Gluonic *B* and J/ψ decays into the η' meson

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Inclusive and exclusive *B* decays into the η' meson plus others are investigated in a model based on the QCD anomaly. The invariant mass distribution is discussed. A constraint on the effective coupling is obtained from data on the exclusive decay modes. The branching ratio of $J/\psi \rightarrow \eta' \eta \gamma$ is predicted to be as large as 5.4×10^{-5} , which can be tested in the forthcoming CLEO *c* experiments.

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I. INTRODUCTION

A few years ago, unexpectedly large branching ratios of *B* decaying into final states with an η' meson such as *B* $\rightarrow \eta' X_s$ and $B \rightarrow \eta' K$ were observed by the CLEO Collaboration $\lceil 1,2 \rceil$ and recently confirmed by the BaBar $\lceil 3 \rceil$ and Belle [4] Collaborations. This stimulated much theoretical activity in understanding the special role of the η' meson in *B* decays. As the contribution of traditional four-quark operators from the effective Hamiltonian in the standard model (SM) is far below the data [5,6], various exotic mechanisms were introduced, such as a large coupling between the gluon and η' through the QCD anomaly $[7-11]$, intrinsic $\overline{c}c$ content inside η' [12,13], and positive interference between several contributions in the SM $[14,15]$, etc. The large contribution arising from new physics was also discussed $[8,16]$. Among those theoretical efforts, the possible enhancement from the QCD anomaly is of particular interest since it is well known that the η' meson plays a very special role in the dynamics of low energy QCD $[17]$.

The mass eigenstates η' and η are related to flavor octet and singlet states η_8 , η_0 through a mixing matrix:

$$
\eta = \eta_s \cos \theta - \eta_0 \sin \theta, \quad \eta' = \eta_s \cos \theta + \eta_0 \sin \theta, \quad (1)
$$

where θ is the mixing angle and η_8, η_0 have the flavor content $\eta_8 = (1/\sqrt{6})(\bar{u}u + \bar{d}d - 2\bar{s}s)$ and η_0 $= (1/\sqrt{3})(\bar{u}u + \bar{d}d + \bar{s}s)$. The associated axial currents are $j_5^{\mu}{}^8 = (1/\sqrt{6})(\bar{u}\gamma^{\mu}\gamma_5 u + \bar{d}\gamma^{\mu}\gamma_5 d - 2\bar{s}\gamma^{\mu}\gamma_5 s)$ and $j_5^{\mu}{}^0$ $=(1/\sqrt{3})(\overline{u}\gamma^{\mu}\gamma_5u+\overline{d}\gamma^{\mu}\gamma_5d+\overline{s})$ respectively. Through the QCD anomaly, the divergence of the flavor singlet axial current is nonzero and is given by

$$
\partial_{\mu} j_{5}^{\mu 0} = \frac{1}{\sqrt{3}} \left(2i \sum_{q=u,d,s} m_{q} \overline{q} \gamma_{5} q + \frac{3 \alpha_{s}}{4 \pi} G_{\mu \nu} \widetilde{G}^{\mu \nu} \right) \tag{2}
$$

where $\tilde{G}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$ is the dual of $G_{\mu\nu}$. This breaks the global $U(1)$ ^A symmetry for massless particles and prevents the flavor singlet state η_0 from being a Goldstone boson of chiral $SU(3)_L \otimes SU(3)_R$ symmetry. The QCD anomaly gives its main contribution to the large mass of $\eta'(m_{\eta'}=0.958 \text{ GeV})$ which is much heavier than the other flavor octet states such as π and *K* and suggests a large gluon content in η' .

The QCD anomaly indicates a strong coupling between η' and the gluon field. It is then natural to understand the large branching ratio of $B \rightarrow \eta' X_s$ through the QCD anomaly. In the literature there are two different ways to handle this problem. One is through the two-body decay process $b \rightarrow s \eta'$ from some effective Hamiltonian due to the QCD anomaly [9]. The other is through the three-body process $b \rightarrow s g \eta'$ [7,8]. In the first step of the decay, the *b* quark decays into the *s* quark and a virtual gluon *g**, then *g** decays into η' and an on-shell gluon *g*. This model has some advantages in explaining the spectrum of invariant mass distribution of recoiling hadrons. However, the effective $gg^*\eta'$ vertex seems to be too small from various approaches $[8,18-$ 20]. In these approaches, the effective coupling between η' and the gluon may contain complicated nonperturbative quark-gluon interactions. It is then better to treat it as a free phenomenological parameter rather than to evaluate it from perturbative calculations $[9]$.

In this note we focus on the phenomenological analysis of the first mechanism. The effective Lagrangian in this model is given by $[9]$

$$
H_{eff} = a\alpha_s G_F \overline{s}_L b_R G_{\mu\nu} \widetilde{G}^{\mu\nu} + \text{H.c.},\tag{3}
$$

where α_s and G_F are the strong coupling constant and Fermi constant, and *a* is the effective coupling. From this effective Hamiltonian, the decay $B \rightarrow \eta' X_s$ arises from the subprocess $b \rightarrow s \eta'$. The evaluation of matrix elements is straightforward:

$$
\langle s \eta' | H_{eff} | b \rangle = a \alpha_s G_F \langle s | \overline{s}_L b_R | b \rangle \langle \eta' | G_{\mu\nu} \widetilde{G}^{\mu\nu} | 0 \rangle. \tag{4}
$$

Applying the relation $\langle 0 | j_{5}^{\mu 8(0)} | \eta_{8(0)} \rangle = i f_{8(0)} P^{\mu}$ to the divergences of both flavor singlet and octet axial currents and ignoring small *u*,*d* quark masses, the matrix elements $\langle 0|G_{\mu\nu}\tilde{G}^{\mu\nu}|\eta'\rangle$ can be rewritten as

$$
\langle 0 | \alpha_s G_{\mu\nu} \tilde{G}^{\mu\nu} | \eta' \rangle = \frac{4\pi}{3} \sqrt{\frac{3}{2}} m_{\eta'}^2 (f_8 \sin \theta + \sqrt{2} f_0 \cos \theta),
$$

$$
\langle 0 | \alpha_s G_{\mu\nu} \tilde{G}^{\mu\nu} | \eta \rangle = \frac{4\pi}{3} \sqrt{\frac{3}{2}} m_{\eta}^2 (f_8 \cos \theta - \sqrt{2} f_0 \sin \theta).
$$
 (5)

In the *b* quark rest frame, the decay branching ratio is given by

$$
BR(B \to \eta' X_s) = \frac{\pi}{12} \tau_B a^2 G_F^2 m_{\eta'}^4 (f_8 \sin \theta + \sqrt{2} f_0 \cos \theta)^2
$$

$$
\times \frac{(m_b^2 - m_{\eta'}^2)^2}{m_b^3},
$$
(6)

$$
BR(B \to \eta X_s) = \frac{\pi}{12} \tau_B a^2 G_F^2 m_{\eta}^4 (f_8 \cos \theta - \sqrt{2} f_0 \sin \theta)^2
$$

$$
\times \frac{(m_b^2 - m_{\eta}^2)^2}{m_b^3},\tag{7}
$$

where τ_B is the lifetime of a *B* meson.

II. RECOIL MASS DISTRIBUTION

For two-body-like subprocesses such as $b \rightarrow \eta' s$, the invariant mass is directly related to the energy E_{η} of the η ['] meson through the relation $m_X^2 = m_B^2 + m_{\eta'}^2 - 2m_B E_{\eta'}$, where m_B and $m_{\eta'}$ are the masses of the *B* and η' mesons. The small *s* quark mass has been ignored. In the two-body decay of $b \rightarrow \eta' s$, the energy of η' is fixed from energymomentum conservation. A typical value of the pole mass m_b =4.8 GeV will lead to a narrow peak with the invariant mass of $m_{X_s} \approx 1.5$ GeV. This seems to be disfavored by the current data since the experiment reported a peak at about 2 GeV with a relatively large width in the recoil mass distribution $|1,3|$.

However, the above estimation may be too naive. Note that in the two-body decay process, the exact distribution of the recoil mass strongly depends on the wave function of the *B* meson, which is hard to estimate theoretically. It is too early to draw the conclusion that the current data already disfavor all the two-body models.

To illustrate the nonperturbative bound state effects here we adopt a simple model proposed by Altarelli *et al.* [21] a number of years ago which is based on the Fermi motion of the *b* quark inside a *B* meson. The basic idea of this model is that the Fermi motion of the *b* quark and the spectator quark *q* in the *B* meson make them have back-to-back relative three-momenta **p** in the *B* rest frame. The momentum is assumed to obey a Gaussian distribution as follows:

$$
\phi(p) = \frac{4}{\sqrt{\pi p_p^3}} e^{-p^2/p_p^2}, \quad p = |\mathbf{p}|,\tag{8}
$$

where $\phi(p)$ is normalized as $\int_0^{\infty} \phi(p) p^2 dp = 1$. The mean value of *p* is $\langle p \rangle = \frac{3}{2} p_F$. In this model the spectator quark *q* is always handled as on shell while the *b* quark is treated as off shell. Through energy-momentum conservation, the effective mass *W* of the *b* quark is determined as

$$
W^2 = m_B^2 + m_q^2 - 2m_B\sqrt{m_q^2 + p^2}
$$
 (9)

and the energy of the *b* quark is $E_W = \sqrt{W^2 + p^2}$. Here, one parameter p_F is introduced which specifies both the distribution width and the mean value. As p_r is linked to the average

FIG. 1. Recoil mass distribution from process $b \rightarrow s \eta'$ with Fermi motion included. The solid, dashed, and dotted curves correspond to p_F = 0.6,0.5, and 0.4 GeV. The value of m_q is fixed at 0.15 GeV. The shadowed area indicates the acceptance cut of m_X ,2.35 GeV from the CLEO experiment.

energy of the *b* quark inside the *B* meson, in principle it can be calculated from theories based on nonperturbative methods or from some models. For example, calculations from the QCD sum rule give $p_F = 0.58 \pm 0.06$ GeV [22], and the value from the relativistic quark model is 0.54 ± 0.16 GeV [23]. The value of p_F can also be extracted directly from the data. A fit to the $B \rightarrow X_s \gamma$ photon energy spectrum gives a value of about 0.45 GeV [24] while the fits to semileptonic *B* decays and $B \rightarrow J/\psi X$ give a value of 0.57 GeV [25,26]. Thus the value of p_F is likely to lie in the range $0.4 \leq p_F$ ≤ 0.6 GeV. After including the Fermi motion, the differential decay width $d\Gamma(m_b)/dm_X$ should be replaced by

$$
\frac{d\Gamma}{dm_X} = \int_0^{p_{max}} dp \, \phi(p) p^2 \cdot \frac{d\Gamma(W)}{dm_X},\tag{10}
$$

where p_{max} is the allowed maximum value of p and $d\Gamma(W)/dm_X$ is the differential decay rate in the *B* meson rest frame, which is linked to the one in the *b* quark rest frame through a Lorentz boost $[25,27]$. In Fig. 1 the invariant mass distribution is generated in this model with different values of p_F . Here we use $(1/\Gamma)(d\Gamma/dm_X)$, which is normalized to unity and independent of the value of *a*. It can be clearly seen that the p_F dependence is rather strong. The peak of the distributions shifts significantly from around \simeq 1.4 GeV (for p_F =0.4 GeV) to \simeq 1.8 GeV (for p_F =0.6 GeV). Considering the considerable uncertainties in both theory and experiment data, there is no significant disagreement in the recoil mass distribution of $B \rightarrow \eta' X_s$.

III. BOUND ON *a* **FROM INCLUSIVE AND EXCLUSIVE** *B* **DECAYS**

The value of *a* could be constrained from the exclusive decay modes $B \to \eta'(\eta) K^{(*)}$. Note that, although predictions of the standard effective Hamiltonian approach are too low to account for the data of inclusive decay modes, the disagreement in the exclusive decay modes is smaller $[6,15]$. Furthermore, the effective Hamiltonian approach can reproduce the correct patterns of $BR(B \to \eta' K) \geqslant BR(B \to \eta K)$ and $BR(B \to \eta' K^*) \leq BR(B \to \eta K^*)$ which are observed in the experiments. This implies that in exclusive decays modes it may still play an important role, and the interference between different contributions may also be important $|28,29|$.

Nevertheless, by saturating the current data on exclusive decays, the upper bound of the parameter *a* can still be obtained. The decay amplitudes of decay modes $B \to \eta' K^{(*)}$ and $B \to \eta K^{(*)}$ in this model read

$$
\mathcal{M}(B^{\pm,0}\to\eta'K^{\pm,0}) = aG_F \frac{4\pi}{3} \sqrt{\frac{3}{2}} m_{\eta'}^2 (f_8 \sin\theta + \sqrt{2}f_0 \cos\theta) \frac{m_B^2 - m_K^2}{2(m_b - m_s)} F_0^{BK}(m_{\eta'}^2),
$$

$$
\mathcal{M}(B^{\pm(0)}\to\eta K^{\pm(0)}) = aG_F \frac{4\pi}{3} \sqrt{\frac{3}{2}} m_{\eta}^2 (f_8 \cos\theta - \sqrt{2}f_0 \sin\theta) \frac{m_B^2 - m_K^2}{2(m_b - m_s)} F_0^{BK}(m_{\eta}^2),
$$

$$
\mathcal{M}(B^{\pm(0)}\to\eta'K^{*\pm(0)}) = -aG_F \frac{4\pi}{3} \sqrt{\frac{3}{2}} m_{\eta'}^2 (f_8 \sin\theta + \sqrt{2}f_0 \cos\theta) \frac{|P_{\eta'K^*}|m_B}{(m_b + m_s)} A_0(m_{\eta'}^2),
$$

$$
\mathcal{M}(B^{\pm(0)}\to\eta K^{*\pm(0)}) = -aG_F \frac{4\pi}{3} \sqrt{\frac{3}{2}} m_{\eta}^2 (f_8 \cos\theta - \sqrt{2}f_0 \sin\theta) \frac{|P_{\eta'K^*}|m_B}{(m_b + m_s)} A_0(m_{\eta'}^2),
$$

(11)

where $|P_{\eta' K^*}| \approx |P_{\eta K^*}| \approx \frac{1}{2} m_B$. $F_0^{BK}(q^2)$ and $A_0(q^2)$ are the form factors for $B \rightarrow K$ and $B \rightarrow K^*$ transitions with momentum transfer q^2 . The value of m_b is taken to be the effective one, i.e., $m_b^{-2} \approx \int W^{-2} \phi(p) p^2 dp$. In the calculations, we take m_b = 4.65 GeV, which corresponds to p_F = 0.5 GeV and $m_q = 0.15$ GeV.

The corresponding branching ratio can be evaluated through the relation

$$
BR = \frac{\tau_B|P|}{8\pi m_B^2} |\mathcal{M}|^2,\tag{12}
$$

where $|P|$ is the momentum of one of the final state mesons in the *B* rest frame.

It is useful to define two kinds of ratio that are independent of the parameter *a*.

(1) The ratio between $B \rightarrow \eta' X$ and $B \rightarrow \eta X(X=X_{s}, K, \text{ or }$ *K**). This ratio is independent of the value of *a* and sensitive only to the η' - η mixing. In this model one finds [30]

$$
R = \frac{BR(B \to \eta X_s)}{BR(B \to \eta' X_s)} = \frac{BR(B \to \eta K)}{BR(B \to \eta' K)} = \frac{BR(B \to \eta K^*)}{BR(B \to \eta' K^*)}
$$

$$
= \frac{m_{\eta}^4}{m_{\eta'}^4} \left(\frac{f_s \cos \theta - \sqrt{2} f_0 \sin \theta}{f_s \sin \theta + \sqrt{2} f_0 \cos \theta} \right)^2.
$$
(13)

In the following numerical calculations we take $\theta = -17$ ° and $f_8 = f_0 = 1.06 f_\pi$ [31] as an illustration. This leads to a value of $R \approx 0.16$. Considering the CLEO data of *R* $\leq 0.1-0.8$ [2], it follows that with the constraints from *B* $\rightarrow \eta K$, this model can account for at most $\sim 60\%$ of the observed $B \rightarrow \eta' K$ branching ratio. Note that the exact value of *R* may vary with different sets of parameters θ , f_8 , and f_0 ; the constraints from *R* are only an order of magnitude estimate.

(2) The ratio between $B \rightarrow P K^*$ and $B \rightarrow P K$ ($P = \eta'$ or η). In this model it is independent of *both* the value of *a* and the details of the η' - η mixing.

$$
R' = \frac{BR(B \to \eta' K^*)}{BR(B \to \eta' K)}
$$

=
$$
\frac{(m_B^2 + m_{K^*}^2 - m_{\eta}^2)^2 - 4m_B^2 m_{K^*}^2}{(m_B^2 - m_K^2)^2} \cdot \left(\frac{A_0(m_{\eta'}^2)}{F_0^{BK}(m_{\eta'}^2)}\right)^2.
$$
 (14)

The values of F_0 and A_0 in the Bauer-Stech-Wirbel model [32] are $F_0 = 0.38, A_0 = 0.32$, which correspond to $R = 0.84$, while from the light cone QCD sum rule [33,34] $F_0 = 0.35$ ± 0.05 , A_0 = 0.39 \pm 0.1, and *R* = 1.1 \pm 0.3. Thus if this model gives the dominant contribution to these modes, the value of *R* should be around 1. However, the current data give a value of $R' \le 0.5$ –0.4 [2]. This is a clearer and stronger constraint than the one from *R*. With the observed small value of R' , this model can explain at most half of the branching ratio of $B \rightarrow \eta'(\eta)K$ and therefore is not the dominant mechanism of these processes. In Figs. $2(c) - 2(f)$ the numerical results for branching ratios as a function of the effective coupling *a* are given and compared with the data. As some inclusive decay modes have not yet been observed by the BaBar and Belle Collaborations, only the CLEO data are used in the numerical evaluations. It can be seen from the figure that the data for the exclusive decay modes $B \rightarrow \eta' K^*$ and $B \rightarrow \eta K$ im-

FIG. 2. Branching ratios for inclusive and exclusive decay modes as a function of *a*. (a) For decay mode $B \rightarrow \eta' X_s$, the dark and light shadows represent the 1 σ (2 σ) ranges allowed by current data. (b) For decay mode $B \rightarrow \eta X_s$, the light shadows represent the 90% allowed range. (c) For decay mode $B \rightarrow \eta' K^+$, the dark and light shadows represent the 1 σ (2 σ) allowed ranges. (d) For decay mode $B \rightarrow \eta K^+$, the light shadows represent the 90% allowed range. (e) For decay mode $B \rightarrow \eta' K^{*+}$, the light shadows represent the 90% allowed range. (f) For decay mode $B \rightarrow \eta K^{*+}$, the dark and light shadows represent the $1\sigma (2\sigma)$ allowed ranges.

pose strong constraints on the effective coupling. With these constraints, the maximum value of *a* lies in the range

$$
a \le (8-9) \times 10^{-3} \text{ GeV}^{-1}.
$$
 (15)

From Eqs. (6) and (7) , the branching ratio of inclusive decays $B \rightarrow \eta'(\eta)X_s$ as a function of *a* is plotted in Figs. $2(a)$ and $2(b)$ and compared with the data. In the decay *B* $\rightarrow \eta' X_s$ the acceptance cut effect is taken into account, which leads to a 19% reduction from the calculation in Eq. (6) . Given the upper bound of *a* in Eq. (15) this model can still successfully reproduce the $B \rightarrow \eta' X_s$ branching ratio within the 1σ range.

IV. PREDICTION OF RADIATIVE DECAY $J/\psi \rightarrow \eta' \eta \gamma$

From the effective Hamiltonian in Eq. (3) , this model can also contribute to the radiative J/ψ decays into η' . Using the relation Eq. (5), the ratio between $J/\psi \rightarrow \eta' \gamma$ and J/ψ $\rightarrow \eta \gamma$ can be predicted and is found to be the same as in Ref. $[31]$:

$$
\frac{\Gamma(J/\psi \to \eta' \gamma)}{\Gamma(J/\psi \to \eta \gamma)} = \left| \frac{\langle 0 | G \tilde{G} | \eta' \rangle}{\langle 0 | G \tilde{G} | \eta \rangle} \right|^2 \cdot \frac{(1 - m_{\eta'}^2)^3}{(1 - m_{\eta'}^2)^3}, \quad (16)
$$

which is in good agreement with the data.

Furthermore, given the value of the effective coupling *a* the decay rate of $J/\psi \rightarrow \eta' \eta \gamma$ can be predicted. To this end let us first define the ratio

$$
r(\eta') = \Gamma(B \to \eta' X_s) / \Gamma(B \to g^* X_s), \tag{17}
$$

which can be understood as the size of $b \rightarrow s \eta'$ relative to $b \rightarrow sg$. Taking $\Gamma(B \rightarrow g^*X_s) \sim 1\%$ and $a \leq 0.008$ GeV⁻¹, which comes from the bounds from exclusive decays, as an example, one finds

$$
r(\eta') \le 0.045. \tag{18}
$$

Note that the strong coupling constant in the effective Hamiltonian has been separated from the effective coupling *a* and absorbed in the matrix element of $\langle 0|\alpha_sG\overline{G}|\eta'(\eta)\rangle$. It is expected that there is no significant running of the value of *a* from the energy scale m_B to $m_{I/dc}$.

Since the radiative decay of $J/\psi \rightarrow \gamma X$ is dominated by the process $J/\psi \rightarrow g^*g^*\gamma$, the branching ratio of J/ψ $\rightarrow \eta'$ η' γ can be simply estimated as

$$
\frac{BR(J/\psi \to \eta' \eta' \gamma)}{BR(J/\psi \to \gamma X)} \simeq r(\eta')^2.
$$
 (19)

Observation of the process $J/\psi \rightarrow \gamma X$ gives $BR(J/\psi \rightarrow \gamma X)$ $=$ (17.0 \pm 2.0) \times 10⁻². Thus, taking $r(\eta')$ = 0.045, the maximum branching ratio of $J/\psi \rightarrow \eta' \eta' \gamma$ is estimated as

$$
BR(J/\psi \rightarrow \eta' \eta' \gamma) \approx 3.4 \times 10^{-4}.
$$
 (20)

The decay rate of $J/\psi \rightarrow \eta' \eta \gamma$ can be estimated as follows:

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$$
\frac{BR(J/\psi \to \eta' \eta \gamma)}{BR(J/\psi \to \eta' \eta' \gamma)} = R.
$$
 (21)

Using the value of $R=0.16$ from Eq. (13) one finds for the maximum branching ratio for $J/\psi \rightarrow \eta' \eta \gamma$

$$
BR(J/\psi \to \eta' \eta \gamma) \approx 5.4 \times 10^{-5}.
$$
 (22)

Considering that the detection efficiency of η' is about a few percent (through $\eta' \rightarrow \eta \gamma \gamma$), it may be hard to find a signal of such a decay mode in Beijing Spectrometer (BES) at Beijing Electron Position Collider (BEPC) due to limited statistics (in BES $5 \times 10^7 J/\psi$ samples are collected). But in the forthcoming CLEO *c* project $1 \times 10^9 J/\psi$ samples are planned to be produced. It will then be promising to search for the signal and test the predictions from this model in the CLEO *c* experiment.

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