

D_s decays into ϕ and $f_0(980)$ mesons

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We consider the nonleptonic and semileptonic decays of D_s mesons into ϕ and $f_0(980)$ mesons. QCD sum rules are used to calculate the form factors associated with these decays and the corresponding decay rates. On the basis of data on $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$, which goes dominantly via the transition $D_s^+ \rightarrow \pi^+ f_0(980)$, we conclude that there is space for a sizeable light quark component on $f_0(980)$.

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I. INTRODUCTION

The interpretation of the nature of the lightest scalar mesons has been controversial since their first observation over 30 years ago. Because of the complications of nonperturbative strong interactions, there is still no general agreement about their structure. Actually, the observed light scalar states are too numerous [1] to be accommodated in a single $q\bar{q}$ multiplet, and, therefore, it has been suggested that some of them escape the quark model interpretation. It is not known whether there is necessarily a glueball among the light scalars, and whether some of the too numerous scalars are multiquark or some meson-meson bound states, or even admixtures of quarks and gluons [2].

In particular, the structure of the meson $f_0(980)$ has been extensively debated. It has been interpreted as a $s\bar{s}$ state [3–5], as a four quark $s\bar{s}q\bar{q}$ state [6], as a bound state of hadrons [7,8], and as a result of a process known as hadronic dressing [3,4,9].

The recently measured relative weight of the reaction $D_s^+ \rightarrow f_0(980) \pi^+ \rightarrow \pi^+ \pi^- \pi^+$ [10], may serve as a tool for the estimation of the $s\bar{s}$ component of the meson $f_0(980)$. As a matter of fact, if $f_0(980)$ has a pure strangeness component [$f_0(980) = s\bar{s}$], the dominant $D_s^+ \rightarrow f_0(980) \pi^+$ decay proceeds via the spectator mechanism, as shown in Fig. 1. However, in the four quark scenario [$f_0(980) = s\bar{s}(u\bar{u} + d\bar{d})/2$] the decay $D_s^+ \rightarrow f_0(980) \pi^+$ is expected to proceed through a much more complicated recombination.

Since the spectator mechanism provides a strong production of the $\phi(1020)$ meson in the decay $D_s^+ \rightarrow \phi \pi^+$, in this work we consider the ratio

$$R = \frac{\Gamma(D_s^+ \rightarrow f_0(980) \pi^+)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} \quad (1)$$

to evaluate the importance of the $s\bar{s}$ component in the $f_0(980)$ meson. This same ratio was evaluated in recent calculations by using the spectral integration technique [11] and the constituent quark meson model [12]. In both calculations, the authors concluded that the $s\bar{s}$ component dominates the

$f_0(980)$ meson and, therefore, the spectator mechanism dominates the $D_s^+ \rightarrow f_0(980) \pi^+$ decay. Here we use the QCD sum rules method to evaluate the ratio in Eq. (1), as well as the branching ratios for the nonleptonic $D_s^+ \rightarrow \phi \pi^+$ and semileptonic $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$ and $D_s^+ \rightarrow f_0(980) \ell^+ \nu_\ell$ decays. The two branching ratios involving the meson ϕ will be used to check the reliability of the method, since these two branching ratios are known experimentally [1]:

$$\mathcal{B}^{exp}(D_s^+ \rightarrow \phi \pi^+) = (3.6 \pm 0.9)\%, \quad (2)$$

$$\mathcal{B}^{exp}(D_s^+ \rightarrow \phi \ell^+ \nu_\ell) = (2.0 \pm 0.5)\%. \quad (3)$$

II. DECAY WIDTHS

The decay width of the nonleptonic process $D_s^+ \rightarrow M \pi^+$, where M stands for the ϕ or $f_0(980)$ mesons, is given by

$$\Gamma(D_s^+ \rightarrow M \pi^+) = \frac{1}{16\pi m_{D_s}^3} |\mathcal{M}|^2 \sqrt{\lambda(m_{D_s}^2, m_M^2, m_\pi^2)}, \quad (4)$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. The QCD factorization formula (in the limit $m_\pi^2 \rightarrow 0$) gives for $f_0(980)$,

$$\begin{aligned} |\mathcal{M}(D_s^+ \rightarrow f_0(980) \pi^+)|^2 &= \frac{G_F^2}{2} |V_{cs}|^2 |V_{ud}|^2 \left(c_1 + \frac{c_2}{3} \right)^2 f_\pi^2 (m_{D_s}^2 - m_{f_0}^2)^2 f_+^2(0), \end{aligned} \quad (5)$$

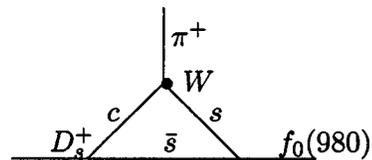


FIG. 1. Schematic picture of the spectator mechanism for the decay $D_s^+ \rightarrow f_0(980) \pi^+$.

where f_+ is the $D_s \rightarrow f_0(980)$ form factor defined as

$$\begin{aligned} \langle f_0(p') | \bar{s} \gamma_\mu (1 - \gamma_5) c | D_s(p) \rangle \\ = i(f_+(t)(p+p')_\mu + f_-(t)q_\mu), \end{aligned} \quad (6)$$

with $t = q^2$ and $q = p - p'$. For ϕ , we have

$$\begin{aligned} |\mathcal{M}(D_s^+ \rightarrow \phi \pi^+)|^2 = & \frac{G_F^2}{8m_\phi^2} |V_{cs}|^2 |V_{ud}|^2 \left(c_1 + \frac{c_2}{3} \right)^2 \\ & \times f_\pi^2 \lambda(m_{D_s}^2, m_\phi^2, m_\pi^2) \\ & \times [(m_{D_s} + m_\phi)A_1(0) \\ & - (m_{D_s} - m_\phi)A_2(0)]^2, \end{aligned} \quad (7)$$

where A_1 and A_2 are $D_s \rightarrow \phi$ form factors defined as [13]

$$\begin{aligned} \langle \phi(p') | \bar{s} \gamma_\mu (1 - \gamma_5) c | D_s(p) \rangle = & -i(m_{D_s} + m_\phi)A_1(t)\epsilon_\mu^\phi \\ & + i \frac{A_2(t)}{m_{D_s} + m_\phi} \epsilon^\phi p(p+p')_\mu \\ & + i \frac{A_3(t)}{m_{D_s} + m_\phi} \epsilon^\phi p q_\mu \\ & + \frac{2V(t)}{m_{D_s} + m_\phi} \epsilon_\mu^{\nu\rho\sigma} \epsilon_\nu^\phi p_\rho p'_\sigma. \end{aligned} \quad (8)$$

In Eqs. (5) and (7) the coefficients c_1 and c_2 are the Wilson coefficients entering the effective weak Hamiltonian evaluated at the normalization scale μ [14]:

$$H_W = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[\left(c_1(\mu) + \frac{c_2(\mu)}{3} \right) O_1 + \dots \right], \quad (9)$$

with $O_1 = [\bar{u} \gamma_\mu (1 - \gamma_5) d][\bar{s} \gamma^\mu (1 - \gamma_5) c]$. Therefore, in calculating the ratio in Eq. (1) we are free from the uncertainties in the Wilson coefficients and in the Cabibbo-Kobayashi-Mashawa transition elements.

In the case of the semileptonic decays $D_s^+ \rightarrow M \ell^+ \nu_\ell$, the differential decay rates are given by

$$\frac{d\Gamma}{dt} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, t) f_+^2(t), \quad (10)$$

for $D_s^+ \rightarrow f_0(980) \ell^+ \nu_\ell$. The decay rate for the decay $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$ is written in terms of the helicity amplitudes

$$H_\pm(t) = (m_{D_s} + m_\phi) A_1(t) \mp \frac{\lambda^{1/2}(m_{D_s}^2, m_\phi^2, t)}{m_{D_s} + m_\phi} V(t), \quad (11)$$

$$\begin{aligned} H_0(t) = & \frac{1}{2m_\phi \sqrt{t}} \left((m_{D_s}^2 - m_\phi^2 - t)(m_{D_s} + m_\phi) A_1(t) \right. \\ & \left. - \frac{\lambda(m_{D_s}^2, m_\phi^2, t)}{m_{D_s} + m_\phi} A_2(t) \right), \end{aligned} \quad (12)$$

so that

$$\frac{d\Gamma_\pm}{dt} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} t \lambda^{1/2}(m_{D_s}^2, m_\phi^2, t) |H_\pm(t)|^2, \quad (13)$$

$$\frac{d\Gamma_L}{dt} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} t \lambda^{1/2}(m_{D_s}^2, m_\phi^2, t) |H_0(t)|^2, \quad (14)$$

$$\frac{d\Gamma_T}{dt} = \frac{d}{dt} (\Gamma_+ + \Gamma_-), \quad \frac{d\Gamma}{dt} = \frac{d}{dt} (\Gamma_L + \Gamma_T). \quad (15)$$

III. SUM RULES

The D_s^+ meson in the initial state is interpolated by the pseudoscalar current

$$j_{D_s}(x) = \bar{s}(x) i \gamma_5 c(x), \quad (16)$$

where c and s are the fields of the charmed and the strange quark, respectively. Summation over spinor and color indices being understood but not indicated explicitly. The final hadronic state M is interpolated by the current

$$j_M(x) = \bar{s}(x) \Gamma_M s(x), \quad (17)$$

where

$$\Gamma_M = \begin{cases} 1 & \text{for } M = f_0(980) \\ \gamma_\alpha & \text{for } M = \phi. \end{cases} \quad (18)$$

Using the QCD sum rule technique [15], the form factors in Eqs. (6) and (8) can be evaluated from the time ordered product of the two interpolating fields in Eqs. (16) and (17) and the weak current $j_\mu^W = \bar{s} \gamma_\mu (1 - \gamma_5) c$,

$$T_{\mu M}(p, p') = i \int d^4x d^4y \langle 0 | T [j_M(x) j_\mu^W(y) j_{D_s}^\dagger(0)] | 0 \rangle \times e^{i(p'x - qy)}. \quad (19)$$

In order to evaluate the phenomenological side we insert intermediate states for D_s and M , we use the definitions in Eqs. (6) and (8), and obtain the following relations:

$$T_\mu^{phen}(p, p') = m_{f_0} f_{f_0} \frac{m_{D_s}^2 f_{D_s}}{m_c + m_s} \frac{f_+(t)(p+p')_\mu + f_-(t)q_\mu}{(p^2 - m_{D_s}^2)(p'^2 - m_{f_0}^2)} + (\text{contributions of higher resonances}), \quad (20)$$

for $f_0(980)$, and

$$T_{\mu\alpha}^{phen}(p, p') = m_\phi f_\phi \frac{m_{D_s}^2 f_{D_s}}{m_c + m_s} \frac{1}{(p^2 - m_{D_s}^2)(p'^2 - m_\phi^2)} \times \left(-(m_{D_s} + m_\phi) A_1(t) g_{\mu\alpha} + \frac{A_2(t)}{(m_{D_s} + m_\phi)} \times (p+p')_\mu p_\alpha - 2i \frac{V(t)}{(m_{D_s} + m_\phi)} \times \epsilon_{\mu\alpha\rho\sigma} p_\rho p'_\sigma + \dots \right) + (\text{contributions of higher resonances}), \quad (21)$$

for ϕ , where we have shown only the structures important for the evaluation of the form factors A_1 , A_2 , and V .

In the above equations the coupling of the $f_0(980)$ to the scalar current $j_s = \bar{s}s$ was parametrized in terms of the constant f_{f_0} as

$$\langle 0 | \bar{s}s | f_0 \rangle = m_{f_0} f_{f_0}, \quad (22)$$

and we have used the standard definitions of the couplings of D_s and ϕ with the corresponding currents:

$$\langle 0 | j_{D_s} | D_s \rangle = \frac{m_{D_s}^2 f_{D_s}}{m_c + m_s}, \quad (23)$$

$$\langle 0 | \bar{s} \gamma_\alpha s | \phi \rangle = m_\phi f_\phi \epsilon_\alpha. \quad (24)$$

The three-point function, Eq. (19), can be evaluated by perturbative QCD if the external momenta are in the deep Euclidean region

$$p \ll (m_c + m_s)^2, \quad p'^2 \ll 4m_s^2, \quad t \ll (m_c + m_s)^2. \quad (25)$$

In order to approach the not-so-deep Euclidean region and to get more information on the nearest physical singularities, nonperturbative power corrections are added to the perturbative contribution. In practice, only the first few condensates contribute significantly, the most important ones being the three-dimension, $\langle \bar{s}s \rangle$, and the five-dimension, $\langle \bar{s}g_s \sigma Gs \rangle$, condensates. For each invariant structure, i , we can write

$$T_i^{theor}(p^2, p'^2, t) = \frac{-1}{4\pi^2} \int_{(m_c + m_s)^2}^\infty ds \int_{4m_s^2}^\infty du \times \frac{\rho_i(s, u, t)}{(s - p^2)(u - p'^2)} + T_i^{D=3} \langle \bar{s}s \rangle + T_i^{D=5} \langle \bar{s}g_s \sigma Gs \rangle + \dots \quad (26)$$

The perturbative contribution is contained in the double discontinuity ρ_i .

In order to suppress the condensates of higher dimension and at the same time reduce the influence of higher resonances, the series in Eq. (26) is Borel improved, leading to the mapping

$$f(p^2) \rightarrow \hat{f}(M^2), \quad \frac{1}{(p^2 - m^2)^n} \rightarrow \frac{(-1)^n}{(n-1)!} \frac{e^{-m^2/M^2}}{(M^2)^n}. \quad (27)$$

Furthermore, we make the usual assumption that the contributions of higher resonances are well approximated by the perturbative expression

$$\frac{-1}{4\pi^2} \int_{s_0}^\infty ds \int_{u_0}^\infty du \frac{\rho_i(s, u, t)}{(s - p^2)(u - p'^2)} \quad (28)$$

with appropriate continuum thresholds s_0 and u_0 . By equating the Borel transforms of the phenomenological expression for each invariant structure in Eqs. (20), (21) and that of the ‘‘theoretical expression,’’ Eq. (26), we obtain the sum rules for the form factors (at the order m_s):

$$\begin{aligned}
C_{f_0} e^{-m_{D_s}^2/M^2} e^{-m_{f_0}^2/M'^2} f_+(t) &= \frac{-1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0} ds \int_0^{u_0} du [e^{-s/M^2} e^{-u/M'^2} \rho_+(s,u,t)] + \frac{\langle \bar{s}s \rangle}{2} e^{-m_c^2/M^2} \left[-m_c + 2m_s + \frac{m_c^2 m_s}{2M^2} \right] \\
&+ \langle \bar{s}g_s \sigma Gs \rangle e^{-m_c^2/M^2} \left[\frac{m_c^2(m_c - m_s)}{8M^4} - \frac{2m_c - m_s}{6M^2} + \frac{m_c^2(4m_c - 3m_s) - 2t(m_c - m_s)}{24M^2 M'^2} \right. \\
&\left. - \frac{m_c - 2m_s}{6M'^2} + \frac{m_c^2 m_s - 2t(m_c - m_s)}{24M^2 M'^2} \right], \tag{29}
\end{aligned}$$

$$\begin{aligned}
C_\phi (m_{D_s} + m_\phi)^2 e^{-m_{D_s}^2/M^2} e^{-m_\phi^2/M'^2} A_1(t) &= \frac{-1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0} ds \int_0^{u_0} du [e^{-s/M^2} e^{-u/M'^2} \rho_1(s,u,t)] + \frac{\langle \bar{s}s \rangle}{2} e^{-m_c^2/M^2} \left[t - m_c^2 - \frac{5}{2} m_c m_s \right. \\
&\left. - \frac{m_c m_s}{2M^2} (t - m_c^2) \right] + \langle \bar{s}g_s \sigma Gs \rangle e^{-m_c^2/M^2} \left[-\frac{1}{6} + \frac{m_c^2(m_c^2 + 2m_c m_s - t)}{8M^4} \right. \\
&\left. - \frac{2m_c^2 + 3m_c m_s - 2t}{12M'^2} - \frac{3m_c^2 + 9m_c m_s - 4t}{12M^2} \right. \\
&\left. + \frac{m_c(2m_c^3 + 3m_c m_s + t m_s)/2 - t(2m_c^2 + 3m_c m_s/2 - 2t)}{6M^2 M'^2} \right], \tag{30}
\end{aligned}$$

$$\begin{aligned}
C_\phi e^{-m_{D_s}^2/M^2} e^{-m_\phi^2/M'^2} A_2(t) &= \frac{-1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0} ds \int_0^{u_0} du [e^{-s/M^2} e^{-u/M'^2} \rho_2(s,u,t)] + \frac{\langle \bar{s}s \rangle}{2} e^{-m_c^2/M^2} \left(1 - \frac{m_c m_s}{2M^2} \right) \\
&- \langle \bar{s}g_s \sigma Gs \rangle e^{-m_c^2/M^2} \left[\frac{1}{6M^2} + \frac{m_c^2}{8M^4} + \frac{2m_c^2 - m_c m_s - 2t}{12M^2 M'^2} \right], \tag{31}
\end{aligned}$$

and

$$\begin{aligned}
-2C_\phi e^{-m_{D_s}^2/M^2} e^{-m_\phi^2/M'^2} V(t) &= \frac{-1}{4\pi^2} \int_{(m_c+m_s)^2}^{s_0} ds \int_0^{u_0} du [e^{-s/M^2} e^{-u/M'^2} \rho_V(s,u,t)] + \langle \bar{s}s \rangle e^{-m_c^2/M^2} \left(1 - \frac{m_c m_s}{2M^2} \right) \\
&- \langle \bar{s}g_s \sigma Gs \rangle e^{-m_c^2/M^2} \left[-\frac{1}{3M^2} + \frac{m_c^2}{4M^4} + \frac{2m_c^2 - m_c m_s - 2t}{6M^2 M'^2} \right], \tag{32}
\end{aligned}$$

where

$$C_{f_0} = \frac{m_f f_{f_0} m_{D_s}^2 f_{D_s}}{m_c + m_s}$$

and

$$C_\phi = \frac{m_\phi f_\phi m_{D_s}^2 f_{D_s}}{(m_c + m_s)(m_{D_s} + m_\phi)}. \tag{33}$$

The decay constants f_{D_s} , f_{f_0} , and f_ϕ defined in Eqs. (22), (23), and (24), and appearing in the constants C_{f_0} and C_ϕ , can also be determined by sum rules obtained from the appropriate two-point functions. The explicit expressions for the two-point sum rules and for the double discontinuities in Eqs. (29)–(32) are given in Appendixes A and B, respectively.

IV. EVALUATION OF THE SUM RULES AND RESULTS

In the complete theory, the form factors on the right hand side of Eqs. (29)–(32) should not depend on the Borel variables M^2 and M'^2 . However, in a truncated treatment there will always be some dependence left. Therefore, one has to work in a region where the approximations made are supposedly acceptable and where the result depends only moder-

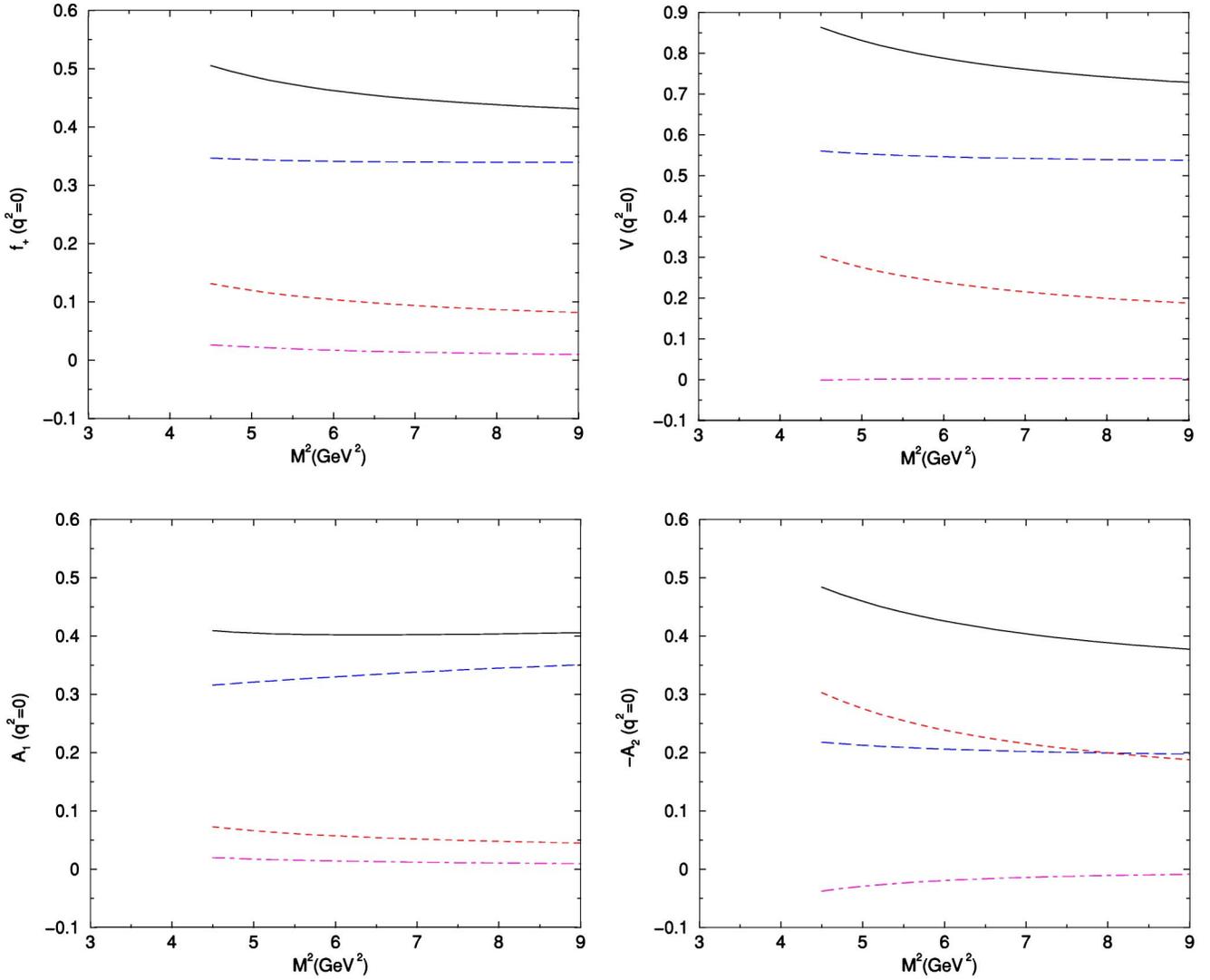


FIG. 2. Various contributions to the operator product expression OPE of the form factors as a function of the Borel parameter M^2 . Solid curve, total contribution; long-dashed line, perturbative; dashed line, quark condensate; dot-dashed line mixed condensate contribution.

ately on the Borel variables. To decrease the dependence of the results on the Borel variables M^2 , we take them in the two-point functions at half the value of the corresponding variables in the three-point sum rules [13,16]. We furthermore choose

$$\frac{M^2}{M'^2} = \frac{m_{D_s}^2 - m_c^2}{m_M^2}. \quad (34)$$

If the momentum transfer t is larger than a critical value t_{cr} , non-Landau singularities have to be taken into account [13]. Since we have to stay away from the physical region, i.e., we must have $t \ll (m_c + m_s)^2$, we limit our calculation to the region $-0.5 < t < 0.4$ GeV. In this range the t dependence can be obtained from the sum rules directly. It can be fitted by a monopole and extrapolated to the full kinematical region.

Since we do not take into account radiative corrections we choose the QCD parameters at a fixed renormalization scale

of about 1 GeV²: the strange and charm mass $m_s = 0.14$ GeV, $m_c = 1.3$ GeV. We take for the strange quark condensate $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ with $\langle \bar{q}q \rangle = -(0.24)^3$ GeV³ and for the mixed quark-gluon condensate $\langle \bar{s}g_s \sigma G s \rangle = m_0^2 \langle \bar{s}s \rangle$ with $m_0^2 = 0.8$ GeV².

For the continuum thresholds we take the values discussed in Appendix A: $s_0 = 7.7 \pm 1.1$ GeV² and

$$u_0 = \begin{cases} 1.6 \pm 0.1 \text{ GeV} & \text{in Eq. (29)} \\ 2.0 \pm 0.1 \text{ GeV} & \text{in Eqs. (30)–(32)}. \end{cases} \quad (35)$$

We evaluate our sum rules in the range $4.5 \leq M^2 \leq 9.0$ GeV², which is compatible with the Borel ranges used for the two-point functions in Appendix A. In Fig. 2 we show the different contributions to the form factors f_+ , A_1 , A_2 and V at zero momentum transfer, from the sum rules in Eqs. (29), (30), (31), and (32), as a function of the Borel variable M^2 , using the continuum thresholds $s_0 = 8.8$ GeV² and $u_0 = 1.6$ GeV² or 2.0 GeV² for f_0 or ϕ , respectively. We see

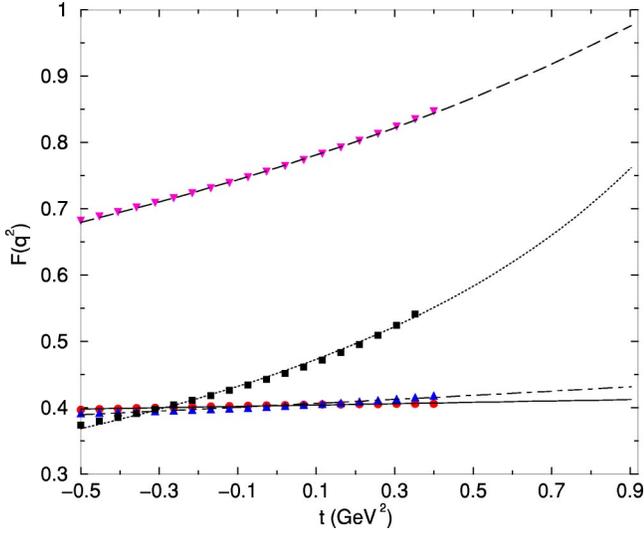


FIG. 3. t dependence of the form factors. The solid, dot-dashed, dotted, and dashed lines give the monopole parametrization, in Eq. (37), to the QCD sum rule results for $-A_2$ (circles), A_1 (triangles up), f_+ (squares), and V (triangles down).

that $A_2(0)$ gets a big contribution from the quark condensate, while the perturbative contribution is the largest one for all other form factors. Such a kind of behavior had been already obtained in the $D \rightarrow K^*$ semileptonic decay studied in [13]. The mixed condensate contribution is negligible for all four form factors, and the stability is quite satisfactory in the Borel range studied. Varying the continuum threshold s_0 in the range discussed in Appendix A, and also evaluating the sum rules used or the expressions given in Appendix A for the meson decay constants, or its numerical values, we get for the form factors at $t=0$,

$$\begin{aligned} 0.40 \leq f_+(0) \leq 0.48, \quad 0.71 \leq V(0) \leq 0.89, \\ 0.32 \leq A_1(0) \leq 0.42, \quad -0.43 \leq A_2(0) \leq -0.37. \end{aligned} \quad (36)$$

The value obtained for $f_+(0)$ is smaller than the value obtained for the same form factor in Ref. [12] by using the constituent quark meson model.

The t dependence of the form factors evaluated at $M^2 = 7 \text{ GeV}^2$ is shown in Fig. 3. In the range $-0.5 \leq t \leq 0.4 \text{ GeV}^2$, no non-Landau singularities occur for our choices of the continuum thresholds. The QCD sum rules results can, in this t range, be very well approximated by a monopole expression

$$F(t) = \frac{F(0)}{1 - \frac{t}{M_P^2}}, \quad (37)$$

for all four form factors. The result of the fit is also shown in Fig. 3. The different values for the pole mass, M_P , for the different form factors are given by

$$M_P = \begin{cases} (1.6 \pm 0.2) \text{ GeV} & \text{for } f_+(t) \\ (4.2 \pm 0.5) \text{ GeV} & \text{for } A_1(t) \\ (8.0 \pm 2.0) \text{ GeV} & \text{for } A_2(t) \\ (1.95 \pm 0.10) \text{ GeV} & \text{for } V(t). \end{cases} \quad (38)$$

In the case of A_2 we get a very high M_P showing a very weak t dependence. This approximate t independence stems from a mutual cancellation in the sum rule of an increase in the perturbative and a decrease in the quark condensate contributions. Even for A_1 the t dependence is much weaker than for f_+ and V . It is also interesting to notice that $M_P^{(V)}$ is of the same order as that for the semileptonic $D \rightarrow K^* \ell \nu_\ell$ found in [13], and $M_P^{(f_+)}$ is compatible with those found for the D_s decays into η [17] and $D \rightarrow \kappa \ell \nu_\ell$ [18].

Having the form factors we can evaluate the decay widths for the $D_s^+ \rightarrow \phi \pi^+$ and $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$ decays, given by Eqs. (4) and (15), respectively. We obtain the following branching ratios:

$$\mathcal{B}(D_s^+ \rightarrow \phi \pi^+) = (2.8 \pm 0.7)\% \quad (39)$$

and

$$\mathcal{B}(D_s^+ \rightarrow \phi \ell^+ \nu) = (1.4 \pm 0.4)\%, \quad (40)$$

where we have used $V_{ud} = V_{cs} = 0.975$, $f_\pi = 0.132$ and the values $c_1(m_c) = 1.263$ and $c_2(m_c) = -0.513$, corresponding to the results for the Wilson coefficients obtained at the leading order in renormalization group improved perturbation theory at $\mu \approx 1.3 \text{ GeV}$ [17]. The errors in the above results were estimated only by taking into account the uncertainties in the form factors in Eq. (36) and should be understood as limiting values for the branching ratios.

We see that, within the errors, our results are compatible with the experimental results given in Eqs. (2) and (3). Therefore, in the case of the D_s decay into ϕ , we can say that the factorization approximation works well. From this, it seems reasonable to suppose that, if $f_0(980)$ has a dominant $\bar{s}s$ component, the factorization approximation should also work well for the D_s decay into $f_0(980)$. Using Eqs. (4), (5), and (7), and the values for the form factors at $t=0$ given in Eq. (36), we get

$$\frac{\Gamma(D_s^+ \rightarrow f_0(980) \pi^+)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} = 0.44 \pm 0.18. \quad (41)$$

In the recently measured spectra from the reaction $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$ [10], the relative weight of the channel $\pi^+ f_0(980)$ is evaluated to be

$$\frac{\mathcal{B}(D_s^+ \rightarrow \pi^+ f_0(980)) \mathcal{B}(f_0(980) \rightarrow \pi^+ \pi^-)}{\mathcal{B}(D_s^+ \rightarrow \pi^+ \pi^+ \pi^-)} = (56.5 \pm 6.4)\%, \quad (42)$$

and the ratio of yields

$$\frac{\Gamma(D_s^+ \rightarrow \pi^+ \pi^+ \pi^-)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} = 0.245 \pm 0.028_{-0.012}^{+0.019} \quad (43)$$

is measured. Taking into account the results in Eqs. (42) and (43), one gets

$$R = \frac{\Gamma(D_s^+ \rightarrow f_0(980) \pi^+)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} = \frac{0.140 \pm 0.046}{\mathcal{B}(f_0 \rightarrow \pi^+ \pi^-)}. \quad (44)$$

Using the branching ratio $\mathcal{B}(f_0(980) \rightarrow \pi^+ \pi^-) \approx 53\%$, the authors in Ref. [11] have estimated the ratio be $R = 0.275(1 \pm 0.25)$. In a different way E791, using couple-channel Breit-Wigner function [10], found a nonsignificant g_K , which means indirectly a nonsignificant contribution for the decay channel $f_0(980) \rightarrow KK$. Thus if we assume that $\mathcal{B}(f_0(980) \rightarrow \pi\pi) \sim 1$ which implies $\mathcal{B}(f_0(980) \rightarrow \pi^+ \pi^-) \sim 2/3$ ($2/3$ being the isospin factor), using this in Eq. (44), we get

$$\frac{\Gamma(D_s^+ \rightarrow f_0(980) \pi^+)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} = 0.210 \pm 0.069. \quad (45)$$

Therefore, from our result in Eq. (41), we conclude that there is a significant deviation from the factorization approximation for the $D_s^+ \rightarrow f_0(980) \pi^+$ decay. This could be an indication that there is a sizable nonstrange component in the $f_0(980)$ meson, or even that the $f_0(980)$ structure is more complex than indicated by the naive quark model.

It is interesting to notice that the result obtained in Ref. [12] for the ratio in Eq. (1) is very similar to our result in Eq. (41). However, the authors in [12] concluded that their result supports a description of $f_0(980)$ as a $s\bar{s}$ state with a possible virtual $K\bar{K}$ cloud, but with no substantial mixture of $u\bar{u}$, $d\bar{d}$. We believe that this conclusion was reached because the authors in Ref. [12] misinterpreted the experimental result [10]. In their words, the E791 collaboration measured $R = 0.62$, with a very small error. From the Particle Data Group [1] we only know that $\mathcal{B}(f_0 \rightarrow \pi\pi)$ is dominant without knowing the exact number, there is still an indetermination in the ratio, Eq. (44). As explained above, if we consider $\mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) \sim 2/3$, we arrive at the result in Eq. (45), which is smaller than our result in Eq. (41), leading us to an opposite conclusion compared with [12].

One possible way to test if there really is a sizable nonstrange component in the $f_0(980)$ is through the measurement of the semileptonic $D_s^+ \rightarrow f_0(980) \ell^+ \nu$ decay, since in this decay we do not have problems with the factorization approximation. Our prediction for the branching ratio obtained from Eq. (10), by supposing $f_0(980)$ as a $\bar{s}s$ state, is

$$\mathcal{B}(D_s^+ \rightarrow f_0(980) \ell^+ \nu) = (0.55 \pm 0.10)\%. \quad (46)$$

Any significant deviation from that will definitively imply in a sizable nonstrange component in the $f_0(980)$ meson, which could or could not be accommodated in the naive quark model. Therefore, we urge the experimentalists to search for this decay.

V. SUMMARY AND CONCLUSIONS

We have presented a QCD sum rule study of the D_s^+ decays to final states containing ϕ and $f_0(980)$ mesons. We have evaluated the t dependence of the form factors $f_+(t)$, $A_1(t)$, $A_2(t)$, and $V(t)$ in the region $-0.5 \leq t \leq 0.4 \text{ GeV}^2$. The t dependence of the form factors could be fitted by a monopole form and extrapolated to the full kinematical region. The axial-vector form factors A_1 and A_2 have a much weaker t dependence than the form factors f_+ and V .

The form factors were used to evaluate the branching ratios for the decays $D_s^+ \rightarrow \phi \pi^+$ and $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$ and we have obtained a good agreement with experimental data. Since the evaluation of the decay width, in the nonleptonic decay, is based on the factorization approximation, our first conclusion is that the factorization approximation works well in the case of the decay $D_s^+ \rightarrow \phi \pi^+$.

We have also evaluated the ratio $D_s^+ \rightarrow f_0(980) \pi^+ / (D_s^+ \rightarrow \phi \pi^+)$ and we got a result greater than the estimate based on experimental data. Based on the fact that factorization approximation works well in the case of the decay $D_s^+ \rightarrow \phi \pi^+$, this result can be interpreted as an indication that there is a sizable nonstrange component in the $f_0(980)$ meson, or even that the $f_0(980)$ structure is more complex than indicated by the naive quark model. This hypothesis could be tested by the measurement of the semileptonic $D_s^+ \rightarrow f_0(980) \ell^+ \nu$ decay, since there is no problem with the factorization approximation in the semileptonic decays.

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APPENDIX A: TWO-POINT SUM RULES

In Ref. [9] the two-point sum rule for the $f_0(980)$ meson was evaluated by considering f_0 as a $\bar{s}s$ state. They got

$$\begin{aligned} m_{f_0}^2 f_{f_0}^2 e^{-m_{f_0}^2/M^2} &= \frac{3}{8\pi^2} \int_{4m_s^2}^{u_0} duu \left(1 - \frac{4m_s^2}{u} \right)^{3/2} e^{-u/M^2} \\ &+ m_s e^{-m_s^2/M^2} \left[\langle \bar{s}s \rangle \left(3 + \frac{m_s^2}{M^2} \right) \right. \\ &\left. + \frac{\langle \bar{s}g_s \sigma G_s \rangle}{M^2} \left(1 - \frac{m_s^2}{2M^2} \right) \right]. \quad (A1) \end{aligned}$$

Considering M^2 in the interval $1 \leq M^2 \leq 2 \text{ GeV}^2$, $u_0 = 1.6 \pm 0.1 \text{ GeV}^2$ they got

$$f_{f_0} = (0.180 \pm 0.015) \text{ GeV}. \quad (A2)$$

If we consider $m_s^2 = 0$, the result for f_{f_0} does not change significantly, and we get $f_{f_0} = (0.19 \pm 0.02) \text{ GeV}$.

The sum rule for ϕ is given by [19]

$$f_\phi^2 e^{-m_\phi^2/M^2} = \frac{1}{4\pi^2} \int_{4m_s^2}^{u_0} du \frac{(u+2m_s^2)\sqrt{u-4m_s^2}}{u^{3/2}} e^{-u/M^2} + \frac{2m_s\langle\bar{s}s\rangle}{M^2} + \frac{\langle g^2 G^2 \rangle}{48\pi^2 M^2}. \quad (\text{A3})$$

Considering m_s at most linearly and using $u_0=2.0 \pm 0.1 \text{ GeV}^2$, we get

$$f_\phi = (0.232 \pm 0.010) \text{ GeV} \quad (\text{A4})$$

in the interval $1 \leq M^2 \leq 2 \text{ GeV}^2$, in very good agreement with the experimental value $f_\phi^{exp} = 0.234 \text{ GeV}$ [1].

For f_{D_s} the two-point sum rule is given by

$$\frac{m_{D_s}^4 f_{D_s}^2}{(m_c + m_s)^2} e^{-m_{D_s}^2/M^2} = \frac{3}{8\pi^2} \int_{(m_c+m_s)^2}^{s_0} ds \left(1 - \frac{(m_c - m_s)^2}{s} \right) \times \sqrt{\lambda(s, m_c^2, m_s^2)} e^{-s/M^2} + \langle \bar{s}s \rangle e^{-m_c^2/M^2} \left(-m_c + \frac{m_s}{2} + \frac{m_s m_c^2}{M^2} \right) - \frac{m_c \langle \bar{s}g_s \sigma G s \rangle}{2M^2} e^{-m_c^2/M^2} \times \left(1 - \frac{m_c^2}{2M^2} \right). \quad (\text{A5})$$

Considering m_s at most linearly and using $s_0 = 8.8 \text{ GeV}^2$, we get

$$f_{D_s} = (0.22 \pm 0.02) \text{ GeV} \quad (\text{A6})$$

in the interval $2.3 \leq M^2 \leq 4.5 \text{ GeV}^2$, in good agreement with the value quoted in Ref. [20], and also with recent lattice determination [21]: $f_{D_s} = (0.252 \pm 0.009) \text{ GeV}$.

APPENDIX B: PERTURBATIVE CONTRIBUTIONS TO THE THREE-POINT FUNCTIONS

In all this work we take into account the mass of the strange quark at most linearly. We have checked that the contribution of terms proportional to m_s^2 and higher powers are negligible. The perturbative contributions for the sum rules defined in Sec. III are

$$\rho_+(s, u, t) = \frac{3}{\lambda^{3/2}(s, u, t)} \{ u[2m_c m_s (2m_c^2 - s - t + u) + m_c^2 (s - t + u) + s(-s + t + u)] - (2m_c^2 - s - t + u)(su + m_c m_s (s - t + u)) \} \times \theta(s - s_M), \quad (\text{B1})$$

$$\rho_1(s, u, t) = -\frac{3}{2\lambda^{3/2}(s, u, t)} \{ m_c u [\lambda(s, u, t) + 2m_c^2 (m_c^2 - s - t + u) + 2st] + m_s [s^3 - t^3 + 2ut^2 + ut(2m_c^2 - u) - 2m_c^2 u (m_c^2 + u) - s^2(3t + 2u) + s(3t^2 - 2tu + u(2m_c^2 + u))] \} \theta(s - s_M), \quad (\text{B2})$$

$$\rho_2(s, u, t) = \frac{3}{2\lambda^{5/2}(s, u, t)} \{ m_c u (s - t - u) [\lambda(s, u, t) + 6st + 6m_c^2 (m_c^2 - s - t + u)] + m_s [s^4 + t^4 - t^3 u + 2m_c^2 u^2 (3m_c^2 + u) - ut^2 (10m_c^2 + u) - s^3 (4t + 3u) + tu(6m_c^4 + 8m_c^2 u + u^2) + s^2 (6t^2 - tu + u(2m_c^2 + 3u)) - s(4t^3 - 5t^2 u - 4tu(2m_c^2 + u) + u(6m_c^4 + 4m_c^2 u + u^2))] \} \theta(s - s_M), \quad (\text{B3})$$

$$\rho_V(s, u, t) = \frac{3}{2\lambda^{3/2}(s, u, t)} \{ m_c u (2m_c^2 - s - t + u) + m_s [s^2 + t^2 - ut - 2um_c^2 - s(2t + u)] \} \theta(s - s_M), \quad (\text{B4})$$

where

$$s_M = m_c^2 + \frac{m_c^2 u}{m_c^2 - t}. \quad (\text{B5})$$

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