Inverting the seesaw formula

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By inverting the seesaw formula we determine the heavy neutrino mass matrix. The impact on baryogenesis via leptogenesis and the radiative lepton decays in supersymmetric models is described. Links to neutrinoless double beta decay are also briefly discussed. The analysis leads to two distinct matrix models. One has small mixing while the other one has maximal mixing. Both cannot give a sufficient amount of baryon asymmetry. Then we also comment on a different form of the Dirac neutrino mass matrix, which does provide sufficient baryon asymmetry. In a supersymmetric scenario the branching ratios of radiative lepton decays are enhanced for this model.

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I. INTRODUCTION

The seesaw mechanism [1] is a simple framework which can explain the smallness of neutrino mass. It requires only a modest extension of the minimal standard model, namely the inclusion of the heavy right-handed neutrino, but can be well realized within left-right models [2], partial unified models [3], and grand unified SO(10) theories [4], where the right-handed neutrino does exist. Then the effective neutrino mass matrix M_L is given by the seesaw formula

$$M_L \simeq M_\nu M_R^{-1} M_\nu^T, \tag{1}$$

where M_R is the mass matrix of the right-handed neutrino and M_ν is the Dirac neutrino mass matrix. The master formula (1) is valid when the eigenvalues of M_R are much larger than the elements of M_ν and in such a case the eigenvalues of M_L come out very small with respect to those of M_ν. Indeed, unlike M_ν, the generation of M_R is not related to electroweak symmetry breaking and thus its scale may be very large. Moreover, M_R is a Majorana mass matrix and as a consequence M_L also is a Majorana mass matrix of left-handed neutrinos (see, for example [5]). This fact is related to the violation of total lepton number at a high scale [6], which should produce important phenomena such as baryogenesis via leptogenesis [7] and the neutrinoless double beta decay [8]. Lepton flavors are also violated, but in the nonsupersymmetric theory, due to the smallness of neutrino mass, such processes are so suppressed to be unobservable [9], apart from neutrino oscillations. The situation is different in the supersymmetric theory, even with universal soft breaking terms, where some of these processes may be observable [10].

Both lepton number and lepton flavor violations depend on the mass matrices M_ν and M_R. On the other hand, we have information on the effective neutrino mass matrix, coming from neutrino oscillations and more generally from neutrino experiments. Therefore, it is reasonable, relating M_ν to the charged fermion's mass matrices, to obtain information on M_R by inverting the seesaw formula:

$$M_R \simeq M_\nu^T M_L^{-1} M_\nu. \tag{2}$$

As a consequence, we should be able to determine also the impact on baryogenesis via leptogenesis, the neutrinoless double beta decay, and, for example, the radiative lepton decays in some supersymmetric models. The seesaw formula is valid above the M_R scale, so that one should determine M_L at that scale. Although in several cases the effect is not relevant, we must take care of the renormalization issue (see the recent paper [11]).

In Sec. II we discuss the Dirac mass matrices of quarks and leptons. In Sec. III we describe the effective neutrino mass matrix and in particular its element M_{ee}, related to neutrinoless double beta decay. In Sec. IV we determine the mass matrix of right-handed neutrinos. In Secs. V and VI, respectively, we study the consequences for baryogenesis via leptogenesis and the radiative lepton decays in supersymmetry. Finally, we present a discussion.

II. DIRAC MASS MATRICES

A symmetric form of the quark mass matrices, in agreement with the phenomenology of quark masses and mixing, is described in Refs. [12,13], and given by

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b, \end{pmatrix}, \tag{3}$$

$$M_u \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t \end{pmatrix}. \tag{4}$$

Moreover, in Ref. [12], the charged lepton mass matrix has an analogous form

$$M_e \simeq \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau \end{pmatrix}. \tag{5}$$

Since the hierarchy and scale of charged lepton masses are similar to the hierarchy and scale of down quark masses (see, for example [14]), one also has the relation $M_e \sim M_d$. Then a natural assumption is $M_\nu \sim M_u$, in which case the Dirac neutrino mass matrix can be written in the form

$$M_\nu \simeq \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & 1 \end{pmatrix} m_t, \tag{6}$$

where $a \ll b \sim c \ll 1$. The relation $b \simeq c$ in quark mass matrices is discussed in Ref. [15]. We take b and c different but of the same order. In fact, also matrices (3) and (5) can be written in the form (6), with overall scales m_b and m_τ, respectively.

III. NEUTRINO PHENOMENOLOGY

Neutrino oscillation data imply that the lepton mixing matrix is given by

$$U \simeq \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \epsilon e^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, 1), \tag{7}$$

where $\epsilon < 0.16$, and the square mass differences among effective neutrino masses m_1, m_2, m_3 are

$$\Delta m_{32}^2 = m_3^2 - m_2^2 \simeq 3 \times 10^{-3} \ \ \mathrm{eV}^2, \tag{8}$$

$$\Delta m_{21}^2 = m_2^2 - m_1^2 \simeq 7 \times 10^{-5} \ \ \mathrm{eV}^2. \tag{9}$$

In the basis where M_e is diagonal, M_L is obtained by the transformation

$$M_L = U^* D_L U^\dagger, \tag{10}$$

with $D_L = \mathrm{diag}(m_1, m_2, m_3)$. The presence of phases φ_1, φ_2 in the mixing matrix (7) is due to the Majorana nature of effective neutrinos. In the lepton mixing matrix, $U_{\mu 3}$ is maximal, U_{e2} is large, and U_{e3} is small. This is in contrast to the small quark mixings.

Since $\Delta m_{21}^2 \ll \Delta m_{32}^2$, we may consider four kinds of neutrino spectra: the normal hierarchy $m_1 \ll m_2 \ll m_3$, with $m_3^2 \simeq \Delta m_{32}^2$ and $m_2^2 \simeq \Delta m_{21}^2$, the partial degeneracy $m_1 \simeq m_2 \ll m_3$, with $m_3^2 \simeq \Delta m_{32}^2$, the inverse hierarchy $m_1 \simeq m_2 \gg m_3$, with $m_{1,2}^2 \simeq \Delta m_{32}^2$, and the almost degenerate spectrum $m_1 \simeq m_2 \simeq m_3 \simeq 1$ eV. The elements of M_L are given by

$$M_{ee} \simeq \epsilon^2 m_3 + \frac{m_2}{3} + 2\frac{m_1}{3},$$

$$M_{e\mu} \simeq \epsilon \frac{m_3}{\sqrt{2}} + \frac{m_2}{3} - \frac{m_1}{3},$$

$$M_{e\tau} \simeq \epsilon \frac{m_3}{\sqrt{2}} - \frac{m_2}{3} + \frac{m_1}{3},$$

$$M_{\mu\tau} \simeq \frac{m_3}{2} - \frac{m_2}{3} - \frac{m_1}{6},$$

$$M_{\mu\mu} \simeq M_{\tau\tau} \simeq \frac{m_3}{2} + \frac{m_2}{3} + \frac{m_1}{6},$$

where phases are inserted by $\epsilon \to \epsilon e^{i\delta}$, $m_1 \to m_1 e^{i\varphi_1}$, $m_2 \to m_2 e^{i\varphi_2}$, and the relation $M_{\mu\mu} \simeq M_{\tau\tau}$ leads to the nearly maximal mixing $U_{\mu 3}$.

Let us consider in particular the element M_{ee}, which is related to neutrinoless double beta decay. For the normal hierarchy we obtain (values in eV) $10^{-3} < M_{ee} \sim \sqrt{\Delta m_{21}^2} < 10^{-2}$, for the partial degeneracy $10^{-3} < M_{ee} \sim 10^{-1} \sqrt{\Delta m_{32}^2} < 10^{-2}$, for the inverse hierarchy $10^{-2} < M_{ee} \sim \sqrt{\Delta m_{32}^2} < 10^{-1}$, and for the degenerate spectrum $10^{-1} < M_{ee} < 1$. Hence, different spectra give quite a distinct prediction for M_{ee}. There is a claim of evidence for the process [16], with $M_{ee} = 0.05$–0.86 eV, in agreement with the degenerate spectrum and also the inverse hierarchy. However, this result is controversial.

IV. THE HEAVY NEUTRINO MASS MATRIX

In this section we determine the right-handed neutrino mass matrix by means of the inverse seesaw formula (2). We need M_L^{-1}, which is easily achieved, since $M_L^{-1} = U D_L^{-1} U^T$. We stress that the difference of U_{e2} from the maximal mixing could be ascribed to M_e [17] and/or to renormalization [18]. Therefore, at the high scale and in the basis, where M_e is given by Eq. (5), we use the nearly bimaximal mixing in the seesaw,

$$U \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon e^{-i\delta} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, 1), \tag{11}$$

with $\epsilon \simeq 0$. Then the elements of M_L^{-1} are given by

$$M_{ee}^{-1} \simeq \frac{1}{2m_1} + \frac{1}{2m_2} + \frac{\epsilon^2}{m_3},$$

$$M_{e\mu}^{-1} \simeq -\frac{1}{2\sqrt{2} m_1} + \frac{1}{2\sqrt{2} m_2} + \frac{1}{\sqrt{2}} \frac{\epsilon}{m_3},$$

$$M_{e\tau}^{-1} \simeq \frac{1}{2\sqrt{2} m_1} - \frac{1}{2\sqrt{2} m_2} + \frac{1}{\sqrt{2}} \frac{\epsilon}{m_3},$$

$$M^{-1}_{\mu\tau}\simeq-\frac{1}{4m_1}-\frac{1}{4m_2}+\frac{1}{2m_3},$$

$$M^{-1}_{\mu\mu}\simeq M^{-1}_{\tau\tau}\simeq\frac{1}{4m_1}+\frac{1}{4m_2}+\frac{1}{2m_3},$$

where phases are inserted by $m_1\rightarrow m_1e^{-i\varphi_1}$, $m_2\rightarrow m_2e^{-i\varphi_2}$, $\epsilon\rightarrow\epsilon e^{-i\delta}$. Now, we determine the forms of M_R according to the four kinds of mass spectra for the effective neutrinos. We consider two extreme cases, that is $\varphi_2\simeq\varphi_1$ and $\varphi_2\simeq\varphi_1+\pi$. The other cases should be intermediate between these two.

For the normal hierarchy we obtain

$$M_R\simeq\begin{pmatrix} a^2 & a(b-c) & -a \\ a(b-c) & (b-c)^2 & -(b-c) \\ -a & -(b-c) & 1\end{pmatrix}\frac{m_t^2}{4m_1}. \quad (12)$$

An overall phase $e^{i\varphi_1}$ will be always absorbed. The corresponding approximate form of M_L at the low scale is given by

$$M_L\sim\begin{pmatrix} m_2 & m_2 & m_2 \\ m_2 & m_3 & m_3 \\ m_2 & m_3 & m_3\end{pmatrix}.$$

For the partial degeneracy, the case $\varphi_2\simeq\varphi_1$ leads to M_R, the double of that in Eq. (12). Instead, $\varphi_2\simeq\varphi_1+\pi$ leads to the special form

$$M_R\simeq\begin{pmatrix} 0 & a & 0 \\ a & 2(c-b) & 1 \\ 0 & 1 & 0\end{pmatrix}\frac{am_t^2}{2\sqrt{2}m_1}. \quad (13)$$

The corresponding approximate forms of M_L at the low scale are given by

$$M_L\sim\begin{pmatrix} m_{1,2} & m_2-m_1 & m_2-m_1 \\ m_2-m_1 & m_3 & m_3 \\ m_2-m_1 & m_3 & m_3\end{pmatrix},$$

with $m_2-m_1\sim\Delta m^2_{21}/m_{1,2}$, and

$$M_L\sim\begin{pmatrix} m_{1,2} & m_{1,2} & m_{1,2} \\ m_{1,2} & m_3 & m_3 \\ m_{1,2} & m_3 & m_3\end{pmatrix}.$$

For the inverse hierarchy both cases $\varphi_2\simeq\varphi_1$ and $\varphi_2\simeq\varphi_1+\pi$ give

$$M_R\simeq\begin{pmatrix} a^2 & a(b+c) & a \\ a(b+c) & (b+c)^2 & (b+c) \\ a & (b+c) & 1\end{pmatrix}\frac{m_t^2}{2m_3}. \quad (14)$$

Note that while for the normal hierarchy the difference $(b-c)$ appears, for the inverse hierarchy, instead, the sum $(b+c)$ appears. At the low scale we have

$$M_L\sim\begin{pmatrix} m_{1,2} & m_2-m_1 & m_2-m_1 \\ m_2-m_1 & m_{1,2} & m_{1,2} \\ m_2-m_1 & m_{1,2} & m_{1,2}\end{pmatrix},$$

with $m_2-m_1\sim\Delta m^2_{2,1}/m_{1,2}$, and a form of M_L with all entries of the order of $m_{1,2}$.

For the degenerate spectrum we get in the case $\varphi_2\simeq\varphi_1$,

$$M_R\simeq\begin{pmatrix} a^2 & ab & ac \\ ab & b^2+c^2 & c \\ ac & c & 1\end{pmatrix}\frac{m_t^2}{m_3}. \quad (15)$$

For $\varphi_2\simeq\varphi_1+\pi$ we have the same form as Eq. (14). At the low scale M_L is of the same kind as the inverse hierarchy case.

In the following sections we will consider, in a simplified approach, the impact of M_ν and M_R on the baryogenesis via leptogenesis and the radiative lepton decays in some supersymmetric models. We first take $M_\nu\sim M_u$, so that [14]

$$M_\nu\sim\begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ 0 & \lambda^4 & 1\end{pmatrix}m_t, \quad (16)$$

where $\lambda=0.22$ is the Cabibbo parameter. Since $b\sim c$, we take only two forms for M_R, one for the normal, inverse, and degenerate case, and the other for the partial degenerate case (13), that is,

$$M_R\sim\begin{pmatrix} \lambda^{12} & \lambda^{10} & \lambda^6 \\ \lambda^{10} & \lambda^8 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1\end{pmatrix}\frac{m_t^2}{m_k}, \quad (17)$$

with eigenvalues $M_1/M_2\sim\lambda^4$, $M_1/M_3\sim\lambda^{12}$, and

$$M_R\sim\begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 1 \\ 0 & 1 & 0\end{pmatrix}\lambda^6\frac{m_t^2}{m_1}, \quad (18)$$

with eigenvalues $M_1/M_2\sim\lambda^6$, $M_1/M_3\sim\lambda^6$. Note that the scale of matrix (18) is smaller by several orders with respect to the scale of matrix (17). Defining $M_D=M_\nu U_R$, where U_R diagonalizes M_R (M_D is the Dirac mass matrix in the basis where M_R is diagonal), we obtain $M_D^\dagger M_D$, which appears both in the formula for leptogenesis and in that for radiative decays in supersymmetry,

$$M_D^\dagger M_D\sim\begin{pmatrix} \lambda^{12} & \lambda^{10} & \lambda^6 \\ \lambda^{10} & \lambda^8 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1\end{pmatrix}m_t^2, \quad (19)$$

$$M_D^\dagger M_D \sim \begin{pmatrix} \lambda^{12} & \lambda^{10} & \lambda^{10} \\ \lambda^{10} & 1 & 1 \\ \lambda^{10} & 1 & 1 \end{pmatrix} m_t^2. \quad (20)$$

In the first case, matrix (17), we have U_R near the identity and $M_D^\dagger M_D \sim M_R m_k$. In the other case, matrix (18), U_R is nearly unimaximal. Therefore, in the matrix model made of Eqs. (16) and (17), M_ν and M_R give small mixing, so that large mixing in M_L is produced through a matching between M_ν and M_R within the seesaw formula. Instead, in the matrix model made of Eqs. (16) and (18), the maximal mixing in M_L comes from M_R. The structures (17) and (18) agree with the results of Ref. [19], where it was realized that the seesaw enhancement of lepton mixing can be achieved by strong mass hierarchy or large off-diagonal elements in the heavy neutrino mass matrix.

V. BARYOGENESIS VIA LEPTOGENESIS

The baryogenesis via leptogenesis mechanism [7] is a well-known mechanism for baryogenesis, related to the seesaw mechanism, where the decays of heavy right-handed neutrinos produce a lepton asymmetry which is partly transformed in a baryon asymmetry by electroweak sphaleron processes [20]. The amount of baryon asymmetry is then given by the expression

$$Y_B \simeq \frac{1}{2} \frac{1}{g^*} d\epsilon_1, \quad (21)$$

where ϵ_1 can be written as

$$\epsilon_1 \simeq \frac{3}{16\pi}\left[\frac{(Y_D^\dagger Y_D)_{12}^2}{(Y_D^\dagger Y_D)_{11}}\frac{M_1}{M_2} + \frac{(Y_D^\dagger Y_D)_{13}^2}{(Y_D^\dagger Y_D)_{11}}\frac{M_1}{M_3}\right], \quad (22)$$

see, for instance, Ref. [21]. In these formulas Y_D are Yukawa matrices, $g^* \simeq 100$, and $d<1$ is a dilution factor, which depends especially on the quantity

$$\tilde{m}_1 = \frac{(M_D^\dagger M_D)_{11}}{M_1}. \quad (23)$$

Moderate dilution is present when $\tilde{m}_1$ is in the range of the effective neutrino masses [22]. The allowed value for the baryon asymmetry is $Y_B \simeq 9\times 10^{-11}$, see Ref. [23]. Yukawa matrices are obtained by dividing mass matrices by their overall scale.

For the two matrix models described in the previous section we get, respectively,

$$\epsilon_1 \simeq \frac{3}{16\pi}\left(\frac{\lambda^{20}}{\lambda^{12}}\cdot\lambda^4 + \frac{\lambda^{12}}{\lambda^{12}}\cdot\lambda^{12}\right) \sim \frac{3}{16\pi}\lambda^{12} \sim 10^{-10}, \quad (24)$$

with $\tilde{m}_1 \sim m_k$, and

$$\epsilon_1 \simeq \frac{3}{16\pi}\left(\frac{\lambda^{20}}{\lambda^{12}}\cdot\lambda^6 + \frac{\lambda^{20}}{\lambda^{12}}\cdot\lambda^6\right) \sim \frac{3}{16\pi}\lambda^{14} \sim 10^{-12}, \quad (25)$$

with $\tilde{m}_1 \sim m_1$. Note that the two terms are comparable. Moreover, it is clear that both models cannot provide a sufficient amount of baryon asymmetry.

VI. RADIATIVE LEPTON DECAYS

In supersymmetric seesaw models with universality above the heavy neutrino mass scale, lepton flavor violations are produced by running effects from the universality scale M_U to the scale M_R [10]. The branching ratio for radiative lepton decays is given by the approximate formula [24]

$$\mathrm{Br}(l_i \to l_j \gamma) \sim \frac{\alpha^3}{G_F^2 m_S^8}\left(\frac{3m_0^2 + A_0^2}{8\pi^2}\log\frac{M_U}{M_R}\right)^2 (Y_D^\dagger Y_D)_{ij}^2 \tan^2\beta, \quad (26)$$

with $l_1 = e$, $l_2 = \mu$, $l_3 = \tau$. Here, m_0 is the universal scalar mass, A_0 is the universal trilinear coupling, and m_S is the average slepton mass at the weak scale, which can be quite different from m_0. The experimental upper bounds are $\mathrm{Br}(\mu \to e\gamma) < 1.2\times 10^{-11}$, $\mathrm{Br}(\tau \to e\gamma) < 2.7\times 10^{-6}$, and $\mathrm{Br}(\tau \to \mu\gamma) < 1.1\times 10^{-6}$. The first and third results are expected to be lowered by almost three orders in the future.

Assuming $m_0 = m_S = 100$ GeV, $A_0 = 0$, and $\tan\beta = 50$, we obtain for the first matrix model the values 10^{-18}, 10^{-12}, 10^{-9}, and for the second matrix model the values 10^{-18}, 10^{-18}, 10^{-3}. Due to large uncertainties in supersymmetric parameters, we cannot make definite predictions, so that previous numbers represent the effect of distinct matrix models, which is our main interest here. However, the element $(Y_D^\dagger Y_D)_{32} \sim 1$ in matrix (20) seems critical.

VII. DISCUSSION

By inverting the seesaw formula we have calculated the heavy neutrino mass matrix, and the implications for baryogenesis via leptogenesis and radiative lepton decays in certain supersymmetric models. The analysis leads to two distinct matrix forms, that is, a nearly diagonal model and a nearly off-diagonal model, which cannot provide sufficient baryon asymmetry. For recent related studies, see Ref. [25].

We have assumed $M_\nu \sim M_u$. However, this assumption can be changed. Indeed, the main feature of the Dirac neutrino mass matrix within the seesaw mechanism is that its overall scale is of the order of m_t. For example, we can take $M_\nu \simeq M_d m_t/m_b$, which means that it has the same overall scale of M_u, but the internal hierarchy of M_d,

$$M_\nu \sim \begin{pmatrix} 0 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} m_t. \quad (27)$$

In this case, sufficient baryon asymmetry is achieved, especially for

$$M_R \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \frac{m_t^2}{m_k}. \quad (28)$$

The branching ratios of lepton decays are also enhanced to 10^{-10}, 10^{-7}, 10^{-6}. However, these strongly depend on the mechanism of supersymmetry breaking. In fact, in the previous section we have adopted a gravity mediated breaking, where $M_U > M_R$, while for a gauge mediated breaking $M_U < M_R$ and running effects are not induced.

An indication towards the existence of the seesaw mechanism would be the evidence for neutrinoless double beta decay. For the moment we predict (in eV) $10^{-3} < M_{ee} < 0.86$. While the upper part of this range will be checked rather soon, the lower part is more difficult to reach.

In conclusion, assuming baryogenesis from leptogenesis, we are led towards a Dirac neutrino mass hierarchy similar to the down quark and charged lepton mass hierarchy. In some supersymmetric scenarios, this model may be checked by measurements of radiative lepton decays.

[1] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); S. L. Glashow, in *Quarks and Leptons*, edited by M. Levy *et al.* (Plenum, New York, 1980); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[2] R.N. Mohapatra and J.C. Pati, Phys. Rev. D **11**, 566 (1975); **11**, 2558 (1975); P. Langacker and S. Uma Sankar, *ibid.* **40**, 1569 (1989).

[3] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).

[4] H. Georgi, in *Particles and Fields*, edited by C. Carlson (AIP, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. **93**, 193 (1975).

[5] S.M. Bilenky, C. Giunti, and W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999).

[6] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[7] M. Fukugita and T. Yanagida, Phys. Lett. **174B**, 45 (1986).

[8] W.H. Furry, Phys. Rev. **56**, 1184 (1939).

[9] S.T. Petcov, Yad. Fiz. **25**, 641 (1977) [Sov. J. Nucl. Phys. **25**, 340 (1977)].

[10] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).

[11] M. Frigerio and A.Yu. Smirnov, J. High Energy Phys. **02**, 004 (2003).

[12] J.L. Chkareuli and C.D. Froggatt, Phys. Lett. B **450**, 158 (1999).

[13] H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D **60**, 013006 (1999).

[14] J.K. Elwood, N. Irges, and P. Ramond, Phys. Rev. Lett. **81**, 5064 (1998); M.S. Berger and K. Siyeon, Phys. Rev. D **63**, 057302 (2001); H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979).

[15] H. Fritzsch and Z. Xing, Phys. Lett. B **555**, 63 (2003).

[16] H.V. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney, and I.V. Krivosheina, Mod. Phys. Lett. A **16**, 2409 (2001).

[17] C. Giunti and M. Tanimoto, Phys. Rev. D **66**, 053013 (2002); **66**, 113006 (2002).

[18] S. Antusch, J. Kersten, M. Lindner, and M. Ratz, Phys. Lett. B **544**, 1 (2002).

[19] A.Yu. Smirnov, Phys. Rev. D **48**, 3264 (1999).

[20] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).

[21] D. Falcone and F. Tramontano, Phys. Rev. D **63**, 073007 (2001).

[22] W. Buchmuller and M. Plumacher, Int. J. Mod. Phys. A **15**, 5047 (2000).

[23] R.H. Cyburt, B.D. Fields, and K.A. Olive, astro-ph/0302431.

[24] J.A. Casas and A. Ibarra, Nucl. Phys. **B618**, 171 (2001).

[25] M. Jezabek and Y. Sumino, Phys. Lett. B **440**, 327 (1998); H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D **60**, 013006 (1999); G. Altarelli, F. Feruglio, and I. Masina, Phys. Lett. B **472**, 382 (2000); M. Hirsch and S.F. King, Phys. Rev. D **64**, 113005 (2001); C. Liu and J. Song, *ibid.* **65**, 057303 (2002).