The amplitude for branes wrapping factorizable three-cycles on  $T^6$  is then the product of the amplitudes for the three  $T^2$  factors. In terms of the complexified coordinates  $X = X^1 + iX^2$ ,  $\overline{X} = X^1 - iX^2$  on  $T^2$  the boundary conditions read

$$\partial X + \overline{\partial} \overline{X} = 0, \quad \partial \overline{X} + \overline{\partial} X = 0, \quad \text{on}$$

$$(-\infty, x_1) \cup (x_2, x_3) \cup (x_4, +\infty),$$

$$e^{i\pi\nu} \partial X + e^{-i\pi\nu} \overline{\partial} \overline{X} = 0,$$

$$e^{-i\pi\nu} \partial \overline{X} + e^{i\pi\nu} \overline{\partial} X = 0, \quad \text{on} \quad (x_1, x_2) \cup (x_3, x_4). \quad (4)$$

These conditions define the operator product expansions (OPEs) of the embedding fields with the twist operators, namely,

$$\partial X(z)\sigma_{\nu}(x) \sim (z-x)^{\nu-1}\tau_{\nu}(x) + \cdots,$$
  

$$\partial \overline{X}(z)\sigma_{\nu}(x) \sim (z-x)^{-\nu}\tau_{\nu}'(x) + \cdots,$$
  

$$\overline{\partial} X(\overline{z})\sigma_{\nu}(x) \sim -(\overline{z}-x)^{-\nu}\tau_{\nu}'(x) + \cdots,$$
  

$$\overline{\partial} \overline{X}(\overline{z})\sigma_{\nu}(x) \sim -(\overline{z}-x)^{\nu-1}\tau_{\nu}(x) + \cdots,$$
(5)

and similarly for  $\sigma_{-\nu}(x)$ . To evaluate the correlation function of four twist fields  $\langle \sigma_{\nu}(x_1)\sigma_{-\nu}(x_2)\sigma_{\nu}(x_3)\sigma_{-\nu}(x_4)\rangle$  we consider the correlators

g(z,w)

$$=\frac{\left\langle -\frac{1}{\alpha'}\partial X(z)\partial \bar{X}(w)\sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle}{\left\langle \sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle},$$
(6)

 $h(\overline{z},w)$ 

$$=\frac{\left\langle -\frac{1}{\alpha'}\overline{\partial}X(\overline{z})\partial\overline{X}(w)\sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle}{\left\langle \sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle},$$
(7)

 $\overline{g}(z,w)$ 

$$=\frac{\left\langle -\frac{1}{\alpha'}\partial \overline{X}(z)\partial \overline{X}(w)\sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle}{\left\langle \sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle},$$
(8)

and

$$\overline{h}(\overline{z},w)$$

$$=\frac{\left\langle -\frac{1}{\alpha'}\overline{\partial}\overline{X}(\overline{z})\partial\overline{X}(w)\sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle}{\left\langle \sigma_{\nu}(x_{1})\sigma_{-\nu}(x_{2})\sigma_{\nu}(x_{3})\sigma_{-\nu}(x_{4})\right\rangle}.$$
(9)



FIG. 3. World-sheet contours. The contours  $C_1$  and  $C_2$  [Fig. 3(a)] are the two topologically inequivalent contours leading to two independent conditions. The contours in Fig. 3(b) define the global monodromy conditions used in Sec. III.

The OPEs (5) together with the conditions

$$g(z,w) \sim (z-w)^{-2}, \quad h(\overline{z},w) \sim \text{regular}$$
 (10)

as  $z \rightarrow w$  uniquely determine

$$g(z,w) = \omega_{1-\nu}(z)\omega_{\nu}(w)$$

$$\times \left[ (1-\nu)\frac{(z-x_{1})(z-x_{3})(w-x_{2})(w-x_{4})}{(z-w)^{2}} + \nu\frac{(z-x_{2})(z-x_{4})(w-x_{1})(w-x_{3})}{(z-w)^{2}} + A(\{x_{i}\}) \right]$$
(11)

and

$$h(\overline{z},w) = -\omega_{\nu}(\overline{z})\omega_{\nu}(w)B(\{x_i\}), \qquad (12)$$

where

$$\omega_{\nu}(z) = (z - x_1)^{-\nu} (z - x_2)^{\nu - 1} (z - x_3)^{-\nu} (z - x_4)^{\nu - 1}.$$
(13)

Here  $A(\{x_i\})$  and  $B(\{x_i\})$  are functions of the twist field positions to be determined. The boundary conditions and holomorphicity imply

$$\overline{h}(z,w) = -g(z,w), \quad \overline{g}(z,w) = -h(z,w).$$
(14)

In order to determine the functions A and B we impose appropriate monodromy conditions which will ensure that the quantum fluctuations  $X_{qu}$  are local. This is guaranteed if

$$\int_{C_i} dX = \int_{C_i} d\bar{X} = 0 \tag{15}$$

where  $C_i$  is any nontrivial world-sheet contour. In the case at hand there are two topologically inequivalent contours  $C_1$ joining the intervals  $(x_2, x_3)$  and  $(-\infty, x_1)$  and  $C_2$  joining the intervals  $(x_3, x_4)$  and  $(x_1, x_2)$  [see Fig. 3(a)]. One can save some effort, however, by noticing that the contours can be analytically continued to the world-sheet boundary along which only one particular linear combination of target space fields satisfies Neumann boundary conditions and can therefore have nontrivial displacement (along the boundary of the world sheet  $d = d\tau \partial_{\tau}$  and hence the fields satisfying Dirichlet boundary conditions give no contribution). The nontrivial conditions are

$$\int_{C_1} d(e^{i\pi\nu}X - e^{-i\pi\nu}\bar{X}) = 0, \qquad (16)$$

$$\int_{C_2} d(X - \overline{X}) = 0. \tag{17}$$

When inserted into the four twist field correlation functions these lead to monodromy conditions for the correlators g,  $\overline{g}$ , h, and  $\overline{h}$ . For example, condition (17) implies

$$\int_{C_2} dz [g(z,w) - \bar{g}(z,w)] + \int_{C_2} d\bar{z} [h(\bar{z},w) - \bar{h}(\bar{z},w)] = 0.$$
(18)

Now, using relations (14) we can trade h and  $\overline{h}$  for g and  $\overline{g}$ . Moreover,  $\overline{z}$  can be traded for z by integrating along the mirror image of contour  $C_2$  about the real axis, call it contour  $\widetilde{C}_2$ . Taking into account the phases of g and  $\overline{g}$  on each of the contours one sees

$$\int_{\tilde{C}_2} dzg(z,w) = \int_{C_2} dzg(z,w), \tag{19}$$

$$\int_{\tilde{C}_2} dz \bar{g}(z, w) = \int_{C_2} dz \bar{g}(z, w), \qquad (20)$$

and hence

$$\int_{C_2} dz [g(z,w) - \bar{g}(z,w)] = 0.$$
(21)

Similarly we derive

$$\int_{C_1} dz [e^{i\pi\nu}g(z,w) - e^{-i\pi\nu}\bar{g}(z,w)] = 0.$$
(22)

Invoking  $SL(2,\mathbb{R})$  invariance we fix  $x_1=0, x_2=x, x_3 = 1, x_4 \rightarrow \infty$ . Dividing then by  $\omega_{\nu}(w)$  and letting  $w \rightarrow \infty$  we get

$$g(z,w) \rightarrow [\nu(z-x) + A(x)]\widetilde{\omega}_{1-\nu}(z)$$
(23)

$$\overline{g}(z,w) \to B(x)\widetilde{\omega}_{\nu}(z),$$
 (24)

where A(x) and B(x) have been redefined appropriately to absorb the singularity arising from  $x_4 \rightarrow \infty$  and

$$\widetilde{\omega}_{\nu}(z) = (z - x_1)^{-\nu} (z - x_2)^{\nu - 1} (z - x_3)^{-\nu}.$$
 (25)

Conditions (22) and (21) then give

$$\nu \int_{C_2} dz (z-x) \widetilde{\omega}_{1-\nu}(z) + A(x) \int_{C_2} dz \widetilde{\omega}_{1-\nu}(z) - B(x)$$
$$\times \int_{C_2} dz \widetilde{\omega}_{\nu}(z) = 0$$
(26)

and

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$$\nu \int_{C_1} dz (z-x) \widetilde{\omega}_{1-\nu}(z) + A(x) \int_{C_1} dz \widetilde{\omega}_{1-\nu}(z)$$
$$-e^{-2\pi i \nu} B(x) \int_{C_1} dz \widetilde{\omega}_{\nu}(z) = 0.$$
(27)

Evaluating the contour integrals we find

$$-\nu \int_{C_i} dz (z-x) \widetilde{\omega}_{1-\nu}(z) = x(1-x) \frac{d}{dx} \int_{C_i} dz \widetilde{\omega}_{1-\nu}(z),$$
(28)

$$\int_{C_1} dz \,\widetilde{\omega}_{1-\nu}(z) = F(x), \qquad (29)$$

and

$$\int_{C_2} dz \,\tilde{\omega}_{1-\nu}(z) = e^{i\pi\nu} F(1-x), \tag{30}$$

where

$$F(x) \equiv B(\nu, 1-\nu)F(\nu, 1-\nu; 1; x)$$
  
=  $\int_0^1 dy y^{-\nu} (1-y)^{\nu-1} (1-xy)^{-\nu}.$  (31)

 $B(\nu,1-\nu)$  is the Euler beta function and F(a,b;c;x) is the hypergeometric function. Solving for A(x) we finally obtain

$$A(x) = \frac{1}{2}x(1-x)\partial_x \log[F(x)F(1-x)].$$
 (32)

The quantum contribution  $Z_{qu}(x)$  to the four twist field correlation function can now be extracted from the OPE

$$\langle T(z)\rangle = \frac{h_{\sigma}}{(z-x)^2} + \frac{1}{z-x}\partial_x \log Z_{qu}(x) + \dots$$
(33)

as  $z \rightarrow x$ . Evaluating the left-hand side by taking the limit  $z \rightarrow x$  in the expression

$$\langle T(z) \rangle = \lim_{w \to z} \left( g(z, w) - \frac{1}{(z - w)^2} \right)$$

$$= \frac{A(\{x_i\})}{(z - x_1)(z - x_2)(z - x_3)(z - x_4)} + \frac{1}{2}\nu(1 - \nu)$$

$$\times \left( \frac{1}{(z - x_1)} + \frac{1}{(z - x_2)} + \frac{1}{(z - x_3)} + \frac{1}{(z - x_4)} \right)^2,$$

$$(34)$$

we obtain

$$Z_{qu}(x) = \lim_{x_4 \to \infty} |x_4|^{\nu(1-\nu)} \langle \sigma_{\nu}(0) \sigma_{-\nu}(x) \sigma_{\nu}(1) \sigma_{-\nu}(x_4) \rangle$$
$$= \frac{\text{const}}{[x(1-x)]^{\nu(1-\nu)} [F(x)F(1-x)]^{1/2}}.$$
(35)

### B. Evaluation of the classical contribution

The path integral over the target space fields  $X^i$  includes a sum over topologically inequivalent configurations from strings wrapping around the compact directions of the torus. The main contribution comes from configurations  $X_{cl}^i$  satisfying the classical equations of motion while the effect of fluctuations about these classical configurations is encoded in  $X_{qu}$  and was calculated in the previous section using conformal field theory techniques.

In this section we first determine the classical configurations satisfying the equation of motion subject to the boundary conditions (4), dictated by the D-brane setup. This is a straightforward boundary value problem for the Laplace operator in two dimensions and the solutions can be expressed in terms of holomorphic or antiholomorphic maps from the disk onto the target space manifold. We then evaluate the on-shell action and sum over the toroidal lattice to obtain the world-sheet instanton contribution to the four-point function.

The solutions to the above boundary value problem are

$$\partial X(z) = a \omega_{1-\nu}(z) \equiv \tilde{a} e^{-i\pi\nu} \tilde{\omega}_{1-\nu}(z),$$
  

$$\overline{\partial} \overline{X}(\overline{z}) = -a \omega_{1-\nu}(\overline{z}) \equiv -\tilde{a} e^{i\pi\nu} \tilde{\omega}_{1-\nu}(\overline{z}),$$
  

$$\partial \overline{X}(z) = b \omega_{\nu}(z) \equiv \tilde{b} e^{i\pi(\nu-1)} \tilde{\omega}_{\nu}(z),$$
  

$$\overline{\partial} X(\overline{z}) = -b \omega_{\nu}(\overline{z}) \equiv -\tilde{b} e^{-i\pi(\nu-1)} \tilde{\omega}_{\nu}(\overline{z}),$$
(36)

where the coefficients a and b are the only free parameters to be determined. These parameters reflect the freedom in specifying the length of the two independent sides of the parallelogram. The classical contribution to the path integral is then

$$Z_{cl} = e^{-S_{cl}},\tag{37}$$

where

$$S_{cl} = \frac{1}{4\pi\alpha'} \int_{C_+} d^2 z (\partial X \overline{\partial} \overline{X} + \partial \overline{X} \overline{\partial} X)$$
$$= -\frac{1}{2\pi\alpha'} \sin(\pi\nu) F(x) F(1-x) (\widetilde{a}^2 + \widetilde{b}^2) \qquad (38)$$

where we have used

$$\int_{C_{+}} d^{2}z |\tilde{\omega}_{\nu}(z)|^{2} = \int_{C_{+}} d^{2}z |\tilde{\omega}_{1-\nu}(z)|^{2}$$
$$= 2\sin(\pi\nu)F(x)F(1-x).$$
(39)

To determine the coefficients  $\tilde{a}$  and  $\tilde{b}$  we impose the monodromy conditions<sup>2</sup>

<sup>&</sup>lt;sup>2</sup>We assume for simplicity that the branes wrap cycles along the two-torus lattice vectors.

$$\int_{C_1} ds = \frac{2n_1 \pi R_1}{\sin(\pi \nu)}, \quad \int_{C_2} ds = 2n_2 \pi R_2.$$
(40)

Since  $X^2 = \cot(\pi \nu) X^1$  along  $C_1$ 

$$ds^{2} = (dX^{1})^{2} + (dX^{2})^{2} = \left(\frac{dX^{1}}{\sin(\pi\nu)}\right)^{2}.$$
 (41)

Similarly,  $ds^2 = (dX^2)^2$  along  $C_2$ . A similar calculation as for the quantum monodromy conditions then gives

$$\widetilde{a} = i \pi \left( \frac{n_1 R_1}{\sin(\pi \nu) F(x)} + \frac{n_2 R_2}{F(1-x)} \right),$$
  
$$\widetilde{b} = i \pi \left( \frac{n_1 R_1}{\sin(\pi \nu) F(x)} - \frac{n_2 R_2}{F(1-x)} \right).$$
 (42)

Hence,

$$S_{cl} = \frac{2\pi}{\alpha'} \sin(\pi\nu) F(x) F(1-x) \left[ \left( \frac{n_1 R_1}{\sin(\pi\nu) F(x)} \right)^2 + \left( \frac{n_2 R_2}{F(1-x)} \right)^2 \right].$$
 (43)

The full amplitude is now of the form

$$Z(x) \equiv \lim_{x_4 \to \infty} |x_4|^{\nu(1-\nu)} \langle \sigma_{\nu}(0) \sigma_{1-\nu}(x) \sigma_{\nu}(1) \sigma_{1-\nu}(x_4) \rangle$$
$$= Z_{qu} \sum_{m_1, m_2} e^{-S_{cl}(m_1, m_2)}, \qquad (44)$$

where  $Z_{qu}$  is determined in Eq. (36) [up to an overall constant and  $S_{cl}$  is defined in Eq. (43)].

Note, in the limit  $R_1, R_2 \rightarrow \infty$ 

$$\sum_{m_1,m_2} e^{-S_{cl}(m_1,m_2)} \to 1$$
 (45)

and hence the four twist amplitude receives no instanton corrections as expected.

### C. Canonical form of the amplitude and generalizations

The above calculation was performed for the amplitude of two intersecting branes wrapping two canonical homology cycles [a] and [b], respectively, of the  $T^2$  with the complex structure specified by  $\nu$ . We can, however, reexpress this amplitude in terms of a four-point twist amplitude for two branes wrapping two general cycles specified by the wrapping numbers  $(n_1, m_1)$  and  $(n_2, m_2)$  on  $T^2$  with the trivial complex structure, first, as

$$Z(x) = \text{const} \times [x(1-x)]^{-\nu(1-\nu)} [F(x)F(1-x)]^{-1/2}$$
$$\times \sum_{r_1, r_2} \exp{-\frac{1}{2\pi\alpha'}\sin(\pi\nu)F(x)F(1-x)}$$
$$\times \left[ \left(\frac{r_1L_1}{F(x)}\right)^2 + \left(\frac{r_2L_2}{F(1-x)}\right)^2 \right], \tag{46}$$

where  $L_i$  are the lengths of the corresponding one-cycles and can be expressed in terms of the wrapping numbers and the radii of the torus as  $L_i = 2\pi \sqrt{(n_i R_1)^2 + (m_i R_2)^2}$ . On the other hand,  $\sin(\pi\nu)$  can be reexpressed in terms of invariant quantities such as the intersection number  $I_{12} \equiv n_1 m_2$  $-n_2 m_1$  and the lengths  $L_1$ ,  $L_2$  of the one-cycles as

$$\sin(\pi\nu) = \frac{(2\pi)^2 I_{12} R_1 R_2}{L_1 L_2} = \frac{I_{12} \chi}{\sqrt{n_1^2 + \chi^2 m_1^2} \sqrt{n_2^2 + \chi^2 m_2^2}},$$
(47)

where  $\chi \equiv R_2/R_1$  is the complex structure modulus. As expected, the angle is insensitive to the overall scale, and depends only on the wrapping numbers and the complex structure modulus.

It is straightforward to generalize this result to a  $T^2$  with a nontrivial complex structure  $\tau$ . In this case it is efficient to parametrize the wrapping numbers in terms of  $m_i \rightarrow \tilde{m}_i \equiv m_i + \tau n_i$  (see, for example, [13]). The complete result takes the form (46), but with  $m_i$ 's replaced with  $\tilde{m}_i$ .

Of course, a generalization of the amplitude to the case of  $T^{2n} = T^2 \times T^2 \cdots$  (we assume the Kähler structure to be a product of the Kähler structures of each  $T^2$ ) is straightforward. In this case each twist field is just a product of individual twist fields for each  $T^2$ , and the four-twist amplitude is a product of individual twist amplitudes (46). The most interesting examples where the above calculations can be applied are cases of type IIA string theory on  $T^6 = T^2 \times T^2 \times T^2$  with intersecting D6-branes wrapping a product of three one-cycles associated with each  $T^2$ .

For the purpose of performing complete string amplitude calculations it is instructive to write down the complete vertex operators for physical bosonic states  $\chi$  and  $\chi^*$  which in the (-1) conformal ghost  $(\phi)$  picture:<sup>3</sup>

$$V_{-1;\chi} = e^{-\phi} \prod_{j=1}^{3} \sigma_{1-\nu}^{j} e^{i(1-\nu_{j})H_{j}} e^{ik_{\mu}X^{\mu}},$$
$$V_{-1;\chi} = e^{-\phi} \prod_{j=1}^{3} \sigma_{\nu}^{j} e^{-i(1-\nu_{j})H_{j}} e^{ik_{\mu}X^{\mu}},$$
(48)

where  $H_i$  corresponds to the bosonized world-sheet fermion  $\psi^i$  (world-sheet superpartner of the *i*th toroidal coordinate  $X^i$ ). Here we chose to write explicitly the complete vertex

<sup>&</sup>lt;sup>3</sup>Here for clarity we have denoted the bosonic antitwist operators by  $\sigma_{1-\nu}$  modifying the notation  $\sigma_{-\nu}$  which was used before for the same operators.

operator for the bosonic states in four dimensions; they appear in the internal space at the intersection of D6-branes wrapping a product of three one-cycles on  $T^6 = T^2 \times T^2 \times T^2$ . In the case of supersymmetry the intersection angles satisfy the condition [13]  $\sum_{j=1}^{3} \pi \nu_i = 2\pi$  which ensures that these vertex operators correspond to massless bosonic states, which now become superpartners of massless fermionic states with the following (-1/2) superconformal ghost picture vertex operators [7]:

$$V_{-1/2;\chi} = e^{-\phi/2} S_{\alpha} \prod_{j=1}^{3} \sigma_{1-\nu}^{j} e^{i[(1/2) - \nu_{j}]H_{j}} e^{ik_{\mu}X^{\mu}},$$
  
$$V_{-1/2;\chi^{*}} = e^{-\phi/2} \widetilde{S}_{\alpha} \prod_{j=1}^{3} \sigma_{\nu}^{j} e^{-i[(1/2) - \nu_{j}]H_{j}} e^{ik_{\mu}X^{\mu}}.$$
 (49)

Here  $S_{\alpha} = e^{\pm (1/2)\mathcal{H}_1 \pm (1/2)\mathcal{H}_2}$  and  $\tilde{S}_{\alpha} = e^{\pm (1/2)\mathcal{H}_1 \mp (1/2)\mathcal{H}_2}$  represent the spin fields with respective positive and negative chirality. [ $\sim e^{\mathcal{H}_{1,2}}$  are bosonized world-sheet fermions  $\psi^a$  with *a* the four-dimensional (complexified) indices.] Note that in the case of supersymmetry the vertex operators (48) for  $\chi$  and  $\chi^*$  have the N=2 world-sheet charge  $H \equiv \sum_{i=1}^{3} H_i$ , +1 and -1, respectively, and thus correctly represent the vertex operators for the bosonic component of the chiral superfield and its complex conjugate, respectively [34]. Similarly, the world-sheet charge H for the fermionic vertex operators (49) are, respectively,  $-\frac{1}{2}$  and  $\frac{1}{2}$ , again in accordance with N=2 world-sheet supersymmetry representing the fermionic components of the chiral superfield and its complex supersymmetry representing the fermionic components of the chiral superfield and its complex supersymmetry representing the fermionic components of the chiral superfield and its complex supersymmetry representing the fermionic components of the chiral superfield and its complex supersymmetry representing the fermionic components of the chiral superfield and its complex supersymmetry representing the fermionic components of the chiral superfield and its complex supersymmetry representing the fermionic components of the chiral superfield and its complex conjugate, respectively.

In the above expressions we have suppressed the Chan-Paton factors; however, they are straightforward to incorporate. The states transform as  $N \otimes \overline{M}$  under the  $U(N) \times U(M)$  gauge symmetry of the two intersecting branes (see, e.g., [13] for details).

The orientifold projection of type IIA theory involves along with the world-sheet parity projection also the mirror symmetry projection, say along the horizontal n plane of  $T^{2n}$ . Note that this projection restricts the value of  $\tau$  to be either 0 or  $\frac{1}{2}$ . Since each brane now also has an orientifold image, obtained by a map  $(n_i, \tilde{m}_i) \rightarrow (n_i, -\tilde{m}_i)$ , one can now consider four amplitudes of states appearing at the intersection of a brane, denoted by *i*, and another one, denoted by  $i^*$ , that is an orientifold image of a brane denoted by *i*. Note that the calculation of the four amplitudes proceeds analogously as above. It is possible to calculate the four-point amplitude of the states appearing at the intersection of, say, brane *i*, with its own orientifold image  $i^*$ . Such states can appear as symmetric or antisymmetric representations of the  $U(N_i)$  (for details see [13]). Again, calculation proceeds along the same lines. In the following we shall determine the crucial normalization constant of the quantum part of the correlation function.

#### D. Normalization of the amplitude

The overall normalization of the four-point amplitude can be determined by factorizing the amplitude in the limits x  $\rightarrow 0$  or  $x \rightarrow 1$  in which the four-point amplitude reduces to a product of the two three-point amplitudes. Namely, in these limits, the four-point amplitude contains a dominant contribution from the exchanges of the intermediate open string winding states around the compact directions. In the effective field theory description the zero winding states correspond to the exchange of gauge bosons living on the brane along the cycle which is not collapsed by the limiting process. As  $x \rightarrow 0$  brane 2 contributes while as  $x \rightarrow 1$  brane 1 contributes.

For the sake of concreteness we shall focus on a fourdimensional example with D6-branes wrapping a product of three-cycles. The physical states at the intersections are represented by the vertex operators (48) and (49). For that purpose we shall evaluate the four-point disk amplitude  $S_4(k_1,k_2,k_3,k_4)$  of two bosonic states  $\chi$  and two  $\chi^*$  at the intersections. We shall relate this amplitude to the product of two three-point functions  $S_3(k_1,k_2;k_3)$  of  $\chi$  and  $\chi^*$  states, and the gauge boson  $A_{\mu}$  via the unitarity condition:

$$S_{4}(k_{1},k_{2},k_{3},k_{4}) = i[g_{YM}^{0}]^{2} \int \frac{d^{4}k}{(2\pi)^{4}} \times \frac{S_{3}(k_{1},k_{2};k)S_{3}(k_{3},k_{4};-k)}{-k^{2}+i\epsilon}.$$
 (50)

With these preliminaries we now proceed with the calculations of the physical string amplitudes. The three-point amplitude takes the form

$$S_{3}(k_{1},k_{2};k_{3}) \equiv \langle V_{-1;\chi}V_{-1;\chi}*V_{0,A_{\mu}} \rangle$$
  
=  $i \frac{C_{D2}g_{0}^{3}}{\sqrt{2\alpha'}} (2\pi)^{4} \delta^{(4)} \left(\sum_{i=1}^{3} k_{i}\right) \alpha'(k_{1}-k_{2}) \cdot e.$   
(51)

This is the standard three-point amplitude, since we have taken into account that  $\langle \sigma_{\nu}(0)\sigma_{1-\nu}(1)\rangle = 1$ , and that in the picture changing procedure of the gauge-boson vertex the internal part of the fermionic stress energy tensor does not contribute. We have introduced the disk coupling  $C_{D2}$  and the coupling  $g_0$  of each vertex operator. The additional factor of  $(2\alpha')^{-1/2}$  is due to the picture changing procedure of the gauge-field vertex. The gauge-field polarization vector is denoted by  $e_{\mu}$ . [For simplicity we calculated the amplitude only for the U(1) gauge field; generalization to U(N) is straightforward.] The factorization of the four-point gauge boson amplitude onto the product of two three-point gauge boson amplitudes yields the standard relationship between  $C_{D2}$  and  $g_0: C_{D2} = 1/(g_0^2 \alpha')$ . Note that this relationship also automatically ensures that on both sides of Eq. (50) the dependence on  $g_0$  drops out; namely,  $[C_{D2}g_0^3]^2 = C_{D2}g_0^4/\alpha'$ . We have chosen  $g_0 = e^{\Phi/2}$  which allows one to write the

We have chosen  $g_0 = e^{\Phi/2}$  which allows one to write the effective kinetic energy action for the gauge fields with the prefactor  $1/[g_{YM}^0]^2$  and the kinetic energy for the  $\chi$  fields to

be canonical. Here  $g_{YM}^0$  is defined in terms of the full gauge coupling for the specific brane as<sup>4</sup>

$$[g_{YM}^{0}]^{2} \equiv e^{-\Phi}g_{YM}^{2} = 2\pi \prod_{i=1}^{3} 2\pi \sqrt{\alpha'}L_{i}^{-1}.$$
 (52)

Thus the unitarity condition (50) appears with an extra factor  $[g_{YM}^0]^2$  on the right-hand side (RHS) of the equation.

When evaluating the four-point amplitude we chose to picture change the vertex operators for both  $\chi$  fields, which in turn ensures that there is no contribution from the internal part of the fermionic stress energy contribution. The upshot is the following form of the amplitude:

$$S_{4}(k_{1},k_{2},k_{3},k_{4}) = i \frac{C_{D2}g_{0}^{4}}{2\alpha'} (2\pi)^{4} \delta^{(4)} \left(\sum_{i=1}^{4} k_{i}\right) 4\alpha'^{2}k_{1} \cdot k_{3}$$

$$\times \left(\int_{0}^{1} dx x^{-\alpha's-1} (1-x)^{-\alpha't-1} \right) \times \prod_{j=1}^{3} [x(1-x)]^{\nu_{j}(1-\nu_{j})} Z_{j}(x) + s \leftrightarrow t$$
(53)

where the  $Z_j$  is the four-twist amplitude defined in Eq. (46) with  $\nu = \nu_i$  while *s*, *t* are the Mandelstam variables.

In order to compare the left-hand side (LHS) and the RHS of Eq. (50) and thus determine the constant we shall evaluate the amplitude (46) in the limit  $x \rightarrow 0$  first. As  $x \rightarrow 0$ ,  $F(x) \sim B(\nu, 1-\nu)$  and  $F(1-x) \sim -\log(x/\delta)$ , where  $\log \delta \equiv 2\psi(1) -\psi(\nu) - \psi(1-\nu)$  and  $\psi(z) \equiv d \log \Gamma(z)/dz$ . Therefore, to take the limit  $x \rightarrow 0$  we must do a Poisson resummation over  $r_2$ . This gives

$$Z(x) = \frac{\pi\sqrt{2\alpha'}}{L_2} \frac{\operatorname{const}}{\sqrt{\sin(\pi\nu)}} [x(1-x)]^{-\nu(1-\nu)} F(x)^{-1}$$

$$\times \sum_{m_1,m_2} \exp(-\pi \frac{F(1-x)}{F(x)} \left[ \left( \frac{m_1 L_1}{\pi\sqrt{2\alpha'}} \right)^2 \right]$$

$$\times \sin(\pi\nu) + \left( \frac{\pi m_2 \sqrt{2\alpha'}}{L_2} \right)^2 \frac{1}{\sin(\pi\nu)} \right]$$

$$\sim \frac{2\pi\sqrt{\alpha'}}{L_2} \frac{\operatorname{const} \times \sqrt{\sin(\pi\nu)}}{\sqrt{2\pi}} x^{-\nu(1-\nu)}$$

$$\times \sum_{m_1,m_2} \left( \frac{x}{\delta} \right)^{[(m_1 L_1 \sin(\pi\nu)/\pi\sqrt{2\alpha'})^2 + (\pi m_2 \sqrt{2\alpha'}/L_2)^2]}.$$
(54)

The prefactors  $2\pi\sqrt{\alpha'/L_2}$  from each of the  $Z_j$  contribution combine precisely into  $[g_{YM}^0]^2/2\pi$  for brane 2 [see Eq. (52)]. Therefore the contribution of  $g_{YM}^0$  on both sides of Eq. (50) cancels. Evaluating the four-point amplitude near x=0yields a pole associated with *s*-channel exchange of the corresponding gauge field. Equating the LHS and RHS of Eq. (50) in turn then determines

$$\operatorname{const} = 2 \pi \prod_{j=1}^{3} \frac{\sqrt{2} \pi}{\sqrt{\sin(\pi \nu_j)}}.$$
(55)

The limit  $x \rightarrow 1$  gives a contribution from the *t* channel exchange of gauge bosons associated with brane 1. In this case the resummation is over  $r_1$  in Eq. (46) which again factorizes to  $[g_{YM}^0]^2$  associated with brane 1 in the four-point amplitude (53) and thus cancels the same gauge coupling contribution on the RHS of Eq. (50). Of course the rest of the calculation is consistent with the values of const in Eq. (55).

#### E. Generalization of the lattice summation

In the amplitude (46) we assumed that the four twist fields were coming from the *same* intersection and therefore the summation over all possible parallelograms reduced simply to a sum over multiples of the lengths,  $L_1$  and  $L_2$ , of the two cycles the branes wrap. We would like to generalize this amplitude to four twist fields coming from more than just one intersection.

First let us consider the correlation function of a twistantitwist pair from intersection *i* and a twist-antitwist pair from intersection *j*. Obviously, the fields coming from the same intersection must be separated by a lattice translation and therefore the distance between them is again a multiple of the length of one of the two cycles, namely,  $L_1$  or  $L_2$ . However, the minimum distance between fields from different intersections is not zero. In particular, it depends on the total number  $I_{12}$  of intersections between the two branes and the lengths of the one-cycles they wrap as we show next.

By translating the one-cycles by all possible lattice vectors in the covering space  $\mathbb{C}$  of  $T^2$  one observes that along one complete cycle each fixed point appears only once and that the one-cycle is divided into  $I_{12}$  equal intervals of length  $L/I_{12}$ , where L stands for either  $L_1$  or  $L_2$  depending on the cycle under consideration. Therefore, the minimum distance between two different intersections is generally an integer multiple of  $L_1/I_{12}$  or  $L_2/I_{12}$ . One must first decide on the labelling of the  $I_{12}$  intersections. There are obviously two evident but equivalent labellings, namely, we can index the intersections in increasing order, starting from 0, along cycle 1 or cycle 2. Let us, for concreteness, label them along cycle 1. The minimum distance between intersections i and j on cycle 1 is then obvious, but the minimum distance between the same intersections on cycle 2 is not because the intersection points are ordered differently along this cycle. So one must first determine how the fixed points are ordered along the second cycle.

To this end consider the cyclic group of order  $I_{12}$ , namely,

<sup>&</sup>lt;sup>4</sup>See, for example, [35] Vol. II, Eq. (13.3.25).



FIG. 4. The fundamental domain for a brane with wrapping numbers (3,1) (solid line) and a brane with wrapping numbers (1,2) (broken line). There are five intersection points labeled in increasing order starting from 0 along the solid brane. Starting from 0 and moving along the second brane (broken line) one first meets fixed point 2. This is the integer *k* that generates the automorphism (58) in this example.

$$\mathbb{Z}_{I_{12}} \equiv \{e, c, c^2, \dots, c^{I_{12}-1} | c^{I_{12}} = e\},$$
(56)

where c is the generator of the group. To each fixed point we can uniquely associate an element of this group by the rule

$$j \leftrightarrow c^j$$
. (57)

It can then be shown that the ordering of the fixed points along cycle 2 is given by the automorphism

$$g \mapsto g^k, \quad \forall g \in \mathbb{Z}_{I_{12}},$$
 (58)

where k is an integer between 1 and  $I_{12}-1$  which depends on the wrapping numbers of the two cycles. For this map to be an automorphism obviously k must not divide  $I_{12}$  for then the map is not injective. A general expression for k as a function of the four wrapping numbers has proved difficult to find so far apart for special cases of wrapping numbers. We emphasize that whatever this expression might be it must guarantee that k does not divide  $I_{12}=n_1m_2-n_2m_1$ . Until such an expression is known one can always determine this



FIG. 5. The two configurations for a twist-antitwist pair at intersection i and a twist-antitwist pair at intersection j. Both configurations must be included in the string amplitude.

number k by drawing the cycles in the fundamental domain of the torus. k is given by the fixed point closest to 0 along cycle 2 (see Fig. 4).

The four-point function with twist fields from intersections *i* and *j* gets contributions from the two lattice configurations in Fig. 5. If  $d_1(i,j) \propto L_1/I_{12}$  and  $d_2(i,j) \propto L_2/I_{12}$  are the minimal distances between fixed points *i* and *j* along cycles 1 and 2, respectively, the four-point amplitude takes the form

$$S_{4}(k_{1},k_{2},k_{3},k_{4}) = i \frac{C_{D2}g_{0}^{4}}{2\alpha'} (2\pi)^{4} \delta^{(4)} \left(\sum_{i=1}^{4} k_{i}\right) 4\alpha'^{2}k_{1} \cdot k_{3}$$

$$\times \left(\int_{0}^{1} dx x^{-\alpha' s-1} (1-x)^{-\alpha' t-1} \right)^{-\alpha' t-1} \times \prod_{j=1}^{3} [x(1-x)]^{\nu_{j}(1-\nu_{j})} Z_{j}^{(1)}(x)$$

$$+ \int_{0}^{1} dx x^{-\alpha' t-1} (1-x)^{-\alpha' s-1} \times \prod_{j=1}^{3} [x(1-x)]^{\nu_{j}(1-\nu_{j})} Z_{j}^{(2)}(x) , \quad (59)$$

where

$$Z^{(1)}(x) = \operatorname{const} \times [x(1-x)]^{-\nu(1-\nu)} [F(x)F(1-x)]^{-1/2}$$
$$\times \sum_{r_1, r_2} \exp{-\frac{1}{2\pi\alpha'}\sin(\pi\nu)F(x)F(1-x)}$$
$$\times \left[ \left(\frac{r_1L_1}{F(x)}\right)^2 + \left(\frac{d_2(i,j)+r_2L_2}{F(1-x)}\right)^2 \right]$$
(60)

and

$$Z^{(2)}(x) = \operatorname{const} \times [x(1-x)]^{-\nu(1-\nu)} [F(x)F(1-x)]^{-1/2}$$
$$\times \sum_{r_1, r_2} \exp{-\frac{1}{2\pi\alpha'}\sin(\pi\nu)F(x)F(1-x)}$$
$$\times \left[ \left(\frac{d_1(i,j) + r_1L_1}{F(x)}\right)^2 + \left(\frac{r_2L_2}{F(1-x)}\right)^2 \right].$$
(61)

The summation of these two lattice contributions in the case of i=j gives the two terms in the amplitude (59). For distinct *i* and *j*, however, there is no *t*-channel massless exchange since twist fields from different intersections do not couple. This can be seen explicitly from the string amplitude. As  $x \rightarrow 0$  or  $x \rightarrow 1$  only one of the terms gives a massless exchange after Poisson resummation. In particular, for the  $x \rightarrow 0$  limit one needs to do a Poisson resummation in  $r_2$  to see that only the  $Z^{(1)}$  term survives in this limit. The second

term, which contains  $d_1(i,j)$ , goes to zero even for  $r_1=0$  in this limit. Analogously, only the  $Z^{(2)}$  term contributes in the  $x \rightarrow 1$  limit.

To determine the overall normalization of the amplitude we proceed as in the previous section. Instead of two *s*-channel poles and two *t*-channel poles from gauge bosons living, respectively, on branes 1 and 2, we now get in the amplitude (53) just two *s*-channel poles, one for each type of gauge bosons. The normalization constant is still given by Eq. (55), however.

One can ask if similar results hold for four-point amplitudes of twist fields coming from more than two intersections. Clearly an amplitude of a twist-antitwist pair from intersection i with a twist from intersection j and an antitwist from intersection k is not possible since the fields coming from intersection *i* must be separated by a lattice translation which forces k = j. However, four-point amplitudes of fields coming from four different intersections are possible. In this case there will be a minimum nonzero distance between each pair of twist fields which will depend on the particular brane configuration. At most one lattice configuration exists for a given set of twist fields all coming from different intersections. The necessary and sufficient condition for a nonvanishing amplitude of two twist fields from intersections *i* and j and two antitwist fields from intersections k and l is i-k= l - j. Such amplitudes do not contain massless exchanges, though, and so their overall normalization cannot be determined directly by the above method. Nevertheless, this normalization constant is part of the quantum amplitude, which is independent of the global effects of the lattice, and hence it must be also given by Eq. (55).

## III. FOUR- AND THREE-POINT FUNCTIONS WITH TWO INDEPENDENT ANGLES

The above method can be directly applied to the problem of a four-point amplitude with two independent angles (Fig. 2). The boundary conditions now read

$$\partial X + \partial X = 0,$$

$$\partial \overline{X} + \overline{\partial} X = 0, \text{ on } (-\infty, x_1) \cup (x_2, x_3) \cup (x_4, +\infty),$$
$$e^{i\pi\nu} \partial \overline{X} + e^{-i\pi\nu} \overline{\partial} \overline{X} = 0,$$
$$e^{-i\pi\nu} \partial \overline{X} + e^{i\pi\nu} \overline{\partial} X = 0, \text{ on } (x_1, x_2),$$
$$e^{-i\pi\lambda} \partial X + e^{i\pi\lambda} \overline{\partial} \overline{X} = 0,$$

$$e^{i\pi x}\partial x + e^{-i\pi x}\partial x = 0$$
, on  $(x_3, x_4)$ . (62)

In the Appendix we evaluate the quantum amplitude  $\langle \sigma_{\nu}(x_1)\sigma_{-\nu}(x_2)\sigma_{-\lambda}(x_3)\sigma_{\lambda}(x_4) \rangle$ . The result is

$$Z_{qu}(x) = \operatorname{const} \times x^{-\nu(1-\nu)} (1-x)^{-\nu\lambda} I(x)^{-1/2}, \quad (63)$$

$$I(x) \equiv (1-x)^{1-\nu-\lambda} [B(\nu,\lambda)F_1(1-x)K_2(x) + B(1-\nu,1-\lambda)F_2(1-x)K_1(x)].$$
(64)

 $B(\nu,\lambda)$  is the Euler beta function and  $F_i$ ,  $K_i$  are hypergeometric functions defined in the Appendix.

From the boundary conditions (62) we determine the classical solutions

ē

$$\partial X(z) = a \omega_{1-\nu,\lambda}(z) \equiv \tilde{a} e^{i\pi(\lambda-1)} \tilde{\omega}_{1-\nu,\lambda}(z)$$

$$\bar{\partial} \bar{X}(\bar{z}) = -a \omega_{1-\nu,\lambda}(\bar{z}) \equiv -\tilde{a} e^{-i\pi(\lambda-1)} \tilde{\omega}_{1-\nu,\lambda}(\bar{z})$$

$$\partial \bar{X}(z) = b \omega_{\nu,1-\lambda}(z) \equiv \tilde{b} e^{-i\pi\lambda} \tilde{\omega}_{\nu,1-\lambda}(z)$$

$$\bar{\partial} X(\bar{z}) = -b \omega_{\nu,1-\lambda}(\bar{z}) \equiv -\tilde{b} e^{i\pi\lambda} \tilde{\omega}_{\nu,1-\lambda}(\bar{z}).$$
(65)

Again the parameters a and b are arbitrary and reflect the freedom in specifying the lengths  $d_1$  and  $d_2$  of the four-sided polygon. However, to obtain a three-point amplitude one must take the limit  $x_2 \rightarrow x_3$ . Unless  $1 - \nu - \lambda = 0$ , which is precisely the case of one independent angle considered above, one of the two linearly independent solutions becomes singular in this limit. For  $1 - \nu - \lambda > 0$ , as we will assume without loss of generality,  $\omega_{\nu,1-\lambda}(z) = (z-x_1)^{-\nu}(z)$  $(z-x_2)^{\nu-1}(z-x_3)^{\lambda-1}(z-x_4)^{-\lambda}$  develops a nonintegrable singularity at  $z=x_3$  in the limit  $x_2 \rightarrow x_3$ . Therefore, the fourpoint amplitude that reduces to the three-point amplitude must have b=0. This is to be expected since the distance  $d_2$ cannot be an independent parameter if one wants to get a three-point amplitude. In fact if b is set to zero  $d_2$  becomes a function of  $d_1$  and  $x_2$  which tends to zero as  $x_2 \rightarrow x_3$  as required. To keep the discussion general, though, we first consider the problem with arbitrary a and b.

The classical action is given by

$$S_{cl} = -\frac{1}{4\pi\alpha'} \left[ \tilde{a}^2 \int_{\mathbb{C}_+} d^2 z |\tilde{\omega}_{1-\nu,\lambda}(z)|^2 + \tilde{b}^2 \int_{\mathbb{C}_+} d^2 z |\tilde{\omega}_{\nu,1-\lambda}(z)|^2 \right]$$
$$= -\frac{I(x)}{4\alpha'} [\tilde{a}^2 + (1-x)^{-2(1-\nu-\lambda)} \tilde{b}^2], \qquad (66)$$

where the integrals have been evaluated using the method of [36] to factorize closed string amplitudes into a product of open string amplitudes. To determine the coefficients  $\tilde{a}$  and  $\tilde{b}$  we impose the global monodromy conditions

$$\int_{C_1'} dX^2 = d_1, \quad \int_{C_2'} dX^2 = d_2, \quad (67)$$

where the contours  $C'_1$  and  $C'_2$  are shown in Fig. 3(b). The two other lengths of the polygon are automatically deter-

where

mined and no other contours provide any additional information.<sup>5</sup> Solving these conditions for  $\tilde{a}$  and  $\tilde{b}$  we obtain

$$\widetilde{a} = i[(1-x)^{-(1-\nu-\lambda)}B(\nu,\lambda)F_1(1-x)d_1 +B(1-\nu,1-\lambda)F_2(1-x)d_2]/J(x)$$
(68)

and

$$\widetilde{b} = i[B(\nu,\lambda)F_1(1-x)d_2 + (1-x)^{(1-\nu-\lambda)} \\
\times B(1-\nu,1-\lambda)F_2(1-x)d_1]/J(x),$$
(69)

where

$$J(x) \equiv (1-x)^{-(1-\nu-\lambda)} [B(\nu,\lambda)F_1(1-x)]^2 + (1-x)^{(1-\nu-\lambda)} [B(1-\nu,1-\lambda)F_2(1-x)]^2.$$
(70)

Although the general four-point amplitude with arbitrary  $d_1$  and  $d_2$  is interesting itself, we want to evaluate the threepoint amplitude which gives the instanton corrections to the Yukawa couplings. *b* then must be set to zero and the monodromy conditions give instead

$$\tilde{a} = \frac{id_1}{B(\nu,\lambda)F_1(1-x)} \tag{71}$$

while  $d_2$  is x dependent

$$d_{2}(x) = d_{1}(1-x)^{1-\nu-\lambda} \frac{B(1-\lambda,1-\nu)F_{2}(1-x)}{B(\lambda,\nu)F_{1}(1-x)} \to 0$$
(72)

as  $x \rightarrow 1$ , which therefore correctly produces a three-point amplitude in this limit. The on-shell action becomes

$$S_{cl} = \frac{I(x)}{4\alpha'} \left(\frac{d_1}{B(\nu,\lambda)F_1(1-x)}\right)^2$$
(73)

and the complete four-point amplitude takes the form

$$Z_{4}(x) = \text{const} \times x^{-\nu(1-\nu)} (1-x)^{-\nu\lambda} I(x)^{-1/2} \\ \times \sum_{m} \exp \left(\frac{I(x)}{4\alpha'} \left(\frac{d_{1}(m)}{B(\nu,\lambda)F_{1}(1-x)}\right)^{2}\right).$$
(74)

Here  $d_1(m) = L_0 + mL$  is the most general form the distance  $d_1$  can take when the polygon is embedded in a lattice.  $L_0$  and the cycle length *L* will generically depend on the lattice and the D-brane configuration. The overall normalization constant can be determined by the same method used to fix the normalization of the four-point amplitude with one inde-

pendent angle. We specialize to  $T^6$  since the amplitude of most interest is the one involving factorizable three-cycles on  $T^6$ . We find

const=
$$16\pi^{5/2}$$
. (75)

The limit  $x \rightarrow 1$  of the four-point amplitude produces the full expression for the three-point amplitude. In particular, since the conformal weights of the twist fields satisfy  $h_{\nu}+h_{\lambda}$  $-h_{\nu+\lambda}=\nu\lambda$ , the latter contains the correct singularity in this limit in agreement with the operator product expansion

$$\mathcal{O}_{i}(z_{1})\mathcal{O}_{j}(z_{2}) \sim \sum_{k} C_{ij}^{k} \mathcal{O}_{k}(z_{2})(z_{2}-z_{1})^{h_{k}-h_{j}-h_{i}}.$$
 (76)

One can then show that the full three-point amplitude for branes wrapping factorizable three-cycles on  $T^6$  takes the form

$$Z_{3} = 2 \pi \prod_{j=1}^{3} \sqrt{\frac{4 \pi B(\nu_{j}, 1 - \nu_{j})}{B(\nu_{j}, \lambda_{j}) B(\nu_{j}, 1 - \nu_{j} - \lambda_{j})}} \sum_{m} \exp{-\frac{A_{j}(m)}{2 \pi \alpha'}}$$
$$= 2 \pi \prod_{j=1}^{3} \sqrt{\frac{4 \pi \Gamma(1 - \nu_{j}) \Gamma(1 - \lambda_{j}) \Gamma(\nu_{j} + \lambda_{j})}{\Gamma(\nu_{j}) \Gamma(\lambda_{j}) \Gamma(1 - \nu_{j} - \lambda_{j})}}}$$
$$\times \sum_{m} \exp{-\frac{A_{j}(m)}{2 \pi \alpha'}}, \tag{77}$$

where  $A_j(m)$  is the area of the triangle formed by the three intersecting branes on the *j*th torus. Note that the amplitude is completely symmetric in all three angles of the triangle as it should.

The above three-point correlation function of bosonic twist fields (78) is the key contribution to the physical Yukawa coupling of two fermionic and one bosonic field. The full three-point amplitude of these fields can be determined by employing the normalization factor for the disk amplitude,  $C_{D2} = 1/(g_0^2 \alpha')$ , and the bosonic string vertex operator in -1 picture [Eq. (48)],  $g_0$ , as determined in Sec. II D. In addition, the normalization factor of the fermionic vertex operators in the -1/2 picture [Eq. (49)] turns out to be  $2^{1/4}\sqrt{\alpha'}g_0$ . (This normalization can be determined from the string amplitudes of fermionic fields to gauge vector bosons, along the same lines as described for the corresponding bosonic fields in Sec. II D.) Thus the final expression for the physical Yukawa couplings is given by  $\sqrt{2}g_0 \times Z_3$ .

A comprehensive analysis of the triangles that contribute to this amplitude for a given set of intersections in diverse brane configurations and including the effect of nontrivial complex structure or Wilson lines has been done in [33].

#### **IV. CONCLUSIONS**

We have applied conformal field theory techniques to obtain three-point and four-point correlation functions of twist fields from D-branes wrapping factorizable *n*-cycles of  $T^{2n} = T^2 \times T^2 \cdots$  and intersecting at points in the internal  $T^{2n}$ . The method allows for a complete determination of the

<sup>&</sup>lt;sup>5</sup>Note, however, that these two conditions become linearly dependent when  $1 - \nu - \lambda = 0$  because  $d_1 = d_2$ . In that case, which is the problem of one independent angle discussed earlier, one should take the contours of Fig. 3(a).

amplitude, including the quantum contribution. Its most interesting application is to the three-point function calculation of intersecting D6-branes wrapping factorizable three-cycles of  $T^6$ , which in turn gives the complete Yukawa coupling of two four-dimensional chiral fermions to the Higgs field.

The method also applies to the study of the three-point and four-point twist field correlation functions in models with orientifold and orbifold projections, as discussed in Sec. II C. Due to the mirror symmetry projection along an *n*-plane in  $T^{2n}$  the branes can now intersect with an orientifold image of another brane. However, the amplitude calculation for states at the intersection proceeds analogously. As for the  $Z_2 \times Z_2$  orbifold projection, we mention that the combined orbifold and orientifold action maps cycles into themselves; thus each intersection is accompanied by a combination of the orientifold and orbifold images that have to be carefully taken into account. We hope to return to the detailed discussion of these contributions in the future work.

The formalism can be applied to the calculation of three and four fermion couplings of classes of type II orientifold compactifications with intersecting D-branes. Among them the four-dimensional type IIA orientifold compactification with intersecting D6-branes is interesting; supersymmetric compactifications of this type are directly related to  $G_2$  compactification of M theory. In addition to the Yukawa couplings, the four-point amplitude of chiral fermions can provide low energy corrections to the effective four-fermion coupling and may be of phenomenological interest. We plan to address further these effects in concrete semirealistic N=1 supersymmetric models, such as those constructed in, e.g., [12,13,17,18].

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## APPENDIX: EVALUATION OF QUANTUM FOUR-POINT AMPLITUDE WITH TWO INDEPENDENT ANGLES

Here we evaluate the four-point correlation function  $\langle \sigma_{\nu}(x_1)\sigma_{-\nu}(x_2)\sigma_{-\lambda}(x_3)\sigma_{\lambda}(x_4)\rangle$ . The calculation closely parallels the calculation for closed string amplitudes on orbifolds [37,38]. As before we introduce the auxiliary correlators g(z,w),  $\bar{g}(z,w)$ ,  $h(z,w) = -\bar{g}(z,w)$  and  $\bar{h}(z,w) = -g(z,w)$  as defined in Sec. II A. In terms of

$$\omega_{\nu,\lambda}(z) = (z - x_1)^{-\nu} (z - x_2)^{\nu - 1} (z - x_3)^{-\lambda} (z - x_4)^{\lambda - 1}$$
(A1)

these take the form

$$g(z,w) = \omega_{1-\nu,\lambda}(z)\omega_{\nu,1-\lambda}(w)\left(\frac{P(z,w)}{(z-w)^2} + A(\{x_i\})\right)$$
(A2)

$$h(\overline{z},w) = \omega_{\nu,1-\lambda}(\overline{z})\omega_{\nu,1-\lambda}B(x_i), \qquad (A3)$$

where

$$P(z,w) = \sum_{i,j=0}^{2} a_{ij} w^{i} z^{j}$$
(A4)

and the condition

$$g(z,w) \sim (z-w)^{-2}$$
 (A5)

determines all coefficients  $a_{ij}$  except for  $a_{20}$ ,  $a_{11}$ , and  $a_{02}$  for which it provides two linear equations. Solving these for  $a_{02}$  and  $a_{11}$  we find

$$\langle T(z) \rangle = \lim_{w \to z} \left( g(z,w) - \frac{1}{(z-w)^2} \right)$$

$$= \frac{A(\{x_i\}) + z^2 + a_{21}z + a_{20}}{(z-x_1)(z-x_2)(z-x_3)(z-x_4)} + \frac{1}{2}\nu(1-\nu)$$

$$\times \left( \frac{1}{(z-x_1)} - \frac{1}{(z-x_2)} \right)^2 + \frac{1}{2}\lambda(1-\lambda)$$

$$\times \left( \frac{1}{(z-x_3)} - \frac{1}{(z-x_4)} \right)^2 - \left( \frac{\nu}{(z-x_1)} + \frac{1-\nu}{(z-x_2)} \right)$$

$$\times \left( \frac{1-\lambda}{(z-x_3)} + \frac{\lambda}{(z-x_4)} \right),$$
(A6)

where  $a_{21} = -[(1 - \nu)x_1 + \nu x_2 + \lambda x_3 + (1 - \lambda)x_4]$ . The freedom in  $a_{20}$  does not affect the final result since  $A(\{x_i\})$  will be determined after  $a_{20}$  is fixed. Fixing

$$a_{20} = \frac{1}{2} [(1 - \nu + \lambda)x_1x_3 + (1 - \nu - \lambda)x_1x_4 - (1 - \nu - \lambda)x_2x_3 + (1 + \nu - \lambda)x_2x_4]$$
(A7)

and using  $SL(2,\mathbb{R})$  invariance to set  $x_1=0, x_2=x, x_3 = 1, x_4 \rightarrow \infty$  we find

$$\partial_{x} Z_{qu}(x) = \frac{1}{2} \frac{(1-\nu-\lambda)}{x-1} - \frac{\nu(1-\nu)}{x} - \frac{(1-\nu)(1-\lambda)}{x-1} + \frac{A(x)}{x(x-1)},$$
(A8)

where

$$A(x) = \lim_{x_4 \to \infty} -x_4^{-1} A(\{x_i\})$$
(A9)

and

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$$Z_{qu}(x) = \lim_{x_4 \to \infty} |x_4|^{\lambda(1-\lambda)} \langle \sigma_{\nu}(0)\sigma_{-\nu}(x)\sigma_{-\lambda}(1)\sigma_{\lambda}(x_4) \rangle.$$
(A10)

As before A(x) is determined by imposing the quantum monodromy conditions

$$\int_{C_i} dX = \int_{C_i} d\overline{X} = 0, \qquad (A11)$$

leading again to conditions (22) and (21). We insert

$$g(z,w) \rightarrow \left[\frac{1}{2}(1-\nu-\lambda)x + (1-\lambda)(z-x) + A(x)\right] \widetilde{\omega}_{1-\nu,\lambda}(z)$$
(A12)

$$\overline{g}(z,w) \rightarrow B(x)\widetilde{\omega}_{\nu,1-\lambda}(z)$$
 (A13)

and arrive at the two equations

$$\left[\frac{1}{2}(1-\nu-\lambda)x+A(x)\right]\int_{C_1} dz\widetilde{\omega}_{1-\nu,\lambda}(z)+(1-\lambda)$$
$$\times \int_{C_1} dz(z-x)\widetilde{\omega}_{1-\nu,\lambda}(z)-e^{-2\pi i\nu}B(x)$$
$$\times \int_{C_1} dz\widetilde{\omega}_{1-\nu,\lambda}(z)=0$$
(A14)

and

$$\left[\frac{1}{2}(1-\nu-\lambda)x+A(x)\right]\int_{C_2} dz \,\widetilde{\omega}_{1-\nu,\lambda}(z)+(1-\lambda)$$

$$\times \int_{C_2} dz (z-x) \,\widetilde{\omega}_{1-\nu,\lambda}(z)-B(x) \int_{C_2} dz \,\widetilde{\omega}_{1-\nu,\lambda}(z)=0.$$
(A15)

To solve these we need the integrals

$$\int_{C_1} dz \,\widetilde{\omega}_{1-\nu,\lambda}(z) = e^{i\pi(1-\nu-\lambda)} B(\nu,1-\nu) K_1(x),$$
(A16)

$$\int_{C_2} dz \,\widetilde{\omega}_{1-\nu,\lambda}(z) = e^{i\pi(1-\lambda)} B(1-\lambda,1-\nu) \\ \times (1-x)^{1-\nu-\lambda} F_2(x), \quad (A17)$$

where

$$K_1(x) \equiv F(\nu, \lambda; 1, x), \qquad (A18)$$

$$K_2(x) \equiv F(1 - \nu, 1 - \lambda; 1; x),$$
 (A19)

$$F_1(x) \equiv F(\nu, \lambda; \nu + \lambda, x), \qquad (A20)$$

and

$$F_2(x) \equiv F(1-\nu, 1-\lambda; 2-\nu-\lambda, x).$$
 (A21)

One also needs the following identities:

$$(1-\nu)(1-\lambda)F(\nu,\lambda;2,x) = (1-\nu-\lambda)K_{1}(x) + (1-x)\partial_{x}K_{1}(x),$$
(A22)

$$(1-\nu)(1-\lambda)F(1-\lambda,1-\nu;3-\nu-\lambda;1-x) = -(2-\nu-\lambda)\partial_x F_2(1-x),$$
(A23)

$$B(1-\nu,1-\lambda)(1-x)^{1-\nu-\lambda}F_{2}(1-x)$$
  
=  $B(\nu,\lambda)F_{1}(1-x) - B(1-\nu,1-\lambda)$   
 $\times B(\nu,\lambda)(1-\nu-\lambda)K_{1}(x),$  (A24)

and

$$K_1(x) = (1-x)^{1-\nu-\lambda} K_2(x).$$
 (A25)

After some algebra we arrive at the desired result

$$2A(x) = x(1-x)\partial_x \log[B(\nu,\lambda)F_1(1-x)K_2(x) + B(1-\nu,1-\lambda)F_2(1-x)K_1(x)].$$
(A26)

Hence

$$Z_{qu}(x) = \text{const} \times x^{-\nu(1-\nu)} (1-x)^{-\nu\lambda} I(x)^{-1/2}, \quad (A27)$$

where

$$I(x) \equiv (1-x)^{1-\nu-\lambda} [B(\nu,\lambda)F_1(1-x)K_2(x) + B(1-\nu,1-\lambda)F_2(1-x)K_1(x)].$$
(A28)

For completeness let us also give the result for B(x), namely,

$$B(x) = \frac{1}{2}e^{-2\pi i\lambda}x(1-x)^{(2-\nu-\lambda)}\partial_x \log\left[\frac{B(\nu,\lambda)F_1(1-x)K_2(x) + B(1-\nu,1-\lambda)F_2(1-x)K_1(x)}{(K_2(x))^2}\right].$$
 (A29)

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