Neutrinoless double β decay and effective field theory

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We analyze neutrinoless double β decay ($0\nu\beta\beta$ decay) mediated by heavy particles from the standpoint of effective field theory. We show how symmetries of the $0\nu\beta\beta$-decay quark operators arising in a given particle physics model determine the form of the corresponding effective, hadronic operators. We classify the latter according to their symmetry transformation properties as well as the order at which they appear in a derivative expansion. We apply this framework to several particle physics models, including R-parity violating supersymmetry (RPV SUSY) and the left-right symmetric model (LRSM) with mixing and a right-handed Majorana neutrino. We show that, in general, the pion exchange contributions to $0\nu\beta\beta$ decay dominate over the short-range four-nucleon operators. This confirms previously published RPV SUSY results and allows us to derive new constraints on the masses in the LRSM. In particular, we show how a nonzero mixing angle ζ in the left-right symmetry model produces a new potentially dominant contribution to $0\nu\beta\beta$ decay that substantially modifies previous limits on the masses of the right-handed neutrino and boson stemming from constraints from $0\nu\beta\beta$ decay and vacuum stability requirements.

I. INTRODUCTION

The study of neutrinoless double beta decay ($0\nu\beta\beta$ decay) is an important topic in particle and nuclear physics (for recent reviews, see Refs. [1–3]). The discovery of neutrino oscillations in atmospheric, solar and reactor neutrino experiments proves the existence of a nonvanishing neutrino mass [4–6]. While oscillation experiments provide information on mass-squared differences, they cannot by themselves determine the magnitude of the neutrino masses nor determine if neutrinos are Majorana particles. If the neutrino sector of an "extended" standard model includes massive, Majorana neutrinos, then $0\nu\beta\beta$ decay provides direct information on the Majorana masses. Indeed, since Majorana neutrinos violate lepton number (L), Feynman graphs such as the one depicted in Fig. 1(a) are nonvanishing. In particular, if the e, μ, τ neutrinos have nonvanishing Majorana masses, an analysis of $0\nu\beta\beta$ coupled with data from neutrino oscillations provides limits on the absolute value of these light neutrino masses [7].

Neutrinoless $\beta\beta$ decay can also be a probe for heavy mass scales. For example, in the left-right symmetric model [2,8,9], a heavy right-handed neutrino also contributes to the process; it can even be dominant depending on the values of the elements of the mixing matrix. Thus, $0\nu\beta\beta$ can be a tool for the exploration of energy scales beyond the electroweak symmetry breaking scale. Alternatively, the L-violating interactions responsible for $0\nu\beta\beta$ decay may not involve Majorana neutrinos directly. For example, semileptonic, R parity-

violating (RPV) supersymmetric (SUSY) interactions, involving exchange of charged-lepton superpartners [an example of which is given in Fig. 1(b)] rather than Majorana neutrinos, can give rise to $0\nu\beta\beta$ decay [10–12]. Here again $0\nu\beta\beta$ decay provides a probe of the heavy SUSY mass scale and imposes constraints on RPV SUSY parameters [13]. Furthermore, these alternative scenarios for $0\nu\beta\beta$ decay are relevant for the study of Majorana neutrinos since any $0\nu\beta\beta$-decay mechanism will generate Majorana masses for the neutrinos [14].

The left-right symmetric model and RPV SUSY are but two of a number of models that involve a heavy mass scale $\Lambda_{\beta\beta}$ that characterizes the heavy, L-violating physics. Although the effects of these mechanisms will typically be suppressed by some inverse power of $\Lambda_{\beta\beta}$, $0\nu\beta\beta$ decay mediated by light neutrinos can also be suppressed since the amplitude is proportional to the neutrino effective mass. Thus, it is important to analyze systematically the potentially comparable contributions stemming from L-violating mechanisms mediated by heavy particles. Since $\Lambda_{\beta\beta}$ is far heavier than any hadronic scale that would enter the problem, there exists a clear separation of scales in this case. For the analysis of such situations, effective field theory (EFT) is the tool of choice.

In what follows, we systematically organize the $0\nu\beta\beta$-decay problem using EFT, focusing on L-violation mediated by heavy physics (for other efforts along these lines, see Refs. [15–17]). Since the particle physics dynamics of this heavy physics occur primarily at short-distance,
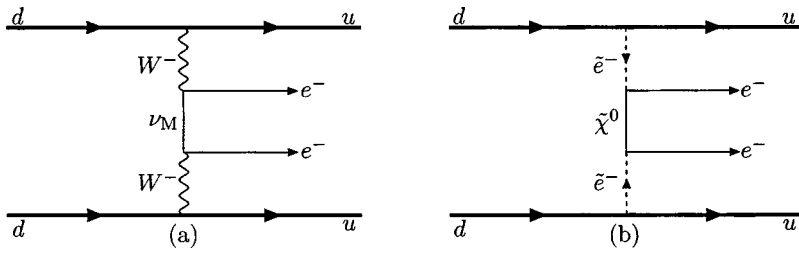
FIG. 1. (a) $0\nu\beta\beta$ through the exchange of a Majorana neutrino. (b) $0\nu\beta\beta$ through the exchange of two selectrons and a neutralino in RPV SUSY.

one may "integrate out" the heavy degrees of freedom, leaving an effective theory of quarks and leptons; these quark-lepton operators in turn generate hadron-lepton operators that have the same transformation properties under various symmetries. In this work, only the lightest quarks are considered, with the relevant symmetries being parity and strong $SU(2)_L \times SU(2)_R$ [chiral $SU(2)$]. The effective hadron-lepton Lagrangian for this theory, $\mathcal{L}_{\text{EFF}}^{0\nu\beta\beta}$, contains an infinite tower of nonrenormalizable operators, which may be systematically classified in powers of p/Λ_H, $p/\Lambda_{\beta\beta}$ and $\Lambda_H/\Lambda_{\beta\beta}$. Here, p denotes any small quantity, such as m_π or the energy of the dilepton pair and $\Lambda_H \sim 1$ GeV is a hadronic mass scale. While the coefficients of the effective operators in $\mathcal{L}_{\text{EFF}}^{0\nu\beta\beta}$ are unknown,[1] the symmetry properties of the underlying short-distance physics may require that certain operator coefficients vanish.

These symmetry properties can have significant consequences for the size of $0\nu\beta\beta$-decay nuclear matrix elements and, thus, for the short-distance mass scale deduced from experimental limits. Specifically, the hadronic vertices appearing in $\mathcal{L}_{\text{EFF}}^{0\nu\beta\beta}$ will be of the type $NNNNee$, $NN\pi ee$ and $\pi\pi ee$, etc. They stem from quark-lepton operators having different transformation properties under parity and chiral $SU(2)$; as such, they will contribute to different orders in the p/Λ_H expansion.

Traditionally, the short-range $NNNNee$ contribution to $0\nu\beta\beta$ decay has been analyzed using a form-factor approach [18] where the finite size of the nucleon is taken into account with the use of a dipole form-factor. The form-factor overcomes the short-range repulsive core in NN interactions that would otherwise prevent the nucleons from ever getting close enough to exchange the heavy particles that mediate $0\nu\beta\beta$ decay. The disadvantage of a form-factor model is that the error introduced by the modeling cannot be estimated systematically in contrast to the EFT approach. A discussion of the $NNNNee$ vertex within the framework of EFT will appear later in this paper.

In contrast to the short range contribution to $0\nu\beta\beta$ decay, the long range contributions involve the exchange of pions [19] through the $NN\pi ee$ and $\pi\pi ee$ vertices. Although these long range contributions have been analyzed in the form-factor approach [20], they are more systematically analyzed within the context of EFT because of the separation of scales: $m_\pi < \Lambda_H \ll \Lambda_{\beta\beta}$. As noted in Ref. [21], for example, the matrix elements associated with the long range pionic

effects allowed under RPV SUSY scenarios can be dominant. However, we show that the dominance of pion exchange in $0\nu\beta\beta$ decay mediated by heavy physics is a more general result not limited to RPV SUSY. These pionic effects can be considerably larger than those obtained using the conventional form factor model for the short-range $NNNNee$ process. For these reasons, the analysis of the long range contributions to $0\nu\beta\beta$ decay in EFT will be the main focus of this paper.

The various types of L-violating operators that contribute to the long range contributions of $0\nu\beta\beta$ decay appear at different orders in the p/Λ_H expansion with $p \sim m_\pi$, and the order at which they appear depends on their symmetry properties. It is therefore important to delineate clearly the symmetry properties of $\mathcal{L}_{\text{EFF}}^{0\nu\beta\beta}$ for various types of L-violating operators and use these symmetries to relate the hadron-lepton operators to the underlying quark-lepton operators. Carrying out this classification constitutes the first component of this study. In doing so, we also comment on the standard approach to deriving $0\nu\beta\beta$-decay nuclear operators and correct some errors appearing in the literature.

The second step in our treatment involves deriving $0\nu\beta\beta$-decay nuclear operators from $\mathcal{L}_{\text{EFF}}^{0\nu\beta\beta}$ and expressing the rate in terms of corresponding nuclear matrix elements. For any $\beta\beta$-decay mode to occur, the final nucleus must be more bound than any other prospective single β-decay daughter nucleus. Such β-forbidden but $\beta\beta$-allowed nuclei only occur for sufficiently heavy nuclei. Thus, the extraction of the short-distance physics that gives rise to $0\nu\beta\beta$ decay (at present, only upper limits on the decay rates exist) depends on a proper treatment of the many-body nuclear physics. Having in hand the appropriate set of nuclear operators (for a given L-violation scenario), one could in principle compute the relevant nuclear matrix elements. Unfortunately, it is not yet possible to do so in a manner fully consistent with EFT. This problem has been studied extensively in the case of the NN and three-nucleon systems, where the state-of-the art involves use of chiral symmetry to organize (and renormalize) the relevant nuclear operators [22–25]. Out of necessity, we follow the same philosophy here. Nonetheless, the organization of various $0\nu\beta\beta$-decay operators based on symmetry considerations and EFT power counting should represent an improvement over present treatments of the nuclear problem.

As a final step, we relate the various nuclear operators obtained from $\mathcal{L}_{\text{EFF}}^{0\nu\beta\beta}$ to different particle physics models for L-violation. Doing so allows us to determine which nuclear mechanisms dominate the rate for a given particle physics model. For example, in both the RPV SUSY and the left-right symmetric model with mixing of the gauge bosons, the

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[1]The computation of these coefficients from the underlying quark-lepton interaction introduces some degree of uncertainty—a problem we will not address in this work.
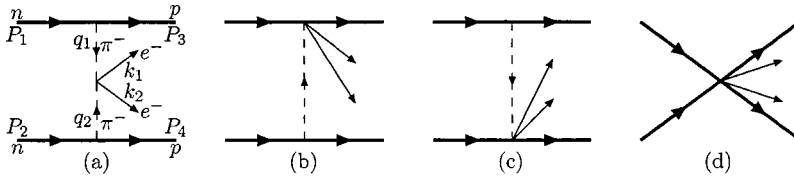
FIG. 2. Diagrams that contribute to $0\nu\beta\beta$ at tree level. The exchange diagrams are not included.

$\pi\pi ee$ contribution to the $0\nu\beta\beta$-decay amplitude is significantly larger than that of the short range $NNNNee$ contribution. In contrast, for left-right symmetric models with no mixing, these contributions are of a similar magnitude. We also show how this large $\pi\pi ee$ contribution to $0\nu\beta\beta$ decay substantially affects the relationship between the masses of the right-handed neutrino and gauge boson including a new correlation between the minimum mass of the right-handed neutrino and the $W_L - W_R$ mixing angle. In short, the sensitivity of the $0\nu\beta\beta$ decay searches is strongly affected by the symmetry transformation properties of the operators contained in a given particle physics model.

The remainder of our paper is organized as follows. In Sec. II, we classify the operators in $\mathcal{L}_{\mathrm{EFF}}^{0\nu\beta\beta}$ according to their symmetry properties and p/Λ counting and we tabulate the various quark-lepton operators according to the hadron lepton operators they can generate. In Sec. III we use the leading operators to derive nonrelativistic nuclear operators and compare their structure with those appearing in conventional treatments. In Sec. IV we work out the particle physics implications under various scenarios, namely RPV SUSY and the left-right symmetric model and compare them to each other. We summarize our conclusions in Sec. V.

II. EFFECTIVE $0\nu\beta\beta$-DECAY OPERATORS

The classification of the operators in $\mathcal{L}_{\mathrm{EFF}}^{0\nu\beta\beta}$ relies on two elements:

(i) The use of symmetry to relate effective lepton-hadron $0\nu\beta\beta$-decay operators to those involving quarks and leptons. The relevant symmetries are parity and chiral SU(2). Indeed, because the lepton-hadron effective operators are generated from the quark-lepton operators through strong interactions, they should retain the same parity and chiral structure.

(ii) The organization of these effective lepton-hadron operators in an expansion in powers of a small momentum p.

To organize the nonstandard model (NSM) operators in powers of p, consider first the long range π-exchange contributions to $0\nu\beta\beta$ decay of Figs. 2(a)–2(c). The fact that pions are Goldstone bosons allows us to use chiral perturbation theory [26,27] to classify the NSM hadronic operators in terms of a p/Λ_H expansion, with $\Lambda_H = 4\pi f_\pi \sim 1$ GeV and $p \sim m_\pi$ where $f_\pi \simeq 92.4$ MeV is the pion decay constant. The leading order (LO) quark operators should therefore induce effective hadronic operators that do not involve derivatives of the pion fields or pion mass insertions,[2] the next-to-leading order (NLO) operators would involve a single de-

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[2]At tree level, the pion mass insertions always have the form m_π^2 and therefore do not contribute at LO or NLO.

rivative of the pion field, the next-to-next-to-leading order (NNLO) would involve two derivatives or pion mass insertions and so on. This approach to $0\nu\beta\beta$ decay is similar to the application of effective field theory to purely hadronic $\Delta S = 0$ parity-violating operators that was done in [28] and the same notation will be used.

The power counting for the long-range $0\nu\beta\beta$-decay operators will involve the chiral order of the standard model (SM) operators as well as the chiral order of the NSM operators. For the SM operators, these counting rules are as follows: (i) a pion propagator is $\mathcal{O}(1/p^2)$ while (ii) each derivative of the pion field and the LO strong πNN vertex is $\mathcal{O}(p)$.

As for the short range operators [Fig. 2(d)], the hadronic part is constructed from a 4-nucleon vertex. This vertex can also be expanded in powers of the nucleon's 3-momentum. However, the chiral counting suggests that the leading $\mathcal{O}(p^0)$ four-nucleon vertex is already strongly suppressed relative to the long range $0\nu\beta\beta$-decay operators such that the 4-nucleon vertex can be neglected to lowest order. Indeed, with these rules, the chiral counting of the $0\nu\beta\beta$-decay operators of Figs. 2(a)–2(d) is

$$\text{Fig. 2(a)} \sim K_{\pi\pi} p^{-2},$$

$$\text{Figs. 2(b),2(c)} \sim K_{NN\pi} p^{-1},$$

$$\text{Fig. 2(d)} \sim K_{NNNN} p^0, \tag{1}$$

where the K_i denote the order of the NSM hadronic vertices. In general, the LO vertex in each diagram is $\mathcal{O}(p^0)$, though in certain cases symmetry considerations require that the leading order vertex vanish (see below). Thus, the long range $0\nu\beta\beta$-decay operators of Figs. 2(a)–2(c) are enhanced by $1/p^2$ and $1/p$, respectively, relative to the short-range operator of Fig. 2(d). In what follows, we will consider contributions generated by all of the diagrams in Fig. 2. Since the LO contribution from Fig. 2(d) is $\mathcal{O}(p^0)$, we must include contributions from Figs. 2(a)–2(c) through this order as well. Consequently, we consider all terms in $K_{\pi\pi}$ and $K_{NN\pi}$ to $\mathcal{O}(p^2)$ and $\mathcal{O}(p)$, respectively.

A. Quark-lepton Lagrangian

In order to construct the hadron-lepton operators, we begin by writing down the quark-lepton Lagrangian for $0\nu\beta\beta$ decay. This is done by considering all the nonvanishing, inequivalent, lowest-dimension quark-lepton operators that are Lorentz-invariant and change lepton number by two units,

$$\mathcal{L}^q_{0\nu\beta\beta} = \frac{G_F}{\Lambda_{\beta\beta}} \{ (o_1 \mathcal{O}^{++}_{1+} + o_2 \mathcal{O}^{++}_{2+} + o_3 \mathcal{O}^{++}_{2-} + o_4 \mathcal{O}^{++}_{3+} + o_5 \mathcal{O}^{++}_{3-}) \bar{e} e^c + (o_6 \mathcal{O}^{++}_{1+} + o_7 \mathcal{O}^{++}_{2+} + o_8 \mathcal{O}^{++}_{2-} + o_9 \mathcal{O}^{++}_{3+}$$

$$+ o_{10} \mathcal{O}^{++}_{3-}) \bar{e} \gamma^5 e^c + (o_{11} \mathcal{O}^{++,\mu}_{4+} + o_{12} \mathcal{O}^{++,\mu}_{4-} + o_{13} \mathcal{O}^{++,\mu}_{5+} + o_{14} \mathcal{O}^{++,\mu}_{5-}) \bar{e} \gamma_\mu \gamma^5 e^c + \text{H.c.} \}, \qquad (2)$$

where

$$\mathcal{O}^{ab}_{1+} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_R \tau^b \gamma_\mu q_R), \qquad (3)$$

$$\mathcal{O}^{ab}_{2\pm} = (\bar{q}_R \tau^a q_L)(\bar{q}_R \tau^b q_L) \pm (\bar{q}_L \tau^a q_R)$$
$$\times (\bar{q}_L \tau^b q_R), \qquad (4)$$

$$\mathcal{O}^{ab}_{3\pm} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_L \tau^b \gamma_\mu q_L)$$
$$\pm (\bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_R \tau^b \gamma_\mu q_R), \qquad (5)$$

$$\mathcal{O}^{ab,\mu}_{4\pm} = (\bar{q}_L \tau^a \gamma^\mu q_L \mp \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R$$
$$- \bar{q}_R \tau^b q_L), \qquad (6)$$

$$\mathcal{O}^{ab,\mu}_{5\pm} = (\bar{q}_L \tau^a \gamma^\mu q_L \pm \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R$$
$$+ \bar{q}_R \tau^b q_L). \qquad (7)$$

The $q_{L,R} = (u,d)_{L,R}$ are left-handed and right-handed isospinors and the τ's are Pauli matrices in isospace. When $a = b$, the operators with subscript $+$ $(-)$ are even (odd) eigenstates of parity as can be verified by noting that the parity operator simply interchanges left-handed spinors with right-handed spinors. This list of nine operators was arrived at by inspection.[3] Other operators that could have been written down are either equivalent to those in Eqs. (3)–(7) or vanish as shown in Appendix A. In particular, all operators proportional to $\bar{e} \sigma^{\mu\nu} e^c$, $\bar{e} \gamma^5 \sigma^{\mu\nu} e^c$ and $\bar{e} \gamma^\mu e^c$ vanish since these leptonic currents are identically zero as can be verified with the use of Fierz transformations. Some of these vanishing leptonic currents were erroneously taken as nonzero in Ref. [17]. Similarly, a quark operator, like $\bar{q} \sigma^{\mu\nu} \tau^\pm q \bar{q} \sigma_{\mu\nu} \tau^\pm q$, can be reexpressed in terms of $\mathcal{O}^{\pm\pm}_{2\pm}$ by applying a Fierz transformation despite the color indices since the hadronic matrix elements of four-quark operators only select their color singlet part.[4]

Recalling that fermion fields have mass dimension 3/2, note that the operators appearing in $\mathcal{L}^q_{0\nu\beta\beta}$ have mass dimension nine. Therefore, the overall coefficients have dimensions $[\text{Mass}]^{-5}$. In Eq. (2), this scale factor is expressed as $G_F^2 / \Lambda_{\beta\beta}$ where $\Lambda_{\beta\beta}$ remains to be determined. Derivative

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[3] In writing down Eqs. (3)–(7), we suppressed the color indices since EFT only relates color-singlet quark operators to hadronic operators.

[4] The projection onto color singlet states introduces a new factor that can ultimately be absorbed in the o_i's.

quark operators are suppressed by extra powers of $\Lambda_{\beta\beta}$ and need not be considered further.

The operators in $\mathcal{L}^q_{0\nu\beta\beta}$ can be generated by various particle physics models, but not all of them are necessarily generated in a single model. For example, the left-right symmetric model always involves the product of left-handed and/or right-handed currents, while only $\mathcal{O}^{ab}_{1+}$ and $\mathcal{O}^{ab}_{3\pm}$ are of that form. Thus, $\mathcal{O}^{ab}_{2\pm}$, $\mathcal{O}^{ab,\mu}_{4\pm}$ and $\mathcal{O}^{ab,\mu}_{5\pm}$ cannot appear in the left-right symmetric model. Another example is a minimal extension of the standard model with only left-handed currents and Majorana neutrinos; in this scenario, only $\mathcal{O}^{ab}_{3\pm}$ could appear. On the other hand, these operators all appear in RPV SUSY. This observation will allow a classification of these particle physics models later in this paper.

Since $0\nu\beta\beta$ decay always requires $a = b = \pm$, the $\mathcal{O}$'s have definite transformation properties. Using the quark field transformation properties under chiral SU(2),

$$\text{under SU(2)}_L \times \text{SU(2)}_R: \quad q_L \to L q_L, \quad q_R \to R q_R, \quad (8)$$

where the L and R transformation matrices have the form $\exp\{P_{L,R}\theta_{L,R}\}$ and

$$\theta_{L,R} \equiv \frac{1}{2} \vec{\tau} \cdot \vec{\theta}_{L,R}, \quad P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma^5), \qquad (9)$$

we derive the transformation properties of the $\mathcal{O}^{ab(\mu)}_{i\pm}$ under chiral SU(2),

$$\mathcal{O}^{ab}_{1+} \to (\bar{q}_L L^\dagger \tau^a \gamma^\mu L q_L)(\bar{q}_R R^\dagger \tau^b \gamma_\mu R q_R), \qquad (10)$$

$$\mathcal{O}^{ab}_{2\pm} \to (\bar{q}_R R^\dagger \tau^a L q_L)(\bar{q}_R R^\dagger \tau^b L q_L)$$
$$\pm (\bar{q}_L L^\dagger \tau^a R q_R)(\bar{q}_L L^\dagger \tau^b R q_R), \qquad (11)$$

$$\mathcal{O}^{ab}_{3\pm} \to (\bar{q}_L L^\dagger \tau^a \gamma^\mu L q_L)(\bar{q}_L L^\dagger \tau^b \gamma_\mu L q_L)$$
$$\pm (\bar{q}_R R^\dagger \tau^a \gamma^\mu R q_R)(\bar{q}_R R^\dagger \tau^b \gamma_\mu R q_R), \qquad (12)$$

$$\mathcal{O}^{ab,\mu}_{4\pm} \to (\bar{q}_L L^\dagger \tau^a \gamma^\mu L q_L \mp \bar{q}_R R^\dagger \tau^a \gamma^\mu R q_R)$$
$$\times (\bar{q}_L L^\dagger \tau^b R q_R - \bar{q}_R R^\dagger \tau^b L q_L), \qquad (13)$$

$$\mathcal{O}^{ab,\mu}_{5\pm} \to (\bar{q}_L L^\dagger \tau^a \gamma^\mu L q_L \pm \bar{q}_R R^\dagger \tau^a \gamma^\mu R q_R)$$
$$\times (\bar{q}_L L^\dagger \tau^b R q_R + \bar{q}_R R^\dagger \tau^b L q_L). \qquad (14)$$

We observe that $\mathcal{O}^{ab}_{1+}$ belongs to the $(3_L, 3_R)$ representation of SU(2)$_L \times$ SU(2)$_R$ (from here on, the subscripts L, R are dropped) in the sense that the first superscript a transforms like a triplet under SU(2)$_L$ while the second superscript b

transforms like a triplet under $SU(2)_R$. Note that only $\mathcal{O}_{1+}^{ab}$ belongs to a representation of chiral $SU(2)$. The other $\mathcal{O}_{i\pm}^{ab(,\mu)}$'s are superpositions of operators that have different transformation properties under chiral $SU(2)$. This is not surprising since the generators of chiral $SU(2)$ do not commute with the parity operator as they involve γ^5. For instance, $(\bar{q}_L\tau^\pm\gamma^\mu q_L)(\bar{q}_L\tau^\pm\gamma_\mu q_L)$ changes isospin by two units and is a singlet under $SU(2)_R$ such that it belongs to $(5,1)$ while $(\bar{q}_R\tau^\pm\gamma^\mu q_R)(\bar{q}_R\tau^\pm\gamma_\mu q_R)$ belongs to $(1,5)$. Hence, $\mathcal{O}_{3\pm}^{\pm\pm}$ belongs to $(5,1)\oplus(1,5)$.

B. Hadron-lepton Lagrangian

Let us now turn to the derivation of the $\pi\pi ee$ vertex from the quark operators. This will be followed by a similar analysis for the $NN\pi ee$ and $NNNNee$ vertices.

1. $\pi\pi ee$ vertex

To derive the hadronic vertex, first consider parity. The product of two pion fields being even under parity, only positive parity operators can contribute. Secondly, note that $\mathcal{O}_{4+}^{\pm\pm,\mu}$ and $\mathcal{O}_{5+}^{\pm\pm,\mu}$ must give rise to an operator of the form

$$\pi^+\partial^\mu\pi^+\bar{e}\gamma_\mu\gamma^5 e^c + \text{H.c.} \tag{15}$$

A partial integration shows that this operator is suppressed by one power of the electron mass, and is therefore negligible.

Thus, the only terms in $\mathcal{L}_{0\nu\beta\beta}^q$ that contribute are

$$\frac{G_F^2}{\Lambda_{\beta\beta}}\{\mathcal{O}_{1+}^{++}\bar{e}(o_1+o_6\gamma^5)e^c + \mathcal{O}_{2+}^{++}\bar{e}(o_2+o_7\gamma^5)e^c$$
$$+ \mathcal{O}_{3+}^{++}\bar{e}(o_4+o_9\gamma^5)e^c + \text{H.c.}\}. \tag{16}$$

The hadronic operators that stem from these quark operators must have the same transformation properties and can be written down by introducing the following fields [28]:

$$X_R^a=\xi\tau^a\xi^\dagger, \quad X_L^a=\xi^\dagger\tau^a\xi, \quad X^a=\xi\tau^a\xi, \tag{17}$$

$$\xi=\exp(i\pi/f_\pi)$$
$$=\exp\left[\frac{i}{\sqrt{2}f_\pi}\left(\tau^+\pi^+ + \tau^-\pi^- + \frac{1}{\sqrt{2}}\tau^3\pi^0\right)\right] \tag{18}$$

$$\pi^\pm=\frac{1}{\sqrt{2}}(\pi^1\mp i\pi^2), \quad N: \text{ Nucleon field.} \tag{19}$$

The transformation properties of the above fields under parity are

$$\pi\rightarrow-\pi, \quad \xi\leftrightarrow\xi^\dagger, \quad X_R^a\leftrightarrow X_L^a, \quad X^a\leftrightarrow X^{\dagger a}, \quad N\rightarrow\gamma^0 N, \tag{20}$$

while under $SU(2)_L\times SU(2)_R$ they transform as

$$\xi\rightarrow L\xi U^\dagger=U\xi R^\dagger \tag{21}$$

$$X^a\rightarrow U\xi \ R^\dagger\tau^a L \ \xi U^\dagger \tag{22}$$

$$X_L^a\rightarrow U\xi^\dagger \ L^\dagger\tau^a L \ \xi U^\dagger \tag{23}$$

$$X_R^a\rightarrow U\xi \ R^\dagger\tau^a R \ \xi^\dagger U^\dagger \tag{24}$$

$$N\rightarrow UN. \tag{25}$$

The transformation matrix U only depends on the τ's and the pion field.

At LO (no derivatives), the two-pion operator stemming from the $\mathcal{O}_{1+}^{\pm\pm}$ operator is

$$\mathcal{O}_{1+}^{\pm\pm}\rightarrow\text{tr}[\Phi_{1+}^{\pm\pm}]\equiv\text{tr}[X_L^\pm X_R^\pm + X_R^\pm X_L^\pm]$$
$$=\frac{4}{f_\pi^2}\pi^\mp\pi^\mp + \cdots, \tag{26}$$

while the one generated by $\mathcal{O}_{2+}^\pm$ is

$$\mathcal{O}_{2+}^\pm\rightarrow\text{tr}[\Phi_{2+}^{\pm\pm}]\equiv\text{tr}[X^\pm X^\pm + X^{\dagger\pm}X^{\dagger\pm}]$$
$$=-\frac{4}{f_\pi^2}\pi^\mp\pi^\mp + \cdots. \tag{27}$$

Here, $\Phi_{1,2\pm}^{\pm\pm}$ are defined

$$\Phi_{1\pm}^{\pm\pm}\equiv X_L^\pm X_R^\pm \pm X_R^\pm X_L^\pm,$$
$$\Phi_{2\pm}^{\pm\pm}\equiv X^\pm X^\pm \pm X^{\dagger\pm}X^{\dagger\pm}, \tag{28}$$

and the $\pm$ subscript refers to the transformation properties of the $\Phi_{i\pm}^{\pm\pm}$'s under parity.

Note that when the traces of $\Phi_{1+}^{\pm\pm}$ and $\Phi_{2+}^{\pm\pm}$ are expanded up to two powers of the pion field, they are physically indistinguishable since the relative minus sign can be absorbed in an operator coefficient referred to as a low energy constant (LEC).

Now consider the case of the two-pion operator generated by $\mathcal{O}_{3+}^{\pm\pm}$; to LO the hadronic operator should be

$$\text{tr}[X_L^+ X_L^+ + X_R^+ X_R^+]=0. \tag{29}$$

Thus, there exists no $(5,1)\oplus(1,5)$ hadronic operator with no derivatives.

The LO Lagrangian for the $\pi\pi ee$ vertex is therefore

$$\mathcal{L}_{(0)}^{\pi\pi ee}=\frac{G_F^2}{\Lambda_{\beta\beta}}\{\text{tr}[\Phi_{1+}^{++}]\bar{e}(a+b\gamma^5)e^c$$
$$+\text{tr}[\Phi_{1+}^{--}]\bar{e}^c(a+b\gamma^5)e + \text{tr}[\Phi_{2+}^{++}]\bar{e}(a'+b'\gamma^5)e^c$$
$$+\text{tr}[\Phi_{2+}^{--}]\bar{e}^c(a'+b'\gamma^5)e\}, \tag{30}$$

where a,b,a',b' are LEC's. Note that although there are nominally four LEC's, once the traces of the $\Phi_i^\pm$'s are expanded, there are in practice only two: $a-a'$ and $b-b'$.

In contrast to the o_i's, the a,b,a',b' are dimensionful. It is useful to express them in terms of dimensionless parameters (denoted in this work by Greek letters) with the aid of a

TABLE I. Cross-reference table between nucleon and quark operators. The X indicates that the quark operator cannot generate the corresponding nucleon operator while the $\sqrt{}$ indicates that it can.

$NNNN$ ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\mathfrak{N}_{1+}^{\pm\pm}$	$\sqrt{}$	$\sqrt{}$	X	$\sqrt{}$	X	X	X	X	X
$\mathfrak{N}_{2+}^{\pm\pm}$	$\sqrt{}$	$\sqrt{}$	X	$\sqrt{}$	X	X	X	X	X
$\mathfrak{N}_{3+}^{\pm\pm}$	$\sqrt{}$	$\sqrt{}$	X	$\sqrt{}$	X	X	X	X	X
$\mathfrak{N}_{4+}^{\pm\pm,\mu}$	X	X	X	X	X	$\sqrt{}$	X	$\sqrt{}$	X
$\mathfrak{N}_{4-}^{\pm\pm,\mu}$	X	X	X	X	X	X	$\sqrt{}$	X	$\sqrt{}$

scaling rule. In a scaling rule, the hadronic operators are divided by the relevant scales such that their coefficients are dimensionless and of a "natural" size. We follow the näive dimensional analysis (NDA) scaling rules given in Ref. [29] and modified here to account for the lepton bilinears:[5]

$$\left(\frac{\bar{N}N}{\Lambda_H f_\pi^2}\right)^k \left(\frac{\partial^\mu}{\Lambda_H}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{f_\pi^2 G_F^2}{\Lambda_{\beta\beta}} \bar{e}e^c\right)$$
$$\times (\Lambda_H f_\pi)^2. \quad (31)$$

Justification for this scaling rule is given in Appendix B. Note that the scaling factor $(\pi/f_\pi)^m$ is already properly accounted for in the definition of ξ and need not be applied again in Eq. (30) after expanding the Φ's to two pions. For the nonderivative $\pi\pi ee$ vertex, we have $(k,l,m)=(0,0,2)$ and

$$\mathcal{L}_{(0)}^{\pi\pi ee} = \frac{G_F^2 \Lambda_H^2 f_\pi^2}{\Lambda_{\beta\beta}} \{\pi^-\pi^-\bar{e}(\beta_1+\beta_2\gamma^5)e^c$$
$$+ \pi^+\pi^+\bar{e}^c(\beta_1-\beta_2\gamma^5)e\}. \quad (32)$$

Consider now the higher order contributions to the $\pi\pi ee$ vertex. As discussed below Eq. (15), there is no NLO contribution. Hence, $\mathcal{L}_{(1)}^{\pi\pi ee}=0$.

At NNLO, not only do $\mathcal{O}_{1+}^{\pm\pm}$ and $\mathcal{O}_{2+}^{\pm\pm}$ generate two-derivative hadronic operator, but so does $\mathcal{O}_{3+}^{\pm\pm}$

$$\mathcal{O}_{3+}^{\pm\pm} \to \frac{1}{2}\text{tr}[\mathcal{D}^\mu X_L^\pm \mathcal{D}_\mu X_L^\pm + \mathcal{D}^\mu X_R^\pm \mathcal{D}_\mu X_R^\pm], \quad (33)$$

where the chiral covariant derivative is given by

$$\mathcal{D}_\mu = \partial_\mu - i\mathcal{V}_\mu, \quad \mathcal{V}_\mu = \frac{1}{2}i(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi). \quad (34)$$

The operator $\mathcal{D}_\mu X_{L,R}$ has the same transformation properties under chiral SU(2) as $X_{L,R}$.

The only other contribution stems from quark mass insertions that always generate squared pion mass insertions. Writing the NNLO contributions directly in terms of pion fields, we obtain

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[5]We neglect electromagnetic effects.

$$\mathcal{L}_{(2)}^{\pi\pi ee} = \frac{G_F^2 f_\pi^2}{\Lambda_{\beta\beta}} \{\partial_\mu\pi^- \partial^\mu\pi^-\bar{e}(\beta_3+\beta_4\gamma^5)e^c$$
$$+ m_\pi^2 \pi^-\pi^-\bar{e}(\beta_5+\beta_6\gamma^5)e^c+\text{H.c.}\}. \quad (35)$$

Note that the $\beta_{5,6}$ terms constitute corrections to $\beta_{1,2}\to\beta_{1,2}+m_\pi^2\beta_{5,6}$ that can be ignored in particle physics models where the LO operators contribute since $\beta_{1,2}$ must be measured.[6]

2. $NN\pi ee$ vertex

We analyze the $NN\pi ee$ vertex of Figs. 2(b) and 2(c) using similar logic as in the foregoing discussion. The LO Lorentz-scalar $NN\pi$ operator is $\bar{N}\tau^\pm\pi^\mp N$ which is odd under parity. Therefore, $\mathcal{O}_{1+}^{\pm\pm}$, $\mathcal{O}_{2+}^{\pm\pm}$ and $\mathcal{O}_{3+}^{\pm\pm}$ cannot contribute since they are parity even. As for $\mathcal{O}_{3-}^{\pm\pm}$, notice that as in the $\pi\pi ee$ case, the LO contribution $(X_L^\pm X_L^\pm - X_R^\pm X_R^\pm)$ vanishes.

The operator $\bar{N}\tau^\pm\pi^\mp N$ can only be induced by $\mathcal{O}_{2-}^{\pm\pm}$. The result is

$$\mathcal{O}_{2-}^{\pm\pm} \to \bar{N}\Phi_{2-}^{\pm\pm}N. \quad (36)$$

It is straightforward to verify that $\bar{N}\Phi_{2-}^{\pm\pm}N$ transforms precisely like $\mathcal{O}_{2-}^{\pm\pm}$ under $\text{SU}(2)_L\times\text{SU}(2)_R$.

In addition, $\mathcal{O}_{4\pm}^{\pm\pm,\mu}$ and $\mathcal{O}_{5\pm}^{\pm\pm,\mu}$ also generate LO contributions to the $NN\pi$ operator,

$$\mathcal{O}_{4+}^{\pm\pm,\mu}, \mathcal{O}_{5+}^{\pm\pm,\mu} \to \bar{N}\gamma^\mu\gamma^5\Phi_{3-}^{\pm\pm}N,$$
$$\mathcal{O}_{4-}^{\pm\pm,\mu}, \mathcal{O}_{5-}^{\pm\pm,\mu} \to \bar{N}\gamma^\mu\Phi_{3-}^{\pm\pm}N, \quad (37)$$

where

$$\Phi_{3-}^{\pm\pm} = (X_L^\pm + X_R^\pm)(X^\pm - X^{\pm\dagger}), \quad (38)$$

as can be checked explicitly by considering the transformation properties under chiral SU(2) and parity. The $NN\pi ee$ LO Lagrangian can now be written down,

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[6]As discussed in Ref. [30], EFT relates the two-derivative $\pi\pi ee$ operator to the **27**-plet $K\to 2\pi$ decays indicating the possible existence of an extra suppression factor beyond that deduced from power counting.

$$\mathcal{L}_{(0)}^{NN\pi ee} = \frac{G_F^2}{\Lambda_{\beta\beta}}\{\bar{N}\Phi_{2-}^{++}N\bar{e}(c+d\gamma^5)e^c+\bar{N}\gamma^\mu(f_1+f_2\gamma^5)\Phi_{3-}^{++}N\bar{e}\gamma_\mu\gamma^5 e^c+\text{H.c.}\}$$

$$\cong \frac{G_F^2\Lambda_H f_\pi}{\Lambda_{\beta\beta}}\{\bar{N}\tau^+\pi^-N\bar{e}(\zeta_1+\zeta_2\gamma^5)e^c+\bar{N}\gamma^\mu(\zeta_3+\zeta_4\gamma^5)\tau^+\pi^-N\bar{e}\gamma_\mu\gamma^5 e^c+\text{H.c.}\}, \qquad (39)$$

where the ζ_i are dimensionless LEC's introduced using Eq. (31) with $(k,l,m)=(1,0,1)$ and where we have expanded the Φ's to one pion.

At NLO, $\mathcal{O}_{1+}^{\pm\pm}$, $\mathcal{O}_{2+}^{\pm\pm}$, $\mathcal{O}_{3-}^{\pm\pm}$ and $\mathcal{O}_{3+}^{\pm\pm}$ contribute to the $NN\pi$ operator,

$$\mathcal{O}_{1+}^{\pm\pm}\rightarrow\bar{N}\gamma^5\Phi_{1-}^{\pm\pm}N, \qquad (40)$$

$$\mathcal{O}_{2+}^{\pm\pm}\rightarrow\bar{N}\gamma^5\Phi_{2-}^{\pm\pm}N, \qquad (41)$$

$$\mathcal{O}_{3-}^{\pm\pm}\rightarrow\bar{N}\{\gamma^\mu[X_L^\pm(-i\mathcal{D}_\mu X_L^\pm)-X_R^\pm(i\mathcal{D}_\mu X_R^\pm)]\}N, \qquad (42)$$

$$\mathcal{O}_{3+}^{\pm\pm}\rightarrow\bar{N}\{\gamma^\mu\gamma^5[X_L^\pm(-i\mathcal{D}_\mu X_L^\pm)-X_R^\pm(i\mathcal{D}_\mu X_R^\pm)]\}N. \qquad (43)$$

The first thing to note is that a term like $\bar{N}\gamma^5\pi N$ is subleading because in the nonrelativistic reduction, the γ^5 couples small and large components of the nucleon spinors. Secondly, we observe that Eqs. (40), (41) and (43) are physically indistinguishable on shell when expanded to one pion and to the order we are considering, as seen from the equations of motion. Thirdly, Eq. (42) is negligible even at NLO because the equations of motion can be used to show that $\bar{N}\slashed{\partial}\pi N$ is proportional to the electron momentum. Therefore, $\mathcal{O}_{3-}^{\pm\pm}$ does not contribute to the $NN\pi ee$ vertex.

Other contributions to $\mathcal{O}(p)$ include terms normally neglected at LO in the nonrelativistic reduction of Eq. (39), namely the terms proportional to ζ_3 and ζ_4 with $\mu=1,2,3$ and $\mu=0$, respectively, where LO and NLO components of the nucleon spinors are coupled. These are the only contributions to the $NN\pi ee$ vertex since the m_π^2 insertions are of $\mathcal{O}(p^2)$ and excluded as discussed below Eq. (1). Hence, the only new contribution to $\mathcal{O}(p)$ is

$$\mathcal{L}_{(1)}^{NN\pi ee}=\frac{G_F^2\Lambda_H f_\pi}{\Lambda_{\beta\beta}}\bar{N}\gamma^5\tau^+\pi^-N\bar{e}(\zeta_5+\zeta_6\gamma^5)e^c+\text{H.c.} \qquad (44)$$

where the scaling rule in Eq. (31) was used with $(k,l,m)=(1,1,1)$. $\mathcal{L}_{(1)}^{NN\pi ee}$ is subleading because the γ^5 couples the large and small components of the nucleon spinors and the

result is proportional to p/M where M is the nucleon mass and p is the magnitude of the nucleon three-momentum [31–35].

3. NNNNee vertex

To identify the quark operators that generate the $0\nu\beta\beta$-decay four-nucleon operators, we insert the hadronic fields $X_{LR}^{\pm\pm}$, $X^{\pm\pm}$, $X^{\dagger\pm\pm}$ in all possible ways into $\bar{N}\Gamma N\bar{N}\Gamma'N$ and use their transformation properties under chiral SU(2) to relate them to the $\mathcal{O}_{i\pm}^{\pm\pm(,\mu)}$. The four-nucleon operators are then obtained by expanding these hadronic fields to LO and ignoring all contributions from pion loops. Thus, it is not necessary to insert these hadronic fields in all possible ways; we only need to show that a particular quark operator can generate a particular nucleon operator with the same transformation properties under parity and chiral SU(2).

For example, the LO operator $(\bar{N}\tau^\pm N)^2$ can be generated by $\mathcal{O}_{1+}^{\pm\pm}$. The latter transforms the same way under parity and chiral SU(2) as the hadronic operator

$$(\bar{N}X_L^\pm N)(\bar{N}X_R^\pm N). \qquad (45)$$

At zero pion order, the $X_L^\pm$ and $X_R^\pm$ both become $\tau^\pm$, so that the operator in Eq. (45) just becomes $(\bar{N}\tau^\pm N)^2$. In a similar fashion, it can be easily shown that the following five operators:

$$\mathfrak{N}_{1+}^{\pm\pm}=(\bar{N}\tau^\pm N)^2, \quad \mathfrak{N}_{2+}^{\pm\pm}=(\bar{N}\tau^\pm\gamma^\mu N)(\bar{N}\tau^\pm\gamma_\mu N),$$

$$\mathfrak{N}_{3+}^{\pm\pm}=(\bar{N}\tau^\pm\gamma^5\gamma^\mu N)(\bar{N}\tau^\pm\gamma^5\gamma_\mu N),$$

$$\mathfrak{N}_{4+}^{\pm\pm,\mu}=(\bar{N}\tau^\pm\gamma^\mu N)(\bar{N}\tau^\pm N),$$

$$\mathfrak{N}_{4-}^{\pm\pm,\mu}=(\bar{N}\tau^\pm\gamma^5\gamma^\mu N)(\bar{N}\tau^\pm N), \qquad (46)$$
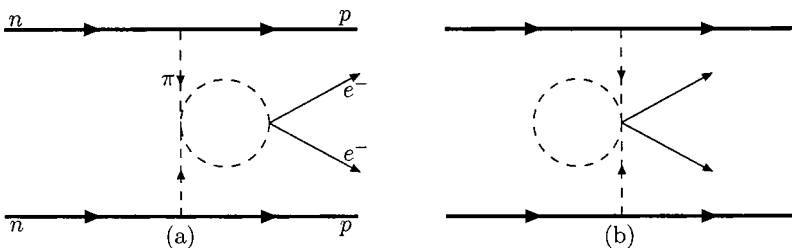

FIG. 3. (a) Example of a graph that renormalizes the LEC's that multiplies the m_π^2 and two-derivative $\pi\pi ee$ vertex. (b) Example of a new vertex ($\pi\pi\pi\pi ee$) that contributes to $0\nu\beta\beta$ at NNLO.

exhaust the list of possible LO four-nucleon operators[7] that can be generated by the checked $\mathcal{O}_{i\pm}^{\pm\pm(,\mu)}$'s in Table I.

The LO four-nucleon Lagrangian is therefore given by

$$\mathcal{L}_0^{NNNNee} = \frac{G_F^2}{\Lambda_{\beta\beta}} \{(\xi_1 \mathfrak{N}_{1+}^{++} + \xi_2 \mathfrak{N}_{2+}^{++} + \xi_3 \mathfrak{N}_{3+}^{++}) \bar{e} e^c + (\xi_4 \mathfrak{N}_{1+}^{++}$$
$$+ \xi_5 \mathfrak{N}_{2+}^{++} + \xi_6 \mathfrak{N}_{3+}^{++}) \bar{e} \gamma^5 e^c + (\xi_7 \mathfrak{N}_{4+}^{++,\mu}$$
$$+ \xi_8 \mathfrak{N}_{4-}^{++,\mu}) \bar{e} \gamma^5 \gamma_\mu e^c + \text{H.c.}\}, \qquad (47)$$

where the ξ_i's are dimensionless.

In concluding this section, we discuss a few issues that will require future work. The first involves the application of EFT to heavy nuclei. As pointed out earlier, no fully consistent treatment for such situations has yet been developed. In principle, one could imagine following a program similar in spirit to the EFT treatment of few-body systems. In that case, there has been recent progress in developing a consistent power counting for EFT with explicit pions [24,25]. The approach involves including the LO π-exchange contribution to the NN potential, expanding it about the chiral limit ($m_\pi^2 \to 0$), and obtaining two-body wave functions by solving the Schrödinger equation with the chirally expanded potential. To be consistent, operators would also be expanded to the same chiral order as the potential and matrix elements computed using the corresponding wave functions. This approach appears to reproduce the consistent momentum power counting obtained with perturbative pions in the $^1S_0$ channel and the convergence obtained with nonperturbative pions in the $^3S_1$-$^3D_1$ channel. In going to more complex nuclei, one might explore a marriage of the chiral expansion with traditional many-body techniques (e.g., shell model or RPA), in which case one would require a corresponding chiral counting of nuclear operators. In organizing the $0\nu\beta\beta$-decay hadronic operators according to both the derivative and chiral expansion, we have taken one step in this direction. For the moment, however, we will have to content ourselves with using these operators along with wave functions obtained from traditional many-body techniques.

A second issue is the presence of higher partial waves in the two-body transition matrix elements appearing in $0\nu\beta\beta$ decay. A fully consistent treatment would, therefore, require that one include the corresponding higher-order operators—a task that is clearly impractical at present. Fortunately, in our case, there is reason to believe our qualitative conclusions about the dominance of long-range, pion-exchange operators are fairly insensitive to this issue. For the cases where the LO $\pi\pi ee$ are not forbidden by the symmetries of the quark-lepton operators, the LO π-exchange operator arising from Fig. 2(a) will always give the LO contribution to the transition matrix element, regardless of the partial wave decomposition of the two-nucleon initial and final states. In general, then, we expect that matrix elements of these operators should always be enhanced relative to those involving the

four-nucleon contact operators or π-exchange operators obtained with higher-order pionic vertices. Indeed, some evidence to this effect is given by the computation of Ref. [39], where the relative importance of the LO π-exchange operators and short-range operators was compared for RPV SUSY.[8]

Finally, when NNLO and NLO interactions are included at tree level, loop graphs must also be included to be consistent with the power counting (examples of which are given in Fig. 3). These loop graphs are handled according to the chiral perturbation theory prescription by which the divergences renormalize the LEC's that multiply the m_π^2 and two-derivative $\pi\pi ee$ vertex of Eq. (35). In this context, loop graphs that renormalize the $NN\pi ee$ vertex are N^3LO and can be ignored. Indeed, this can be demonstrated using power counting where each loop involves a factor of p^4 while nucleon propagators count as p^{-1} [36–38].

When loops are included, new lepton-violating tree level vertices can contribute inside the loop graphs, such as the $\pi\pi\pi ee$ vertex of Fig. 3(b). Other new vertices that could potentially contribute at the one loop level are $NN\pi\pi ee$ and $\pi\pi ee$ vertices. In short, a large number of Feynman diagrams may need to be calculated at NNLO. We defer a discussion of such loop contributions to a subsequent study.

To summarize the conclusions of the analysis, Table II lists the quark-lepton operators that contribute to the various hadron operators at LO. One important result indicated in the table is the fact that if the short-distance physics responsible for $0\nu\beta\beta$ decay belongs to a representation of $SU(2)_L \times SU(2)_R$, only operators that belong to the (3,3) and $(5,1)\oplus(1,5)$ can generate $0\nu\beta\beta$ decay and therefore, only $\mathcal{O}_{1+}^{\pm\pm}$ and $\mathcal{O}_{3+}^{\pm\pm}$ can contribute. For example, the left-right symmetric model with mixing between left- and right-handed gauge bosons induces operators belonging to the (3,3) as well as the $(5,1)\oplus(1,5)$. From Table II, the LO $0\nu\beta\beta$-decay operator that contributes in this case is generated by Fig. 2(a) and is $\mathcal{O}(p^{-2})$.

Alternatively, consider a short-distance model involving products of two left-handed currents or two right-handed currents only. Such a situation arises, for instance, in the left-right symmetric model when the W_L and W_R bosons do not mix. For this scenario, only $\mathcal{O}_{3+}^{\pm\pm}$ contributes, and there are no LO contributions to the $\pi\pi ee$ and $NN\pi ee$ vertices. The first nonzero contributions to the hadronic part of these vertices are given by Eqs. (33) and (43) as well as contributions that include m_π^2 insertions. The resulting contribution to the amplitude is $\mathcal{O}(p^0)$. In this case, both the long- and short-range nuclear operators occur at the same order.

III. NUCLEAR OPERATORS TO LO AND NLO

In the calculation of the $0\nu\beta\beta$-decay amplitude, the Feynman diagrams of Fig. 2 must be calculated to $\mathcal{O}(p^0)$, where p is the small momentum used as an expansion pa-

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[7]Since $\bar{N}\gamma^5 N$ and $(\bar{N}\gamma^5\gamma^\mu N)(\bar{N}\gamma_\mu N)$ are proportional to p/M, they are subleading in the nonrelativistic limit.

[8]However, in that work, the traditional, form factor approach was used to compute short-range effects.

TABLE II. Leading order $0\nu\beta\beta$-decay hadronic-lepton operators generated by the various quark-lepton operators.

$0\nu\beta\beta$-decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\pi\pi ee$ LO	√	√	X	X	X	X	X	X	X
$\pi\pi ee$ NNLO	√	√	X	√	X	X	X	X	X
$NN\pi ee$ LO	X	X	√	X	X	√	√	√	√
$NN\pi ee$ NLO	X	√	X	√	X	√	√	√	√
$NNNN ee$ LO	√	√	X	√	X	√	√	√	√

rameter. As discussed below Eq. (1), this implies that we need to include NNLO $\pi\pi ee$ operators, NLO $NN\pi ee$ operators and LO $NNNN ee$ operators.

From the $\pi\pi ee$ Lagrangian of Eq. (32), the LO $0\nu\beta\beta$-decay amplitude of Fig. 2(a) is calculated to be

$$M_0^{\pi\pi} = -\frac{g_A^2 G_F^2 \Lambda_H^2 M^2}{\Lambda_{\beta\beta}} \frac{8(\bar{u}_{p3}\gamma^5 u_{n1})(\bar{u}_{p4}\gamma^5 u_{n2})}{(q_1^2 - m_\pi^2 + i\epsilon)(q_2^2 - m_\pi^2 + i\epsilon)}$$
$$\times \bar{u}_{e1}\gamma^2\gamma^0(\beta_1 + \beta_2\gamma^5)\bar{u}_{e2}^{\mathrm{T}}, \quad (48)$$

where $q_1 = P_1 - P_3$, $q_2 = P_2 - P_4$ as defined in Fig. 2(a) and $g_A = 1.27$ is the usual axial pion-nucleon coupling related to $g_{\pi NN}$ by the Goldberger-Treiman relation.

As for the NLO, recall from Eq. (15) and the discussion that followed that the $\pi\pi ee$ vertex has no NLO contributions. Thus, the NLO $0\nu\beta\beta$-decay nuclear operators are given by Figs. 2(b) and 2(c). Note that experiments planned and under way involve mainly ground state to ground state transitions $0^+ \to 0^+$ which are favored by phase space considerations. The nuclear matrix elements of all the operators of $\mathcal{L}_0^{NN\pi ee}$ [Eq. (39)] vanish for this transition by parity.[9] There are therefore no NLO contributions for the $0^+ \to 0^+$ transition and $M_0^{\pi\pi}$ is the only nonvanishing amplitude through $\mathcal{O}(p)$. Nevertheless, we provide the expressions for the NLO nuclear operators in Appendix C for completeness.

Taking the nonrelativistic limit of Eq. (48) and Fourier transforming to coordinate space yields

$$\text{F.T.}M_0^{\pi\pi} \simeq \frac{1}{12\pi}\frac{g_A^2 G_F^2 \Lambda_H^2}{\Lambda_{\beta\beta}}\bar{u}_{e1}\gamma^2\gamma^0(\beta_1 + \beta_2\gamma^5)$$
$$\times \bar{u}_{e2}^{\mathrm{T}}\mathcal{O}_0^{\pi\pi}(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4), \quad (49)$$

where the nuclear operator is given by

$$\mathcal{O}_0^{\pi\pi}(\vec{x}_1, \ldots, \vec{x}_4)$$
$$= -\delta(\vec{x}_1 - \vec{x}_3)\delta(\vec{x}_2 - \vec{x}_4)(\chi_{3,\alpha}^\dagger \chi_{1,\beta})$$
$$\times (\chi_{4,\phi}^\dagger \chi_{2,\delta})\frac{1}{\rho}[\mathrm{F}_1\vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\phi\delta} + \mathrm{F}_2 T_{\alpha\phi,\beta\delta}], \quad (50)$$

and

$$T_{\alpha\phi,\beta\delta} \equiv 3\vec{\sigma}_{\alpha\beta}\cdot\hat{\rho}\,\vec{\sigma}_{\phi\delta}\cdot\hat{\rho} - \vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\phi\delta}. \quad (51)$$

The form-factors F_1 and F_2 were first introduced in Ref. [20]

$$\mathrm{F}_1(x) = (x-2)\mathrm{e}^{-x}, \quad \mathrm{F}_2(x) = (x+1)\mathrm{e}^{-x}, \quad (52)$$

where $x = m_\pi\rho$, $\rho = |\vec{x}_1 - \vec{x}_2|$ is the distance between the nucleons, and $\hat{\rho} = \vec{\rho}/\rho$. However, in Ref. [20], these form-factors were derived within a minimal extension of the standard model with only left-handed currents and heavy Majorana neutrinos; as was shown above by considering the possible representations to which the product of two left-handed weak currents can belong, this minimal extension cannot give rise to the LO $\pi\pi ee$ vertex that yields these form-factors. In contrast, the derivation of F_1 and F_2 was performed here by considering the symmetry properties of the quark operators that could generate the hadronic $0\nu\beta\beta$-decay operators without specifying the short-distance physics responsible for $0\nu\beta\beta$ decay.

Up to NLO, the $0\nu\beta\beta$-decay half-life is therefore

$$\frac{1}{T_{1/2}} = \frac{\hbar c^2}{144\pi^5\ln 2}\frac{g_A^4}{R^2}\frac{\Lambda_H^4 G_F^4}{\Lambda_{\beta\beta}^2}\int_{m_e}^{E_{\beta\beta}-m_e}\mathrm{d}E_1$$
$$\times F(Z+2,E_1)F(Z+2,E_2)\frac{1}{2}[(\beta_1^2 + \beta_2^2)p_1 E_1 p_2 E_2$$
$$- (\beta_1^2 - \beta_2^2)p_1 p_2 m_e^2]|\mathcal{M}_0|^2, \quad (53)$$

where $F(Z,E)$ is the usual Fermi function describing the Coulomb effect on the outgoing electrons with

$$\mathcal{M}_0 = \langle\Psi_{A,Z+2}|\sum_{ij}\frac{R}{\rho_{ij}}[F_1(x_{ij})\vec{\sigma}_i\cdot\vec{\sigma}_j$$
$$+ F_2(x_{ij})T_{ij}]\tau_i^+\tau_j^+|\Psi_{A,Z}\rangle, \quad (54)$$

$$T_{ij} = 3\vec{\sigma}_i\cdot\hat{\rho}_{ij}\vec{\sigma}_j\cdot\hat{\rho}_{ij} - \vec{\sigma}_i\cdot\vec{\sigma}_j, \quad (55)$$

$$E_2 = E_{\beta\beta} - E_1, \quad p_i = \sqrt{E_i^2 - m_e^2}. \quad (56)$$

Here ρ_{ij} is the distance between the ith and jth neutrons in the initial nucleus $|\Psi_{A,Z}\rangle$ or the distance between two protons in the final state $|\Psi_{A,Z+2}\rangle$, m_e is the mass of the electron, R is a scale taken to be of the order of the nuclear

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[9]Recall from above that $\bar{N}\gamma^5 N$ and $\bar{N}\gamma^i N$, $i = 1,2,3$, are NNLO operators that couple the large and small components of the nucleon spinors.

radius,[10] $\vec{\sigma}_{i(j)}$ acts on the spin of the $i(j)$th neutron and the isospin matrix $\tau_{i(j)}^+$ turns the $i(j)$th neutron into a proton. Note that independently of the nuclear matrix element, the $\beta_1^2 - \beta_2^2$ part of the rate in Eq. (53) is always considerably smaller (by at least a factor of ~ 10 from the kinematics) than the $\beta_1^2 + \beta_2^2$ part which is the only one usually considered.

A. Long range operators At NNLO

Consider now the long range operators at NNLO. We are interested in comparing the LO and NNLO tree-level long range contributions and for simplicity we will ignore contributions from loops, m_π^2 insertions and the four-nucleon vertex which also contribute at NNLO.[11] Thus, we only need the hadronic operators of Eqs. (35) and (44) rewritten here

$$M_2 = \frac{G_F^2}{\Lambda_{\beta\beta}} \{ f_\pi^2 \partial_\mu \pi^- \partial^\mu \pi^- \bar{e}(\beta_3 + \beta_4 \gamma^5)e^c + f_\pi \Lambda_H \bar{N} \gamma^5 \tau^+ \pi^- N \bar{e}(\zeta_5 + \zeta_6 \gamma^5)e^c + \text{H.c.} \}. \quad (57)$$

The diagrams of Figs. 2(a)–2(c) can be evaluated using the operators of Eq. (57). The Fourier transform of the final result is

$$M_2 = \frac{1}{8\pi} g_A^2 \frac{G_F^2}{\Lambda_{\beta\beta}} [\bar{u}_{e1} \gamma^2 \gamma^0 (\beta_3 + \beta_4 \gamma^5) \bar{u}_{e2}^{\mathrm{T}} \mathcal{O}_2^{\pi\pi}$$
$$\times (\vec{x}_1, \dots, \vec{x}_4) + \bar{u}_{e1} \gamma^2 \gamma^0 (\zeta_5 + \zeta_6 \gamma^5)$$
$$\times \bar{u}_{e2}^{\mathrm{T}} \mathcal{O}_2^{\pi NN}(\vec{x}_1, \dots, \vec{x}_4)], \quad (58)$$

with

$$\mathcal{O}_2^{\pi\pi}(\vec{x}_1, \dots, \vec{x}_4) = -\delta(\vec{x}_1 - \vec{x}_3)\delta(\vec{x}_2 - \vec{x}_4)(\chi_{3,\alpha}^\dagger \chi_{1,\beta})$$
$$\times (\chi_{4,\phi}^\dagger \chi_{2,\delta}) \frac{1}{\rho^3} (G_1^{\pi\pi} \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\phi\delta}$$
$$+ G_2^{\pi\pi} T_{\alpha\phi,\beta\delta}) \quad (59)$$

$$\mathcal{O}_2^{\pi NN}(\vec{x}_1, \dots, \vec{x}_4) = -\frac{\sqrt{2}\Lambda_H}{g_A M} \delta(\vec{x}_1 - \vec{x}_3) z \delta(\vec{x}_2 - \vec{x}_4)$$
$$\times (\chi_{3,\alpha}^\dagger \chi_{1,\beta})$$
$$\times (\chi_{4,\phi}^\dagger \chi_{2,\delta}) \frac{1}{\rho^3} (G_1^{\pi NN} \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\phi\delta}$$
$$+ G_2^{\pi NN} T_{\alpha\phi,\beta\delta}), \quad (60)$$

and ($x = m_\pi \rho$ as before)

$$G_1^{\pi\pi} = -\frac{x^2}{3}(4-x)e^{-x}, \quad (61)$$

$$G_2^{\pi\pi} = -\left[2 + 2x + \frac{1}{3}x^2 - \frac{1}{3}x^3\right]e^{-x}, \quad (62)$$

$$G_1^{\pi NN} = -\frac{1}{3}x^2 e^{-x}, \quad (63)$$

$$G_2^{\pi NN} = -\left(1 + x + \frac{1}{3}x^2\right)e^{-x}. \quad (64)$$

The new form-factors $G_1^{\pi\pi}$ and $G_2^{\pi\pi}$ stem from the $\pi\pi ee$ vertex while $G_1^{\pi NN}$ and $G_2^{\pi NN}$ (also given in Ref. [39]) stem from the $NN\pi ee$ vertex. In contrast to the zero-derivative case, the amplitudes stemming from these two vertices are of the same order in this minimal extension of the standard model.

The corresponding half-life, assuming that Eq. (58) represents the only decay amplitude, is

$$\frac{1}{T_{1/2}} = \frac{1}{64\pi^5 \ln 2} \left(\frac{\hbar c}{R}\right)^6 \frac{g_A^4}{\hbar} \frac{G_F^4}{\Lambda_{\beta\beta}^2 c^4} \int_{m_e}^{E_{\beta\beta} - m_e} dE_1 F(Z+2, E_1) F(Z+2, E_2) \frac{1}{2} \left\{ \left[\left| \beta_3 \mathcal{M}_2^{\pi\pi} + \frac{\sqrt{2}\Lambda_H}{g_A M} \zeta_5 \mathcal{M}_2^{\pi NN} \right|^2 + \left| \beta_4 \mathcal{M}_2^{\pi\pi} \right. \right. \right.$$
$$\left. + \frac{\sqrt{2}\Lambda_H}{g_A M} \zeta_6 \mathcal{M}_2^{\pi NN} \right|^2 \right] p_1 E_1 p_2 E_2 - \left[\left| \beta_3 \mathcal{M}_2^{\pi\pi} - \frac{\sqrt{2}\Lambda_H}{g_A M} \zeta_5 \mathcal{M}_2^{\pi NN} \right|^2 - \left| \beta_4 \mathcal{M}_2^{\pi\pi} - \frac{\sqrt{2}\Lambda_H}{g_A M} \zeta_6 \mathcal{M}_2^{\pi NN} \right|^2 \right] p_1 p_2 m_e^2 \right\},$$
$$(65)$$

with

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[10]This scale is inserted to make the operator in Eq. (54) dimensionless. It is canceled by a corresponding factor of $1/R^2$ in the rate.
[11]We also ignore recoil order corrections from the amplitude of Fig. 2(a) where $K_{\pi\pi}$ is of $\mathcal{O}(p^0)$. In this case, the rate will be dominated by terms in Eq. (53).

$$\mathcal{M}_2^{\pi\pi(\pi NN)} = \langle \Psi_{A,Z+2} | \sum_{ij} \left(\frac{R}{\rho_{ij}} \right)^3 [G_1^{\pi\pi(\pi NN)}(x_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j + G_2^{\pi\pi(\pi NN)}(x_{ij})T_{ij}]\tau_i^+ \tau_j^+ | \Psi_{A,Z} \rangle. \tag{66}$$

We can compare the rates of Eq. (53) and Eq. (65) by assuming that all dimensionless constants are of the order of unity with $1/\rho_{ij} \sim m_\pi$ and $\Lambda_H \sim 1$ GeV, and that the nuclear matrix elements cancel in the ratio

$$\frac{\text{Eq. (53)}}{\text{Eq. (65)}} \sim \frac{\Lambda_H^4}{m_\pi^4} \approx 10^3. \tag{67}$$

Note that this ratio agrees with our expectation based on power counting. We end this subsection by emphasizing that Eq. (65) is not the general formula for the $0\nu\beta\beta$-decay half-life at NNLO (which must include all contributing terms including loops, recoil effects, *NNNNee* terms and m_π^2 corrections) since the LO contributions should be added if they do not vanish from symmetry considerations before squaring the amplitude.

IV. PARTICLE PHYSICS MODELS

While our discussion so far has been quite general and independent of the underlying physics of the lepton-number violation, we apply in this section our EFT analysis to two particle physics models: RPV SUSY and the left-right symmetric (LRS) model.

A. RPV SUSY

R-parity-violating supersymmetry can contribute to $0\nu\beta\beta$ decay through diagrams like the one in Fig. 1(b). Since supersymmetric particles are heavy, their internal lines can be shrunk to a point in tree level diagrams yielding operators that involve only quarks and leptons. When the RPV superpotential is expanded to yield a lepton number violating Lagrangian, and a Fierz transformation is used to separate leptonic from quark currents, the result is [21]

$$\mathcal{L}_{qe} = \frac{G_F^2}{2M}\bar{e}(1+\gamma^5)e^c\left[(\eta_{\tilde{q}} + \eta_{\tilde{f}})(J_P J_P + J_S J_S)\right.$$
$$\left. - \frac{1}{4}\eta_{\tilde{q}}J_T^{\mu\nu}J_{T\mu\nu}\right], \tag{68}$$

where

$$J_P = \bar{q}\gamma^5\tau^+ q, \quad J_S = \bar{q}\tau^+ q,$$

$$J_T^{\mu\nu} = \bar{q}\sigma^{\mu\nu}(1+\gamma^5)\tau^+ q, \tag{69}$$

and $\eta_{\tilde{q}}, \eta_{\tilde{f}}$ are quadratic functions of the RPV SUSY parameter, λ'_{111} defined in Ref. [21]:

$$\eta_{\tilde{k}} = \frac{2\pi}{9}\frac{|\lambda'_{111}|^2 M}{G_F^2 m_{\tilde{q}}^4}\left[2\alpha_s\frac{1}{m_{\tilde{g}}} + \cdots\right],$$
$$\text{with} \quad \tilde{k} = \tilde{q}, \tilde{f}. \tag{70}$$

Here M is the nucleon mass, $m_{\tilde{q}}$ is a first generation squark mass, $m_{\tilde{g}}$ is the gluino mass, α_s is the running SU(3)$_C$ coupling, and the $+\cdots$ indicate contributions involving the first generation sleptons and lightest neutralino.[12] Note that the dependence on G_F and M cancels from Eq. (68), so that the effective lepton-quark $0\nu\beta\beta$-decay operator depends on five inverse powers of SUSY masses.

It is useful to rewrite Eq. (68) in terms of our operators $\mathcal{O}_{i\pm}^{++}$:

$$\mathcal{L}_{qe} = \frac{G_F^2}{2M}\bar{e}(1+\gamma^5)e^c\left[\frac{1}{2}(\eta_{\tilde{q}} + \eta_{\tilde{f}})\mathcal{O}_{2+}^{++} - \frac{3}{14}\eta_{\tilde{q}}(\mathcal{O}_{2+}^{++}\right.$$
$$\left. - \mathcal{O}_{2-}^{++})\right]. \tag{71}$$

The first thing to note is that $\mathcal{O}_{2-}^{++}$ can be neglected for $0^+ \to 0^+$ nuclear transitions. Secondly, from Table II we see that $\mathcal{O}_{2+}^{++}$ gives rise to LO $\pi\pi ee$ and NLO $NN\pi ee$ operators and therefore contributes to the long range $0\nu\beta\beta$-decay operator of Fig. 2(a) that is enhanced relative to the short range interaction of Fig. 2(d) as observed by direct calculation in Ref. [21], but derived with different assumptions about the scaling of the LEC.

From Eqs. (16), (30) and (32), it follows that the LO $\pi\pi ee$ operator contributes dominantly to the $0\nu\beta\beta$ decay in RPV SUSY. The corresponding half-life formula is Eq. (53) with $\beta_1 = \beta_2$ and with the substitution

$$\frac{1}{\Lambda_{\beta\beta}} \to \frac{1}{4M}\left(\frac{4}{7}\eta_{\tilde{q}} + \eta_{\tilde{f}}\right). \tag{72}$$

Obviously, a lower limit on the half-life can be interpreted as an upper limit on the coupling constants $\eta_{\tilde{q}}$ and $\eta_{\tilde{f}}$. Making further assumptions about masses of SUSY particles, one can ultimately obtain model-dependent upper limits on the coupling constant λ'_{111} as discussed in Ref. [39].

Next, let us compare the scaling rules used here and in Refs. [21] and [39]. In the previous section, we used NDA to extract the relevant scales out of the dimensionful LEC's by

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[12]The slepton/neutralino terms—which have complicated expressions—cause $\eta_{\tilde{q}} \neq \eta_{\tilde{f}}$. We have only shown the gluino contributions for illustrative purposes.

using the scaling rule Eq. (31). The alternative method used in Ref. [21] was to calculate the quark operator matrix element in the vacuum insertion approximation (VIA) and match the result to the hadron operator matrix element.

Specifically, for the LO $\pi\pi ee$ operator of Eq. (30) we found that the dimensionful LEC scaled as $\Lambda_H^2 f_\pi^2$ while the VIA would predict[13]

$$\text{LEC's} \sim \langle\pi^+|J_P J_P|\pi^-\rangle \approx \langle\pi^+|J_P|0\rangle\langle 0|J_P|\pi^-\rangle$$

$$= -2f_\pi^2 \frac{m_\pi^4}{(m_u+m_d)^2}, \quad (73)$$

where $m_{u,d}$ are the light quark masses. Taking $\Lambda_H = \Lambda_\chi = 4\pi f_\pi$, the chiral symmetry breaking scale, and $m_u + m_d = 11.6$ MeV we find

$$\frac{\text{NDA}}{\text{VIA}} \sim \frac{(4\pi f_\pi)^2 f_\pi^2}{2f_\pi^2 \dfrac{m_\pi^4}{(m_u+m_d)^2}} = 0.7. \quad (74)$$

The NDA scaling is thus slightly smaller than that obtained from the VIA. Although they give results of the same order, VIA has proved to be unreliable in other contexts (see, e.g., the study of rare kaon decays in Ref. [40]). We will therefore use NDA in what follows.

Referring to Table II, it follows that there should be additional, subdominant contributions from the operator $\pi\pi ee$ and from the $NN\pi ee$ operator at NNLO. The NNLO contributions from the $NN\pi ee$ vertex were considered in Ref. [39] where detailed numerical evaluations showed that they contribute on average about thirty times less than the LO contribution. Our systematic analysis leads to the same qualitative conclusion (namely with regards to the NNLO suppression of p^2/Λ_H^2 with respect to the LO), but differs from Ref. [39] in some respects.

First of all, not all NNLO contributions were included. In particular, as pointed out above, the NNLO $\pi\pi ee$ operator contributes to $0\nu\beta\beta$ decay at the same order as the $NN\pi ee$ operator (called 1π in Ref. [39]) and the form-factors $G_{1,2}^{\pi\pi}$ should be included.

Secondly, our analysis shows that the $NNNNee$ operator (the only one considered previously in this type of analysis) gives contributions at NNLO.[14] In Ref. [21] the suppression of that operator relative to the LO $\pi\pi ee$ contribution was

only by a factor of ten for ^{76}Ge which is larger than what would be expected from our power counting (see also Ref. [42]). However, this suppression is still in qualitative agreement with our analysis keeping in mind that considerable uncertainty remains in the evaluation of nuclear matrix elements. Furthermore, although the traditional method of calculating the short-range $0\nu\beta\beta$-decay operator using dipole form-factors [20] may yield results of the correct order, the method is unsystematic with uncontrollable errors that cannot be easily estimated.

B. Left-right symmetric model

We consider LRS models that contain a heavy right-handed neutrino, and mixing between the right-handed and left-handed gauge bosons with $g_L \approx g_R = g$ where g_L and g_R are the left-handed and right-handed gauge couplings. The LRS Lagrangian is taken to be invariant under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ where B,L are the baryon, lepton numbers, respectively. We will not be concerned with the CP-violating phases of the mixing matrix U^R of the right-handed quark generations (the right-handed equivalent of the Cabbibo-Kobayashi-Maskawa matrix, denoted here U^L) nor the precise nature of the relationship between U^R and U^L (e.g., manifest versus pseudo-manifest LRS model) as the order of magnitude of the constraints obtained from experiments are broadly robust to the different possibilities [43–46]. We will use the standard Higgs sector composed of a left-handed triplet, Δ_L, a right-handed triplet, Δ_R, and a multiplet, Φ, that respectively transform under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ according to $(L,R,Y) = (3,1,2)$, $(1,3,2)$ and $(2,2,0)$. Their vacuum expectation values are

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 \\ 0 \\ \Delta_L^0 \end{pmatrix}, \quad \langle\Delta_R\rangle = \begin{pmatrix} 0 \\ 0 \\ \Delta_R^0 \end{pmatrix},$$

$$\langle\Phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}. \quad (75)$$

Assume the following relation between the gauge and the mass eigenstates (ignoring the possibility of a CP-violating phase):

$$W_L = \cos\zeta W_1 + \sin\zeta W_2$$

$$W_R = -\sin\zeta W_1 + \cos\zeta W_2, \quad (76)$$

where ζ is a small mixing angle between the mass eigenstates and,

$$M_{W_1}^2 \cong \frac{g^2}{2}(\kappa^2 + \kappa'^2), \quad (77)$$

$$M_{W_2}^2 \cong \frac{g^2}{2}(\kappa^2 + \kappa'^2 + 2\Delta_R^{0^2}), \quad (78)$$

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[13]Note that we do not take into account the color factor 8/3 of Ref. [21] since it is a number of $\mathcal{O}(1)$ which does not involve any mass scale. It can therefore be absorbed in the LEC's which are undetermined. See also the footnote below Eq. (7).

[14]We note that the long-range operators considered in Ref. [41] through the induced pseudoscalar coupling terms of the nucleon current correspond to the NNLO contributions of Eq. (35). The results presented by the authors of Ref. [41] in the form-factor approach are compatible with the EFT analysis given here since they only considered left-handed hadronic currents.
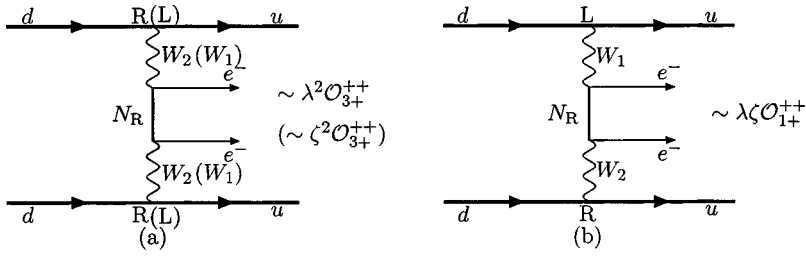
FIG. 4. Left-right symmetric model graphs. (a) involves the interaction of two right-handed (left-handed) currents while (b) depicts the interaction of left-handed and right-handed currents.

$$\zeta \cong \frac{\kappa\kappa'}{\Delta_R^{02}}, \tag{79}$$

where $M_{W_{1,2}}$ are the masses of $W_{1,2}$. From these equations and the fact that $|\kappa^2 + \kappa'^2|/2 \gg |\kappa\kappa'|$, we immediately obtain the important relation first derived in Ref. [47],[15]

$$\lambda \equiv \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 \geq \zeta. \tag{80}$$

Turning to experimental bounds on the masses and mixing angles, we will use for the lower limit on the right handed gauge boson $M_{W_2} > 715$ GeV [48], which corresponds roughly to

$$\lambda < 10^{-2}. \tag{81}$$

To put limits on the mixing angle, we use recent results from superallowed $0^+ \to 0^+$ β-decay in Ref. [49] that imply a violation of the unitarity of the CKM matrix at the 95% confidence level. In the LRS model, unitarity can be restored by taking a positive value for the mixing angle with magnitude

$$\zeta = 0.0016 \pm 0.0007, \tag{82}$$

given that one has

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9968 \pm 0.0014, \tag{83}$$

in the standard model only [49]. A range of $2 \times 10^{-4} \leq \zeta \leq 3 \times 10^{-3}$ is allowed at 95% confidence level. Note that the discrepancy in the unitarity condition cannot be resolved by adjusting λ because it enters the ordinary β-decay amplitude quadratically and, thus, produces a correction smaller than 10^{-4} [see Eq. (81)]. In what follows, we will consider the range $0 \leq \zeta \leq 3 \times 10^{-3}$ and use the central value of Eq. (82) for some specific estimates. Note that for the central value of ζ of Eq. (82), we obtain an upper limit on M_{W_2} from Eq. (80) of

$$M_{W_2} \leq M_{W_1}/\sqrt{\zeta} \to M_{W_2} \leq 2 \text{ TeV for } \zeta = 0.0016. \tag{84}$$

With these bounds on M_{W_2} and ζ, we can now estimate the relative order of magnitude of the graphs of Fig. 4.

When the right-handed neutrino and $W_{L,R}$ are integrated out, the amplitude of Fig. 4(a) reduces to an operator of the form $\mathcal{O}_{3+}^{++}$ while Fig. 4(b) reduces to an operator of the form $\mathcal{O}_{1+}^{++}$.[16] In previous treatments of $0\nu\beta\beta$ decay, only graph 4(a) with right-handed interacting currents is considered and the impact of W_L-W_R mixing is neglected. Our analysis of the previous sections implies that the hadronic operators generated by $\mathcal{O}_{3+}^{++}$ are suppressed by a factor of $p^2/\Lambda_H^2 \sim 10^{-2}$ relative to those generated by $\mathcal{O}_{1+}^{++}$. Hence, taking into account the fact that the coupling of a (right)left-handed current with a $(W_1)W_2$ involves a suppression factor of ζ while a W_2 internal line involves a suppression factor of λ, we expect the $\pi\pi$ operators generated by these quark operators to scale as

$$M_{4(a)}^{(LL)} \sim \zeta^2 \frac{p^2}{\Lambda_H^2} < 10^{-8}, \quad M_{4(a)}^{(RR)} \sim \lambda^2 \frac{p^2}{\Lambda_H^2} < 10^{-6},$$

$$M_{4(b)}^{(LR)} \sim \lambda\zeta < 10^{-5}, \tag{85}$$

with all else assumed equal. Therefore, even if ζ is ten times smaller than the central value in Eq. (82), the contribution stemming from the mixing of left-handed and right-handed gauge bosons is still non-negligible. It may even be dominant.

Such analysis may modify two constraints that relate the right-handed weak boson and neutrino masses, M_{W_2} and M_{N_R}, respectively.[17]

The first constraint stems from the requirement that the vacuum expectation value of the Higgs field Δ_R be a true minimum of the Higgs potential that generates the masses of the right-handed particles [50]. The vacuum is then stable against collapse. This imposes stringent constraints on the one-loop corrections to the effective potential [51–53]. In particular, the loop corrections will involve terms of the form $k\Delta_R^4 \ln(\Delta_R^2/\Delta_R^{02})$ where k is a constant that depends on the particle masses. For the vacuum to be stable at large values of Δ_R, k must be positive to ensure that the minimum at the VEV is a true minimum and not simply a local minimum. The condition $k > 0$ is equivalent to a condition on the masses. Following this formalism allows us to derive a relationship between M_{W_2} and M_{N_R}:

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[15]From here on, ζ will exclusively denote the magnitude of the mixing angle.

[16]Recall that the parity-odd LL/RR operator $\mathcal{O}_{3-}^{++}$ is suppressed at NNLO.

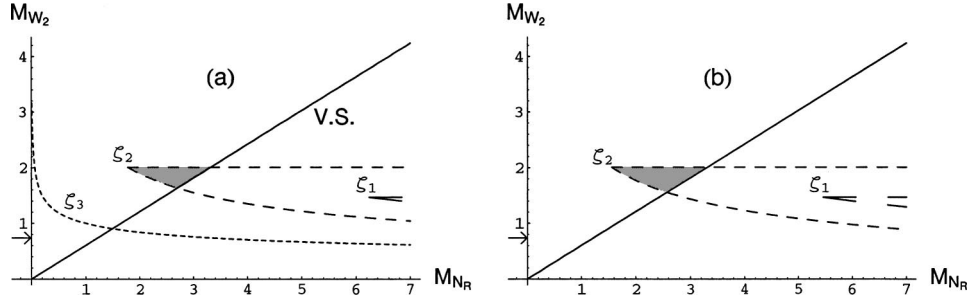[17]For illustrative purposes, we assume the existence of only one right-handed neutrino.

FIG. 5. Constraints on the right-handed weak boson and neutrino masses (in TeV) in the LRS model. The solid lines stem from the vacuum stability (V.S.) constraint of Eq. (86) while the hyphenated lines correspond to limits imposed from $0\nu\beta\beta$ decay and Eq. (80) with the following values of mixing angle from longest to shortest dashes: $\zeta_i = \{3 \times 10^{-3}, 1.6 \times 10^{-3}, 0\}$ with $i = 1,2,3$. Graphs (a) and (b) correspond to cases 1 and 2 of the text, respectively. Note that the value of the mixing angle $\zeta_3 = 0$ cannot occur for case 2 without simultaneously taking M_{W_2} to infinity, while ζ_2 corresponds to the central value obtained from CKM unitarity. The arrows indicate the lower bound $M_{W_2} \gtrsim 715$ GeV imposed by direct searches. The shaded, triangular regions in the graphs are the allowed values of the masses if the mixing angle is ζ_2.

$$1.65 M_{W_2} \gtrsim M_{N_R}. \tag{86}$$

This constraint is represented in the graphs of Fig. 5 by the fact that no value of (M_{N_R}, M_{W_2}) below the solid lines is allowed.[18]

A second relationship constraining M_{W_2} and M_{N_R} in the LRS model with mixing can be inferred from experimental limits on $0\nu\beta\beta$ decay [50,55] from Eq. (53) with $\Lambda_{\beta\beta} = M_{N_R}$ and choosing $\beta_1 = \beta_2 = 1$

$$|\zeta\lambda \pm \delta(\lambda^2 + \zeta^2)|^2$$

$$< \frac{9}{2} \frac{M_{N_R}^2}{\Lambda_H^4 G_{0\nu}^{(A,Z)} |\mathcal{M}_0^{(A,Z)}|^2 T_{1/2}^{(A,Z)}}$$

$$\equiv \nu^{(A,Z)2}, \tag{87}$$

$$G_{0\nu}^{(A,Z)} = (G_F \cos\theta_C g_A)^4 \left(\frac{\hbar c}{R}\right)^2 \frac{1}{32\pi^5 \hbar \ln 2}$$

$$\times \int_{m_e}^{E_{\beta\beta} - m_e} dE_1 F(Z+2, E_1) F(Z+2, E_2) p_1 E_1 p_2 E_2, \tag{88}$$

where $\nu^{(A,Z)}$ is defined by Eq. (87), $\Lambda_H \approx 1$ GeV, $T_{1/2}^{(A,Z)}$ is the current limit on the half-life of the $0\nu\beta\beta$-decay transition of a nucleus (A,Z) and where the functions $G_{0\nu}^{(A,Z)}$ were tabulated in Ref. [56] for various nuclei. The matrix element $\mathcal{M}_0^{(A,Z)}$ is defined in Eq. (54).[19] In Eq. (87) we have made explicit the scaling factors of Eq. (85) and also introduced a factor δ which parametrizes the p^2/Λ_H^2 suppression of the NNLO $0\nu\beta\beta$-decay operators relative to the LO operators. As mentioned above, the numerical evaluations in Ref. [39] suggest that $\delta \approx 1/30$ which is the conservative number we

will use. Thus, the λ^2 term stems from the exchange of two W_2's while the ζ^2 term comes from the exchange of two W_1's where ζ, being the magnitude of the mixing angle, is always positive. The relative sign between the $\zeta\lambda$ and $\delta(\lambda^2 + \zeta^2)$ terms on the LHS of Eq. (87) cannot be predicted by EFT since we do not know the sign of the LEC's.

For the values of half-life, $G_{0\nu}^{(A,Z)}$ and $\mathcal{M}_0^{(A,Z)}$, we will use the ones determined for [76]Ge

$$T_{1/2}^{Ge} \gtrsim 1.9 \times 10^{25} \text{ yrs}, \quad (G_{0\nu}^{Ge})^{-1} = 4.09$$

$$\times 10^{25} \text{eV}^2 \text{ yrs}, \quad \mathcal{M}_0^{Ge} = 2, \tag{89}$$

where we extracted the value of $\mathcal{M}_0^{Ge}$ from the value of $\mathcal{M}^{2\pi}$ calculated in Ref. [39] and the limit on the half-life is at 90% confidence level [57]. With these numbers, Eq. (87) becomes

$$|\zeta\lambda \pm \delta(\lambda^2 + \zeta^2)| < \nu^{Ge} = \sqrt{\frac{9}{3.8}} \left(\frac{M_{N_R}}{\text{TeV}}\right) 10^{-6}. \tag{90}$$

In the limit $\zeta \to 0$ we obtain

$$M_{W_2} > \left(\delta \sqrt{\frac{3.8}{9}} \frac{\text{TeV}}{M_{N_R}} 10^6\right)^{1/4}$$

$$M_{W_1} \cong \left(\frac{\text{TeV}}{M_{N_R}}\right)^{1/4} \text{TeV}. \tag{91}$$

Our result is slightly smaller than the result obtained in Refs. [50,55] for zero mixing angle. In Ref. [50] this constraint was calculated with the short range NNLO *NNNNee* operator of Fig. 2(d) using the dipole form factor approach. Note that we can reproduce exactly the values given in Refs. [50,55] by slightly adjusting the unknown constants β_1, β_2 in Eq. (53).

To extract the constraint imposed by Eq. (90) on M_{N_R} and M_{W_2}, we need to consider three cases:

(1) The LO and NNLO terms have the same sign which corresponds to taking the plus sign in Eq. (90).

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[18]In Ref. [50], the constraint that appears is $0.95 M_{W_2} \gtrsim M_{N_R}$, the result of a typo [54].

[19]From here on, we take $\cos\theta_C = 1$.

(2) They have opposite signs with $\zeta\lambda > \delta(\lambda^2+\zeta^2)$.

(3) They have opposite signs with $\zeta\lambda < \delta(\lambda^2+\zeta^2)$.

We note that in all three cases, the upper limit on M_{W_2} for $\zeta>0$ implied by Eq. (80) always holds.

Case 1. When solving the quadratic equation in λ, we must keep the root that has the same limit as Eq. (90) when $\delta,\zeta\rightarrow 0$,

$$\zeta\leqslant\lambda\leqslant\frac{1}{2\delta}[-\zeta+\sqrt{(1-4\delta^2)\zeta^2+4\delta\nu}], \qquad (92)$$

where we used Eq. (80) to obtain the first inequality. The first thing to note is that Eqs. (90)–(92) impose a lower-limit on the mass of the right-handed neutrino

$$\frac{M_{N_R}}{\text{TeV}} > \sqrt{\frac{3.8}{9}}10^6(1+2\delta)\zeta^2\cong 1.8, \qquad (93)$$

assuming the central value of Eq. (82). This lower limit only depends on the mixing angle since δ can in principle be calculated. In Fig. 5(a), the constraint Eq. (92) is plotted for three values of the mixing angle $\zeta_i=\{3\times 10^{-3},1.6\times 10^{-3},0\}$.

In Fig. 5(a), we see that the larger the mixing angle, the larger the parameter space that is ruled out. In particular, for ζ_1, the largest angle that we are considering, the region allowed by Eqs. (92) and (93) is located below the constraint imposed by vacuum stability. Hence, a value of the mixing angle as large as ζ_1 is excluded. In contrast, the central mixing angle value from CKM unitarity, ζ_2, allows for a triangular region [bordered by the vacuum stability curve and Eq. (92)] of possible values for the masses. In particular, for ζ_2, we note that not only do we have the upper-limit of Eq. (84), but we also have $M_{W_2}\geqslant 1.6$ TeV and $M_{N_R}\leqslant 3.2$ TeV, which would constitute more stringent limits than that obtained from direct searches so far. For zero mixing angle, the entire region that is simultaneously above the vacuum stability curve and the curve stemming from Eq. (91) is allowed. Thus, in general, as the mixing angle increases, the allowed region of parameter space shrinks while the minimum value of M_{W_2} increases. The maximum mixing angle that results in a nonvanishing allowed region[20]

$$\zeta\leqslant 2.2\times 10^{-3} \quad \text{with}$$

$$M_{W_2}\cong 1.7 \text{ TeV}, \quad M_{N_R}\cong 2.8 \text{ TeV}. \qquad (94)$$

Case 2. The condition of validity for this case, $\zeta\lambda > \delta(\lambda^2+\zeta^2)$, rules out the positive root of the quadratic equation in λ, Eq. (90). The limits on λ are then

$$\zeta\leqslant\lambda\leqslant\frac{1}{2\delta}[\zeta-\sqrt{(1-4\delta^2)\zeta^2-4\delta\nu}]. \qquad (95)$$

We note that Eq. (95) imposes upper and lower limits on both M_{N_R} and M_{W_2},

$$\sqrt{\frac{3.8}{9}10^6(1-2\delta)\zeta^2}\leqslant\frac{M_{N_R}}{\text{TeV}}$$

$$\leqslant\sqrt{\frac{3.8}{9}10^6\frac{1}{4\delta}(1-4\delta^2)\zeta^2},$$

$$\sqrt{\frac{2\delta M_{W_1}^2}{\zeta}}\leqslant M_{W_2}. \qquad (96)$$

For ζ_2, we obtain in particular, 1.6 TeV$\leqslant M_{N_R}\leqslant 12$ TeV and $M_{W_2}\geqslant 0.51$ TeV. Note that the upper limit on M_{N_R} for ζ_2 is well above the constraint stemming from vacuum stability, Eq. (86), combined with the upper limit on M_{W_2} given in Eq. (84). Equations (96) also imply a new relationship between M_{N_R} and M_{W_2} applicable only to case 2,

$$M_{W_2}\leqslant\left(\sqrt{\frac{3.8}{9}}\frac{10^6\text{TeV}}{4\delta M_{N_R}}\right)^{1/4}M_{W_1}$$

$$\cong 3.8\left(\frac{\text{TeV}}{M_{N_R}}\right)^{1/4}\text{TeV}, \qquad (97)$$

where we neglected the $4\delta^2$ term.

From the plot in Fig. 5(b), the same analysis as in case 1 follows: as the mixing angle increases, the region of allowed values for the masses shrinks. As in case 1, ζ_1 is already excluded while ζ_2 allows for a triangular region of possible values for the masses. We note that Eq. (97) does not further constrain the allowed region of parameter space and has been included here for completeness. For this case, the maximum mixing angle is calculated to be,

$$\zeta\leqslant 2.1\times 10^{-3} \quad \text{with}$$

$$M_{W_2}\cong 1.8 \text{ TeV}, \quad M_{N_R}\cong 2.9 \text{ TeV}, \qquad (98)$$

which are similar to the values found for case 1.

Case 3. For the case $\zeta\lambda < \delta(\lambda^2+\zeta^2)$, we must keep the root that gives the correct upper-limit when $\zeta\rightarrow 0$ since now the limit $\delta\rightarrow 0$ cannot be taken. With the constraint on λ stemming from the condition of validity of this case, $\zeta\lambda < \delta(\lambda^2+\zeta^2)$, the inequalities satisfied by λ are

$$\lambda\leqslant\frac{\zeta}{2\delta}(1-\sqrt{1-4\delta^2}),$$

$$\frac{\zeta}{2\delta}(1+\sqrt{1-4\delta^2})\leqslant\lambda$$

$$\leqslant\frac{1}{2\delta}[\zeta+\sqrt{(1-4\delta^2)\zeta^2+4\delta\nu}]. \qquad (99)$$

Thus, values of λ located between the roots $\lambda_\pm=\zeta/(2\delta)(1\pm\sqrt{1-4\delta^2})$ are excluded.[21] Note that for the two nonzero

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[20]Actually, a point in this case.

[21]Since $1/(2\delta)[\zeta+\sqrt{(1-4\delta^2)\zeta^2+4\delta\nu}]\gg\zeta/(2\delta)(1-\sqrt{1-4\delta^2})$ for all nonzero values of ζ and ν, we need only be concerned with the λ_- upper limit on λ.

TABLE III. Order at which the left-right symmetric models with/without mixing and RPV SUSY contribute to the $0\nu\beta\beta$-decay operators of Fig. 2.

Models	Fig. 2(a)	Figs. 2(b),2(c)	Fig. 2(d)
LRSM $\zeta=0$	p^0	p^0	p^0
LRSM $\zeta\neq0$	p^{-2}	p^0	p^0
RPV SUSY	p^{-2}	p^{-1}	p^0

angles considered in Fig. 5, the ranges defined by $\lambda\geq\lambda_+$ have already been ruled out by direct searches of right-handed bosons [44] and we are left with the first constraint of Eqs. (99) which does not depend on limits from $0\nu\beta\beta$ decay. However case three appears to be entirely ruled out by Eq. (80). Indeed, approximating the remaining constraint of Eq. (99) to $\lambda<\delta\zeta$, we see that both constraints cannot be satisfied simultaneously.

From Fig. 5 and the three cases considered above, it follows that the effect of mixing on the mass constraint can be very important—a point not recognized previously. In particular, we see that nonzero mixing angles will generally exclude much of the parameter space by imposing much more stringent constraints on the masses and that the mass of the right-handed neutrino is bounded from below. We also note that quite generally, the mixing angle is constrained to be $\leq2.2\times10^{-3}$.

We conclude this section by briefly comparing the left-right symmetric model and RPV SUSY. We observe that although both models can contribute to $\mathcal{O}(p^{-2})$ to the operator of Fig. 2(a), only RPV SUSY contributes to Figs. 2(b) and 2(c) to $\mathcal{O}(p^{-1})$ as discussed in the previous section. These results are summarized in Table III.

V. CONCLUSIONS

Neutrinoless double beta decay will continue to probe "new" physics scenarios that violate lepton number for some time to come. The existence of such scenarios is intimately related to the nature of the neutrino, namely, whether or not it is a Majorana particle. If a significant signal for $0\nu\beta\beta$ decay were to be observed, one would know that the neutrino is a Majorana particle. However, one would not know whether the rate is dominated by the exchange of a light Majorana neutrino or by some other L-violating process that is also responsible for generation of the Majorana mass. Such L-violating processes could involve mass scales ($\Lambda_{\beta\beta}$) well above the weak scale. Thus, it is important to study the implications of $0\nu\beta\beta$ decay for such scenarios—a task which we have undertaken in the present paper.

In doing so, we have applied the ideas of EFT, which is appropriate in this case because there is a clear distinction of scales: $\Lambda_{\beta\beta}\gg\Lambda_H\gg p$. We wrote down all nonequivalent quark-lepton operators of dimension nine that contribute to $0\nu\beta\beta$ decay, and showed how to match them to hadron-lepton operators by using their transformation properties under parity and chiral SU(2). We then organized the hadron-lepton operators ($\pi\pi ee$, $NN\pi ee$ and $NNNNee$) in powers of p/Λ_H and discussed how the symmetries determine the type of hadronic operators that can be generated by each quark operator. In particular, we demonstrated that the hadronic operators generated by the interaction of two left-handed or two right-handed quark currents are always of NNLO. We also showed that EFT can classify particle physics models of $0\nu\beta\beta$ decay in terms of the hadron-lepton operators they can generate and to what order these operators enter. In particular, we found that left-right symmetric models with mixing can potentially and considerably modify existing constraints on the masses of the right-handed particles. Indeed, a nonzero mixing angle gives far more stringent constraints on the allowed values of the masses of right-handed particles including a correlation between the mass of the right-handed neutrino and the mixing angle. We also found that a necessary condition for the existence of a region of allowed values of M_{W_2} and M_{N_R} is $\zeta\leq2.2\times10^{-3}$. For RPV SUSY models, we have also confirmed the previous conclusion that the dominant contribution stems from the $\pi\pi ee$ operator which leads to more severe constraints on the corresponding RPV SUSY parameters than traditionally believed. More generally, with this EFT analysis and using Table II, it can be immediately known what hadron-lepton operators can be generated by any quark-lepton operators appearing in any particle physics model that gives rise to $0\nu\beta\beta$ decay, and to what order these hadron-lepton operators will contribute. Finally, we note that deriving detailed information about a given scenario for L-violation will require combining information from a variety of measurements. As our analysis of the left-right symmetric model shows, using studies of $0\nu\beta\beta$ decay in conjunction with precision electroweak measurements (e.g., light quark β decay) and collider experiments can more severely constrain the particle physics parameter space than can any individual probe alone. Undertaking similar analysis for other new physics scenarios and other probes of L-violation constitutes an interesting problem for future study.

APPENDIX A: EQUIVALENT AND VANISHING QUARK OPERATORS

All operators proportional to $\bar{e}^c\gamma_\mu e$ and $\bar{e}^c\sigma_{\mu\nu}e$ vanish identically by virtue of the fact that the electron fields are Grassmann variables. For example

$$\bar{e}^c\gamma_\mu e = ie_\alpha\gamma^0_{\alpha\beta}\gamma^2_{\beta\sigma}\gamma^\mu_{\sigma\delta}e_\delta$$

$$= -ie_\delta(\gamma^\mu_{\delta\sigma})^{\mathrm{T}}\gamma^2_{\sigma\beta}\gamma^0_{\beta\alpha}e_\alpha$$

$$= i e^{\mathrm{T}} \gamma^2 \gamma^0 \gamma^\mu e$$

$$= -\bar{e}^c \gamma_\mu e$$

$$= 0. \tag{A1}$$

Note also that $\gamma^5 \sigma^{\mu\nu} = 2i\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$ implies that $\bar{e}^c \gamma^5 \sigma_{\mu\nu} e$ also vanish identically. In Ref. [17], these operators were incorrectly included in their super-formula.[22]

Other color singlet operators that could potentially contribute to $0\nu\beta\beta$ decay are

$$\mathcal{O}_{6+}^{++} = (\bar{q}_L^a \tau^+ q_{R,a})(\bar{q}_R^b \tau^+ q_{L,b}) = \frac{1}{6}\mathcal{O}_{1+}^{++}, \tag{A2}$$

$$\mathcal{O}_{7\pm}^{++} = (\bar{q}_R^a \tau^+ \sigma^{\mu\nu} q_{L,a})(\bar{q}_R^b \tau^+ \sigma_{\mu\nu} q_{L,b})$$
$$\pm (\bar{q}_L^a \tau^+ \sigma^{\mu\nu} q_{R,a})(\bar{q}_L^b \tau^+ \sigma_{\mu\nu} q_{R,b})$$
$$= \frac{12}{7}\mathcal{O}_{2\pm}^{++}, \tag{A3}$$

$$\mathcal{O}_{8+}^{++} = (\bar{q}_L^a \tau^+ \sigma^{\mu\nu} q_{R,a})(\bar{q}_R^b \tau^+ \sigma_{\mu\nu} q_{L,b}) = 0, \tag{A4}$$

$$\mathcal{O}_{9\pm}^{++,\mu} = (\bar{q}_L^a \tau^+ \sigma^{\mu\nu} q_{R,a} + \bar{q}_R^a \tau^+ \sigma^{\mu\nu} q_{L,a})$$
$$\times (\bar{q}_L^b \tau^+ \gamma_\nu q_{L,b} \pm \bar{q}_R^b \tau^+ \gamma_\nu q_{R,b})$$
$$= \frac{-i}{1\pm8}\mathcal{O}_{4\pm}^{++,\mu}, \tag{A5}$$

$$\mathcal{O}_{10\pm}^{++,\mu} = (\bar{q}_L^a \tau^+ \sigma^{\mu\nu} q_{R,a} - \bar{q}_R^a \tau^+ \sigma^{\mu\nu} q_{L,a})$$
$$\times (\bar{q}_L^b \tau^+ \gamma_\nu q_{L,b} \mp \bar{q}_R^b \tau^+ \gamma_\nu q_{R,b})$$
$$= \frac{-i}{1\mp8}\mathcal{O}_{5\pm}^{++,\mu}, \tag{A6}$$

where the Latin indices denote color and terms that involve the product of color octet currents are ignored (see below). Using Fierz transformations and the following formula,

$$\delta_{ab}\delta_{cd} = \frac{1}{3}\delta_{ad}\delta_{cb} + \frac{1}{2}\sum_{i=1}^{8}\lambda_{ad}^i\lambda_{cb}^i, \tag{A7}$$

it is easy to prove Eqs. (A2)–(A6). Note that the second term on the right-hand side of Eq. (A7) represents the product of two color octet currents. This term does not contribute since the asymptotic states are colorless and a completeness relation involving only hadronic states can be inserted between the currents. We therefore neglect this contribution.

Even though two Fierz-related operators can arise due to different short-distance dynamics, they are physically indis-

tinguishable. Note that in Ref. [17], these indistinguishable operators were included as separate operators.

APPENDIX B: NAÏVE DIMENSIONAL ANALYSIS SCALING RULE

To determine the scaling rules of the various fields appearing in the chiral Lagrangian, start with the relation between the axial current and the pion decay constant [40],

$$\langle 0|A^{a,\mu}|\pi^b(p)\rangle = i\delta^{ab}f_\pi p^\mu, \tag{B1}$$

which implies that π is normally normalized by f_π. Recalling that chiral perturbation theory is an expansion in powers of p/Λ_H, we scale pion derivatives by Λ_H noting that pion loop corrections will involve factors of $p^2/(4\pi f_\pi)^2$; this suggests that $\Lambda_H \approx 4\pi f_\pi$.

Since the action is dimensionless, we also have from the kinetic energy term of the pion field

$$\int \mathrm{d}^4 x \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} = \int \mathrm{d}^4 x (\Lambda_H f_\pi)^2 \frac{\partial^\mu}{\Lambda_H}\frac{\vec{\pi}}{f_\pi} \cdot \frac{\partial_\mu}{\Lambda_H}\frac{\vec{\pi}}{f_\pi}. \tag{B2}$$

This shows that we can associate with $\mathrm{d}^4 x$ the scale $(\Lambda_H f_\pi)^2$. This is the origin of the last factor of Eq. (31). From the parity-conserving pion nucleon coupling, we have

$$\int \mathrm{d}^4 x \frac{g_A}{f_\pi}\bar{N}\gamma^5\slashed{\partial}\pi N$$

$$= \int \mathrm{d}^4 x (\Lambda_H f_\pi)^2 \frac{g_A}{\Lambda_H f_\pi^2}\bar{N}\gamma^5\frac{\slashed{\partial}}{\Lambda_H}\frac{\pi}{f_\pi}N. \tag{B3}$$

This shows that we can associate the scale $\Lambda_H f_\pi^2$ with $\bar{N}N$.

Next, we note that since the axial current at the quark level is given by $\bar{q}\gamma^5\gamma^\mu q$ while a contribution to the axial current at the hadronic level is $\bar{N}\gamma^5\gamma^\mu N$, we can also associate with $\bar{q}q$ the scale $\Lambda_H f_\pi^2$. For a $0\nu\beta\beta$-decay quark-lepton operator, this implies

$$\frac{G_F^2}{\Lambda_{\beta\beta}}\int \mathrm{d}^4 x (\bar{q}\Gamma q)(\bar{q}\Gamma' q)(\bar{e}\Gamma'' e^c)$$

$$= \frac{G_F^2 f_\pi^2}{\Lambda_{\beta\beta}}\int \mathrm{d}^4 x (\Lambda_H f_\pi)^2 \frac{\bar{q}\Gamma q}{\Lambda_H f_\pi^2}\frac{\bar{q}\Gamma' q}{\Lambda_H f_\pi^2}\bar{e}\Gamma'' e^c. \tag{B4}$$

Therefore, we can associate the scale $G_F^2 f_\pi^2/\Lambda_{\beta\beta}$ with the lepton bilinears. This explains the origin of the scaling rule in Eq. (31).

APPENDIX C: NLO NUCLEAR OPERATORS

Here we present the results for Figs. 2(b) and 2(c). The Lagrangian Eq. (39) gives

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[22]However, they neglected them in their final analysis because they worked in the s-wave approximation.

$$(b)+(c)=4i\frac{g_A M \Lambda_{H-}}{\sqrt{2}}\bar{u}_{e1}\gamma^2\gamma^0(\zeta_1+\zeta_2\gamma^5)\bar{u}_{e2}^T\left[\frac{(\bar{u}_{p3}u_{n1})(\bar{u}_{p4}\gamma^5 u_{n2})}{(q_2^2-m_\pi^2+i\epsilon)}+\frac{(\bar{u}_{p3}\gamma^5 u_{n1})(\bar{u}_{p4}u_{n2})}{(q_1^2-m_\pi^2+i\epsilon)}\right]+4i\frac{g_A M}{\sqrt{2}f_\pi}\bar{u}_{e1}\gamma_\mu\gamma^2\gamma^0\gamma^5\bar{u}_{e2}^T$$

$$\times\left[\frac{(\bar{u}_{p4}\gamma^5 u_{n2})}{(q_2^2-m_\pi^2+i\epsilon)}\bar{u}_{p3}(\zeta_3+\zeta_4\gamma^5)\gamma^\mu u_{n1}+\frac{(\bar{u}_{p3}\gamma^5 u_{n1})}{(q_1^2-m_\pi^2+i\epsilon)}\bar{u}_{p4}(\zeta_3+\zeta_4\gamma^5)\gamma^\mu u_{n2}\right]. \tag{C1}$$

After taking the nonrelativistic limit and performing a Fourier transform we obtain

$$\text{F.T.}(\text{C1})\simeq\frac{1}{2\pi}\frac{m_\pi}{\sqrt{2}g_A\Lambda_H}\frac{g_A^2\Lambda_H^2}{\Lambda_{\beta\beta}^5}\delta(\vec{x}_1-\vec{x}_3)\delta(\vec{x}_2-\vec{x}_4)\frac{e^{-x}}{\rho}\left(1+\frac{1}{x}\right)\{\bar{u}_{e1}\gamma^2\gamma^0(\zeta_1+\zeta_2\gamma^5)\bar{u}_{e2}^T(\delta_{24}\chi_3^\dagger\vec{\sigma}\cdot\hat{\rho}\chi_1-\delta_{13}\chi_4^\dagger\vec{\sigma}\cdot\hat{\rho}\chi_2)$$

$$+\bar{u}_{e1}\gamma_\mu\gamma^2\gamma^0\gamma^5\bar{u}_{e2}^T[-\chi_3^\dagger(\zeta_3\delta^{\mu 0}-\zeta_4\sigma^i\delta^{\mu i})\chi_1\chi_4^\dagger\vec{\sigma}\cdot\hat{\rho}\chi_2+\chi_4^\dagger(\zeta_3\delta^{\mu 0}-\zeta_4\sigma^i\delta^{\mu i})\chi_2\chi_3^\dagger\vec{\sigma}\cdot\hat{\rho}\chi_1]\}. \tag{C2}$$

One can check explicitly that this nuclear operator is parity-odd and does not contribute to the $0^+\rightarrow 0^+$ nuclear transitions. Note also the extra factor of m_π/Λ_H relative to the LO contribution of Eq. (49) which is consistent with the power counting of Eq. (1).

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