

Supersymmetric flavor models and the $B \rightarrow \phi K_S$ anomaly

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We consider the flavor structure of supersymmetric theories that can account for the deviation of the observed time-dependent CP asymmetry in $B \rightarrow \phi K_S$ from the standard model prediction. Assuming simple flavor symmetries and effective field theory, we investigate possible correlations between sizable supersymmetric contributions to $b \rightarrow s$ transitions and to flavor changing processes that are more tightly constrained. With relatively few assumptions, we determine the properties of minimal Yukawa and soft mass textures that are compatible with the desired supersymmetric flavor-changing effect and constraints. We then present explicit models that are designed (at least approximately) to realize these textures. In particular, we present an Abelian model based on a single $U(1)$ factor and a non-trivial extra-dimensional topography that can explain the CP asymmetry in $B \rightarrow \phi K_S$, while suppressing other supersymmetric flavor changing effects through a high degree of squark-quark alignment.

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I. INTRODUCTION

In the standard model (SM), the direct decay amplitudes for $B \rightarrow J/\psi K_S$ and $B \rightarrow \phi K_S$ have approximately no weak phases in the Wolfenstein parametrization. Thus, the CP asymmetry in both decays is due to the weak phase in $B - \bar{B}$ mixing and hence is determined by $\sin 2\beta$, where β is an angle of the unitarity triangle.¹ Recently, the asymmetric B -factories, BABAR and Belle, announced that $\sin 2\beta$ extracted from CP asymmetry in $B \rightarrow \phi K_S$ decays [assuming the Cabibbo-Kobayashi-Maskawa (CKM) paradigm for CP violation] is smaller by 2.7σ than the measurement using $B \rightarrow J/\psi K_S$ decays—the world averages are -0.39 ± 0.41 and 0.734 ± 0.054 , respectively [3]. The latter measurement of $\sin 2\beta$ is consistent with the result of a global fit to other data.

If this discrepancy persists, then it could be a signal of new physics. While the decay $B \rightarrow J/\psi K_S$ occurs at tree level in the SM, the amplitude for $B \rightarrow \phi K_S$ arises at one-loop (since it involves a $b \rightarrow s s \bar{s}$ transition). One therefore expects that new physics is more likely to affect the latter decay mode. Also, a new physics amplitude in $B - \bar{B}$ mixing cannot account for the anomaly since it will not change the difference of the two CP asymmetries. Hence, we require a new physics contribution to the direct $B \rightarrow \phi K_S$ decay amplitude. This new physics amplitude (a) must be comparable to the SM amplitude and (b) have a new $\mathcal{O}(1)CP$ violating phase, since the experimental CP asymmetry is ~ 0 , while

the SM prediction is $\sim \mathcal{O}(1)$.²

Supersymmetric extensions of the SM have new sources of flavor and CP violation and hence are good candidates for an explanation of this anomaly. Consider the soft supersymmetry (SUSY) breaking masses for the down-type squarks,

$$\begin{aligned} \mathcal{L}_{soft} \ni & \tilde{d}_R^\dagger (\tilde{M}_d^2)_{RR}^{gauge} \tilde{d}_R + \tilde{d}_L^\dagger (\tilde{M}_d^2)_{LL}^{gauge} \tilde{d}_L \\ & + [\tilde{d}_L^\dagger (\tilde{M}_d^2)_{LR}^{gauge} \tilde{d}_R + \text{H.c.}] \end{aligned} \quad (1.1)$$

One may find the unitary transformations U_{D_R} and U_{D_L} that diagonalize the fermion Yukawa matrices,

$$d_i^{gauge} = (U_D)_{ij} d_j^{mass}, \quad (1.2)$$

and apply these transformations to the squarks so that the quark-squark-gluino couplings are flavor diagonal. This defines the super-CKM basis a mass eigenstate basis for quarks in which the gluino couplings are also diagonal. The down squark mass matrices in the super-CKM basis are given by

$$(\tilde{M}_d^2)_{AB}^{super-CKM} = U_{D_A}^\dagger (\tilde{M}_d^2)_{AB}^{gauge} U_{D_B} \quad (1.3)$$

where the subscripts AB represent either LL , LR or RR . In general, the squark mass matrices in the super-CKM basis are *not* diagonal, and provide an origin for flavor changing effects in the low-energy theory.

Flavor violation can be attributed to the “insertion” of an off-diagonal element of one of these matrices (which converts a squark of one flavor to that of another) in a squark propagator. If all the squark masses are of the same order and the off-diagonal elements in these squark mass matrices are small compared to the diagonal elements, then this “mass

²For recent model-independent studies of new physics effects in this decay, see Refs. [4,5].

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¹In the SM, the deviation of the CP asymmetry from $\sin 2\beta$ in $B \rightarrow \phi K_S$ (due to a u -quark penguin) is $\mathcal{O}(\lambda^2) \sim 5\%$, where $\lambda \sim 0.22$ is the Cabibbo angle (see, for example, Ref. [1] and, for a recent study, Ref. [2]); the deviation in the case of $B \rightarrow J/\psi K_S$ is smaller, $\mathcal{O}(\lambda^2) \times$ ratio of penguin to tree amplitudes.

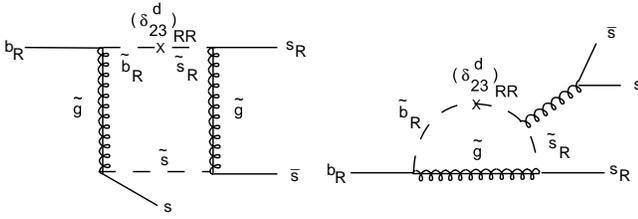


FIG. 1. Typical direct supersymmetric contributions to $b \rightarrow s\bar{s}$ transitions in the mass insertion approximation.

insertion" approximation is accurate for computing SUSY contributions to flavor changing neutral currents (FCNC's). In this case, convenient measures of flavor violation are given by the parameters [6]

$$\delta_{ij} \equiv (\tilde{M}_d^2)_{ij}^{super-CKM} / (\tilde{M}_d^2)_{ii}^{super-CKM}. \quad (1.4)$$

Possible Feynman diagrams contributing directly to $b \rightarrow s\bar{s}$ transitions are shown in Fig. 1. It has been shown that flavor violation (in gluino-mediated Feynman diagrams) can explain the anomaly while satisfying the constraints from $b \rightarrow s\gamma$; correlation with $B_s - \bar{B}_s$ mixing has also been discussed. [7–10].³ Since the new physics amplitude has to be comparable to the SM amplitude to account for $B \rightarrow \phi K_S$ anomaly, it is clear that we need large flavor violation,⁴ e.g., $\delta_{23}^d \sim 1$ with $\mathcal{O}(1)$ phase.⁵

In this paper we explore *correlations* between a large SUSY contribution to $B \rightarrow \phi K_S$ and the SUSY contributions to other FCNC's. In a model independent analysis, the various δ 's can be treated as independent parameters and the stringent bounds from, for example, $K - \bar{K}$ and $B - \bar{B}$ mixing, may be satisfied by assumption. While this approach is important as a first step, it is somewhat *ad hoc*; the nontrivial flavor structure required of the theory strongly hints at some underlying organizing principle. The use of flavor symmetries and effective field theory presents a well-motivated and consistent framework in which the question of FCNC correlations may be addressed. Flavor symmetries explain/postdict the hierarchy of fermion masses and mixings while simultaneously constraining the flavor structure of soft SUSY breaking masses. This in turn yields information on the pattern of SUSY contributions to FCNC's.

While it is impossible to explore the space of possible models in its entirety, we can focus on the typical properties of successful models and let minimality guide us in finding

³For an earlier study of these effects, see Ref. [11]. For other SUSY and non-SUSY explanations, see Ref. [12].

⁴The SM amplitude has a CKM suppression (V_{ts}) which roughly matches the suppression of the SUSY amplitude due to the fact that the superpartner masses are larger than the W , top quark masses.

⁵Strictly speaking, the mass insertion approximation is no longer valid in the presence of such large flavor violation. Instead, we should use the mass eigenstate basis for both quarks and squarks. This point has been emphasized in the present context in Ref. [9]. Since we are not concerned with a detailed numerical analysis in this paper, it will suffice to use the mass insertion approximation.

specific examples. In a wide range of flavor models, one does not expect large contributions to $B \rightarrow \phi K_S$ from $(\delta_{23}^d)_{LL}$ and $(\delta_{23}^d)_{LR}$. One might anticipate $(\delta_{23}^d)_{LL} \sim \lambda^2$, where $\lambda \sim 0.22$ is the Cabibbo angle, since the left-handed quark mixings are of that order. Moreover, the $(\tilde{M}_d^2)_{LR}^{gauge}$ terms and the Yukawa couplings transform identically under flavor symmetries and are often "aligned," i.e., are diagonalized by the same unitary transformations, so that the δ_{LR} 's are suppressed. We therefore focus on the possibility that the $B \rightarrow \phi K_S$ anomaly is explained by a large imaginary part of $(\delta_{23}^d)_{RR}$, the most likely solution if $\tan\beta$, the ratio of Higgs vacuum expectation values (VEVs), is order one. (We do not consider the case of large $\tan\beta$ in this paper.) It is worth noting that the preferred regions of supersymmetry parameter space that follow from the analyses in Refs. [7–10] depend crucially on the renormalization of the Wilson coefficients in the $\Delta B = 1$ effective Hamiltonian. The results are obtained numerically, cover only parts of the multidimensional supersymmetry parameter space, and cannot be summarized usefully in any simple analytic approximation. We therefore use the numerical results in Figs. 3 through 7 of Ref. [9] as a basis for our investigation of more explicit supersymmetric models. In Sec. II we study the properties of minimal mass matrix textures that lead to $(\delta_{23}^d)_{RR} \sim 1$, and determine the correlation with other δ parameters. In Sec. III we present explicit flavor models that approximate these textures.

It is worth pointing out that our analysis is of interest even if the current anomaly in $B \rightarrow \phi K_S$ turns out to be a statistical fluctuation. One possibility is that the SUSY contribution in all $b \rightarrow s$ transitions is large, but *CP-conserving* so that the *CP* asymmetry in $B \rightarrow \phi K_S$ is not affected. In this case, anticipated improvements in the measurement of $B_s - \bar{B}_s$ mixing have the potential of revealing the SUSY contribution. Another possibility is that the SUSY contribution is *CP-violating*, but is *small* in the amplitude for the decay $B \rightarrow \phi K_S$ and *large* in the amplitude for $b \rightarrow s\gamma$ or in $B_s - \bar{B}_s$ mixing. This will result in a direct *CP* asymmetry in $B \rightarrow X_s \gamma$ or a mixing-induced, time-dependent *CP* asymmetry in $B_s \rightarrow J/\psi \phi$, which potentially can be detected in ongoing or future experiments. Whether such effects are consistent with the tight bounds on $s \rightarrow d$ and $b \rightarrow d$ transitions in realistic models is also addressed in our approach.

II. TEXTURE ANALYSIS

In order to obtain a large value for $(\delta_{23}^d)_{RR}$, one must either have (a) squark nondegeneracy and a large 2-3 rotation on right-handed (RH) fields from down-quark Yukawa diagonalization, or (b) large off-diagonal terms in the squark mass matrix in the gauge basis. We consider these possibilities in turn. Of course, combinations of (a) and (b) are also possible, as we will see in Sec. III.

Large RH 23 rotation in down quark Yukawa matrix

For simplicity, we assume that in the gauge basis the off-diagonal terms in the RH down squark mass matrix are small,

$$(\tilde{M}_d^2)_{RR}^{gauge} \approx \begin{pmatrix} m_{d_R}^2 & 0 & 0 \\ 0 & m_{s_R}^2 & 0 \\ 0 & 0 & m_{b_R}^2 \end{pmatrix}, \quad (2.1)$$

so that $(\delta_{23}^d)_{RR} \sim 1$ is obtained only from large rotation between strange and bottom quarks. Such a large mixing might be motivated in grand unified theories as follows [13]: it is possible that the large $\nu_\mu - \nu_\tau$ mixing required to explain the atmospheric neutrino anomaly originates in the left-handed (LH) charged lepton mass matrix. Since right-handed down-type quarks and left-handed leptons are part of the same multiplet, large $b_R - s_R$ mixing is an immediate consequence.

The ‘‘minimal’’ quark texture which gives the desired phenomenology can be obtained as follows. We require $\mathcal{O}(1)$ rotation in the 23 sector for RH down-type quarks (with CP violation) to explain the $B \rightarrow \phi K_S$ anomaly. Also, the size of bottom and strange quark Yukawa couplings renormalized at a high scale are $\mathcal{O}(\lambda^2)$ and $\mathcal{O}(\lambda^4)$, respectively. Thus, the ‘‘minimal’’ quark texture required in the 23 block of the down quark Yukawa matrix is

$$(Y_d)_{2-3 \text{ block}} \sim \begin{pmatrix} \lambda^4 & \lambda^4 \\ \lambda^2 \exp(i\phi) & \lambda^2 \end{pmatrix}. \quad (2.2)$$

Here, we use the notation $\mathcal{L} \ni Y_{dij} \bar{d}_{Li} d_{Rj}$, where d_L and d_R denote the left-handed and right-handed down-type quarks (i, j are generation indices). Also, unless stated otherwise, there are unknown $\mathcal{O}(1)$ fluctuations in the coefficients of all elements of Yukawa/mass matrices. Neglecting the first generation entries, which are typically small, the phase in (3,2) element can be ‘‘transferred’’ to the (2,3) element by field redefinitions so that the (2,3) element has to be $\mathcal{O}(\lambda^4)$ to get the maximum CP violation. The (2,3) element cannot be larger, or it would give $V_{cb} > \lambda^2$.

In order to study the implications of the large 2-3 RH rotation in Y_d for other FCNC’s, we need the texture in the 1-2 block of Y_d , which depends on whether the Cabibbo angle originates in Y_d or in Y_u . We analyze these cases in turn.

Cabibbo angle from Y_d

In this case, we have $(U_{D_L})_{12,21} \sim \lambda$ from the 12 rotation among the LH down-type quarks. If $m_{d_L}^2 \neq m_{s_L}^2$, then it follows from Eqs. (1.3) and (1.4) that $(\delta_{12}^d)_{LL} \sim \lambda$. This is ruled out by the $K - \bar{K}$ mixing bound $(\delta_{12}^d)_{LL} \leq 0.1 - 0.04$, for squark masses ≤ 1 TeV [6]. Hence, to evade this bound, we need approximate degeneracy between first and second generation squarks. Models based on non-Abelian flavor symmetries with a $\mathbf{2} + \mathbf{1}$ representation structure for fields of the three generations have this feature. The simplest models of this type [for example, based on $U(2)$ symmetry [14,15] or one of its discrete subgroups [16]] also lead to the decomposition $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$, where the $\mathbf{3}$ and $\mathbf{1}$ correspond to two-by-two symmetric and antisymmetric matrices. As a result, the symmetry breaking entries in the 1-2 block of Y_d tend to

appear in a symmetric or antisymmetric pattern. Motivated by the minimality of these models and their success in maintaining the squark degeneracy that we require given our present assumptions, we combine a $U(2)$ -like texture in the 1-2 sector with Eq. (2.2):

$$Y_d \sim \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ 0 & \lambda^2 \exp(i\phi) & \lambda^2 \end{pmatrix}. \quad (2.3)$$

Generally, the (1,2) and (2,1) entries have contributions from symmetry breaking fields, flavons, that transform as symmetric or antisymmetric tensors under the flavor group. Thus, unless symmetric and antisymmetric flavon VEVs are fine tuned, the (1,2) and (2,1) entries will be of the same size, namely $\mathcal{O}(\lambda^5)$. These entries provide an origin for the Cabibbo angle, and also yield a down quark Yukawa coupling of the right order, namely λ^6 .

To proceed further in correlating the large 2-3 rotation with other FCNC’s, we obtain the form of U_{D_R} . In general, it is difficult to analytically diagonalize 3×3 matrices. However, if the entries in the Yukawa matrices have a hierarchical structure, then the matrices can be diagonalized perturbatively by three successive rotations in the 2-3, 1-3 and 1-2 subspaces (see, for example, Ref. [17]). Denoting the sines of the rotation angles by $s_{23}^{R(L)}$, $s_{13}^{R(L)}$ and $s_{12}^{R(L)}$ for the RH (LH) quarks, we have

$$Y_d^{diagonal} = (U_{D_L})^\dagger Y_D U_{D_R}, \quad (2.4)$$

where

$$U_{D_R} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & s_{23}^R \\ 0 & -s_{23}^R & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & s_{13}^R \\ 0 & 1 & 0 \\ -s_{13}^R & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & s_{12}^R & 0 \\ -s_{12}^R & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.5)$$

A similar decomposition is valid for U_{D_L} . Given the texture in Eq. (2.3), it is not hard to see that $s_{12}^R \sim \mathcal{O}(\lambda)$, $s_{23}^R \sim \mathcal{O}(1)$ and that s_{13}^R is very small.⁶ Expanding the RHS of Eq. (2.5), we get $(U_{D_R})_{12} \sim s_{12}^R$, $(U_{D_R})_{21} \sim s_{12}^R + s_{13}^R s_{23}^R$, $(U_{D_R})_{23} \sim s_{23}^R$ and $(U_{D_R})_{32} \sim s_{23}^R + s_{13}^R s_{12}^R$ so that $(U_{D_R})_{12,21} \sim \lambda$ and $(U_{D_R})_{23,32} \sim \mathcal{O}(1)$, as one might expect. What is more interesting is the observation that

$$(U_{D_R})_{31} = -s_{13}^R + s_{23}^R s_{12}^R, \quad (2.6)$$

⁶Strictly speaking, although perturbative diagonalization cannot be used in this case since 2-3 mixing is $\mathcal{O}(1)$, it suffices for our semiquantitative analysis.

which is enhanced by the large 2-3 mixing. Using the above values of the 3 rotations, we see that perturbative diagonalization gives $(U_{D_R})_{31} \sim \lambda$. We have confirmed this expectation by a numerical analysis, which also shows that $(U_{D_R})_{31}$ has a phase.⁷ Thus, we obtain the following significant result: $(U_{D_R})_{31} \sim \mathcal{O}(\lambda)$ is a generic consequence of the minimal texture in Eq. (2.3), even though the 1-3 rotation defined by Eq. (2.5) is small.

Let us now estimate the relevant δ parameters. In non-Abelian models with a $2 \oplus 1$ representation structure, one expects that $m_{\tilde{d}_R}^- \approx m_{\tilde{s}_R}^-$ while $m_{\tilde{d}_R}^-$ and $m_{\tilde{b}_R}^-$ are of the same order but not degenerate. Then, using $(U_{D_R})_{32,33} \sim 1$ and $(U_{D_R})_{31} \sim \lambda$ in Eq. (1.3), we find $(\delta_{12}^d)_{RR} \sim \lambda$ (with no phase) and $(\delta_{13}^d)_{RR} \sim \lambda \exp(i\phi)$. On the other hand, the limits from $K-\bar{K}$ and $B-\bar{B}$ mixing require $\text{Re}(\delta_{12}^d)_{RR} \lesssim 0.04$ and $\text{Re}(\delta_{13}^d)_{RR} \lesssim 0.1$, respectively, for squark and gluino masses ≈ 500 GeV [6]. Thus, the SUSY contribution to Δm_K is too large with this texture, but there is no SUSY contribution to ϵ_K . The SUSY contribution to Δm_B is borderline (i.e., comparable to the experimental value) and has an $\mathcal{O}(1)$ phase. Thus, the time-dependent CP asymmetry in $B \rightarrow J/\psi K_S$ measures a combination of angle β and the new phase ϕ .

Clearly, our theory must be modified if the contribution to $K-\bar{K}$ mixing is to be sufficiently suppressed. One could entertain the possibility that the squarks of the first two generations are much heavier than the third; a squark mass spectrum of the form $m_{\tilde{d},\tilde{s}_R}^- \gtrsim 1$ TeV and $m_{\tilde{b}_R}^-$ a few 100 GeV can still explain the $B \rightarrow \phi K_S$ anomaly [9]. To study this case, one must work in the mass eigenstate basis for both the quarks and squarks. Given the assumed form of Eq. (2.1), the gauge and mass bases for the squarks are the same; the quark-squark-gluino coupling in the mass basis is then given by

$$\tilde{g}(\tilde{d}_i^{mass})^\dagger (U_{D_R})_{ij} d_j^{mass}. \quad (2.7)$$

Dominant contributions to FCNC's come from Feynman diagrams involving the exchange of the light \tilde{b}_R . From Eq. (2.7), we see that its couplings to d and s quarks are given by $(U_{D_R})_{31} \sim \lambda$ (with a phase) and $(U_{D_R})_{32} \sim 1$ (with a phase), respectively. Box diagrams involving the exchange of \tilde{b}_R and a gluino generate a CP -conserving, $\Delta S=2$ 4-fermion operator with coefficient $\propto \lambda^2/m_{\tilde{b}_R}^2$, and a $\Delta B=2$ 4-fermion operator with a coefficient of the same order, but with a phase. It is easy to see that this is equivalent to having $(\delta_{12,13}^d)_{RR} \sim \lambda$ and thus gives too large a contribution to Δm_K and a large CP -violating contribution to $B-\bar{B}$ mixing, just as before.

If one, on the other hand, tries to make *all* the quarks heavier than a TeV, than one must take the gluino mass to be

relatively light (a few 100 GeV) to affect $B \rightarrow \phi K_S$ (see Ref. [9]). Thus, $x \equiv m_g^2/m^2 \tilde{q} \ll 1$. Heavy squarks relax the bounds on δ 's, whereas small x tightens the bounds. As a result, the limits on δ 's stay about the same [6]. Thus, the large contributions to $K-\bar{K}$ and $B-\bar{B}$ are not alleviated if we are also to explain the anomaly of interest.

These considerations lead us to the conclusion that we must suppress the (2,1) entry of Eq. (2.3). For U(2)-like textures, with symmetric and antisymmetric contributions to the upper two-by-two block of Y_d , this suggests a mild fine tuning is required. We will construct an explicit model based on this solution in Sec. III.

Cabibbo angle from Y_u

For models in which the Cabibbo angle originates in Y_u , we can choose a minimal block diagonal form for Y_d :

$$Y_d \sim \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^4 & \lambda^4 \\ 0 & \lambda^2 \exp(i\phi) & \lambda^2 \end{pmatrix}. \quad (2.8)$$

To generate the Cabibbo angle, we must introduce a (1,2) entry of the appropriate size in Y_u :

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & 0 \\ 0 & \lambda^4 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.9)$$

Note that the eigenvalues of Y_u are in the correct ratio for the up-type quarks, namely, $1::\lambda^4::\lambda^8$. If we were to follow the same non-Abelian ansatz described earlier, we would also expect an $\mathcal{O}(\lambda^5)$ (2,1) entry in Y_u . However, this would lead to an up quark Yukawa coupling of order λ^6 , about a factor of 25 too large. The fine tuning required to fix this problem is much greater than what we needed to adequately suppress the flavor changing problems in our previous example. So it is more natural in this case to assume that the (2,1) entry in Y_u is absent, and to discard our assumption that the underlying theory is non-Abelian;⁸ unwanted SUSY FCNC's must then be suppressed by a suitable quark-squark alignment. We will show in Sec. III that this can be achieved in Abelian models with multiple U(1) factors (à la Nir-Seiberg [19]), or in a new class of models involving fewer U(1)'s and a nontrivial extra-dimensional topography.

From the texture of Y_d and the form of the squark mass matrix in Eq. (2.1), we see that the first generation is decoupled from the second and third in the down sector. Thus, there are no SUSY contributions to $K-\bar{K}$ and $B-\bar{B}$ mixing, even if the three down-type squarks are not degenerate.

⁷We work in the basis where the quark masses are real and the CKM matrix, i.e., $U_{U_L}^\dagger U_{D_L}$ has the standard form given in the Review of Particle Physics [18].

⁸Of course, the underlying symmetry could still be non-Abelian but of a more complicated form than we have assumed, e.g., products of simple non-Abelian factors. We do not consider this possibility here.

However, since $(U_{U_L})_{12} \sim \lambda$, some mild degeneracy between $m_{\tilde{u}_L}$ and $m_{\tilde{c}_L}$ may be required to satisfy the bounds on D - \bar{D} mixing.

B. Large 2-3 mixing in RH down type squark mass matrix

In this case, the ‘‘minimal’’ texture for the down-type squark mass matrix in the gauge basis is

$$(\tilde{M}_d^2)_{RR}^{gauge} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \exp(i\phi) \\ 0 & \exp(-i\phi) & 1 \end{pmatrix}, \quad (2.10)$$

where, for simplicity, we have assumed that the off-diagonal elements involving down squarks are small. In the discussion that follows immediately below, we assume that large 2-3 mixing is present only in the supersymmetry breaking sector of the theory. However, it should be pointed out that in most realistic models, the form of Eq. (2.10) suggests that the entire 2-3 block is flavor group invariant, and that s_R and b_R have the same flavor charges. In this case, one expects large 2-3 mixing from both the Yukawa and soft mass matrices, as illustrated in the first model of Sec. III.

If the Cabibbo angle comes from the down sector, then as before we need degeneracy of $m_{\tilde{u}_L}$ and $m_{\tilde{s}_L}$ and hence a non-Abelian flavor symmetry. Given our previous assumptions on the symmetry or antisymmetry of the upper two-by-two block of Y_d , we expect generically that $(U_{D_R})_{12,21} \sim \lambda$. However, unlike our earlier example, $(U_{D_R})_{31}$ can be very small [see Eq. (2.6)] since we no longer need an $\mathcal{O}(1)$ 2-3 RH rotation in the quark sector. Using Eq. (1.3) and $(U_{D_R})_{12,21} \sim \lambda$, we get

$$[(\tilde{M}_d^2)_{RR}^{super-CKM}]_{13} \sim \lambda \times [(\tilde{M}_d^2)_{RR}^{gauge}]_{23}, \quad (2.11)$$

resulting in $(\delta_{13}^d)_{RR} \sim \lambda$ (with a phase), and large B - \bar{B} mixing as before. Since $(U_{D_R})_{31}$ and $(U_{D_R})_{32}$ can be small and $m_{\tilde{u}_R} \approx m_{\tilde{s}_R}$ is implied by the non-Abelian flavor symmetry, we can check [using Eq. (1.3)] that $(\delta_{12}^d)_{RR}$ and hence K - \bar{K} mixing can be small, unlike our earlier example.

If the Cabibbo angle originates in the up-quark sector, then the off-diagonal entries of Y_d involving the down quark can be small or zero. Assuming further that the down squark mass matrix is as in Eq. (2.10), we see that the down quark is decoupled from the bottom and strange quarks so that there are no SUSY contributions to K - \bar{K} and B - \bar{B} mixing.

III. MODELS

In this section we will consider simple Abelian and non-Abelian models that can account for the discrepancy in the measured value of $\sin 2\beta$. Our Abelian model will realize the Yukawa textures in Eqs. (2.8) and (2.9) by construction, just as the well-known alignment models of Ref. [19]. However, unlike these models, we rely on localization in extra dimensions rather than holomorphy to achieve an appropriate quark-squark alignment for the lighter generations. This al-

lows us to achieve a viable phenomenology using only one flavor U(1) factor. An interesting feature of our model is that texture zeros can occur in non-holomorphic terms, such as the soft scalar masses, as a consequence of extra-dimensional locality. This leads to somewhat different scalar mass matrix textures exact zero entries for the off-diagonal elements involving the downsquark, where they otherwise would be finite but small. In our model and those of Ref. [19], $(\delta_{23}^d)_{RR}$ is due to a large RH 2-3 rotation in the down quark Yukawa matrix and large 2-3 mixing in the down squark mass matrix. In the non-Abelian example, we will show how a U(2) flavor model may be modified to provide for the desired b - s mixing; large K - \bar{K} mixing can be avoided provided that a mild fine tuning is allowed.

A. An extra-dimensional Abelian model

In this model we assume the horizontal flavor symmetry $G_f = U(1)$; the particle content is that of the minimal supersymmetric standard model with one additional field ϕ that has U(1) charge +1. We assume that the ratio of the VEV of ϕ to the ultraviolet cutoff of the effective theory, M_f , is approximately given by the Cabibbo angle,

$$\frac{\langle \phi \rangle}{M_f} = \lambda \approx 0.22. \quad (3.1)$$

We assume that M_f is generically well below the fundamental gravitational scale M_* and that M_f sets the mass scale for all the operators contributing to the flavor structure of the theory. The U(1) charge assignments of the matter fields are given as follows:

$$\begin{aligned} E \sim Q &\sim (-3, -2, 0), & L \sim D &\sim (-3 - 2 - 2), \\ U \sim &(-5, -2, 0), & H_U, H_D &\sim 0. \end{aligned} \quad (3.2)$$

Yukawa textures originate as higher-dimensional operators involving the MSSM matter fields and powers of $\langle \phi \rangle / M_f$. The model described thus far is not viable since the Cabibbo angle receives an unsuppressed contribution from the diagonalization of the down quark Yukawa matrix (as one can verify from the explicit form for Y_d), leading to $(\delta_{12}^d)_{LL} \sim \lambda$; this exceeds the bounds from K^0 - \bar{K}^0 mixing and ϵ_K if superparticle masses are a few hundred GeV.

We now consider a possible non-trivial, extra-dimensional topography for the model. We assume that there are two extra spatial dimensions, compactified on the orbifold $(S^1/Z_2)^2$. We take the compactification radii to be the same, namely R , so that fixed points exist at $(0,0)$, $(0,\pi R)$, $(\pi R,0)$ and $(\pi R, \pi R)$. The space can be described as a rectangle, with fields confined either to corners (fixed points of both S^1/Z_2 factors), sides (fixed point of one S^1/Z_2 factor), or defined everywhere (the six-dimensional bulk). We choose to localize our fields as shown in Fig. 2. In addition, we assume that the unknown, high-energy dynamics that is responsible for generating M_f -suppressed operators is localized at fixed points of both S^1/Z_2 factors.

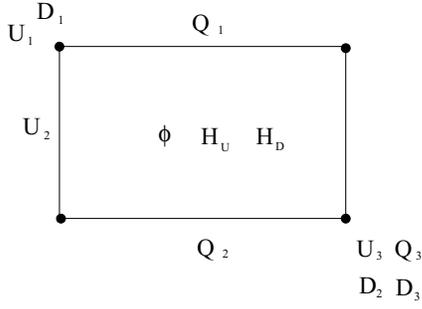


FIG. 2. Extra-dimensional topography for the Abelian model. Corners represent fixed points, sides represent fixed lines, and the interior represents the entire bulk.

Given this construction, the separation of fields at isolated fixed points prevents Yukawa couplings involving these fields in the four-dimensional theory, after dimensional reduction. For the couplings that do remain, one must generally take into account volume suppression factors (i.e., powers of the ratio R^{-1}/M_f) that, to varying degrees, suppress the effective four-dimensional interactions. In our model we will assume that $R^{-1} \sim M_f$ so that these factors are $\mathcal{O}(1)$ and do not alter the analysis. Moreover, we will take R^{-1} to be sufficiently large so that only the physics of the zero modes of the MSSM matter fields will be relevant to the low-energy phenomenology. Finally, a minor technical point: for MSSM matter fields that are confined to 5D subspaces of the 6D bulk, we allow for the introduction of chiral-conjugate mirror fields with opposite parity under the appropriate Z_2 factor so that chiral zero modes can be obtained.

Given the constraints of symmetry and geometry, we find the following textures for the Yukawa matrices:

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & 0 \\ 0 & \lambda^4 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^4 & \lambda^4 \\ 0 & \lambda^2 & \lambda^2 \end{pmatrix}. \quad (3.3)$$

While these are the same as in Ref. [19] (by construction), the soft mass squared matrices are somewhat different:

$$m_Q^2 \sim m^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad m_D^2 \sim m^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

$$m_U^2 \sim m^2 \begin{pmatrix} 1 & \lambda^3 & 0 \\ \lambda^3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3.4)$$

As far as strangeness changing processes are concerned, this model yields a remarkable level of quark-squark alignment; notice that there are no 1-2 down quark rotations (on either left- or right-handed fields), and no 1-2 entries in either m_Q^2 or m_D^2 . The model provides the desired $\mathcal{O}(1)$ 2-3 right-handed down-quark mixing with an irremovable phase (as well as large 2-3 mixing in m_D^2), and like other models of this type predicts $D^0 - \bar{D}^0$ mixing near the experimental limit. It is

worth noting that a choice of M_f closer to M_* may lead to exponentially suppressed, though non-negligible, interactions between fields at isolated fixed points, due to the exchange of string states. Such effects are not relevant here given our choice of scales, but could be of interest in similar flavor models.

B. A non-Abelian example

Non-Abelian flavor symmetries resolve the supersymmetric flavor problem by maintaining a sufficient degree of degeneracy among squarks of the first two generations. Theories of flavor based on $U(2)$ symmetry assume a $2+1$ representation structure for the three generations, and a two-stage symmetry breaking

$$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} \text{nothing}, \quad (3.5)$$

where the $U(1)$ factor represents phase rotations on fields of the first generation, and where ϵ and ϵ' are small parameters defined in analogy to Eq. (3.1). For a detailed discussion of models based on $U(2)$ symmetry and its discrete subgroups we refer the reader to Refs. [14–16]. While conventional $U(2)$ models provide an elegant theory of fermion masses, they do not generally lead to the large 2-3 down quark mixing of interest to us here. In this section we present a minimal modification that yields the desired result while preserving most of the desirable feature of the original $U(2)$ models.

We assume the flavor group $G_f = U(2) \times U(1)_F$, with the three generations of matter fields again embedded in $2+1$ representations of the $U(2)$ factor, and the symmetry breaking pattern given in Eq. (3.5). The additional $U(1)_F$ factor is assumed to break at the first of the two symmetry-breaking scales in Eq. (3.5), and is therefore associated with a small parameter of size ϵ . Conventional $U(2)$ models involve singlet, doublet and triplet flavon fields A^{ab} , ϕ^a and S^{ab} , that obtain a pattern of VEVs that are consistent with Eq. (3.5). In our model, we make the unconventional choices $\epsilon \sim \lambda^2$, $\epsilon' \sim \lambda^6$, and

$$\frac{\langle A \rangle}{M_f} \sim \begin{pmatrix} 0 & \lambda^6 \\ -\lambda^6 & 0 \end{pmatrix}, \quad \frac{\langle \phi \rangle}{M_f} \sim \begin{pmatrix} 0 \\ \lambda^2 \end{pmatrix},$$

$$\frac{\langle S \rangle}{M_f} \sim \begin{pmatrix} \lambda^6 & \lambda^6 \\ \lambda^6 & 0 \end{pmatrix}. \quad (3.6)$$

Unlike the usual $U(2)$ model, the first generation fermions get their Yukawa eigenvalues primarily from the (1,1) entry of the symmetric flavon rather than the off diagonals. Note that the λ^6 entries in the symmetric flavon are completely consistent with Eq. (3.5); in the effective theory below the first symmetry breaking scale, these entries correspond to fields with differing charges under the intermediate $U(1)$ symmetry acquiring order ϵ' VEVs. All the entries are consistent with the dynamical assumption that a given flavon will either get a VEV of the same size as a symmetry-breaking scale, or no VEV at all. In addition, we introduce

the following additional flavons with a nontrivial $U(1)_F$ charge, indicated by a subscript:

$$\frac{\langle \phi_+ \rangle}{M_f} \sim \begin{pmatrix} 0 \\ \lambda^2 \end{pmatrix}, \quad \frac{\langle \sigma_- \rangle}{M_f} \sim \lambda^2, \quad \frac{\langle \sigma_+ \rangle}{M_f} \sim \lambda^2. \quad (3.7)$$

Complete $U(2)$ representation of the matter fields can be assigned different $U(1)_F$ charges. We assign a $U(1)_F$ charge of -1 to the right-handed up-quark superfields of the first two generations and $+1$ to the right-handed down quark superfield of the third generation.⁹ We then obtain the Yukawa textures

$$Y_u \sim \begin{pmatrix} \lambda^8 & \lambda^8 & 0 \\ \lambda^8 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \lambda^6 & 2.5\lambda^6 & 0 \\ 0.5\lambda^6 & 0.5\lambda^4 & \lambda^4 \\ 0 & \lambda^2 & \lambda^2 \end{pmatrix}, \quad (3.8)$$

and the soft scalar masses

$$m_Q^2 \sim m^2 \begin{pmatrix} c_0 + \lambda^{12} & \lambda^{10} & \lambda^8 \\ \lambda^{10} & c_0 + \lambda^4 & \lambda^2 \\ \lambda^8 & \lambda^2 & c_3 \end{pmatrix},$$

$$m_U^2 \sim m_D^2 \sim \begin{pmatrix} c_0 + \lambda^{12} & \lambda^{10} & \lambda^{10} \\ \lambda^{10} & c_0 + \lambda^4 & \lambda^2 \\ \lambda^{10} & \lambda^2 & c_3 \end{pmatrix}. \quad (3.9)$$

Here, since the first two generation squarks are degenerate, we have explicitly shown the $\mathcal{O}(1)$ coefficients in the diagonal entries (the c_0, c_3 's are different for Q, U, D). Also, while we generally show only the order in λ of a given Yukawa entry, we have displayed some of the order one coefficients in Y_d . This is to illustrate that with a mild, $\mathcal{O}(\lambda)$ fine tuning between $\langle A \rangle$ and $\langle S \rangle$ we can obtain the Cabibbo angle from the down quark Yukawa matrix while sufficiently suppressing its 21 entry, thus avoiding the flavor changing problems discussed in Sec. II.

Finally it is worth pointing out that the similarity between the down quark and charged lepton mass hierarchies suggests that under the flavor group $L \sim D$ and $E \sim Q$ or $L \sim Q$ and $E \sim D$, where \sim indicates identical flavor charge assignments. Detailed differences in the mass spectrum can be accommodated using the freedom to adjust order one coefficients. The first (second) choice implies large LH (RH) 2-3 mixing in the charged lepton sector, independent of whether the theory has any grand unified embedding. The bi-large neutrino mixing that is favored by the current data must therefore partly originate from the neutral lepton sector. While we do not explicitly investigate this issue here, it is worth pointing out that Aranda, Carone and Meade [16] have shown that bi-large neutrino mixing can arise entirely from

⁹We follow the usual convention that all matter fields are embedded into left-handed chiral superfields. Our charge assignments apply to these fields.

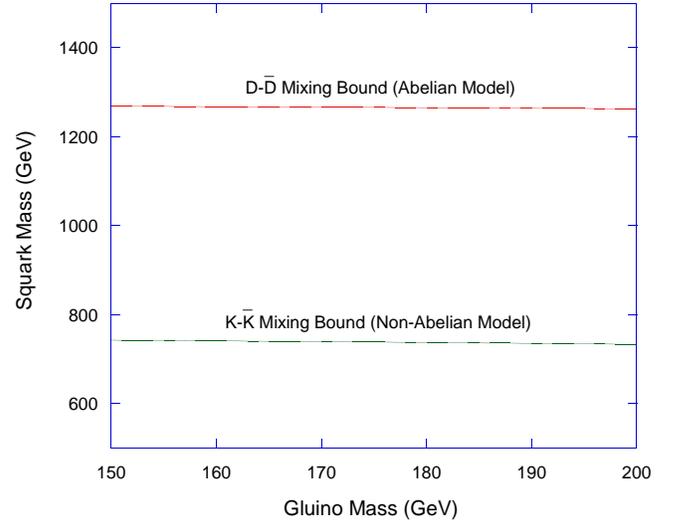


FIG. 3. Typical bounds on the squark masses of the light two generations, assuming that unspecified order one coefficients are exactly 1, and the gluino is light enough to produce a significant effect in $B \rightarrow \phi K_s$.

the neutral lepton sector in non-Abelian models with a $2+1$ representation structure, as a consequence of the structure of the right-handed neutrino mass matrix and the non-linearity of the seesaw mechanism. Non-Abelian models of the type we discuss here can in principle be generalized along these lines. We refer the interested reader to Ref. [16].

To conclude this section, we point out that the models we have constructed can be tested in future collider experiments assuming that these models are relevant for explaining the $B \rightarrow \phi K_s$ anomaly. Both models involve significant right-handed down squark mixing, which implies that the gluino mass must be relatively light, in the 150–200 GeV mass range, based on the numerical results in Fig. 3 of Ref. [9]. Using the off-diagonal squark masses predicted in each of our models, we may obtain lower bound estimates on the mass of first two generation squarks, from $D\text{-}\bar{D}$ and $K\text{-}\bar{K}$ mixing constraints. These are shown in Fig. 3, assuming a common mass for all squarks of the first two generations. While there is uncertainty in the order one coefficients in such predictions, we can conclude qualitatively that the discovery of very light squarks of the first two generation would disfavor the models discussed in this section.

IV. CONCLUSIONS

We have considered the detailed flavor structure of supersymmetric theories that can give large contributions to $b \rightarrow s$ transitions. We have focused on the possibility that such theories may explain the discrepancy between the value of $\sin 2\beta$ measured in $B \rightarrow \phi K_s$ and $B \rightarrow J/\psi K_S$ decays. With relatively few assumptions, we have isolated minimal, preferred textures for the Yukawa couplings and the soft supersymmetry-breaking masses. In the case where the Cabibbo angle originates in the down quark Yukawa matrix, we argued that the need for squark degeneracy among the first two generations suggests an underlying non-Abelian fla-

vor symmetry with a $2 \oplus 1$ representation structure. However, in a wide class of these models with symmetric 3 and anti-symmetric 1 dimensional representations, the required, large 2-3 mixing is correlated with Cabibbo-like 1-2 and 1-3 right-handed mixing angles. This leads generically to unwanted flavor changing effects, given the requirement that some elements of the superparticle spectrum must be light to contribute non-negligibly to the processes of interest. We argue that non-Abelian models of this type may provide a viable solution to the $\sin 2\beta$ anomaly providing that a mild fine tuning of parameters is allowed. On the other hand, if the Cabibbo angle originates in the up quark Yukawa matrix, the same non-Abelian ansatz leads to a value for the up quark mass that is too large. Barring more complicated non-Abelian constructions, the desired phenomenology seems in this case to be realized more naturally in Abelian models that rely on alignment rather than degeneracy to suppress strangeness-changing neutral currents.

We have presented explicit models that reproduce many of the features of the idealized textures that we have dis-

cussed. We showed how the minimal U(2) model, which normally does not have any large mixing angles, may be modified with the help of an additional U(1) factor to yield textures of the desired form. In addition, we presented a new type of alignment model with the desired properties that is based on a single U(1) factor and a non-trivial extra-dimensional topography for the matter content. If the $B \rightarrow \phi K_S$ anomaly turns out to be real, or if large deviations from SM predictions are seen in other $b \rightarrow s$ transitions, these models represent relatively minimal realizations of the desired flavor structure of the MSSM and provide a framework for investigating the correlation between a variety of flavor changing process in both quark and lepton sectors.

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