# $Z_k$ string fluxes and monopole confinement in non-Abelian theories

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Recently we considered $N=2$ super Yang-Mills theory with a $N=2$ mass breaking term and showed the existence of Bogomol'nyi-Prasad-Sommerfield (BPS) $Z_k$ string solutions for arbitrary simple gauge groups which are spontaneously broken to non-Abelian residual gauge groups. We also calculated their string tensions exactly. In doing so, we have considered in particular the hypermultiplet in the same representation as that of a diquark condensate. In the present work, we analyze some of the different phases of the theory and find that the magnetic fluxes of the monopoles are multiples of the fundamental $Z_k$ string flux, allowing for monopole confinement in one of the phase transitions of the theory. We also calculate the threshold length for string breaking. Some of these confining theories can be obtained by adding a deformation term to the $N=2$ or $N=4$ superconformal theories.

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## I. INTRODUCTION

It has long been believed that quark confinement would be dual to a non-Abelian generalization of the Meissner effect, as proposed by 't Hooft and Mandelstam many years ago [1]. Following their ideas, important progress was made by Seiberg and Witten [2], who starting from an $N=2$ $SU(2)$ supersymmetric theory, obtained an effective $N=2$ $U(1)$ super QED with an $N=2$ mass breaking term associated with the point in moduli space where a monopole becomes massless. In this effective theory, the $U(1)$ group is broken to a discrete subgroup. As it happens, the theory develops (solitonic) string solutions and monopole confinement occurs, with these monopoles being identified with electric charges. After that, much interesting work appeared [3], analyzing various aspects of $N=2$ $SU(N_c)$ supersymmetric QCD with $N=2$ $U(1)^{N_c-1}$ theory with an $N=2$ mass breaking term as the effective theory. These effective theories also have string solutions when the gauge group is broken by the Higgs mechanism. These string solutions confine monopoles and carry topological charges in the group $\Pi_1[U(1)^{N_c-1}/Z^{N_c-1}]=Z^{N_c-1}$. On the other hand, recently we considered [4] $N=2$ super Yang-Mills theory with an $N=2$ mass breaking term, with an arbitrary simple gauge group $G$ which in general is broken to a non-Abelian residual gauge group. One spontaneous symmetry breaking is produced by a complex scalar $\phi$ that could be in the symmetric part of the tensor product of $k$ fundamental representations, with $k\geqslant 2$. In particular, if $k=2$, this is the same representation as that of a diquark[1] condensate, and therefore this scalar can be thought as being itself the diquark condensate (or the monopole condensate in the dual theory). Therefore, when $k=2$, we can consider this theory as being an effective theory, as the Abelian-Higgs is an effective theory for BCS theory. In addition to the fact that our "effective" theory has a non-Abelian gauge group $G$, another interesting feature is that it has solitonic monopoles which are solutions of the equations of motion, unlike the Dirac monopoles that usually appear in the Abelian theories. When the scalar $\phi$ acquires an expectation value it breaks the gauge group $G$ into $G_\phi$, such that $\Pi_1(G/G_\phi)=Z_k$, allowing the existence of $Z_k$ string solutions. We showed the existence of Bogomol'nyi-Prasad-Sommerfield (BPS) $Z_k$ string solutions for these theories (for arbitrary $k\geqslant 2$)and calculated their string tensions exactly. In the present work, we analyze some other features of these theories. In Sec. II we show that by continuously varying a mass parameter $m$ we can pass from an unbroken phase to a phase with free monopoles and then to a phase with $Z_k$ strings and confined monopoles. In Sec. III we analyze the phase that has solitonic BPS monopole solutions, which we call the "free-monopole phase." These monopole solutions are expected to fill irreducible representations of the dual unbroken gauge group [5]. In this phase, $N=2$ supersymmetry is recovered and we show that some of these theories are conformal. In Sec. IV we analyze the magnetic fluxes of the BPS strings that appear in the superconducting phase. We show that the magnetic fluxes of the magnetic monopoles are multiples of the fundamental string flux and therefore the monopoles can become confined. We also obtain the threshold length for a string to break into a new monopole-antimonopole pair. These results are obtained considering the theory in the weak coupling regime since, in the "dual Meissner effect picture" for confinement, one wants to map a theory in the weak coupling regime with monopole confinement to a theory in the strong coupling regime with quark confinement, through a duality transformation. The general topological aspects for monopole confinement during a phase transition have been given in [6] (see also [7]). Our aim here is to analyze the monopole confinement in our specific theory. We conclude with a summary of the results.

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[1]By "quark" we mean a fermion in a fundamental representation.

 

## II. PHASES OF THE THEORY

As is quite well known, in the broken phase of the Abelian-Higgs theory in 3+1 dimensions, there exist string solutions with string tension satisfying the inequality

$$T \geq \frac{q_\phi}{2} a^2 |\Phi_{\rm st}|, \tag{1}$$

where $a$ is a breaking parameter which appears in the potential, $\Phi_{\rm st}$ is the string's magnetic flux, which satisfies the quantization condition

$$\Phi_{\rm st} = \frac{2\pi n}{q_\phi}, \quad n \in Z, \tag{2}$$

and $q_\phi$ is the electric charge of the scalar field. Considering that $\phi^\dagger$ is a condensate of electron pairs, then[2] $q_\phi = 2e$. Following 't Hooft and Mandelstam's [1] idea, if one considers a Dirac monopole-antimonopole system in the Abelian-Higgs theory, the magnetic lines cannot spread over space but must rather form a string which gives rise to a confining potential between the monopoles. That is only possible because the Dirac monopole magnetic flux is $\Phi_{\rm mon} = g = 2\pi/e$, which is twice the fundamental string's magnetic flux, allowing one to attach to the monopole two strings with $n=1$.

One of our aims in this work is to generalize some of these ideas on monopole confinement to non-Abelian gauge theories. For simplicity let us consider a gauge group $G$ which is simple, connected, and simply connected, and adopt the same conventions as in [4]. Following our previous work, we shall consider a Yang-Mills theory with a complex scalar $S$ in the adjoint representation and a complex scalar $\phi$ in another particular representation. We consider a scalar in the adjoint representation because in a spontaneous symmetry breaking it can produce an unbroken gauge group with a $U(1)$ factor, which allows the existence of monopole solutions. Additionally, another motivation for having a scalar in the adjoint representation is because with it we can form an $N=2$ vector supermultiplet and, as in the Abelian-Higgs theory, the BPS string solutions appear naturally in a theory with $N=2$ supersymmetry and an $N=2$ mass breaking term. Moreover, in a theory with the field content of $N=2$, the monopole spin is consistent with quark-monopole duality [10], which is another important ingredient in 't Hooft and Mandelstam's ideas. In order to have monopole confinement we need also to have string solutions. A necessary condition for the existence of a (topological) string is to have a nontrivial first homotopy group of the vacuum manifold. One way to produce a spontaneous symmetry breaking satisfying this condition is to introduce a complex scalar $\phi$ in a representation that contains the weight state $|k\lambda_\phi\rangle$ [8], where $k$ is an integer greater than or equal to 2, and $\lambda_\phi$ a fundamental weight. For an arbitrary gauge group $G$ there are at least three possible representations which have this weight state. One is to consider $\phi$ in the representation with $k\lambda_\phi$ as highest weight, which we shall denote $R_{k\lambda_\phi}$. We can also consider $\phi$ to be in the direct product of $k$ fundamental representations with fundamental weight $\lambda_\phi$, which we shall denote $R^{\otimes}_{k\lambda_\phi}$. Finally, a third possibility would be to consider $\phi$ in the symmetric part of $R^{\otimes}_{k\lambda_\phi}$, called $R^{\rm sym}_{k\lambda_\phi}$, which always contains $R_{k\lambda_\phi}$. This last possibility has the extra physical motivation that, if $k=2$, it corresponds to the representation of a condensate of two massless fermions (in the microscopic theory) in the fundamental representation with fundamental weight $\lambda_\phi$, which we shall loosely call quarks. Therefore, for $k=2$, we could interpret $\phi$ as being this diquark condensate. In this case, when $\phi$ takes a nontrivial expectation value, it also gives rise to a mass term for these quarks. In order to have $N=2$ supersymmetry we need another complex scalar to be in the same hypermultiplet as $\phi$. For simplicity's sake, however, we shall ignore it, setting it to zero. Note that, for the gauge group $SU(n)$, the scalar $S$ in the adjoint representation could also be interpreted as a bound state of quark-antiquark, for the quark in the $n$-dimensional representation.

Let us consider the Lagrangian used in [4],

$$L = -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} + \frac{1}{2} (D^\mu S)^*_a (D_\mu S)_a + \frac{1}{2} (D^\mu \phi)^\dagger D_\mu \phi - V(S,\phi) \tag{3}$$

with

$$V(S,\phi) = \frac{1}{2}(Y_a^2 + F^\dagger F) \geq 0 \tag{4}$$

where

$$Y_a = \frac{e}{2}\left\{ (\phi^\dagger T_a \phi) + S_b^* i f_{bca} S_c - m\left(\frac{S_a + S_a^*}{2}\right)\right\}, \tag{5}$$

$$F = e\left(S^\dagger - \frac{\mu}{e}\right)\phi. \tag{6}$$

$T_a$ are the orthogonal Lie algebra generators which satisfy

$$\mathrm{Tr}(T_a T_b) = x_\phi \psi^2 \delta_{ab}, \tag{7}$$

where $x_\phi$ is the Dynkin index of $\phi$'s representation and $\psi^2$ is the length squared of the highest root, which we shall take to be 2. This Lagrangian is the bosonic part of $N=2$ super Yang-Mills theory with one flavor (with one of the aforementioned scalars of the hypermultiplet put equal to zero) and an $N=2$ breaking mass term.[3] The parameter $\mu$ gives a bare mass to $\phi$ and $m$ gives a bare mass to the *real part* of $S$ which therefore breaks $N=2$ to $N=0$. This breaking is dif-

[2]Considering that $\hbar = 1 = c$.

[3]One can check that easily by comparing with the Lagrangian of $N=2$ super Yang-Mills theory written in the Appendix of [4].

ferent from the one considered by Seiberg and Witten [2], which breaks $N=2$ to $N=1$. We shall consider that this theory is in the weak coupling regime.

The vacua are solutions to the equation $V(S,\phi)=0$, which is equivalent to the conditions

$$Y_a=0=F. \tag{8}$$

In order for topological string solutions to exist, we look for vacuum solutions of the form

$$\phi^{\text{vac}}=a|k\lambda_\phi\rangle, \tag{9}$$

$$S^{\text{vac}}=b\lambda_\phi\cdot H, \tag{10}$$

where $a$ is a complex constant, $b$ is a real constant, and $|k\lambda_\phi\rangle$ is a weight state with $\lambda_\phi$ being an arbitrary fundamental weight and $k$ being an integer greater than or equal to 2. If $a\neq 0$, this configuration breaks $G\rightarrow G_\phi$ in such a way that [8] $\Pi_1(G/G_\phi)=Z_k$, which is a necessary condition for the existence of $Z_k$ strings. Let us consider that $\mu>0$. Following [4], from the vacuum conditions (8) one can conclude that

$$|a|^2=\frac{mb}{k},$$

$$\left(kb\lambda_\phi^2-\frac{\mu}{e}\right)a=0.$$

There are three possibilities.

(i) $m<0$. If $a\neq 0$, then $b=\mu/\lambda_\phi^2 ke>0$, which would imply $|a|^2<0$. Therefore we must have $a=0=b$ and the gauge group $G$ remains unbroken.

(ii) $m=0$. Then $a=0$ and $b$ can be an arbitrary constant. In this case, considering $b\neq 0$, $S^{\text{vac}}$ breaks [8]

$$G\rightarrow G_S\equiv[K\times U(1)]/Z_l, \tag{11}$$

where $k$ is the subgroup of $G$ associated with the algebra whose Dynkin diagram is given by removing the dot corresponding to $\lambda_\phi$ from that of $G$. The $U(1)$ factor is generated by $\lambda_\phi\cdot H$ and $Z_l$ is a discrete subgroup of $U(1)$ and $k$. The order $l$ of $Z_l$ is equal to $p_\phi|Z(K)|/|Z(G)|$ where $|Z(G)|$ and $|Z(K)|$ are, respectively, the orders of the centers of the groups $G$ and $K$, and $p_\phi$ is the smallest integer such that $p_\phi 2\lambda_\phi/\alpha_\phi^2$ is in the coroot lattice [9].

(iii) $m>0$. Besides the solution $a=0=b$, we can have

$$|a|^2=\frac{m\mu}{k^2e\lambda_\phi^2}, \tag{12}$$

$$b=\frac{\mu}{ke\lambda_\phi^2}, \tag{13}$$

and $G$ is further broken to [8]

$$G\rightarrow G_\phi\equiv(K\times Z_{kl})/Z_l\supset G_S. \tag{14}$$

In particular, for $k=2$, we can have, for example, the symmetry breaking patterns,

$$\text{Spin}(10)\rightarrow[SU(5)\times Z_{10}]/Z_5,$$

$$SU(3)\rightarrow[SU(2)\times Z_4]/Z_2.$$

Therefore by continuously changing the value of the parameter $m$ we can produce a symmetry breaking pattern $G\rightarrow G_S\rightarrow G_\phi$. It is interesting to note that, unlike the Abelian-Higgs theory, in our theory the bare mass $\mu$ of $\phi$ is not required to satisfy $\mu^2<0$ in order to have spontaneous symmetry breaking. Therefore in the dual formulation, where one could interpret $\phi$ as being the monopole condensate, we do not need to have a monopole mass satisfying the problematic condition $M_{\text{mon}}^2<0$ mentioned by 't Hooft [11].

Let us analyze the last two phases in more detail.

### III. THE $m=0$ OR FREE-MONOPOLE PHASE

When $m=0$, $N=2$ supersymmetry is restored. In this phase $a=0$ and $b$ is an arbitrary constant, which we shall consider to be given by Eq. (13), in order to have the same value as in the case when $m<0$. The vacuum configuration $S^{\text{vac}}$ defines the $U(1)$ direction in $G_S$, Eq. (11), and one can define the corresponding $U(1)$ charge as [12]

$$Q\equiv e\frac{S^{\text{vac}}}{|S^{\text{vac}}|}=e\frac{\lambda_\phi\cdot H}{|\lambda_\phi|}. \tag{15}$$

Since in this phase $\Pi_2(G/G_S)=Z$, $Z$ magnetic monopoles can exist. These solutions can be written in the following form [13]: for each root $\alpha$, such that $2\alpha^V\cdot\lambda_\phi\neq 0$ (where $\alpha^{\text{v}}\equiv\alpha/\alpha^2$), we can define the generators

$$T_1^\alpha=\frac{E_\alpha+E_{-\alpha}}{2},\quad T_2^\alpha=\frac{E_\alpha-E_{-\alpha}}{2i},\quad T_3^\alpha=\frac{\alpha\cdot H}{\alpha^2} \tag{16}$$

which satisfy the $SU(2)$ algebra

$$[T_i^\alpha,T_j^\alpha]=i\epsilon_{ijk}T_k^\alpha.$$

Using spherical coordinates we define the group elements

$$g_p^\alpha(\theta,\phi)\equiv\exp(ip\varphi T_3^\alpha)\exp(i\theta T_2^\alpha)\exp(-ip\varphi T_3^\alpha),$$
$$p\in Z. \tag{17}$$

Let $S=M+iN$, where $M$ and $N$ are real scalar fields. The asymptotic form for the scalars of the $Z$ monopole are obtained by performing a gauge transformation on the vacuum solution (9),(10) by the above group elements. The result at $r\rightarrow\infty$ is

$$M(\theta,\phi)=g_p^\alpha v\cdot H(g_p^\alpha)^{-1}, \tag{18}$$

$$N(\theta,\phi)=0,$$

$$\phi(\theta,\phi)=0, \tag{19}$$

where $v\equiv b\lambda_\phi$. The $U(1)$ magnetic charge of these monopoles is [13]

$$g\equiv\frac{1}{|v|}\int dS_iM^aB_i^a=\frac{4\pi}{e}\frac{pv\cdot\alpha^v}{|v|} \tag{20}$$

where $B_i^a \equiv -\epsilon_{ijk}G_{jk}^a/2$ are the non-Abelian magnetic fields. Due to the $N=2$ supersymmetry, these must be BPS monopoles satisfying the mass formula

$$m_{\rm mon}=|v||g|. \tag{21}$$

Not all of these monopoles are stable. The stable or fundamental BPS monopoles are those with $p=1$ and $2\alpha^V\cdot\lambda_\phi=\pm 1$ [14]. From now on we shall consider only these fundamental monopoles, which are believed to fill representations of the gauge subgroup $K$ [5].

It is interesting to note that, for the particular case where the gauge group is $G=SU(2)$ and $\phi$ is in the symmetric part of the tensor product of two fundamental representations, that correspond to the adjoint representation, the supersymmetry of the theory is enhanced to $N=4$, and the theory has a vanishing $\beta$ function. There are other examples of vanishing $\beta$ functions when $m=0$. In order to see that we must recall that the $\beta$ function of $N=2$ super Yang-Mills theory with a hypermultiplet is given by

$$\beta(e)=\frac{-e^3}{(4\pi)^2}[h^V-x_\phi]$$

where $h^V$ is the dual Coxeter number of $G$ and $x_\phi$ is the Dynkin index of $\phi$'s representation (7). If $\phi$ belongs to $R_{2\lambda_\phi}^{\rm sym}$,

$$x_\phi=x_{\lambda_\phi}(d_{\lambda_\phi}+2),$$

where $x_{\lambda_\phi}$ and $d_{\lambda_\phi}$ are, respectively, the Dynkin index and the dimension of the representation associated with the fundamental weight $\lambda_\phi$. On the other hand, if $\phi$ belongs to the direct product of two fundamental representations, $R_{2\lambda_\phi}^{\otimes}$,

$$x_\phi=2d_{\lambda_\phi}x_{\lambda_\phi},$$

Therefore for $SU(n)$ (which has $h^V=n$), if $\phi$ is in the tensor product of the fundamental representation of dimension $d_{\lambda_{n-1}}=n$ with itself (which has Dynkin index $x_{\lambda_{n-1}}=1/2$), then $x_\phi=n$ and the $\beta$ function vanishes. Therefore in this phase the theory is $N=2$ superconformal (if we take $\mu=0$) and $SU(n)$ is broken to $U(n-1)\sim[SU(n-1)\otimes U(1)]/Z_{n-1}$.

## IV. THE $m>0$ OR SUPERCONDUCTING PHASE

In the "$m>0$" phase, the $U(1)$ factor of $G_S$ [eq. (11)] is broken and, as in Abelian-Higgs theory, the magnetic flux lines associated with this $U(1)$ factor cannot spread over space. Since $G$ is broken in such a way that $\Pi_1(G/G_\phi)=Z_k$, these flux lines may form topological $Z_k$ strings. We indeed showed in [4] the existence of BPS $Z_k$ strings in the limit $m\to 0_+$ and $\mu\to\infty$, with $m\mu=$const. We shall show now that, as in the Abelian-Higgs theory, the $U(1)$ magnetic flux $\Phi_{\rm mon}$ of the above monopoles is a multiple of the fundamental $Z_k$ string magnetic flux, and therefore these $U(1)$ flux lines coming out of the monopole can be squeezed into $Z_k$ strings, which gives rise to a confining potential.

### A. $Z_k$ string magnetic flux

From Eqs. (9) and (15) it follows that

$$Q\phi^{\rm vac}=ek|\lambda_\phi|\ \ \phi^{\rm vac},$$

and therefore the $U(1)$ electric charge of $\phi^{\rm vac}$ is

$$q_\phi=ek|\lambda_\phi|. \tag{22}$$

On the other hand, the string tension satisfies the bound [4]

$$T\geqslant\frac{me}{2}\left|\int d^2xM^aB_3^a\right|=\frac{me|v|}{2}|\Phi_{\rm st}|=\frac{q_\phi}{2}|a|^2|\Phi_{\rm st}|, \tag{23}$$

where $B_i^a\equiv-\epsilon_{ijk}G^{aij}/2$ is the non-Abelian magnetic field and

$$\Phi_{\rm st}\equiv\frac{1}{|v|}\int d^2xM^aB_3^a \tag{24}$$

is the $U(1)$ string magnetic flux, with the integral taken over the plane perpendicular to the string. This flux definition is gauge invariant and consistent with the flux definition for the monopole (20). One notes that Eq. (23) is very similar to the Abelian result (1), but here $q_\phi$ and $a$ are given by Eqs. (22) and (12), respectively. Let us use the string ansatz in [4]:

$$\begin{aligned}
&\phi(\varphi,\rho)=f(\rho)e^{i\varphi L_n}a|k\lambda_\phi\rangle,\\
&mS(\varphi,\rho)=h(\rho)k|a|^2e^{i\varphi L_n}\lambda_\phi\cdot He^{-i\varphi L_n},\\
&W_i(\varphi,\rho)=g(\rho)L_n\frac{\epsilon_{ij}x^j}{e\rho^2},\\
&i,j=1,2\to B_3(\varphi,\rho)=\frac{L_n}{e\rho}\frac{\partial g}{\partial\rho},\\
&W_0(\varphi,\rho)=W_3(\varphi,\rho)=0,
\end{aligned} \tag{25}$$

with the boundary conditions

$$f(\infty)=g(\infty)=h(\infty)=1,$$

$$f(0)=g(0)=0,$$

and considering

$$L_n=\frac{n}{k}\frac{\lambda_\phi\cdot H}{\lambda_\phi^2},\quad n\in Z_k.$$

Then, using the BPS conditions obtained in [4], which are valid in the limit $m\to 0$ and $\mu\to\infty$, gives the result that the functions $f(\rho)$ and $g(\rho)$ satisfy the same differential equations as the BPS strings in the $N=2$ Abelian-Higgs theory. However, that fact does not mean that BPS $Z_k$ strings are solutions of the $N=2$ Abelian-Higgs theory, since in this limit our theory continues to be non-Abelian. Moreover, from the asymptotic configuration we obtain that the "topological classes'' are determined by the first homotopy group $\Pi_1(G/G_\phi)$, which is different from that of the Abelian-Higgs theory.

From the BPS condition $D_\pm S=0$ together with the boundary conditions, we get $h(\rho)=1$. Therefore we obtain that for the BPS $Z_k$ strings

$$\Phi_{\rm st}=\oint dl_i A_i=\frac{2\pi n}{q_\phi}, \quad n\in Z_k, \tag{26}$$

where $A_i\equiv W_i^a M^a/|v|$, $i=1,2$. This flux quantization condition is also very similar to the Abelian result (2), but different due to the value of the electric charge $q_\phi$ given by Eq. (22). This result generalizes, for example, the string magnetic flux for $SU(2)$ [15] and for $SO(10)$ [16] (up to a factor of $\sqrt{2}$). In [17], the magnetic fluxes for the $SU(n)$ theory are also calculated, but with the gauge group completely broken to its center and a different definition of string flux, which is not gauge invariant. We can also rewrite the above result as

$$\Phi_{\rm st}q_\phi=2\pi n, \quad n\in Z_k,$$

which is similar to the magnetic monopole charge quantization condition.

Let us now check that the magnetic flux $\Phi_{\rm mon}$ of the monopoles in the $U(1)$ direction generated by $\lambda_\phi\cdot H$ is a multiple of $\Phi_{st}$. From Eq. (20), using Eq. (22) and the fact that

$$\alpha^V=\sum_{i=1}^{r} m_i\alpha_{\rm i}^V, \quad \alpha_i^V\equiv\frac{\alpha_i}{\alpha_i^2}, \quad m_i\in Z,$$

where $\alpha_i$ are simple roots, it then follows that

$$\Phi_{\rm mon}=g=\frac{2\pi k m_\phi p}{q_\phi}.$$

Therefore, for the fundamental monopoles, which have $p=1$ and $m_\phi=1$, $\Phi_{\rm mon}$ is equal to the flux $\Phi_{\rm st}$ of the string with $n=k$ or $k$ strings with $n=1$. This can be interpreted as meaning that for one fundamental monopole one can attach $kZ_k$ strings with $n=1$. This is consistent with the fact that a set of $kZ_k$ strings with $n=1$ belongs to the trivial first homotopy of the vacuum manifold and therefore can terminate in a magnetic monopole, which also belongs to the trivial first homotopy group. It is important to stress the fact that being in the trivial topological sector does not mean that the set of $kZ_k$ strings with $n=1$ has vanishing flux $\Phi_{\rm st}$.

### B. Monopole confinement

In the $m>0$ phase, it is expected [6] that the monopoles produced in the $m=0$ phase develop a flux line or string and become confined. Naively, we could see this fact in the following way. As usual, in order to obtain the asymptotic scalar configuration of a monopole, starting from the vacuum configuration (9)and (10), one performs the gauge transformation (17) and obtains that at[4] $r\to\infty$

$$S(\theta,\varphi)=g_p^\alpha b\lambda_\phi\cdot H(g_p^\alpha)^{-1}, \tag{27}$$

$$\phi(\theta,\varphi)=g_p^\alpha a|k\lambda_\phi\rangle. \tag{28}$$

However, $\phi(\theta,\varphi)$ is singular. In order to see this, let us consider for simplicity $p=1$, $k=2$, and $\alpha$ to be those positive roots such that $2\lambda_\phi\cdot\alpha^V=1$. In this case, the orthonormal weight states

$$|2\lambda_\phi\rangle, \ |2\lambda_\phi-\alpha\rangle, \ |2\lambda_\phi-2\alpha\rangle$$

form a spin 1 irreducible representation of the $su(2)$ algebra (16), and the orthonormal states

$$|\pm\rangle\equiv\frac{1}{2}(|2\lambda_\phi\rangle\pm i\sqrt{2}|2\lambda_\phi-\alpha\rangle-|2\lambda_\phi-2\alpha\rangle), \tag{29}$$

$$|0\rangle\equiv\frac{1}{\sqrt{2}}(|2\lambda_\phi\rangle+|2\lambda_\phi-2\alpha\rangle), \tag{30}$$

satisfy

$$T_2^\alpha|\pm\rangle=\pm|\pm\rangle,$$

$$T_2^\alpha|0\rangle=0.$$

We can then write

$$|2\lambda_\phi\rangle=\frac{1}{2}(|+\rangle+|-\rangle+\sqrt{2}|0\rangle).$$

Then, Eq. (28) can be written as

$$\phi(\theta,\varphi)=a\left\{\cos^2\frac{\theta}{2}|2\lambda_\phi\rangle-\frac{\sqrt{2}}{2}\sin\theta e^{-i\varphi}|2\lambda_\phi-\alpha\rangle + \sin^2\frac{\theta}{2}e^{-2i\varphi}|2\lambda_\phi-2\alpha\rangle\right\}.$$

Therefore at $\theta=\pi$

$$\phi(\pi,\varphi)=ae^{-2i\varphi}|2\lambda_\phi-2\alpha\rangle,$$

which is singular. This generalizes Nambu's result [18] for the $SU(2)\times U(1)$ case. In order to cancel the singularity we should attach a string on the $z<0$ axis with a zero at the core, similar to our string ansatz (25). One could construct an ansatz for $\phi(r,\theta,\phi)$ by multiplying the above asymptotic configuration by a function $F(r,\theta)$ such that $F(r,\pi)=0$.

Since our theory has solitonic monopoles with masses given by Eq. (21), we can obtain a bound for the threshold length for a string to break, producing a new monopole-antimonopole pair in the following way. From Eqs. (23) and

[4]Note that when we take $m=0 \Rightarrow a=0$ we recover the asymptotic scalar field configuration for the $Z$ monopole in the ''$m=0$ phase'' (18),(19).

(26), it follows that the string tension for a string with $n=k$ or $k$ strings with $n=1$ satisfies[5] the bound

$$T \geqslant k \frac{me|v|}{2} \frac{2\pi}{q_\phi}.$$

Using the monopole masses (21), the threshold length $d^{\mathrm{th}}$ for a string breaking can be derived form the relation

$$2|v| \frac{2\pi k}{q_\phi} = E^{\mathrm{th}} = T d^{\mathrm{th}} \geqslant k \frac{me|v|}{2} \frac{2\pi}{q_\phi} d^{\mathrm{th}},$$

which results in

$$d^{\mathrm{th}} \leqslant \frac{4}{me}.$$

[5]As in the Abelian-Higgs theory, it is expected that when $V(\phi\ S) \geqslant Y_a^2/2$, as we are considering, the tension of a string with $n=k$ should be greater than or equal to the tension of $k$ strings with $n=1$.

## V. SUMMARY AND CONCLUSIONS

In this work we have extended to non-Abelian theories some of the ideas of 't Hooft and Mandelstam on quark confinement. In addition to the fact that our theory has an unbroken non-Abelian gauge group, another interesting feature is that it has solitonic monopoles instead of the Dirac monopoles that appear in Abelian theories. We have considered $N=2$ super Yang-Mills theory with an arbitrary simple gauge group, with one flavor, and with an $N=2$ mass breaking term. We have shown that, by continuously varying the mass breaking parameter $m$, we can pass from an unbroken phase to a phase with free monopoles and then to a phase with $Z_k$ strings. This last phase occurs due to the fact that the scalar $\phi$ acquires a nonvanishing expectation value. When $k=2$, $\phi$ can be interpreted as a diquark condensate. We showed that the magnetic flux of the monopoles is a multiple of the fundamental $Z_k$ string flux and therefore the monopoles can undergo confinement. We also obtained a bound for the threshold length for a string to break into a new monopole-antimonopole pair. Following the ideas of 't Hooft and Mandelstam, one might expect that, in the dual theory, with $\phi$ being a monopole condensate, quark-antiquark confinement will occur.

We have seen that some of our confining theories are obtaining by adding a deformation to $U(N)$ superconformal theories, which breaks the gauge group further to $[SU(N)\otimes Z_2]/Z_{2N}$. It is expected that a confining theory obtained by a deformation of superconformal gauge theory in four dimensions should satisfy gauge/string correspondence [19] (which would be a kind of deformation of the conformal field theory/AdS correspondence [20]). In gauge/string correspondence confining gauge theories with $SU(N)$ or $U(N)$ completely broken to a discrete group are usually considered. Therefore, it would be interesting to know if those theories also satisfy some gauge/string correspondence.

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