Spin-orbit and tensor forces in heavy-quark light-quark mesons: Implications of the new D_s state at 2.32 GeV

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We consider the spectroscopy of heavy-quark light-quark mesons with a simple model based on the nonrelativistic reduction of vector and scalar exchange between fermions. Four forces are induced: the spin-orbit forces on the light and heavy quark spins, the tensor force, and a spin-spin force. If the vector force is Coulombic, the spin-spin force is a contact interaction, and the tensor force and spin-orbit force on the heavy quark to order $1/m_1 m_2$ are directly proportional. As a result, just two independent parameters characterize these perturbations. The measurement of the masses of three p-wave states suffices to predict the mass of the fourth. This technique is applied to the D_s system, where the newly discovered state at 2.32 GeV provides the third measured level, and to the D system. The mixing of the two $J^P = 1^+$ p-wave states is reflected in their widths and provides additional constraints. The resulting picture is at odds with previous expectations and raises new puzzles.

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Mesons composed of one light quark and one heavy quark are quite analogous to the hydrogen atom [1] and can be analyzed using the traditional methods [2]. The fine and hyperfine structures have direct analogue in meson spectroscopy, but the confinement of quarks requires that the potential cannot be purely Coulombic. A convenient phenomenological approach is to postulate that there are two separate static potentials: one arising as the zeroth component of a vector potential and the other as a Lorentz scalar. Asymptotic freedom suggests that the vector potential be Coulombic, while confinement suggests that the scalar be a linear potential. Such models have had reasonable success in describing the spectroscopy of the D, D_s, B, and B_s systems [3–7].

The recently discovered state with a mass of 2.32 GeV decaying to $D_s \pi^0$ [8] appears to be a $c\bar{s}$ p-wave meson with $J^P = 0^+$. Typical predictions for its mass were near 2.5 GeV, above the threshold for the strong decay to DK. Here we analyze the p-wave $c\bar{s}$ and $c\bar{u}/\bar{d}$ mesons in a simple model that is more general, although less predictive, than the models described above. It is more general in that we do not specialize to a scalar potential that is linear.

Our concern here is restricted to the p-wave states. While our primary interest is in the D_s system, we consider as well the analogous nonstrange D mesons. It is not only the mass spectrum that needs to be addressed, but also the pattern of decay widths. The early studies, which predicted a much higher mass for the $J^P = 0^+$ $c\bar{s}$, were successful in explaining the narrow width of the observed $J^P = 1^+$ states in both the D and D_s systems. We find that the combined constraints of the mass spectrum as now known (although lacunae remain) and the decay patterns make it difficult to find a consistent picture of the p-wave charmed strange and nonstrange mesons.

By analogy with the hydrogen atom, a convenient classification of states is given in terms of $\mathbf{j} = \boldsymbol{\ell} + \mathbf{s}_1$, where $\boldsymbol{\ell}$ is the orbital angular momentum and $\mathbf{s}_1$ is the spin of the light quark. The light-quark angular momentum $\mathbf{j}$ is conserved in the limit in which the heavy-quark mass m_2 goes to infinity.

For p-wave states, the values of j are 3/2 and 1/2. These levels are split by the ordinary spin-orbit force. The hyperfine structure is proportional to $1/m_1 m_2$ and includes the spin-orbit coupling of the heavy quark, a spin-spin interaction, and the tensor force. All these contribute to further reducing the degeneracy so that $\mathbf{J} = \mathbf{j} + \mathbf{s}_2$ is conserved, but not $\mathbf{j}$ alone. The result is four distinct states with $J^P = 0^+$, 1^+, 1^+, and 2^+. The two $J^P = 1^+$ states are mixtures of $j = 3/2$ and $j = 1/2$. The $J = 2$ and $J = 0$ states are pure $S = 1$ states, where $\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2$ is the total spin. The $J = 1$ eigenstates are mixtures of spin-triplet and spin-singlet states.

The p-wave D mesons decay by pion emission. The 2^+ state decays through d-wave emission to the ground state $0^-, D$ or to the $1^-, D^*$. The 1^+ states can decay, in principle, by either d-wave or s-wave pion emission. The limited phase space favors the s-wave decay. However, the decay of the $j = 3/2$ state to the D^*, which has $j = 1/2$, is forbidden to the extent that $\mathbf{j}$ is conserved. Indeed, the $J^P = 1^+$ state at 2.422 GeV is narrow.

For p-wave D_s mesons, decay by pion emission is forbidden by isospin invariance. The D_s states at 2.572 and 2.536 GeV decay to DK and D^*K. In a fashion analogous to the pattern in the D system, the $J^P = 1^+$ state at 2.536 GeV is also narrow.

We base our analysis on the quasistatic potential, including the spin-dependent forces, computed from the Lorentz-invariant fermion-antifermion scattering amplitude using Feynman diagrams, replacing the vector or scalar propagators by the Fourier transforms of the postulated potentials $S(r)$ and $V(r)$. In this way, neglecting velocity-dependent but spin-independent terms, we find

$$\mathcal{V}_{\text{quasistatic}} = V + S + \left(\frac{V' - S'}{r} \right) \boldsymbol{\ell} \cdot \left(\frac{\boldsymbol{\sigma}_1}{4m_1^2} + \frac{\boldsymbol{\sigma}_2}{4m_2^2} \right)$$

$$+ \left(\frac{V'}{r} \right) \boldsymbol{\ell} \cdot \left(\frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2m_1 m_2} \right) + \frac{1}{12 m_1 m_2} \left(\frac{V'}{r} - V'' \right) S_{12}$$

$$+ \frac{1}{6 m_1 m_2} \nabla^2 V \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$

where $S_{12}=3\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ is the tensor force operator.

We imagine solving the corresponding differential equation with the potential $V+S$ and then computing the fine structure and hyperfine structure perturbatively. We work consistently only to order $1/m_2$. In general, there are four independent matrix elements to consider, $\langle(V'-S')/r\rangle$, $\langle V'/r\rangle$, $\langle -V''+V'/r\rangle$, and $\langle\nabla^2 V\rangle$. However, if V is Coulombic, $-V''+V'/r=3V'/r$ and $\langle\nabla^2 V\rangle$ vanishes except for s waves, where it gives a contact interaction. It follows that for Coulombic V and for $\ell>0$, there are only two independent matrix elements. Three measured p-wave masses will give two splittings, which will determine the matrix elements and allow us to predict the mass of the fourth p-wave state.

The mass operator for the splittings of the four states of any multiplet of orbital angular momentum different from zero can be written (to order $1/m_2$) as [9]

$$M=\lambda\boldsymbol{\ell}\cdot\mathbf{s}_1+4\tau\boldsymbol{\ell}\cdot\mathbf{s}_2+\tau S_{12} \tag{1}$$

with the notation

$$\lambda=\frac{1}{2m_1^2}\left[\frac{V'}{r}\left(1+\frac{2m_1}{m_2}\right)-\frac{S'}{r}\right],\quad\tau=\frac{1}{4m_1 m_2}\frac{V'}{r}. \tag{2}$$

In practice, we shall apply this operator to p-wave states for the D or D_s system. Henceforth, we will use λ and τ to indicate the expectation values of the quantities in Eq. (2). The values of λ and τ will be different for the D and D_s systems. For the assumed attractive, Coulombic V, the tensor-force energy τ is manifestly positive. However, the spin-orbit energy λ can be either positive or negative, depending on the relation between the potentials V and S.

Because there is no single basis that diagonalizes all the interactions, we need to fix one basis, then calculate the mixing between the two $J=1$ states. We choose the basis in which j^2 is diagonal. The eigenstates $|J,j,m\rangle$ of J^2, j^2, and J_z can be written in terms of eigenstates of J^2, S^2, and J_z, or in terms of eigenstates of J^2, $(\mathbf{j}')^2=(\boldsymbol{\ell}+\mathbf{s}_2)^2$, and J_z. The details are given in the Appendix.

Up to an additive constant common to the four p-wave states, the masses of the $J=2$ and $J=0$ states are $M_2=\frac{1}{2}\lambda+\frac{8}{5}\tau$, $M_0=-\lambda-8\tau$, while the masses of the two $J=1$ state are obtained by diagonalizing the matrix in the $|J,j,m\rangle$ basis:

$$\begin{pmatrix}\frac{1}{2}\lambda-\frac{8}{3}\tau & -2\sqrt{2}\tau/3 \\ -2\sqrt{2}\tau/3 & -\lambda+\frac{8}{3}\tau\end{pmatrix}.$$

The two eigenmasses for $J=1$ are then $M_{1\pm}=-\lambda/4\pm\sqrt{\lambda^2/16+(1/2)(\lambda-4\tau)^2}$. The eigenmasses are shown as functions of λ/τ in Fig. 1. Also shown is the $j=1/2$ fraction of the higher mass $J^P=1^+$ state. The vertical bands correspond to theoretical models and to data, as explained below.

If we define the mass splittings among three of the four states as $D_2=M_2-M_0$, $D_1=M_1-M_0$, we find $\tau=\frac{10}{87}D_2-\frac{2}{29}D_1\pm\{[(10/87)D_2-(2/29)D_1]^2+(5/232)(D_1^2-D_1 D_2)\}^{1/2}$, $\lambda=\frac{2}{3}D_2-\frac{32}{5}\tau$.

A very strong scalar potential S leads to "inversion," namely, the $j=1/2$ states lying above the $j=3/2$ states.
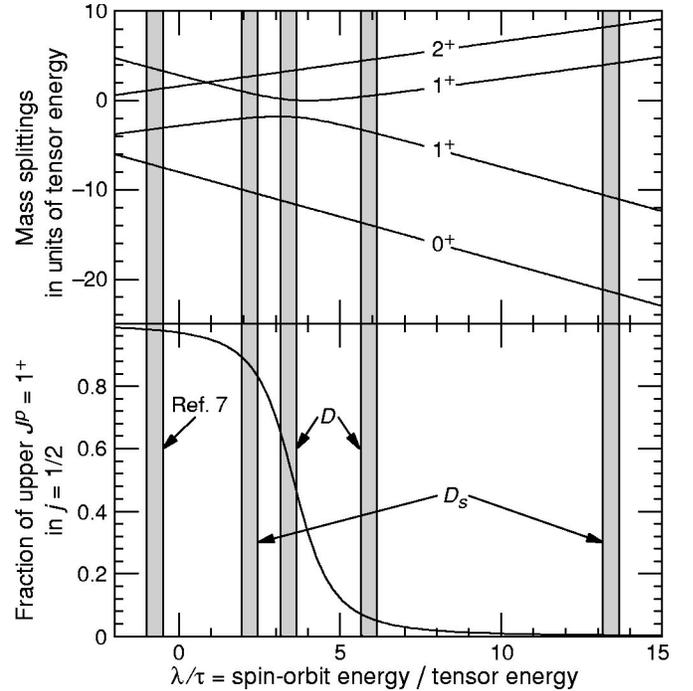
FIG. 1. Upper: the energy levels for the four p-wave states as a function of the ratio of the spin-orbit to tensor energies, in units of the tensor energy. Lower: the percentage of the more massive $J^P=1^+$ state that comes from the $j=1/2$ state. The D masses are from [10,11] and the D_s masses from [8,10].

With the formulas above, we can use the measured masses of three p-wave states to predict the fourth. Alternatively, we can take three masses that are predicted theoretically and confirm that we find the fourth predicted mass. Applying this to the D and D_s systems as given in Ref. [7] we indeed find congruence and determine the values of λ and τ shown in Table I. The leftmost vertical bar in Fig. 1 indicates the range found in Ref. [7]. The negative value of λ is indicative of inversion.

The result of inversion is that the higher mass $J^P=1^+$ state actually lies above the 2^+ state. This prediction is contradicted by the reports from the Belle Collaboration [11] of the $J^P=1^+$ state at 2.400 GeV. Using the well established D states at 2.459 and 2.422 GeV [12] and the reported $J^P=0^+$ at 2.290 GeV in our Eq. (1) leads to two solutions, labeled A and B in Table I. Both give masses near the observed state at 2.400 GeV. From the lower graph in Fig. 1 we see that the solution with the higher value of λ/τ results in the higher mass $J^P=1^+$ state (2.422 GeV) being nearly entirely $j=3/2$, while the lower value (solution A) would give it nearly equal contributions from $j=1/2$ and 3/2.

We can differentiate between the two solutions for both D and D_s by considering the widths. The experimental widths and theoretical estimates are shown in Table II. The theoretical estimates are obtained from [7] after making phase-space corrections. The widths for s- and d-wave decay are shown separately for the $J^P=1^+$ states. The proper combination depends on the mixing of the $j=3/2$ and 1/2 states.

TABLE I. Masses of the various p-wave states in the D and D_s systems and the spin-orbit and tensor energies. The experimental masses for the D states at 2.400 and 2.290 GeV are from Ref. [11]. The mass of the D_s state at 2.317 GeV is from Ref. [8]. The values of λ and τ for Ref. [7] were obtained by fitting their mass spectrum with the ansatz of Eq. (1). The square brackets indicate values that were used as inputs to the fits that determined the remaining mass and the values of λ and τ. At the time of Ref. [7] only the masses of the 2^+ states and the $D_1(2420)$ and $D_{s1}(2535)$ were known.

| | Experiment | Theory | | |
	Refs. [8,10,11]	Solution A	Solution B	Ref. [7]
		D mesons		
$M(2^+)$ (GeV)	2.459	[2.459]	[2.459]	2.460
$M(1^+)$ (GeV)	2.400	2.400	2.385	2.490
$M(1^+)$ (GeV)	2.422	[2.422]	[2.422]	2.417
$M(0^+)$ (GeV)	2.290	[2.290]	[2.290]	2.377
λ (MeV)		39	54	−11
τ (MeV)		11	9	11
		D_s mesons		
$M(2^+)$ (GeV)	2.572	[2.572]	[2.572]	2.581
$M(1^+)$ (GeV)		2.480	2.408	2.605
$M(1^+)$ (GeV)	2.536	[2.536]	[2.536]	2.535
$M(0^+)$ (GeV)	2.317	[2.317]	[2.317]	2.487
λ (MeV)		43	115	−7
τ (MeV)		20	9	11

Referring to Fig. 1 and Table I, we see that the model of [7], which has $\lambda < 0$, has a $J = 1$ state that is nearly pure $j = 1/2$ above the $J = 2$ state. The lower $J = 1$ state is identified with the $D_1(2420)$. Because it is nearly entirely $j = 3/2$ its decay by pion emission must be d wave. This causes a suppression that conforms to the measured width. However, the mass spectrum predicted by Ref. [7] is not in good agree-

TABLE II. Decay widths of p-wave D and D_s states in MeV. The theoretical values are derived from [7] using phase-space corrections to adjust for the masses known now. The widths shown for the $D_2^*(2460)$ states are obtained from the total width of $23 \pm 5\,0^-$ and the measured [10] ratio $\Gamma(D^+\pi^-)/\Gamma(D^{*+}\pi^-) = 2.3 \pm 0.6$. For the $D_{sJ}(2573)$ we have assigned the entire width to the decay to $D(1865)K$ since the decay to $D^*(2007)K$ has not been seen.

| | Experiment [10,11] | Theory | |
		s wave	d wave
	D mesons		
$D_2^*(2460) \to D(1865)\pi$	16 ± 4		16
$D_2^*(2460) \to D^*(2007)\pi$	7 ± 3		9
$D_1(2422) \to D^*(2007)\pi$	$18.9^{+4.6}_{-3.5}$	90	10
$D_1(2400) \to D^*(2007)\pi$	$380 \pm 100 \pm 100$	100	
$D_0^*(2290)$	$305 \pm 30 \pm 25$	100	
	D_s mesons		
$D_2^*(2573) \to D(1865)K$	15^{+5}_{-4}		9
$D_2^*(2573) \to D^*(2007)K$	—		1.4
$D_1(2535) \to D^*(2007)K$	<2.3	100	0.3

ment with the data. Of our two solutions for the D data, only one has a large value of λ/τ as needed to make the lower mass $J = 1$ state broad, as required by the data.

Turning to the D_s system, we find a similar situation. While Ref. [7] would have $\lambda < 0$, the known masses, which now include the 0^+ state at 2.32 GeV, require $\lambda > 0$. Indeed, to suppress the width of the $D_{s1}(2535)$, a large λ is needed. This is the case for solution B, where the mass predicted for the fourth p-wave state is 2408 MeV. We note that in Ref. [8] there is an apparent signal in the invariant mass of $D_s\pi^0\gamma$ 2.46 GeV. This would be consistent with a $J^P = 1^+$ state decaying through $D_{sJ}^*(2317)\gamma$ or through $D_s^*(2112)\pi^0$. The mass fits better with solution A of Table I, but the narrow width of the $D_{s1}(2535)$ favors solution B.

Both the D and D_s systems show deviations from the pattern anticipated by potential models. While "inversion," i.e., $\lambda < 0$, has been a favored prediction, it is not in agreement with the data. This suggests that the ansatz taken for the potentials V and S may not be as simple as assumed. We have allowed S to be any potential, while requiring that V be Coulombic; Ref. [7] required that S be linear. Relaxing that restriction might lead to improved agreement with the data for this highly predictive approach, which actually uses the solutions to the Dirac equation. On the other hand, even with the flexibility allowed in our approach, the resulting picture is not entirely attractive. We expect the confining potential to be quite important since the p-wave states are not concentrated at the origin. This would lead to a suppression of λ through the contribution of the $-S'/r$ term. This is not borne out by the values of λ we deduce.

The discovery of the $D_{sJ}^*(2317)$ has provided an important clue to heavy-quark light-quark spectroscopy by nailing down a p-wave state with $j = 1/2$. Puzzles remain. The anticipated discovery of the accompanying $j = 1/2$ state with $J = 1$ should add important new information, but it is not likely to resolve all the questions we have described.

APPENDIX

The relations between the three bases that diagonalize j^2, j'^2, and S^2 are

$$|J = \ell + 1, j = \ell + 1/2, m\rangle = |J = \ell + 1, S = 1, m\rangle,$$

$$|J = \ell, j = \ell - 1/2, m\rangle = \sqrt{(J+1)/(2J+1)}|J = \ell, S = 1, m\rangle$$
$$- \sqrt{J/(2J+1)}|J = \ell, S = 0, m\rangle,$$

$$|J = \ell, j = \ell + 1/2, m\rangle = \sqrt{J/(2J+1)}|J = \ell, S = 1, m\rangle$$
$$+ \sqrt{(J+1)/(2J+1)}|J = \ell, S = 0, m\rangle,$$

$$|J = \ell - 1, j = \ell - 1/2, m\rangle = |J = \ell - 1, S = 1, m\rangle,$$

$$|J=\ell+1,j'=\ell+1/2,m\rangle=|J=\ell+1,S=1,m\rangle,$$

$$|J=\ell,j'=\ell-1/2,m\rangle=\sqrt{(J+1)/(2J+1)}$$

$$\times|J=\ell,S=1,m\rangle+\sqrt{J/(2J+1)}|J=\ell,S=0,m\rangle,$$

$$|J=\ell,j'=\ell+1/2,m\rangle=\sqrt{J/(2J+1)}|J=\ell,S=1,m\rangle$$

$$-\sqrt{(J+1)/(2J+1)}|J=\ell,S=0,m\rangle,$$

$$|J=\ell-1,j'=\ell-1/2,m\rangle=|J=\ell-1,S=1,m\rangle,$$

$$|J=\ell+1,j=\ell+1/2,m\rangle=|J=\ell+1,j'=\ell+1/2,m\rangle,$$

$$|J=\ell,j=\ell-1/2,m\rangle=\frac{1}{2J+1}|J=\ell,j'=\ell-1/2,m\rangle$$

$$+\frac{2\sqrt{J(J+1)}}{2J+1}|J=\ell,j'=\ell+1/2,m\rangle,$$

$$|J=\ell,j=\ell+1/2,m\rangle=\frac{2\sqrt{J(J+1)}}{2J+1}|J=\ell,j'=\ell-1/2,m\rangle$$

$$-\frac{1}{2J+1}|J=\ell,j'=\ell+1/2,m\rangle,$$

$$|J=\ell-1,j=\ell-1/2,m\rangle=|J=\ell-1,j'=\ell-1/2,m\rangle.$$

In these three bases it is easy to evaluate the matrix elements of $\boldsymbol{\ell}\cdot\mathbf{s}_1$, $\boldsymbol{\ell}\cdot\mathbf{s}_2$

$$\langle Jjm|2\boldsymbol{\ell}\cdot\mathbf{s}_1|Jjm\rangle=j(j+1)-\ell(\ell+1)-3/4,$$

$$\langle Jj'm|2\boldsymbol{\ell}\cdot\mathbf{s}_2|Jj'm\rangle=j'(j'+1)-\ell(\ell+1)-3/4,$$

and of S_{12} (all of whose matrix elements vanish if the initial or final state has $S=0$)

$$\langle J=\ell-1,S=1,m|S_{12}|J=\ell-1,S=1,m\rangle=-2\frac{J+2}{2J+1},$$

$$\langle J=\ell,S=1,m|S_{12}|J=\ell,S=1,m\rangle=+2,$$

$$\langle J=\ell+1,S=1,m|S_{12}|J=\ell+1,S=1,m\rangle=-2\frac{J-1}{2J+1},$$

from which we find

$$\langle J=\ell+1,j=\ell+1/2,m|2\boldsymbol{\ell}\cdot\mathbf{s}_2|J=\ell+1,j=\ell+1/2,m\rangle$$
$$=J-1,$$

$$\langle J=\ell,j=\ell+1/2,m|2\boldsymbol{\ell}\cdot\mathbf{s}_2|J=\ell,j=\ell+1/2,m\rangle$$
$$=-J(2J+3)/(2J+1),$$

$$\langle J=\ell,j=\ell-1/2,m|2\boldsymbol{\ell}\cdot\mathbf{s}_2|J=\ell,j=\ell-1/2,m\rangle$$
$$=(2J-1)(J+1)/(2J+1),$$

$$\langle J=\ell,j=\ell-1/2,m|2\boldsymbol{\ell}\cdot\mathbf{s}_2|J=\ell,j=\ell+1/2,m\rangle$$
$$=-2\sqrt{J(J+1)}/(2J+1),$$

$$\langle J=\ell-1,j=\ell-1/2,m|2\boldsymbol{\ell}\cdot\mathbf{s}_2|J=\ell-1,j=\ell-1/2,m\rangle$$
$$=-J-2,$$

and

$$\langle J=\ell+1,j=\ell+1/2,m|S_{12}|J=\ell+1,j=\ell+1/2,m\rangle$$
$$=-2(J-1)/(2J+1),$$

$$\langle J=\ell,j=\ell+1/2,m|S_{12}|J=\ell,j=\ell+1/2,m\rangle=\frac{2J}{2J+1},$$

$$\langle J=\ell,j=\ell-1/2,m|S_{12}|J=\ell,j=\ell-1/2,m\rangle=\frac{2(J+1)}{2J+1},$$

$$\langle J=\ell,j=\ell-1/2,m|S_{12}|J=\ell,j=\ell+1/2,m\rangle=\frac{2\sqrt{J(J+1)}}{2J+1},$$

$$\langle J=\ell-1,j=\ell-1/2,m|S_{12}|J=\ell-1,j=\ell-1/2,m\rangle$$
$$=-2(J+2)/(2J+1).$$

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