Gauge quintessence

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We discuss a new model of quintessence in which the quintessence field is identified with the extra component of a gauge field in a compactified five-dimensional theory. We show that the extremely tiny energy scale $\sim(3\times10^{-3}\ \text{eV})^4$ needed to account for the present acceleration of the Universe can be naturally explained in terms of high energy scales such as the scale of grand unification.

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There is increasing evidence that the energy density of (baryonic plus dark) matter in the Universe is smaller than the critical density [1]. If the Universe is flat, as predicted by most natural inflation models [2] and confirmed by the recent measurements of the cosmic microwave background anisotropies [3], an additional dark energy density is necessary to reach $\Omega_0=1$. This dark energy seems to be the predominant form of energy in the present Universe, about 70% of the critical energy density, and should possess a negative pressure p. An obvious candidate is represented by the cosmological constant, whose equation of state is $\rho=-p$. If this is the option chosen by nature, particle physicists have to face the Herculean task of explaining why the energy of the vacuum V_0 is of the order of $(3\times10^{-3}\ \text{eV})^4$. Another possibility invokes a mixture of cold dark matter and quintessence [4], a slowly varying, spatially inhomogeneous component with an equation of state $p_Q=w_Q\rho_Q$, with $-1<w_Q\leqslant 0$. The role of quintessence may be played by any scalar field Q which is slowly rolling down its potential $V(Q)$. The slow evolution is needed to obtain a negative pressure, $p_Q=\frac{1}{2}\dot{Q}^2-V(Q)$, so that the kinetic energy density is less than the potential energy density. The quintessence field Q rolls down a potential according to the equation of motion $\ddot{Q}+3H\dot{Q}+V'(Q)=0$, where H is the Hubble constant satisfying the Friedmann equation in a flat universe:

$$H^2=\left(\frac{\dot{a}}{a}\right)^2=\frac{1}{3M_p^2}\left(\frac{1}{2}\dot{Q}^2+V(Q)+\rho_B\right), \tag{1}$$

where a is the scale factor, $M_p=2\times10^{18}$ GeV is the reduced Planck scale, and ρ_B is the remaining background energy density.

Since at present the quintessence field Q dominates the energy density of the Universe, one can write $\frac{1}{2}\dot{Q}^2=\frac{3}{2}(1+w_Q)H^2M_p^2$ and $V(Q)=\frac{3}{2}(1-w_Q)H^2M_p^2$. Let us assume that the quintessence potential has the parametric form

$$V(Q)=V_0\mathcal{V}\left(\frac{Q}{f}\right), \tag{2}$$

where V_0 parametrizes the height of the potential. Let us borrow the notation traditionally adopted in inflation model-building by defining a slow-roll parameter $\epsilon=-\dot{H}/H^2$. If the Universe is suffering an acceleration stage because of the quintessence dynamics, then $\ddot{a}/a=(1-\epsilon)H^2>0$ and the parameter $\epsilon\sim M_p^2(V'/V)^2\sim(M_p/f)^2$ has to be smaller than unity. This implies that the scale f has to be larger than the Planck scale. In turn, the quintessence mass m_Q must be extremely tiny since

$$m_Q^2\sim V''\sim\frac{V_0}{f^2}\sim H^2\left(\frac{M_p}{f}\right)^2. \tag{3}$$

The quintessence field has to roll down its potential with a mass comparable (or smaller) than $H\sim10^{-42}$ GeV.

The extreme flatness of the quintessence field represents a real challenge from the particle physics point of view and there are no completely natural models of quintessence. Supersymmetry is usually invoked to preserve the potential from acquiring large corrections to the mass of the quintessence field. However, the flatness of the potential tends to be spoiled when supergravity [5] corrections are included [6]. The same problem manifests itself in trying to build-up a satisfactory model of inflation [2].

Another possibility is to consider the quintessence field as a pseudo Nambu-Goldstone boson (PNGB) [7], i.e., the underlying theory possesses a nonlinearly realized symmetry and the quintessence field can be parametrized through an angular variable $\theta=Q/f$. In the limit of exact symmetry the quintessence field Q does not have a potential which is generated only in the presence of an explicit breaking term. The same effect could explain why the quintessence field does couple to ordinary matter more weakly than gravity.

If the quintessence field is a PNGB, its Lagrangian can be written as

$$\mathcal{L}=\frac{1}{2}(\partial_\mu Q)^2-V_0\left[1-\cos\left(\frac{Q}{f}\right)\right], \tag{4}$$

where f is the spontaneous breaking scale.

The problem of identifying the quintessence field with a PNGB comes from the fact that the spontaneous breaking scale f has to be comparable to the Planckian scale and, therefore, the effective four-dimensional field theory description is expected to break down due to quantum gravity corrections. One should note, however, that there might be shift symmetries, for example, acting on the model-independent axion, which constrain the form of these quantum corrections to be small in some regions of parameter space (see Kim and Nilles in Refs. [7] and [8]). The existence of such a symmetry is not imposed from a four-dimensional theory, but it is deduced from the string theory.

To summarize, there are two necessary key steps one needs to take in order to build up a successful quintessence particle physics model. One is to explain the reason why the scale V_0—parametrizing the height of the potential—is so tiny and the other is to explain why the overall scale f spanned by the quintessence field may be comparable to the Planckian scale without running into trouble with the four-dimensional description. Motivated by similar recent considerations applied to models of primordial inflation [9–11], in this paper we would like to show that the extra-dimensional generalization of identifying the quintessence field with a PNGB may help in taking both steps.

We consider a five-dimensional model with the extra fifth dimension compactified on a circle of radius R and identify the quintessence field with the fifth component A_5 of an Abelian gauge field $A_M(M=0,1,2,3,5)$ propagating in the bulk (the generalization to the non-Abelian case is straightforward). As such, the quintessence field cannot have a local potential because of the higher-dimensional gauge invariance. However, a nonlocal potential as a function of the gauge-invariant Wilson line

$$e^{i\theta}=e^{i\oint g_5 A_5 dy}, \tag{5}$$

where y is the coordinate along the fifth dimension, $0\leqslant y<2\pi R$, will be generated in the presence of fields charged under the Abelian symmetry [12].

Writing the field A_5 as

$$A_5=\frac{\theta}{2\pi g_5 R}, \tag{6}$$

where g_5 is the five-dimensional gauge coupling constant, at energies below the scale $1/R$, θ looks like a four-dimensional field with Lagrangian

$$\mathcal{L}=\frac{1}{2g_4^2(2\pi R)^2}(\partial_\mu\theta)^2-V(\theta), \tag{7}$$

where $g_4=g_5/(2\pi R)^{1/2}$ is the four-dimensional gauge coupling constant. Comparing Eqs. (4) and (7) one identifies the overall scale $f=1/2\pi g_4 R$ and the field $Q=\theta/2\pi g_4 R$. Therefore one can easily see that the overall scale f may be comparable to the four-dimensional Planckian scale, $f\sim M_p$, if the four-dimensional constant is small enough [9,11]. For instance, requiring that $1/R\sim 10^{16}$ GeV imposes that $g_4\sim 10^{-3}$. The higher-dimensional nature of the theory preserves the quintessence potential from acquiring dangerous corrections, and nonlocal effects must be necessarily exponentially suppressed because the typical length of five-dimensional quantum gravity effects $\sim M_5^{-1}$, where M_5 is the five-dimensional Planck scale, is much smaller than the size of the extra-dimensions.

Let us now turn to the form of the potential. We assume that the potential for the quintessence field is generated radiatively by a set of bulk fields which are charged under the U(1) symmetry with charges q_a. The fundamental hypothesis we make is that these bulk fields possess a bare mass $M_a\gg R^{-1}$ and that there is no charged matter with mass below the compactification scale. The masses M_a may be generated by some gauge symmetry breaking phenomenon at scales larger than R^{-1}. For instance, bulk fields may be charged under another Abelian factor broken at energies larger than the compactification scale. From the four-dimensional point of view, this is equivalent to having a tower of Kaluza-Klein states with squared masses

$$m_a^2=M_a^2+\left(\frac{n}{R}+g_4 q_a Q\right)^2 \quad (n=0,\pm 1,\pm 2,\dots). \tag{8}$$

Borrowing from finite temperature field theory calculations, the Q-dependent part of the potential can be written as [13]

$$V(Q)=\frac{1}{128\pi^6 R^4}\mathrm{Tr}[V(r_a^F,Q)-V(r_a^B,Q)], \tag{9}$$

where the trace is over the number of degrees of freedom, the superscripts F and B stand for fermions and bosons, respectively, and

$$V(r_a,Q)=x_a^2\,\mathrm{Li}_3(r_a e^{-x_a})+3x_a\,\mathrm{Li}_4(r_a e^{-x_a}) +3\,\mathrm{Li}_5(r_a e^{-x_a})+\mathrm{H.c.} \tag{10}$$

We have defined

$$x_a=2\pi R M_a,$$
$$r_a=e^{iq_a Q/f}, \tag{11}$$

and in Eq. (10) the functions $\mathrm{Li}_n(z)$ stand for the polylogarithm functions

$$\mathrm{Li}_n(z)=\sum_{k=1}^{\infty}\frac{z^k}{k^n}. \tag{12}$$

The potential (10) shows many similarities with the potential one obtains in four-dimensional field theories at finite temperature where, in the imaginary time formalism, four-dimensional loop integrals become integrals over the three spatial momenta and a sum over the so-called Matsubara frequencies. The finiteness of the potential at finite temperature is due to the fact that particles with wavelengths smaller than the inverse temperature have Boltzmann (exponentially) suppressed abundances in the plasma. Similarly, the potential (10) is independent of any ultraviolet cutoff. This is because the Wilson line is a global quantity while ultraviolet effects are local.

More crucial for our considerations is the behavior of the potential when the bare masses $M_a \gg R^{-1}$: the overall height of the potential is exponentially suppressed. This can be easily understood by thinking again of the four-dimensional finite temperature case. Particles in the plasma with bare masses M much larger than the temperature T do not contribute to the effective potential apart from tiny exponentially suppressed contributions. In the very same way, bulk fields charged under the Abelian gauge symmetry U(1) give an exponentially suppressed contribution to the potential (10) if their bare mass term is much larger than the effective temperature $T = R^{-1}$.

Let us, for simplicity, assume that all bare masses M_a are equal to a common mass $M \gg R^{-1}$. The potential (10) is well approximated by the form (2) with

$$V_0 \simeq \frac{c}{16\pi^4}\frac{M^2}{R^2}e^{-2\pi MR} = \frac{c}{16\pi^4}\frac{M^2}{R^2}e^{-MM_p^2/M_5^3}, \tag{13}$$

where $c = \mathcal{O}(1)$ is a numerical coefficient depending upon the charges of the bulk fields and in the last passage we have made use of the relation $M_p^2 = 2\pi RM_5^3$. We discover that the extreme smallness of the height of the potential V_0 can be naturally explained with a moderate fine-tuning of the parameter MR. This is the main result of this paper. To give a feeling for the numbers, setting $x_* = M_5/M_p$ and imposing the condition (13) gives

$$\frac{M}{M_5} \simeq x_*^2(270 + 12\ln x_*), \tag{14}$$

which fixes the parameter MR to be

$$MR \simeq 40 + 2\ln x_*. \tag{15}$$

If we do require that the overall scale M is smaller than the five-dimensional Planck scale M_5 in order to avoid dealing with loops of massive excitations of quantum gravity, we have to require $M \ll M_5$. This corresponds to a mild constraint on M_5, $M_5 \ll 6\times 10^{-2} M_p$. The problem of explaining the smallness of the energy scale of the quintessence potential is therefore exponentially reduced and requires only a moderate (logarithmic) fine-tuning.

The idea of identifying the quintessence field with a Wilson line has been briefly discussed and disregarded in Ref. [9] which was devoted to propose an interesting model of primordial inflation where the inflaton field is interpreted as the extra-component of a gauge field in a five-dimensional theory. Reference [9] considered the case in which the charged bulk fields do not possess large bare masses. The quintessence mass-squared m_Q^2 turns out to be of the order of $(f^2R^4)^{-1} \sim g_4^2/R^2$ and an extreme fine-tuning is needed either for the four-dimensional gauge coupling g_4 or for the radius of compactification R. Our findings show that such a fine-tuning can be avoided.

In conclusion we have shown that the extra-component of a gauge field in five-dimensions may be a good candidate for quintessence. The flatness of its potential is protected by gauge invariance in the higher-dimensional world and the tiny scale of the potential needed to accommodate the presently observed accelerating phase of the Universe may be naturally obtained if the nonlocal potential for the quintessence fields is provided by massive bulk fields. Our proposal does not solve, however, the so-called coincidence problem, that is why the amount of dark energy density is of the same order as the energy density stored in dark matter at the present epoch.

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