

Impact of CP phases on neutrinoless double beta decay

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We highlight in a model independent way the dependence of the effective Majorana mass parameter, relevant for neutrinoless double beta decay, on the CP phases of the Pontecorvo-Maki-Nakagawa-Sakata matrix, using the most recent neutrino data including the cosmological Wilkinson Microwave Anisotropy Probe measurement. We perform our analysis with three active neutrino flavors in the context of three kinds of mass spectra: quasidegenerate, normal hierarchical, and inverted hierarchical. If a neutrinoless double beta decay experiment records a positive signal, then assuming that Majorana masses of light neutrinos are responsible for it, we show how it might be possible to discriminate between the three kinds of spectra.

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The importance of looking for neutrinoless double beta decay ($0\nu\beta\beta$) lies in the fact that, if observed, it would establish a violation of the total lepton number, which is otherwise a conserved quantum number in the standard model. Any nonvanishing amplitude for this decay may be inferred as a signal for an effective Majorana mass of the electron neutrino. This way it is sensitive to some kind of an absolute mass of the neutrino, contrary to the oscillation experiments, which can fix only the neutrino mass squared differences. Evidence for this decay has recently been claimed on the basis of results from the Heidelberg-Moscow experiments [1]. This claim has been criticized by authors in [2], which has subsequently been followed by a reply to the criticism [3]. In any case, the currently running NEMO3 experiment [4] and future [5] (Majorana, EXO, CUORICINO, CUORE, GENIUS) ($0\nu\beta\beta$) experiments could either confirm this evidence or put a stronger bound on the amplitude of this decay. The rate of ($0\nu\beta\beta$) is proportional to the square of the (ee) element of the neutrino mass matrix, often called the effective mass parameter m_{ee} . This parameter depends on the absolute neutrino masses, the solar and CHOOZ mixing angles, and two CP phases. A detailed discussion of the dependence of m_{ee} on different parameters may be found, e.g., in [6,7].

The purpose of this paper is to highlight in a model independent way the dependence of m_{ee} on the CP phases, using the most recent oscillation data on mass square splittings and mixing angles [8–13], as well as the recent cosmological bound from the Wilkinson Microwave Anisotropy Probe (WMAP) on the sum of all neutrino masses [14] in conjunction with data from the 2dF galaxy redshift survey (2dFGRS) [15]. We base our analysis on the three possible kinds of mass spectra: quasidegenerate, hierarchical, and inverted hierarchical, in the context of three neutrino generations. The ($0\nu\beta\beta$) experiment in a sense serves to distinguish

between the spectra: due to the present sensitivity, its observation in the ongoing ($0\nu\beta\beta$) experiment, as it would turn out, would only establish a nearly degenerate mass spectrum.

We stress at this point that even though the ($0\nu\beta\beta$) amplitude does depend on the CP phases, this decay does not correspond to a manifest CP -violating phenomenon. The rate of this decay is indeed affected by the phases. But the effect is CP -even, i.e., the rate of this decay in a nucleus will be the same, in principle, to that in an antinucleus. The CP -odd effect that these Majorana phases might cause have been studied in the context of neutrino \leftrightarrow antineutrino oscillation, rare leptonic decays of the K and B mesons, and leptogenesis (for a recent discussion on this issue, see [16]).

Let us now set up our notations in a scenario with three active neutrino flavors. In other words, we keep the Liquid Scintillator Neutrino Detector (LSND) results [17] out of our consideration.¹ We recall that observation of neutrino oscillation implies mixing between the flavors due to the fact that the flavor basis is not parallel to the mass basis. The flavor basis is written as $\nu_{\ell L}$ where $\ell = e, \mu, \tau$, and the mass basis is expressed as ν_{iL} where $i = 1, 2, 3$ (L stands for left-handed). The two bases are related to each other by

$$\nu_{\ell L} = \sum_{i=1}^3 U_{\ell i} \nu_{iL}, \quad (1)$$

where the unitary matrix U is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [20]. A useful parametrization of U is given by [21]

¹Indeed, we know now that miniBoone [18] will either confirm or rule out the LSND signal at the earliest by 2006, see [19].

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}\{e^{i\alpha_1}, e^{i\alpha_2}, 1\}, \quad (2)$$

where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$, δ is the Dirac CP phase, and $\alpha_{1,2}$ are the Majorana phases.

If the $(0\nu\beta\beta)$ amplitude is indeed generated by a $(V - A)$ weak charged current interaction via Majorana neutrino exchange, and if the masses of those neutrinos are less than a typical Fermi momentum (~ 100 MeV) of the nucleons inside a nucleus, then the $(0\nu\beta\beta)$ amplitude is proportional to the effective mass m_{ee} defined as [22]

$$|m_{ee}| = |U_{ei}^2 m_i| = |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{2i\alpha_M} + m_3 |U_{e3}|^2 e^{2i\alpha_{MD}}|, \quad (3)$$

where $\alpha_M = (\alpha_2 - \alpha_1)$ is a pure Majorana type and $\alpha_{MD} = -(\delta + \alpha_1)$ is a mixture of the Majorana and Dirac type CP phases. Without any loss of generality, we can take the mass eigenvalues (m_1, m_2, m_3) to be positive. The effective mass parameter depends on the solar angle θ_{12} , the CHOOZ angle θ_{13} , the masses m_i , and the CP phases. The solar angle measurement has become increasingly precise particularly after the SNO results came out. As regards θ_{13} , there exists only an upper limit from the CHOOZ [12] and Palo Verde [13] neutrino disappearance reactor experiments. The latter angle links the solar and the atmospheric sectors in the PMNS matrix. This angle is also important in the context of future CP violation measurements in the long baseline experiments. For an observable impact of CP violation θ_{13} should not be smaller than 0.2° (the other necessity is a large solar angle which has already been established anyway). More specifically, the future first generation superbeams JHF-SK [23] and NuMI [24] long baseline experiments (JHF-SK to start taking data in 2007) along with possible large reactor experiments will measure $\sin^2 \theta_{13}$ to a few 10^{-3} level [25] and, if luck permits, will also determine some CP asymmetries. Now we turn our attention to the CP phases. As yet, these phases are completely unknown. Only the $(0\nu\beta\beta)$ amplitude offers a unique and direct probe to them. These phases take an active role in determining the size of the $(0\nu\beta\beta)$ amplitude, and the possibility of a likely signal for this decay in the current and foreseeable experiments hangs crucially on the amount of destructive interferences created by these phases.

We now briefly summarize the experimental data which concern the effective mass calculation related to neutrinoless double beta decay.

The post-KamLAND analysis [9] constrain the solar angle, θ_{sol} or θ_{12} , as (b.f. means best fit)

$$0.70 \lesssim \sin^2 2\theta_{\text{sol}} \lesssim 0.96 \quad (95\% \text{ C.L.});$$

$$\sin^2 2\theta_{\text{sol}} \text{ (b.f.)} = 0.82. \quad (4)$$

The CHOOZ experiment [12] constrains the θ_{13} angle as

$$\sin\theta_{13} \lesssim 0.22 \quad (95\% \text{ C.L.}) \quad (5)$$

and a global analysis by Fogli *et al.* [26] led to $|U_{e3}|^2 < 5.0 \times 10^{-2}$ (99.7% C.L.).

The solar [9] and atmospheric [11] squared mass differences are constrained as (95% C.L.)

$$5.8 \times 10^{-5} \lesssim \Delta m_{\text{sol}}^2 \text{ (eV}^2\text{)} \lesssim 9.1 \times 10^{-5},$$

$$\Delta m_{\text{sol}}^2 \text{ (b.f.)} = 7.2 \times 10^{-5} \text{ (eV}^2\text{)}, \quad (6)$$

$$1 \times 10^{-3} \lesssim \Delta m_{\text{atm}}^2 \text{ (eV}^2\text{)} \lesssim 5.0 \times 10^{-3},$$

$$\Delta m_{\text{atm}}^2 \text{ (b.f.)} = 2.5 \times 10^{-3} \text{ (eV}^2\text{)}. \quad (7)$$

The WMAP result [14] in conjunction with the 2dFGRS data [15] constrain the total mass of the active neutrino species (with the assumption that these neutrinos have decoupled while still being relativistic) as

$$\sum_i m_i \lesssim 0.71 \text{ eV} \quad (95\% \text{ C.L.}). \quad (8)$$

Implicitly, the limit in Eq. (8) uses the Ly- α forest data [27] whose interpretation is still controversial. Excluding the latter, one obtains a more robust and conservative bound $\sum_i m_i \lesssim 1.01$ eV [28].

The Heidelberg-Moscow claim on evidence of $(0\nu\beta\beta)$ translates into an effective Majorana mass [1-3]

$$0.11 \lesssim |m_{ee}| \text{ (eV)} \lesssim 0.56 \quad (95\% \text{ C.L.});$$

$$|m_{ee}| \text{ (b.f.)} = 0.39 \text{ eV}. \quad (9)$$

The Mainz [29] and Troitsk [30] Tritium beta decay experiments have put the bound $m_{\nu_e} \lesssim 2.2$ eV on the electron-type neutrino mass. The future KATRIN Tritium beta decay experiment [31], planned to be operative from 2007, has the possibility to probe m_{ν_e} down to the 0.35 eV level.

We perform our analysis on the basis of the usual three kinds of mass hierarchy, and we discuss them one by one. But, before that, we observe that the WMAP limit automatically sets an *upper limit* for the effective mass parameter in neutrinoless double beta decay. In other words, keeping in mind Schwarz inequality, it follows from Eq. (3) that $|m_{ee}| \lesssim 0.71$ eV (or a more conservative upper limit of 1.01 eV *ala* Hannestad [28]). A similar conclusion was also drawn in [32].

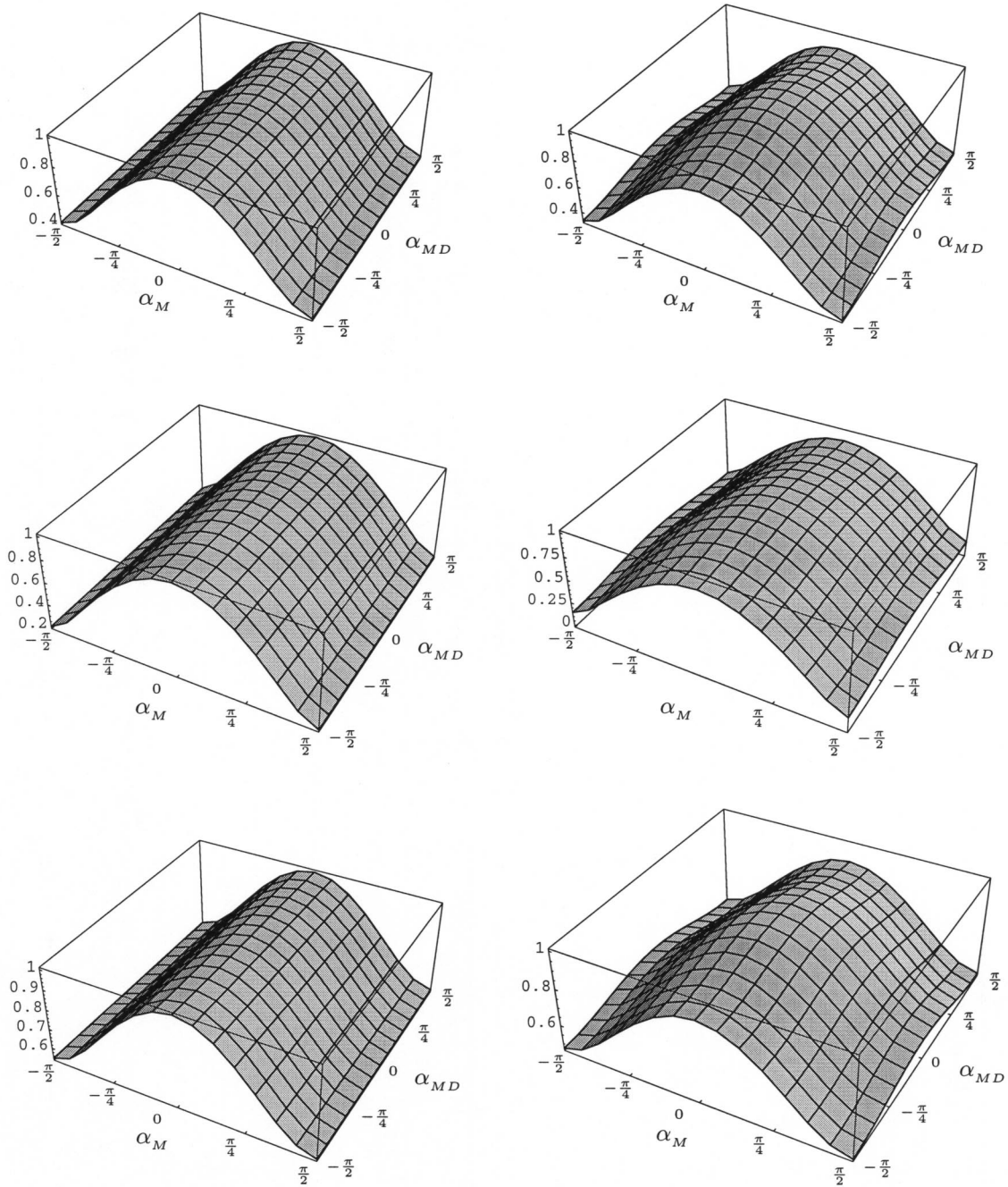


FIG. 1. The quasidegenerate case is represented with the minimum and maximum allowed values for the CHOOZ angle. The left panel corresponds to $\sin \theta_{13}=0$ and the right panel to $\sin \theta_{13}=0.22$. The z axis represents $|m_{ee}|/m_0$ in terms of the two CP phases. The lowest value of $|m_{ee}|/m_0$ is not zero but $|\cos 2\theta_{\text{sol}}|$. The first, second, and third rows correspond to $\sin^2 2\theta_{\text{sol}}=0.82$ (best fit), 0.96 (95% C.L. upper limit), and 0.70 (95% C.L. lower limit), respectively.

(1) Quasidegenerate: The three eigenvalues are $m_1 \approx m_2 \approx m_3 \equiv m_0$. The absolute scale can be made large enough to saturate the WMAP bound, i.e., $m_0 \approx 0.23$ eV. In this case, Eq. (3) turns out to be

$$|m_{ee}| \approx m_0 |c_{12}^2 c_{13}^2 + s_{12}^2 c_{13}^2 e^{2i\alpha_M} + s_{13}^2 e^{2i\alpha_{MD}}|. \quad (10)$$

Since CHOOZ data constrain s_{13} to be small, one would naively throw away the third term in Eq. (10), as has been

the usual practice. But the effect of this term can be significant when there is a cancellation between the first two terms. For $s_{13}=0$, we obtain

$$m_0 |\cos 2\theta_{\text{sol}}| \lesssim |m_{ee}| \lesssim m_0. \quad (11)$$

As a matter of fact, the upper bound m_0 in Eq. (11) holds, thanks to the Schwartz inequality, irrespective of the value of s_{13} . The role of the destructive interference can be seen in Fig. 1 where we have plotted the effective mass parameter

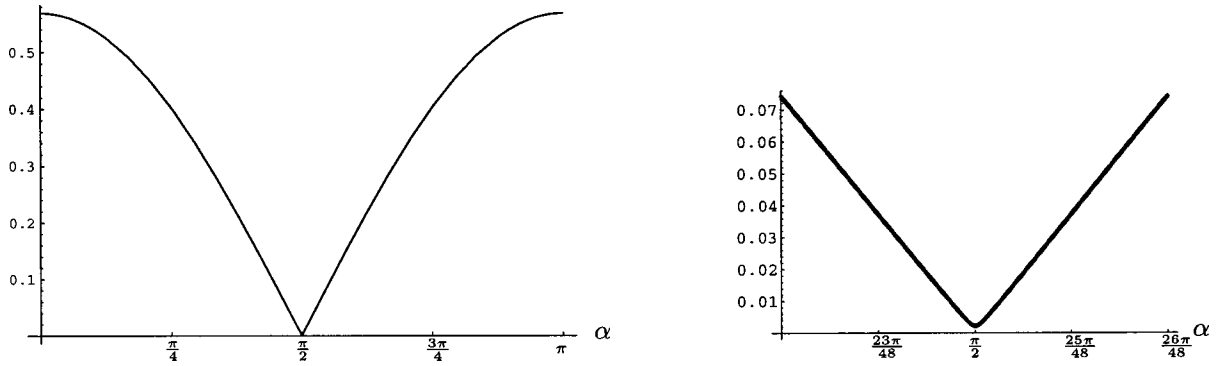


FIG. 2. The normal hierarchy case: the effective mass normalized as $|m_{ee}|/\sqrt{\Delta m_{\text{sol}}^2}$ is plotted against the only CP phase α (see text). We have used the best fit value for the solar angle. In the right panel we zoom the part where there is an extreme cancellation.

for both $s_{13}=0$ (left panel) and the maximum allowed value $s_{13}=0.22$ (right panel). We point out here that the lowest value of $|m_{ee}|/m_0$ in the plots of Fig. 1 is not zero but $|\cos 2\theta_{\text{sol}}|$. The relative importance of the two phases and also the impact of nonvanishing $\sin \theta_{13}$ (right panel) are apparent from Fig. 1. Comparing the left and right panels, we infer that a nonvanishing s_{13} (we put the CHOOZ upper limit of 0.22) somewhat suppresses (by something like 10%) the maximum value m_0 can attain for $\alpha_M=0$ compared to the $s_{13}=0$ scenario.

(2) Normal hierarchy: In this case, $m_1 < m_2 \ll m_3$. As an illustrative example, we can take $m_1 \approx 0$, $m_2 \approx \sqrt{\Delta m_{\text{sol}}^2}$, and $m_3 \approx \sqrt{\Delta m_{\text{atm}}^2}$. Then one can effectively get rid of one of the two CP phases in Eq. (3), and can write ($\alpha \equiv \alpha_{MD} - \alpha_M$)

$$|m_{ee}| = |\sqrt{\Delta m_{\text{sol}}^2} s_{12}^2 c_{13}^2 + \sqrt{\Delta m_{\text{atm}}^2} s_{13}^2 e^{2i\alpha}|. \quad (12)$$

In Fig. 2, we have plotted $|m_{ee}|/\sqrt{\Delta m_{\text{sol}}^2}$ against the CP phase α . We observe that even in the case of maximum cancellation ($\alpha = \pi/2$) the effective mass never vanishes (see the zoom in Fig. 2) and thus corresponds to a lower bound, which is unfortunately much below the present and foreseeable experimental sensitivity. Putting numbers, we obtain within the 95% confidence level from the data

$$|m_{ee}| \lesssim 0.007 \text{ eV}. \quad (13)$$

A nonzero m_1 (but small enough to satisfy $m_1 \ll \sqrt{\Delta m_{\text{sol}}^2}$) can, however, push $|m_{ee}|$ to slightly higher values.

(3) Inverted hierarchy: In this case, $m_1 > m_2 \gg m_3$. One can take $m_1 \approx m_2 \approx \sqrt{\Delta m_{\text{atm}}^2}$ and $m_3 \approx 0$. Again, only one CP phase, the pure Majorana one, enters into the expression for $|m_{ee}|$, given by

$$|m_{ee}| = |\sqrt{\Delta m_{\text{atm}}^2} c_{13}^2 |c_{12}^2 + s_{12}^2 e^{2i\alpha_M}|. \quad (14)$$

This case is very similar to the quasidegenerate scenario, except that the overall mass scale is suppressed by $\sqrt{\Delta m_{\text{atm}}^2}/m_0$ and that the third term in Eq. (3) is even further suppressed. This case is illustrated in Fig. 3 where we have plotted $|m_{ee}|/\sqrt{\Delta m_{\text{atm}}^2}$ as a function of the CP phase α_M . The maximum cancellation holds for $\alpha_M = \pi/2$, as it was for the quasidegenerate case. When $\alpha_M = 0$, which does not nec-

essarily mean that the original Majorana phases α_1 and α_2 in the PMNS matrix individually vanish, we obtain the maximum amplitude. Again putting numbers, we obtain at 95% C.L. from experimental data

$$0.006 \text{ eV} \lesssim |m_{ee}| \lesssim 0.07 \text{ eV}. \quad (15)$$

Thus we may observe that a measurement of $|m_{ee}|$ may serve to distinguish between the spectra. As an example, any measurement of $|m_{ee}|$ reasonably above the maximum $\sqrt{\Delta m_{\text{atm}}^2} \approx 0.07 \text{ eV}$ will conclusively rule in favor of the quasidegenerate spectrum, irrespective of the present uncertainty over the absolute mass upper limit. In future experiments, if the effective mass is found between 0.007 and 0.07 eV, then the spectrum would correspond to the inverted hierarchy pattern, while an observation of $|m_{ee}|$ below 0.006 eV would imply a normal hierarchical pattern. But if $|m_{ee}|$ is observed between 0.006 and 0.007 eV, then the two kinds of hierarchies cannot be discriminated. These divisions are based on the basis of accepting the experimentally allowed regions at 95% C.L. If, instead, one employs 99% C.L. criterion, the lower bound of $|m_{ee}|$ in the case of inverted hierarchy enters *more* into the zone admitted by normal hierarchy. Another point to note is that in the future if the KATRIN Tritium beta decay experiment confirms a large ($\geq 0.35 \text{ eV}$) absolute mass, then a measurement of $|m_{ee}|$ in an ongoing

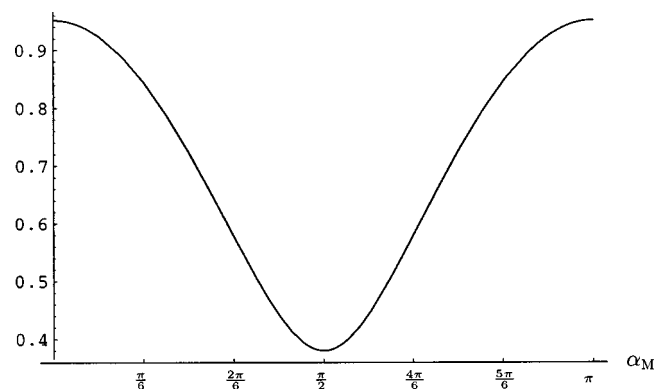


FIG. 3. The inverted hierarchy case: the effective mass normalized as $|m_{ee}|/\sqrt{\Delta m_{\text{atm}}^2}$ is plotted against the only CP phase α_M (see text). We have used the best fit value for the solar angle.

neutrinoless double beta decay experiment would provide an idea about the CP phases. The Heidelberg-Moscow and NEMO3 experiments have been designed to reach a sensitivity of a few 10^{-1} eV. Thus a positive signal in these experiments will only imply a degenerate spectrum. Among the future short term projects, CUORICINO will have a sensitivity of a few 10^{-1} eV, but the Majorana, EXO, and CUORE experiments are expected to attain a sensitivity of a few 10^{-2} eV. Therefore we will be able to distinguish between the inverted hierarchical and the degenerate spectra. On the other hand, if the spectrum is normal hierarchical then we will have to wait for the long term projects, which are expected to reach a sensitivity of a few 10^{-3} eV (e.g., 10t GENIUS). We refer to Ref. [33] for a general discussion about the future direct neutrino mass measurements.

A word of caution is relevant here. Nonzero Majorana masses of light neutrinos are not necessarily the only source behind a nonvanishing $(0\nu\beta\beta)$. Heavy Majorana neutrinos or doubly charged scalars may also contribute to $(0\nu\beta\beta)$, where the contributions are suppressed by their heavy masses. In fact, in the context of a left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, the seesaw generated light Majorana neutrinos, the heavy Majorana neutrino, and the doubly charged scalar all contribute to $(0\nu\beta\beta)$; additionally, there is a fourth contribution arising out of light and heavy neutrino mixing. Nonobservation of $(0\nu\beta\beta)$ can therefore be translated into lower bounds on the relevant heavy masses in the range of a few hundred GeV to a few TeV. In supersymmetric models with broken R -parity, the trilinear λ'_{111} coupling or the product couplings $\lambda'_{11j} \cdot \lambda'_{1j1}$ also drive $(0\nu\beta\beta)$, and again stringent bounds emerge on those couplings. The R -parity violating couplings will have distinct collider signals. So before one interprets a nonzero signal of $(0\nu\beta\beta)$ as a *direct* consequence of light neutrino Majorana masses, one must ensure that all other lepton number violating contributions are comparatively dwarfed. It should, however, be noted that regardless of whatever mechanism is responsible for $(0\nu\beta\beta)$, once there is a lepton number violating interaction, neutrino Majorana masses will be definitely generated at higher loops, even if it is forbidden

at tree level. Thus a nonvanishing $(0\nu\beta\beta)$ amplitude effectively implies a nonvanishing neutrino Majorana mass, directly or indirectly. For an illustrative discussion on different kinds of lepton number violating processes and their contributions to $(0\nu\beta\beta)$, see Ref. [34].

In conclusion, assuming that the Majorana masses of light neutrinos are mainly responsible for $(0\nu\beta\beta)$, the major ingredients for the prediction of $|m_{ee}|$ are the solar and atmospheric mass splittings (for the normal and inverted hierarchical cases), the absolute mass scale (for the degenerate case), the solar mixing angle, the CHOOZ angle, and the CP phases. The ongoing oscillation experiments provide mass squared splittings and mixing angles. In the near future the precision of all the oscillation parameters will be significantly enhanced, which will sharpen the $|m_{ee}|$ prediction. Then the chances of getting a positive signal in the $(0\nu\beta\beta)$ experiments will depend crucially on the CP phases. It was our aim to demonstrate the role of these phases in this context. Here we have not indulged ourselves in the discussion of theoretical uncertainties associated with $|m_{ee}|$ prediction. The uncertainty in the nuclear matrix element calculations is estimated to be roughly $\mathcal{O}(2)$ (for a recent analysis on theoretical and experimental uncertainties associated with $|m_{ee}|$, see, e.g., [7]). Eventually, if a nonzero $(0\nu\beta\beta)$ signal is observed in experiment, then its size will give a hint on the nature of the spectrum. This is an advantage over the oscillation experiments. Additionally, such an event will give us a handle on the magnitude of the CP phases, which might lead to CP odd effects at an observable level [16]. Finally, we point out that following the WMAP results [14] a lot of enthusiasm has been generated towards a close scrutiny of neutrinoless double beta decay (some of these references are contained in [35]).

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