

Gauged dimension bubbles

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Some of the peculiar electro-dynamical effects associated with gauged “dimension bubbles” are presented. Such bubbles, which effectively enclose a region of 5D spacetime, can arise from a 5D theory with a compact extra dimension. Bubbles with thin domain walls can be stabilized against total collapse by the entrapment of light charged scalar bosons inside the bubble, extending the idea of a neutral dimension bubble to accommodate the case of a gauged $U(1)$ symmetry. Using a dielectric approach to the 4D dilaton-Maxwell theory, it is seen that the bubble wall is almost totally opaque to photons, leading to a new stabilization mechanism due to trapped photons. Photon dominated bubbles very slowly shrink, resulting in a temperature increase inside the bubble. At some critical temperature, however, these bubbles explode, with a release of radiation.

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I. INTRODUCTION

An inhomogeneous higher dimensional spacetime compactified to four dimensions (4D) can contain pockets, or, what may be referred to as “dimension bubbles,” where the extra dimensional scale factor becomes large enough that the spacetime has an effective dimensionality within the pocket that is higher than in the surrounding four dimensional spacetime outside of it [1,2]. Here we consider a dimension bubble, arising from the dimensional reduction of a 5D theory, which encloses a 5D region and is surrounded by a region that is effectively 4D, i.e., the extra dimensional scale factor changes rapidly from the interior of the bubble to the exterior. The two regions are, from a 4D perspective, separated by a domain wall generated by a scalar field associated with the scale factor of the extra space dimension. Such a domain wall can result from a Rubin-Roth potential [3] (which includes bosonic as well as fermionic Casimir energies—which can stabilize the compact extra dimension from collapsing due to the gravitational Casimir effect [4]) along with a cosmological constant [2], or, in the case of more than one extra dimension, from higher dimensional Maxwell fields [5] or their generalizations [1]. For simplicity, we take the 4D spacetime to be Minkowski and the extra space dimension to be toroidally compact, so that the 5D spacetime has the topology of $M_4 \times S^1$. (In this sense, the dimension bubbles studied here are simplifications of “gravitational bags,” previously analyzed by Davidson and Guendelman [1], which have an associated nontrivial spacetime geometry.) Although gravitational bags are static solutions where the surface tension pushing the bubble wall inward is equilibrated with a nontrivial pressure from the scalar field inside, the scalar field inside suffers from a singular behavior at the center of the geometry (although there is still finite

energy due to a gravitational effect). There is, however, another way to stabilize the bubble which does not require a singular scalar field at the center of the geometry. The stabilization can be achieved if the inside of the bubble is filled with scalar bosons described by a complex scalar field χ [6].

Previously, it was assumed that the χ field only possessed a global $U(1)$ symmetry [6] giving rise to a conserved number charge Q . Here, we consider the case where the scalar field χ has a *local* $U(1)$ symmetry, i.e., a $U(1)$ “gauged dimension bubble.” Because the extra dimensional scale factor $B(x)$ can take drastically different values inside and outside of the bubble, the effective “dielectric function” of these two regions can also have drastically different values. As a result, there are nontrivial electromagnetic effects associated with the bubble wall and its 5D interior. In particular, the EM contribution to the mass M of the bubble is reduced from what would be expected if the bubble interior were also 4D, that is, if the dilaton associated with the scale factor B were constant everywhere. In addition, the bubble wall is found to be almost perfectly reflecting to photons, so that photons cannot pass through the wall from either direction. Entrapped photons can themselves stabilize a bubble from total collapse. The effects of the extra dimension therefore make the gauged dimension bubble an object that is quite different from other purely 4D nontopological solitons [7], such as gauged Q balls [8–10], charged vacuum bubbles [11], and Fermi balls [12–15], that have been studied previously.

We first present the dielectric approach to the study of electromagnetic effects of dimension bubbles, considering a bubble with a 5D interior (emerging from a 5D theory dimensionally reduced to 4D) as a specific prototype. This dielectric approach is then applied to the case of a charged bubble having a conserved number Q of charged χ bosons trapped inside the bubble. Upon evaluating the various contributions to the bubble’s energy, the bubble’s equilibrium radius R and mass M can be obtained. We shall also make a couple of simplifying assumptions. First, we assume a thin

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walled bubble, i.e., the wall thickness $\delta \ll R$, so that in a simplifying limit we may take the inner radius R_- and outer radius R_+ of the wall to coincide, $R_-, R_+ \rightarrow R$. It is within the wall that the scale factor $B(x)$ varies rapidly, and we assume that B takes on different constant values in the interior and exterior regions, with the interior value being much greater than the exterior one, $B_{in} \gg B_{out}$. This allows for an efficient mechanism of trapping the χ bosons inside the bubble, since the boson mass ($m_\chi \propto B^{-1/2}$), assumed to be small inside, becomes very large outside, $m_{\chi,in} \ll m_{\chi,out}$, and a boson therefore experiences an enormous force $\vec{F} \sim -\nabla m_\chi$ exerted on it by the wall, keeping it in the interior of the bubble.

II. THE MODEL

A. Metric ansatz and dimensional reduction

We begin with a 5D action of the general form

$$S_5 = \frac{1}{2\kappa_{N(5)}^2} \int d^5x \sqrt{\tilde{g}_5} \{ \tilde{R}_5 - 2\Lambda + 2\kappa_{N(5)}^2 \mathcal{L}_5 \} \quad (1)$$

that is defined on a 5D spacetime with a metric described by

$$ds^2 = \tilde{g}_{MN} dx^M dx^N = \tilde{g}_{\mu\nu} dx^\mu dx^\nu + \tilde{g}_{55} dy^2. \quad (2)$$

Here, $x^M = (x^\mu, y)$ is the set of coordinates on a 5D spacetime with topology of $M_4 \times S^1$ that has a toroidally compact spacelike extra dimension $x^5 = y$, which is assumed to be a linear coordinate lying within the range $0 \leq y \leq 2\pi R$. We use the indices $M, N = 0, \dots, 3, 5$ to label the coordinates of the 5D spacetime and the indices $\mu, \nu = 0, \dots, 3$ to label the coordinates of the noncompact 4D spacetime. A zero mode Kaluza-Klein ansatz is assumed where the fields and metric are independent of x^5 , i.e., $\tilde{g}_{MN} = \tilde{g}_{MN}(x^\mu)$ depends only on the 4D coordinates with $\partial_5 \tilde{g}_{MN} = 0$. We also assume that the metric factorizes so that $\tilde{g}_{\mu 5} = 0$. A dimensionless scale factor $B(x^\mu)$, with $\tilde{g}_{55} = -B^2$, is associated with the extra dimension, along with an associated scalar field φ that is related to B by

$$\varphi = \frac{1}{\kappa_N} \sqrt{\frac{3}{2}} \ln B. \quad (3)$$

The constant κ_N is related to the 4D Planck mass M_P by $\kappa_N = \sqrt{8\pi G} = \sqrt{8\pi} M_P^{-1}$. The extra dimensional scale factor can then be written as $B = e^{\sqrt{2/3}\kappa_N\varphi}$.

Determinants of the 4D and 5D parts of the original metric \tilde{g}_{MN} are denoted by $\tilde{g} = \det \tilde{g}_{\mu\nu}$ and $\tilde{g}_5 = \det \tilde{g}_{MN}$, respectively, so that $\sqrt{|\tilde{g}_5|} = \sqrt{-\tilde{g}} \sqrt{|\tilde{g}_{55}|}$. In the action given by Eq. (1), $\kappa_{N(5)} = \sqrt{8\pi G_5}$ represents a 5D gravitational constant, $\tilde{R}_5 = \tilde{g}^{MN} \tilde{R}_{MN}$ is the 5D Ricci scalar built from \tilde{g}_{MN} , \mathcal{L}_5 is a Lagrangian of the 5D theory, and Λ is a 5D cosmological constant. In order to pass to an effective 4D theory we define $\mathcal{L} = (2\pi R)\mathcal{L}_5$ and define $\kappa_{N(5)}$ in terms of κ_N by $\kappa_{N(5)}^2 = (2\pi R)\kappa_N^2$.

The 4D Jordan frame metric is $\tilde{g}_{\mu\nu}$, the $\mu\nu$ part of the 5D metric \tilde{g}_{MN} . A 4D Einstein frame metric $g_{\mu\nu}$ can be defined by $g_{\mu\nu} = B\tilde{g}_{\mu\nu} = e^{\sqrt{2/3}\kappa_N\varphi}\tilde{g}_{\mu\nu}$ and the line element in Eq. (2) can be rewritten in terms of the Einstein frame metric and extra dimensional scale factor as

$$ds^2 = B^{-1} g_{\mu\nu} dx^\mu dx^\nu - B^2 dy^2 \\ = e^{-\sqrt{2/3}\kappa_N\varphi} g_{\mu\nu} dx^\mu dx^\nu - e^{2\sqrt{2/3}\kappa_N\varphi} dy^2. \quad (4)$$

Using Eq. (4), a dimensional reduction of the action given by Eq. (1) gives the effective 4D Einstein frame action

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa_N^2} R + \frac{1}{2} (\nabla\varphi)^2 \right. \\ \left. + e^{-\sqrt{2/3}\kappa_N\varphi} \left[\mathcal{L} - \frac{1}{\kappa_N^2} \Lambda \right] \right\} \quad (5)$$

where $R = g^{\mu\nu} R_{\mu\nu}$ is the 4D Ricci scalar built from the 4D Einstein frame metric $g_{\mu\nu}$ and $g = \det g_{\mu\nu}$.

B. 4D (Einstein frame) effective Lagrangian

Consider a Lagrangian from the 5D theory of a $U(1)$ gauged scalar field χ ,

$$\mathcal{L} = (2\pi R)\mathcal{L}_5 = -\frac{1}{4} \tilde{F}'^{MN} \tilde{F}'_{MN} \\ + (\tilde{D}^M \chi)^* (\tilde{D}_M \chi) - V(|\chi|) \quad (6)$$

which gives rise to an effective 4D (EF) Lagrangian $\mathcal{L}_4 = B^{-1}\mathcal{L}$, and

$$\tilde{F}'_{MN} = \partial_M A'_N - \partial_N A'_M, \quad D_M \chi = (\nabla_M + ie_0 A'_M) \chi \quad (7)$$

with e_0 and A'_M being the charge parameter and gauge field potential, respectively, appearing in the original 5D Lagrangian, and tildes remind us that indices are raised and lowered with the metric \tilde{g}_{MN} , so that $(\tilde{D}^M \chi)^* (\tilde{D}_M \chi) = \tilde{g}^{MN} (D_M \chi)^* (D_N \chi)$. It is assumed that $\partial_5 \chi = 0$ so that $D_5 \chi = ie_0 A'_5 \chi$. We then have, with the assumption that $\partial_5 A'_\mu = 0$,

$$-\frac{1}{4} \tilde{F}'^{MN} \tilde{F}'_{MN} = -\frac{1}{4} B^2 F'^{\mu\nu} F'_{\mu\nu} + \frac{1}{2} B^{-1} (\partial^\mu A'_5 \partial_\mu A'_5). \quad (8)$$

Using $\tilde{g}^{\mu\nu} = B g^{\mu\nu}$, $\tilde{g}^{55} = -B^{-2}$, the scalar field kinetic term is given by

$$(\tilde{D}^M \chi)^* (\tilde{D}_M \chi) = (\tilde{D}^\mu \chi)^* (\tilde{D}_\mu \chi) + (\tilde{D}^5 \chi)^* (\tilde{D}_5 \chi) \\ = B |D_\mu \chi|^2 - e_0^2 B^{-2} A_5'^2 |\chi|^2. \quad (9)$$

The 4D EF effective Lagrangian $\mathcal{L}_4 = B^{-1}\mathcal{L}$ then becomes

$$\begin{aligned} \mathcal{L}_4 = & -\frac{1}{4}BF'^{\mu\nu}F'_{\mu\nu} + \frac{1}{2}B^{-2}(\partial A_5')^2 + |D_\mu\chi|^2 \\ & - e_0^2B^{-3}A_5'^2|\chi|^2 - B^{-1}V(|\chi|). \end{aligned} \quad (10)$$

The φ -dependent potential $U(\varphi)$ (which contains, e.g., the Rubin-Roth potential, V_{RR} , and cosmological constant, Λ , terms), along with the φ kinetic term $\frac{1}{2}(\partial\varphi)^2$ and gravitational term $(1/2\kappa_N^2)R$ can be added to get a total 4D effective Lagrangian

$$\begin{aligned} \mathcal{L}_{eff} = & \mathcal{L}_4 + \frac{1}{2\kappa_N^2}R + \frac{1}{2}(\partial\varphi)^2 - U \\ = & \frac{1}{2\kappa_N^2}R + \frac{1}{2}(\partial\varphi)^2 - \frac{1}{4}BF'^{\mu\nu}F'_{\mu\nu} \\ & + \frac{1}{2}B^{-2}(\partial A_5')^2 + |D_\mu\chi|^2 - W \end{aligned} \quad (11)$$

where we define a total effective potential

$$W = U(\varphi) + B^{-1}V(|\chi|) + e_0^2B^{-3}A_5'^2|\chi|^2. \quad (12)$$

C. Dielectric approach

The 4D Einstein frame effective Lagrangian \mathcal{L}_{eff} above contains a Maxwell term $-\frac{1}{4}BF'^{\mu\nu}F'_{\mu\nu}$ and a gauge covariant derivative $D_\mu\chi = (\partial_\mu + ie_0A'_\mu)\chi$. Let us define a rescaled gauge field $A_M = B_4^{1/2}A'_M$, where B_4 (or B_5) denotes the value of B in a region of 4D (or 5D) spacetime outside (or inside) of a dimension bubble, so that in a region of 4D spacetime the Maxwell term is properly normalized and assumes a canonical form, $-\frac{1}{4}B_4F'^{\mu\nu}F'_{\mu\nu} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$, with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$A_\mu = B_4^{1/2}A'_\mu, \quad A_5 = B_4^{1/2}A'_5. \quad (13)$$

The gauge covariant derivative operator $D_\mu = (\partial_\mu + ie_0A'_\mu)$ can be written as

$$D_\mu = (\partial_\mu + ie_0B_4^{-1/2}A_\mu) = (\partial_\mu + ieA_\mu) \quad (14)$$

where we have defined the 4D effective, or physical, charge

$$e = B_4^{-1/2}e_0. \quad (15)$$

We also define the ‘‘dielectric function’’

$$\kappa(x^\mu) = \frac{B(x^\mu)}{B_4}. \quad (16)$$

In terms of physical 4D quantities, the effective 4D Lagrangian in Eq. (11) now assumes the form

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{1}{2\kappa_N^2}R + \frac{1}{2}(\partial\varphi)^2 - \frac{1}{4}\kappa F_{\mu\nu}F^{\mu\nu} \\ & + \frac{1}{2}\kappa^{-2}B_4^{-3}(\partial A_5)^2 + |D_\mu\chi|^2 - W \end{aligned} \quad (17)$$

where

$$W = U(\varphi) + \kappa^{-1}B_4^{-1}V(|\chi|) + e^2\kappa^{-3}B_4^{-3}A_5^2|\chi|^2. \quad (18)$$

The function κ is seen to play the role of a dielectric function, except that in this case it arises as a consequence of the possible position dependence of the extra dimensional scale factor B . Thus, from a 4D perspective, a region of effectively 5D spacetime is viewed as being endowed with a dielectric property described by κ . We also note that in a 4D region $\kappa \rightarrow 1$, and that $\kappa_5 = B_5/B_4 > 1$, and, in particular, we assume that $\kappa_5 \gg 1$. Using the familiar results of electrostatics, we have that the normal component of the ‘‘displacement’’ field $\vec{D} = \kappa\vec{E}$ and the tangential components of the electric field \vec{E} are continuous at an interface between two media with different dielectric constants.

D. Effective potential, vacuum, and χ boson mass

The total effective potential W of the 4D theory is given by Eq. (18). The vacuum state is obtained by minimizing W with respect to φ (or κ), χ , and A_5 :

$$\begin{aligned} \frac{\partial W}{\partial \kappa} = & \frac{\partial U}{\partial \kappa} - B_4^{-1}\kappa^{-2}V - 3e^2B_4^{-3}\kappa^{-4}A_5^2|\chi|^2 = 0, \\ \frac{\partial W}{\partial \chi^*} = & B_4^{-1}\kappa^{-1}\frac{\partial V}{\partial \chi^*} + e^2B_4^{-3}\kappa^{-3}A_5^2\chi = 0, \quad (19) \\ \frac{\partial W}{\partial A_5} = & 2e^2B_4^{-3}\kappa^{-3}A_5|\chi|^2 = 0. \end{aligned}$$

These equations are solved by $A_5 = 0$, $\partial V/\partial \chi^* = 0$, and $\partial U/\partial \kappa - B_4^{-1}\kappa^{-2}V = 0$. (Later we will consider the case for which $V = \mu_0^2\chi^*\chi$, giving $\chi = 0$, $V = 0$, $\partial U/\partial \kappa = 0$, with A_5 undetermined, and we will assume that $A_5 = 0$.)

The 5D mass parameter for the χ field is denoted by $\mu_0^2 = (\partial^2 V/\partial \chi^* \partial \chi)|_{vac}$, so that the effective χ boson mass in the 4D effective (EF) theory is given by

$$m_\chi^2 = \left(\frac{\partial^2 W}{\partial \chi^* \partial \chi} \right)_{vac} = [B^{-1}\mu_0^2 + e^2B_4^{-3}\kappa^{-3}A_5^2]_{vac}. \quad (20)$$

Assuming the vacuum value of A_5 to vanish, we have simply

$$m_\chi = B^{-1/2}\mu_0 = \kappa^{-1/2}\mu_4 \quad (21)$$

where we define $\mu_4 = B_4^{-1/2}\mu_0$. In this effective 4D dilaton-Maxwell system, the effects of the scale factor (or dilaton φ) become manifest in the 4D mass parameter m_χ . In the 5D region of a dimension bubble interior, where $\kappa \gg 1$, the mass $\kappa_5^{-1/2}\mu_4$ becomes very small or negligible in comparison to

the mass μ_4 in the 4D region outside the bubble. This effective mass dependence of m_χ upon κ gives rise to the entrapment of χ bosons inside the bubble.

III. GAUGED DIMENSION BUBBLE

Consider the case where a domain bubble forms, entrapping a conserved number Q of χ bosons. (For definiteness, we take the electromagnetic charge Qe to be positive and the potential V is taken to be given by $V = \mu_0^2 \chi^* \chi$.) The simplified situation is assumed to exist for which the bosons are nearly massless inside the bubble, so that a gas of ultrarelativistic particles exists inside, and the bosons are massive outside, with the boson mass being m . We assume that the bubble takes a spherical shape at equilibrium. The inner surface of the bubble wall lies at a radius R_- and the outer surface is located at R_+ . We will use a thin wall approximation, in which case $R_- \approx R_+ \approx R$.

The mass M of the bubble gets contributions from the kinetic energy \mathcal{E}_χ of the χ bosons, the energy of the domain wall forming the bubble surface, $\mathcal{E}_W = 4\pi R^2 \sigma$, and the electromagnetic (EM) energy due to the entrapped χ bosons, \mathcal{E}_{em} . In addition, there is a contribution from the φ -dependent potential $U(\varphi)$ in the interior of the bubble. This arises from the fact that we are considering the case [6] where $U(\varphi) \rightarrow 0$ in the 4D vacuum region and $U(\varphi) > 0$ in the 5D vacuum region of the bubble's interior, where φ assumes a large, but finite, value. Denoting the value of $U(\varphi)$ in the bubble's interior by λ , a constant in our approximation, we have the corresponding volume term $\mathcal{E}_V = \frac{4}{3}\pi\lambda R^3$ contributing to the mass M of the bubble. The bubble mass can therefore be written as

$$M = \mathcal{E}_\chi + \mathcal{E}_{em} + \mathcal{E}_W + \mathcal{E}_V. \quad (22)$$

The first term \mathcal{E}_χ in this expression for the mass can be estimated easily. For the ground state kinetic energy of an ultrarelativistic χ boson trapped inside a bubble of radius R , we take the boson wavelength to be roughly equal to the bubble diameter, $\lambda_\chi \approx 2R$. Then the kinetic energy is $\mathcal{E}_{kin} \approx 2\pi/\lambda_\chi \approx \pi/R$. The kinetic energy of Q bosons in the ground state is then approximately $\mathcal{E}_\chi = Q\pi/R$. For the EM energy we need to integrate the electromagnetic energy density $u = \frac{1}{2}\vec{D} \cdot \vec{E} = \frac{1}{2}\kappa E^2$ over all space,

$$\mathcal{E}_{em} = \int u dV = 2\pi \int_0^\infty \kappa E^2 r^2 dr. \quad (23)$$

A. Electric and displacement fields

The EM field satisfies the field equation $\nabla_\mu(\kappa F^{\mu\nu}) = j^\nu = eJ^\nu$, where J^μ is the current per unit charge that generates the χ boson number density, so that $Q = \int J^0 dV$. As a simplifying approximation we assume that B and $\kappa = B/B_4$ take on constant values inside and outside the bubble:

$$B = \begin{cases} B_5, & \text{inside, } (r < R) \\ B_4, & \text{outside, } (r > R) \end{cases},$$

$$\kappa = \begin{cases} \kappa_5 = \frac{B_5}{B_4}, & \text{inside, } (r < R) \\ 1, & \text{outside, } (r > R) \end{cases}. \quad (24)$$

We therefore have $\nabla \cdot \vec{D} = j^0 = \rho$, where $\vec{D} = \kappa \vec{E}$, and by Gauss' law the radial displacement field is $(4\pi r^2)D = q_{en} = \int_V \rho dV$, and $q_{en} = Q_{en}e$, where Q_{en} is the number of bosons enclosed within the volume V . The radial component of the displacement field is therefore

$$D = \kappa E(r) = \frac{q_{en}(r)}{4\pi r^2} = \frac{Q_{en}(r)e}{4\pi r^2}. \quad (25)$$

Inside the bubble we approximate the charge density as a constant so that $\rho = Qe/(\frac{4}{3}\pi R^3)$, and

$$q_{en}(r) = \begin{cases} \frac{Qer^3}{R^3}, & (r \leq R) \\ Qe, & (r \geq R) \end{cases}. \quad (26)$$

We therefore obtain the displacement field

$$D = \begin{cases} \kappa_5 E_5 = \frac{Qer}{4\pi R^3}, & (r < R) \\ E_4 = \frac{Qe}{4\pi r^2}, & (r > R) \end{cases} \quad (27)$$

where E_5 (E_4) denotes the electric field inside (outside) the bubble. The displacement \vec{D} is continuous at the bubble wall, but with our thin wall approximation with an infinitely thin domain wall, the value of the electric field jumps up by a factor of κ_5 on the outer surface of the bubble. The interior of the bubble appears as a dielectric with an enormous dielectric constant $\kappa_5 \gg 1$.

B. Electromagnetic energy \mathcal{E}_{em}

We calculate the EM energy associated with the configuration by integrating the EM energy density $u = \frac{1}{2}\vec{D} \cdot \vec{E}$ over all space. From Eq. (27)

$$u = \begin{cases} \left(\frac{Q^2 \alpha}{8\pi} \right) \frac{r^2}{\kappa_5 R^6}, & (r < R) \\ \left(\frac{Q^2 \alpha}{8\pi} \right) \frac{1}{r^4}, & (r > R) \end{cases} \quad (28)$$

where $\alpha = e^2/4\pi$. The EM configuration energy

$$\begin{aligned} \mathcal{E}_{em} &= \int_0^\infty u 4\pi r^2 dr \\ &= 2\pi \int_0^R \kappa_5 E_5^2 r^2 dr + 2\pi \int_R^\infty E_4^2 r^2 dr \end{aligned} \quad (29)$$

is then given by

$$\mathcal{E}_{em} = \frac{Q^2 \alpha}{2R} \left(1 + \frac{1}{5\kappa_5} \right) = \frac{Q^2 \alpha}{2R} \left(1 + \frac{1}{5} \frac{B_4}{B_5} \right). \quad (30)$$

For $\kappa_5 \gg 1$, the contribution to \mathcal{E}_{em} from the interior region is negligible in comparison to the contribution from the exterior region. If the dielectric constant inside the bubble were unity, the EM energy would be $\frac{3}{5}(Q^2 \alpha/R)$, so that the effect of a macroscopic extra dimension is a reduction of the EM energy of the bubble.

C. Bubble mass and radius

Equation (22) for the bubble mass gives, approximately,

$$M = \mathcal{E}_\chi + \mathcal{E}_{em} + \mathcal{E}_W + \mathcal{E}_V = \frac{Q\pi}{R} \left(1 + \frac{Q\alpha c}{2\pi} \right) + 4\pi\sigma R^2 + \frac{4}{3}\pi\lambda R^3 \quad (31)$$

where

$$c = \left(1 + \frac{1}{5\kappa_5} \right) = \left(1 + \frac{1}{5} \frac{B_4}{B_5} \right). \quad (32)$$

The equilibrium radius is obtained by minimizing the expression for M with respect to R , holding the charge Q fixed. The equilibrium mass of a bubble at its equilibrium radius can then be obtained. We consider two limiting cases allowing us to obtain analytical expressions for the bubble's equilibrium mass and radius: (i) the surface term \mathcal{E}_W is negligible and (ii) the volume term \mathcal{E}_V is negligible.

(i) *Negligible Surface Term \mathcal{E}_W* . For the first case we assume that $\mathcal{E}_W \ll \mathcal{E}_\chi + \mathcal{E}_{em}$ and $\mathcal{E}_W \ll \mathcal{E}_V$, which can be rewritten as the conditions

$$\frac{4\sigma R^3}{Q \left(1 + \frac{Q\alpha c}{2\pi} \right)} \ll 1, \quad \frac{3\sigma}{\lambda R} \ll 1. \quad (33)$$

In this case the bubble mass is approximately $M = \mathcal{E}_\chi + \mathcal{E}_{em} + \mathcal{E}_V$. We get for the equilibrium radius R and equilibrium mass M , respectively,

$$R = \left[\frac{Q \left(1 + \frac{Q\alpha c}{2\pi} \right)}{4\lambda} \right]^{1/4} \quad (34)$$

and

$$M = \pi(4\lambda)^{1/4} \left[Q \left(1 + \frac{Q\alpha c}{2\pi} \right) \right]^{3/4} = \frac{\pi Q}{R} \left(1 + \frac{Q\alpha c}{2\pi} \right). \quad (35)$$

Using Eq. (34) we find that the conditions of Eq. (33) are approximately satisfied for

$$\sigma \ll \left[\lambda^3 Q \left(1 + \frac{Q\alpha c}{2\pi} \right) \right]^{1/4}. \quad (36)$$

(ii) *Negligible Volume Term \mathcal{E}_V* . For the second case we assume that $\mathcal{E}_V \ll \mathcal{E}_\chi + \mathcal{E}_{em}$ and $\mathcal{E}_V \ll \mathcal{E}_W$, which can be rewritten as the conditions

$$\lambda \ll \frac{3\sigma}{R}, \quad \lambda \ll \frac{3Q}{4R^4} \left(1 + \frac{Q\alpha c}{2\pi} \right). \quad (37)$$

The bubble mass in this case is approximately $M = \mathcal{E}_\chi + \mathcal{E}_{em} + \mathcal{E}_W$, and we get for the equilibrium radius R and equilibrium mass M , respectively,

$$R = \frac{1}{2} \left[\frac{Q}{\sigma} \left(1 + \frac{Q\alpha c}{2\pi} \right) \right]^{1/3} \quad (38)$$

and

$$M = 3\pi\sigma^{1/3} \left[Q \left(1 + \frac{Q\alpha c}{2\pi} \right) \right]^{2/3} = \frac{3\pi Q}{2R} \left(1 + \frac{Q\alpha c}{2\pi} \right). \quad (39)$$

Using Eq. (38) we find that the conditions of Eq. (37) are approximately satisfied for

$$\lambda \ll \sigma^{3/4} \left[Q \left(1 + \frac{Q\alpha c}{2\pi} \right) \right]^{-1/3}. \quad (40)$$

We can notice that the ratio

$$\frac{\mathcal{E}_{em}}{\mathcal{E}_\chi} = \frac{Q\alpha c}{2\pi} = \frac{Q\alpha}{2\pi} \left(1 + \frac{1}{5\kappa_5} \right) \quad (41)$$

indicates that \mathcal{E}_χ dominates \mathcal{E}_{em} for $Q \ll 2\pi/\alpha$ and vice versa for $Q \gg 2\pi/\alpha$.

IV. PHOTON OPACITY AND PHOTON STABILIZED BUBBLES

A. Bubble wall opacity

An interesting effect associated with the difference in space dimensionalities inside and outside of a dimension bubble is the opacity of the bubble wall to electromagnetic radiation. From the Lagrangian for the EM field $F_{\mu\nu}$, given by Eq. (17), we have the field equations $\nabla_\mu(\kappa F^{\mu\nu}) = j^\nu$, which represents Maxwell's equations in terms of the displacement field $\vec{D} = \epsilon \vec{E} = \kappa \vec{E}$ and the magnetic field $\vec{H} = \vec{B}/\mu = \kappa \vec{B}$. We then identify the permittivity ϵ , permeability μ , index of refraction $n = \sqrt{\epsilon\mu}$, and "impedance" $Z = \sqrt{\mu/\epsilon}$ of a region of space as

$$\epsilon = \frac{1}{\mu} = \kappa, \quad n = \sqrt{\epsilon\mu} = 1, \quad Z = \sqrt{\mu/\epsilon} = \frac{1}{\kappa}. \quad (42)$$

The reflectivity and transmissivity of light at an interface between two dielectrics with indices ϵ_I, μ_I and ϵ_T, μ_T ,

where I, T represent the incident and transmitting media, respectively, can be obtained from classical electrodynamics. For example, light impinging upon an interface between two media with impedances Z_I and Z_T has an associated reflection ratio given¹ by [16]

$$\mathcal{R} = \frac{E_R}{E_I} = \frac{1 - (Z_T/Z_I)}{1 + (Z_T/Z_I)}. \quad (43)$$

For light impinging upon the bubble wall from either the inside or the outside, we have $|\mathcal{R}| \approx 1 - O(\kappa_5^{-1}) \approx 1$, where $\kappa_5^{-1} = B_4/B_5 \ll 1$. Therefore the bubble wall is almost totally opaque to EM radiation, and photons inside the bubble remain effectively trapped inside the bubble. Photon radiation pressure therefore serves as yet another means of stabilizing a dimension bubble against collapse.

B. Photon stabilized bubble

A dimension bubble may contain a bath of photon radiation in addition to the charged bosons in its interior. Let us focus on a limiting case in which essentially all the energy density of the bubble's contents is due to photons. The photon energy density² is $\rho_\gamma = (\pi^2/15)T^4 = aT^4$, so that the energy of the bubble is

$$\begin{aligned} M &= \mathcal{E}_\gamma + \mathcal{E}_W = \frac{4}{3} \pi \rho_\gamma R^3 + 4 \pi \sigma R^2 \\ &= \frac{4 \pi^3}{45} T^4 R^3 + 4 \pi \sigma R^2. \end{aligned} \quad (44)$$

The bubble, after it forms, will adjust its radius to reach an equilibrium with an adiabatic (isentropic) expansion or contraction. The photon entropy density is $s_\gamma \sim T^3$, and if we assume that the bubble adjusts its radius to reach equilibrium on a very small time scale so that essentially no energy is lost from the bubble during equilibration, we have that

$$RT = \text{const}. \quad (45)$$

Using these expressions, a minimization of the mass function gives an equilibrium radius R and an equilibrium mass M of

$$R = \frac{90}{\pi^2} \frac{\sigma}{T^4}, \quad M = 12 \pi \sigma R^2. \quad (46)$$

(One could also obtain these results by balancing the photon pressure with the pressure associated with the tension in the domain wall.)

To estimate the lifetime of such a bubble, we use the fact that photons slowly leak out through the bubble wall at a rate

¹This result for the reflection ratio \mathcal{R} is true for all angles of incidence. This follows from Snell's law and the fact that the index of refraction is unity everywhere.

²In keeping with our Kaluza-Klein zero mode ansatz, we assume that there are essentially no nonzero momentum states in the direction of the extra dimension.

that depends upon the transmission coefficient $\mathcal{T} \sim O(\kappa_5^{-1})$. Using Poynting's theorem relating the Poynting flux through the bubble wall to the rate of energy decrease inside the bubble, we estimate that the bubble loses energy at a rate

$$\frac{dM}{dt} \approx -S_T(4 \pi R^2) \quad (47)$$

where $S_T = \mathcal{T} S_{inc} = \mathcal{T} \rho_\gamma$ is the Poynting flux transmitted through the bubble wall, \mathcal{T} is the transmission coefficient, S_{inc} is the Poynting flux incident upon the wall, and $\rho_\gamma = aT^4 = (\pi^2/15)T^4$ is the photon energy density. We then have, from Eqs. (46) and (47),

$$\frac{dM}{dt} \approx -(\mathcal{T} a T^4)(4 \pi R^2) \approx 24 \pi \sigma R \frac{dR}{dt} \quad (48)$$

which gives

$$\frac{dR}{dt} \approx -\mathcal{T} \sim -O(\kappa_5^{-1}). \quad (49)$$

At time t a bubble has a radius $R \approx R_0 - \mathcal{T}t$, and the lifetime of the bubble is

$$\tau \approx \frac{R_0}{\mathcal{T}} \sim O(\kappa_5) R_0. \quad (50)$$

A remark is in order for the case of a dimension bubble with a 5D interior, whose domain wall arises, in part, from the low temperature Rubin-Roth potential contribution to the effective potential $U(\varphi)$. In this case the local minimum of $U(\varphi)$ disappears at high temperatures, so that the domain wall itself disappears, or "bursts," at some temperature T_c . Therefore the photon temperature of the bubble's (5D) interior must be restricted to values $T \leq T_c$. At a temperature $T \sim T_c$ the bubble would burst, releasing all of its radiation. From Eq. (46) we see that as the bubble shrinks, the photon temperature increases. When the temperature reaches $T \sim T_c$, the bubble explodes, so that the bubble lifetime is actually $\tau \lesssim O(\kappa_5) R_0$.

V. SUMMARY

The dimension bubble scenario has been extended to include Maxwell fields and sources. Beginning with a 5D Maxwell theory with sources, the reduction to four dimensions leads to an effective 4D dilaton-Maxwell system that, in the dielectric approach, leads to an interpretation of the extra dimensional scale factor $B(x)$ as a dielectric function $\kappa(x) = B(x)/B_4$ that takes drastically different values $\kappa_5 \gg 1$ in the interior of a bubble and $\kappa = 1$ in the exterior. One result of this is that the electromagnetic energy associated with charged scalar bosons confined to the bubble's interior is reduced from what would be the case for a 4D interior. This is due to the fact that the electromagnetic energy density $u = \frac{1}{2} \kappa E^2 = D^2/2\kappa$ is greatly suppressed in a 5D interior with dielectric constant κ_5 .

Another, somewhat striking, result is that the bubble wall possesses a near total reflectivity for $\kappa_5 \gg 1$, making it opaque to photons incident upon it from either side. Photons that are trapped within the bubble at its time of formation cannot easily escape, so that a dimension bubble can be held in a metastable state, supported by entrapped photons alone. The photon temperature, however, must remain below the

critical temperature T_c above which the domain wall disappears and an existing bubble would burst. A bubble filled with photons slowly decreases in size, with a resulting lifetime $\tau \lesssim O(\kappa_5)R_0$. As the bubble shrinks, the temperature T inside increases until it reaches a critical temperature T_c , at which time the bubble explodes, releasing its radiation contents.

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