Neutrino masses with a "zero sum" condition: $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}=0$

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It is well known that the neutrino mass matrix contains more parameters than experimentalists can hope to measure in the foreseeable future even if we impose *CP* invariance. Thus, various authors have proposed *Ansätze* to restrict the form of the neutrino mass matrix further. Here we propose that $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}=0$. With this condition, the absolute neutrino mass can be obtained in terms of the mass-squared differences. When combined with the accumulated experimental data, this condition predicts two types of mass hierarchies, with one of them characterized by $m_{\nu_3}\approx -2m_{\nu_1}\approx -2m_{\nu_2}\approx 0.063$ eV, and the other by $m_{\nu_1}\approx -m_{\nu_2}\approx 0.054$ eV and $m_{\nu_3}\approx 0.0064$ eV. The range predicted for $|m_{\nu_1}|+|m_{\nu_2}|+|m_{\nu_3}|$ is below the cosmological upper bound of 0.69 eV from recent Wilkinson Microwave Anisotropy Probe data and can be probed in the near future. We also point out some implications for direct laboratory measurement of neutrino masses and the neutrino mass matrix.

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There are abundant data [1–6] from solar, atmospheric, laboratory, and long baseline neutrino experiments on the neutrino mass and mixing. The present experimental data, including recent results from KamLAND [5] and K2K [6], can be explained by oscillations between three active neutrinos [7–11].[1] Neutrino oscillations provide direct evidence of nonzero neutrino masses and mixing between different species of neutrinos.

We will assume that the neutrinos are Majorana neutrinos, as favored by some theoretical considerations [7]. The mass matrix M is symmetric due to Fermi statistics. In the weak basis where all charged leptons are already diagonalized, the neutrino mixing matrix V is determined by

$$D=V^T M V, \tag{1}$$

where D is a diagonal matrix. The diagonal entries m_i of D are the mass eigenvalues which can always be made real by an appropriate choice of phase convention.

Although there are a lot of data on neutrinos, more data are needed to determine the detailed properties of neutrinos. There is at present certainly no information on any of the *CP* violating phases, and in the foreseeable future no set of experiments can fully determine all the parameters in the neutrino mass matrix. Certain theoretical inputs have to be employed to reconstruct the neutrino mass matrix [13–19]. Several proposals have been made to reduce the parameters, such as texture zero [15] and determinant zero requirements [16] for the mass matrix.

Here we propose another way of reducing the number of unknown parameters, by imposing a condition on the mass eigenvalues:

$$m_{\nu_1}+m_{\nu_2}+m_{\nu_3}=0. \tag{2}$$

[1]There is additional evidence for oscillation between electron and muon neutrinos from the LSND experiment [12]. If confirmed, more neutrinos are needed to explain all the data.

If there is no *CP* violation in the neutrino mass matrix M, the mass matrix can always be made real and it can be diagonalized by an orthogonal transformation, namely, $V^T V = I$. In this case the "zero sum" condition (2) is equivalent to the traceless condition $\mathrm{Tr}\,M=\mathrm{Tr}(V^\dagger V^* D)=\mathrm{Tr}\,D=0$. If *CP* is not conserved, the "zero sum" and traceless conditions are different. One needs to be careful about the phase definitions [20]. In this paper we simply explore the phenomenological consequences of requiring the neutrino masses to satisfy the "zero sum" condition with *CP* conservation without speculating on its theoretical origin. We note, however, that it holds if $M=[A,B]$, that is, the mass matrix can be expressed as a commutator[2] of two matrices A and B.

Direct measurement of neutrino masses is a very difficult experimental task. If the "zero sum" condition $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}=0$ or equivalently $\mathrm{Tr}\,M=0$ is applied, all the neutrino masses are determined in terms of the mass-squared differences. We have

$$m^2_{\nu_1}=-\frac{1}{3}\Big[2\Delta m^2_{21}+\Delta m^2_{32} - 2\sqrt{(\Delta m^2_{32})^2+\Delta m^2_{21}\Delta m^2_{32}+(\Delta m^2_{21})^2}\Big],$$

$$m^2_{\nu_2}=\frac{1}{3}\Big[\Delta m^2_{21}-\Delta m^2_{32} + 2\sqrt{(\Delta m^2_{32})^2+\Delta m^2_{21}\Delta m^2_{32}+(\Delta m^2_{21})^2}\Big],$$

[2]In the simplest versions of the model proposed in Ref. [13], M results from radiative correction and comes out to be the commutator of a coupling matrix and the mass-squared matrix of the charged leptons. However, there are a number of variations of this model, and the condition $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}=0$ fails to hold in most of them.

TABLE I. Solutions of eigenmasses for the best fit values of $|\Delta m_{21}^2|=7.0\times10^{-5}$ eV^2 and $|\Delta m_{32}^2|=3.0\times10^{-3}$ eV^2.

Δm_{32}^2 (eV^2)	Δm_{21}^2 (eV^2)	m_{ν_1} (eV)	m_{ν_2} (eV)	m_{ν_3} (eV)	$\lvert m_{ee}\rvert$ (eV)
3.0×10^{-3}	7.0×10^{-5}	-0.0313	-0.0324	0.0636	(0.01–0.032)
-3.0×10^{-3}	7.0×10^{-5}	0.0541	-0.0548	6.43×10^{-4}	(0.018–0.054)

$$m_{\nu_3}^2=\frac{1}{3}[\Delta m_{21}^2+2\Delta m_{32}^2 +2\sqrt{(\Delta m_{32}^2)^2+\Delta m_{21}^2\Delta m_{32}^2+(\Delta m_{21}^2)^2}]. \quad (3)$$

The choice of the sign in front of the square root is decided by the requirement that all $m_{\nu_i}^2$ must be larger than or equal to zero. The relative signs of the eigenmasses m_i are determined by the condition (2). We will use a convention such that $m_{\nu_3}\geqslant 0$ in our later discussions.

At present the sign of the measured Δm_{21}^2 is determined to be positive [21], but the sign of Δm_{32}^2 is not determined; there are two possible solutions corresponding to the sign of Δm_{32}^2. In Table I we list all solutions for the best fit values of the mass-squared differences.

We see that the mass eigenvalues exhibit two types of hierarchies:

$$\text{(I)} \quad m_{\nu_3}\approx -2m_{\nu_1}\approx -2m_{\nu_2}\approx 0.064 \text{ eV},$$

$$\text{(II)} \quad m_{\nu_1}\approx -m_{\nu_2}\approx 0.054 \text{ eV} \quad \text{and} \quad m_{\nu_3}\approx 0.00064 \text{ eV}. \quad (4)$$

The sign of Δm_{32}^2 decides which mass hierarchy the solutions belong to. Note that the "natural" sign $\Delta m_{32}^2>0$ corresponds to scenario I, in which the masses are of the same order of magnitude, in contrast to scenario II, in which m_{ν_3} is two orders of magnitude smaller than m_{ν_1} and m_{ν_2}. We would like to suggest that I is more favored than II.

In contrast to oscillation experiments, the contribution of the neutrinos to the energy density of the universe, $\Omega_\nu h^2 \approx \Sigma_i|m_i|/(93.5\text{ eV})$, depends on the absolute values of $|m_i|$ of course. It is interesting to note that the absolute neutrino mass sum $|m_{\nu_1}|+|m_{\nu_2}|+|m_{\nu_3}|$ predicted by the "zero sum" condition is only a few times smaller than the present bound [22] of 0.69 eV obtained from the Wilkinson Microwave Anisotropy Probe (WMAP), and is close to the sensitivity of 0.12 eV of the combined Planck cosmic microwave background (CMB) data with the SDSS sky survey [23]. A future sky survey with an order of magnitude larger survey volume would allow the sensitivity to reach 0.03 eV [24]. The mass ranges predicted by the condition (2) may be tested in the future.

To obtain more information about neutrino properties, one needs to have information from mixing. Before the SNO and KamLAND data, assuming three active neutrino oscillations, one of the solutions that could account for known data was the bimaximal mixing matrix [25] with $|V_{e2}|=|V_{\mu 3}|=1/\sqrt{2}$. Experimental data from SNO and the recent data from KamLAND, however, disfavor the maximal mixing for the V_{e2} entry. We were thus led to propose [26] the following mixing matrix:

$$V=\begin{pmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (5)$$

This mixing matrix (but with the first and second columns interchanged) was first suggested by Wolfenstein more than 20 years ago [27]. It has subsequently been studied extensively by Harrison, Perkins, and Scott [28] and by Xing [29]. Oscillation experiments cannot determine the relative signs of the mass eigenvalues, which implies that one can multiply a phase matrix $P=\mathrm{Diag}(e^{i\rho},e^{i\sigma},1)$ from the right on V. With CP invariance, σ and ρ can take the values of zero or $\pm\pi/2$.

With the above information on the mixing matrix, let us estimate two observables related to neutrino mass measurements, the effective mass electron neutrino mass $\langle m_e\rangle^2$ measured in tritium beta decay end point spectrum experiments, and the effective Majorana electron neutrino mass m_{ee} in neutrinoless double beta decays.

The effective mass $\langle m_e\rangle^2$ is given by $(m_{\nu_1}^2|V_{e1}|^2+m_{\nu_2}^2|V_{e2}|^2+m_{\nu_3}^2|V_{e3}|^2)$. Using the values for the neutrino masses in Table I, we find that $\langle m_e\rangle$ is below the sensitivity of 0.12 eV for the proposed experiment KATRIN [30]. However, neutrinoless double beta decay experiments may be sensitive to the predicted ranges. The amplitude of neutrinoless double beta decay is proportional to the effective electron-neutrino Majorana mass m_{ee}, namely, M_{11}, given by

$$|m_{ee}|=|m_{\nu_1}V_{e1}^{*2}e^{-2i\rho}+m_{\nu_2}V_{e2}^{*2}e^{-2i\sigma}+m_{\nu_3}V_{e3}^{*2}|. \quad (6)$$

From the above expression we see that the value m_{ee} depends on the Majorana phases ρ and σ. We list the allowed ranges in Table I in the last column. The ranges obtained are well below the present upper bounds of 0.4 eV [31], but can be almost fully covered by future experiments [32], such as GENIUS, MAJORANA, EXO, MOON, or COURE, where sensitivity as low as 0.01 eV seems possible.

If the neutrino masses and mixing matrix are known to good precision, one can invert the eigenmasses according to Eq. (1) to obtain the mass matrix *M*. To have some feeling how this may provide important information about the mass matrix, we present some details of the mass matrices that produce the mixing matrix *V* in Eq. (5).

The most general mass matrix that can produce the mixing matrix *V* in Eq. (5) can be specified in the following form by the mass eigenvalues:

$$M=\frac{m_{\nu_1}}{6}\begin{pmatrix}4 & -2 & -2\\ -2 & 1 & 1\\ -2 & 1 & 1\end{pmatrix}+\frac{m_{\nu_2}}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}+\frac{m_{\nu_3}}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{pmatrix}. \qquad (7)$$

Being real symmetric (and so *a fortiori* Hermitian), the above three matrices generate a $U(1)\otimes U(1)\otimes U(1)$ subgroup of $U(3)$.

With the "zero sum" condition $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}=0$, the above matrix can be written as

$$M=\frac{m_{\nu_1}}{3}\begin{pmatrix}2 & -1 & -1\\ -1 & -1 & 2\\ -1 & 2 & -1\end{pmatrix}+\frac{m_{\nu_2}}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & -1/2 & 5/2\\ 1 & 5/2 & -1/2\end{pmatrix}. \qquad (8)$$

Since $|\Delta m^2_{21}/\Delta m^2_{32}|\ll 1$, it is instructive to work with the case $\Delta m^2_{21}=0$ as the first approximation and to see how the obtained mass matrix can be perturbed to produce the desired Δm^2_{21}. The corresponding mass matrices are given by

$$\text{(I)}\quad M_0=a\begin{pmatrix}2 & 0 & 0\\ 0 & -1 & 3\\ 0 & 3 & -1\end{pmatrix};$$

$$\text{(II)}\quad M_0=\frac{1}{3}a\begin{pmatrix}2 & -4 & -4\\ -4 & -1 & -1\\ -4 & -1 & -1\end{pmatrix}. \qquad (9)$$

Note that the unperturbed mass matrix M_0 looks "simpler" in the "natural" hierarchy I than in the "inverted" hierarchy II.

The mass matrix in case I was studied in a previous paper by us [26]. The desired mass-squared difference Δm^2_{21} can be obtained by a small perturbation of the form

$$\delta M_T=\varepsilon a\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix}, \qquad (10)$$

with the perturbed eigenvalues given by $m_{\nu_1}=2a(1-\varepsilon/2)$, $m_{\nu_2}=2a(1+\varepsilon)$, and $m_{\nu_3}=-4a(1+\varepsilon/4)$. The parameter ε to the lowest order is given by $\varepsilon=\Delta m^2_{21}/\Delta m^2_{32}$.

For case II, adding δM_T to M_0, the eigenmasses are given by $m_{\nu_1}=2a(1-\varepsilon/2)$, $m_{\nu_2}=-2a(1-\varepsilon)$, and $m_{\nu_3}=-a\varepsilon$ with $\varepsilon\approx\Delta m^2_{21}/\Delta m^2_{32}$.

The perturbation δM_T preserves the "zero sum" condition in Eq. (2). One can also consider situations where the perturbations break this "zero sum" condition but still produce the mixing matrix *V* in Eq. (5). For example, adding a "democratic" perturbation

$$\delta M_D=\varepsilon a\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} \qquad (11)$$

produces a mixing matrix of the form given by *V* in Eq. (5), but different mass eigenvalues $[m_{\nu_1}=2a,\ m_{\nu_2}=2a(1+3\varepsilon/2),\ m_{\nu_3}=-4a,\ \varepsilon=\Delta m^2_{21}/\Delta m^2_{32}]$ and $[m_{\nu_1}=2a,\ m_{\nu_2}=-2a(1-3\varepsilon/2),\ m_{\nu_3}=0,\ \varepsilon=\Delta m^2_{21}/3\Delta m^2_{32}]$ for cases I and II, respectively.

In conclusion, we have studied the consequences of neutrino masses with the "zero sum" condition $m_{\nu_1}+m_{\nu_2}+m_{\nu_3}=0$. With this condition the neutrino masses can be determined from measured mass-squared differences from oscillation experiments. We find that this condition predicts only two types of neutrino mass hierarchies with one of them characterized by $m_{\nu_3}\approx-2m_{\nu_1}\approx-2m_{\nu_2}\approx 0.063$ eV and the other by $m_{\nu_1}\approx-m_{\nu_2}\approx 0.054$ eV and $m_{\nu_3}\approx 0.0064$ eV. These masses although small, can be probed by experiments from CMB measurements and large scale structure surveys, and can also be probed by neutrinoless double beta decay experiments. In conjunction with information on neutrino mixing, the "zero sum" condition also predicts simple mass matrices for neutrinos.

Note added. The *Ansatz* discussed in this paper was proposed earlier by Black *et al.* [33]. Their theoretical motivation and the values of the masses they obtained are rather different, however.

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