

Hard scale in the exclusive ρ meson production in diffractive deep inelastic scattering

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(Received 15 March 2003; revised manuscript received 22 May 2003; published 8 August 2003)

We reanalyze the issue of the PQCD factorization scale in the exclusive ρ production in diffractive deep inelastic scattering from the k_T -factorization point of view. We find that this scale differs significantly from the widely used value $(Q^2 + m_\rho^2)/4$, and it is a much flatter function of Q^2 . With these results in mind, we discuss the Q^2 shape of the ρ meson production cross section and comment on the recent ZEUS observation of energy-independent ratio $\sigma(\gamma^*p \rightarrow \rho p)/\sigma_{\text{tot}}(\gamma^*p)$.

DOI: 10.1103/PhysRevD.68.032001

PACS number(s): 13.60.Le, 13.85.Fb

I. INTRODUCTION

The exclusive production of vector mesons in diffractive deep inelastic scattering (DIS), $\gamma(^*)p \rightarrow Vp$, turned out to be an ideal testing ground [1,2,3] of many predictions of the famous color-dipole approach [4,5,6]. Within this formalism, the basic quantity is the cross section of the color dipole interaction with the target proton $\sigma_{\text{dip}}(x, r)$, which can be approximately related to the conventional gluon density $G(x_g, \bar{Q}^2)$ of the proton [4]. The hard scale \bar{Q}^2 , at which the gluon density should be taken, is set both by the virtuality and the mass of the vector meson, and in the case of heavy quarkonium is approximately equal to $1/4(Q^2 + m_V^2)$. The latter result can be also obtained in the direct Dokshitzer-Gribov-Lipatov-Altarelli-Parisi- (DGLAP) inspired calculations [7].

Although the identification of the perturbative QCD (PQCD) factorization scale with $1/4(Q^2 + m_V^2)$ is valid only in the case of heavy vector mesons, it is often assumed that the same is true for light vector mesons and large virtualities. It is this assumption that stands behind a remarkable universality: the cross section of light and heavy mesons will exhibit the same Q^2 behavior if plotted against $Q^2 + m_V^2$ rather than Q^2 alone.

On the experimental side, the first data showed that cross sections of ρ , ω , ϕ , and J/ψ mesons, taken at equal values of $Q^2 + m_V^2$ and corrected by corresponding SU(4) factors, were indeed very close to each other [8]. However, new, more accurate data on ρ and J/ψ production proved that this universality was only approximate. Recently, ZEUS concluded [9] that the J/ψ production cross section is typically 40% higher than the cross sections of the light vector meson production. This difference is especially obvious on the plots of σ_L and σ_T separately [8].

It is natural then to ask what is the relevant PQCD factorization scale of the highly virtual ρ electroproduction,¹ and how it differs from $1/4(Q^2 + m_V^2)$. The answer to these questions was given already in [3]. Within the color dipole formalism, the PQCD factorization scales for the longitudinally

and transverse produced ρ mesons were shown to be $\bar{Q}^2(\rho_L) \approx 0.15 \cdot (Q^2 + m_V^2)$ and $\bar{Q}^2(\rho_T) \approx (0.07 - 0.1) \cdot (Q^2 + m_V^2)$. This is notably smaller than $(Q^2 + m_V^2)/4$, and suggests that even the highest values of Q^2 available experimentally correspond to, at most, semiperturbative values of $\bar{Q}^2(\rho)$.

In fact, the exact value of the PQCD factorization scale can be affected by the shape of the unintegrated gluon distribution. When [3] appeared, no numerically reliable parametrizations of dipole cross section or of the unintegrated gluon structure function were available, and one was bound to a semiquantitative guess. The situation changed two years ago, when numerically accurate, simple, and ready-to-use parametrizations of the unintegrated gluon structure function $\mathcal{F}(x_g, \vec{\kappa})$ were obtained from the analysis of proton structure function F_{2p} [10]. These parametrizations were devised for $x_{Bj} < 0.01$ and for the entire domain of relevant Q^2 values. They were put in the basis of the k_T -factorization calculations of both light and heavy vector meson production cross sections and yielded rather good description of the available data [11,12,13]. These fits now allow for a quantitative reanalysis of the hard scale in the ρ meson production. This is performed in the present paper.

The structure of the paper is the following. In Sec. II we briefly review the results of the k_T -factorization approach to the exclusive production of vector mesons in diffractive DIS and show that, at high Q^2 , it is natural to expect $\bar{Q}^2 < (Q^2 + m_V^2)/4$ for light vector mesons. We then conduct numerical analysis and explicitly find $Q^2 \rightarrow \bar{Q}_L^2$ and $Q^2 \rightarrow \bar{Q}_T^2$ mapping. In Sec. III we discuss phenomenological consequences of this mapping. Finally, in Sec. IV we draw conclusions.

II. DETERMINING THE SCALE IN EXCLUSIVE ρ PRODUCTION

The basic formulas for the vector meson production within the k_T -factorization approach are well-known (see details in [13]). Here, we limit ourselves to the forward production of vector mesons. We denote the quark and gluon loop transverse momenta by \mathbf{k} and $\boldsymbol{\kappa}$, respectively (here, the vector sign denotes transverse vectors). The fraction of the photon light cone momentum carried by the quark is denoted by z , while the fractions of the proton light cone momentum carried by the two gluons are x_1 and x_2 . With this notation,

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¹To be definite, we will deal with ρ mesons, but the general conclusions are valid for all light vector mesons.

the imaginary part of the forward amplitude can be written in a compact form:

$$\begin{aligned} \text{Im } \mathcal{A} = s \frac{c_V \sqrt{4\pi} \alpha_{em}}{4\pi^2} \int \frac{d^2 \boldsymbol{\kappa}}{\boldsymbol{\kappa}^4} \alpha_S(q^2) \mathcal{F}(x_1, x_2, \boldsymbol{\kappa}) \\ \times \int \frac{dz d^2 \mathbf{k}}{z(1-z)} \psi_V^*(z, \mathbf{k}) \cdot I(\lambda_V, \lambda_\gamma). \end{aligned} \quad (1)$$

The helicity-dependent integrands $I(\lambda_V, \lambda_\gamma)$ are well-known [13]. The strong coupling constant is taken at $q^2 \equiv \max[\bar{Q}^2 + \mathbf{k}^2, \boldsymbol{\kappa}^2]$, where $\bar{Q}^2 = z(1-z)Q^2 + m_q^2$.

For a very asymmetric gluon pair, the off-forward gluon structure function $\mathcal{F}(x_1, x_2, \boldsymbol{\kappa})$ that enters Eq. (1) can be approximately related to the forward gluon density $\mathcal{F}(x_g, \boldsymbol{\kappa})$ via

$$\mathcal{F}(x_1, x_2 \leq x_1, \boldsymbol{\kappa}) \approx \mathcal{F}(x_g, \boldsymbol{\kappa}).$$

Here $x_g \approx 0.41x_1$; the coefficient 0.41 is just a convenient representation of the off-forward to forward gluon structure function relation found in [14]. Numerical parametrizations of the forward unintegrated gluon density $\mathcal{F}(x_g, \boldsymbol{\kappa})$ for any practical values of x_g and $\boldsymbol{\kappa}^2$ can be found in [10].

The vector meson wave function $\psi_V(z, \mathbf{k})$ describes the projection of the $q\bar{q}$ pair onto the physical vector meson. When choosing the shape of the radial wave function, we followed a pragmatic strategy. We took two simple *Ansätze* for the wave function, namely, the oscillator and the Coulomb wave functions, and compared the results. Since these two wave functions represent the two extremes (very compact and very broad wave functions that still lead to the same value of the electronic decay width), the difference observed should give a reliable estimate of the uncertainty. Details can be found in [13,11].

When analyzing the PQCD scale of the ρ production, we follow the method used in [3]. We change the order of integration in Eq. (1) and introduce weight functions $W_L(\boldsymbol{\kappa}^2)$ and $W_T(\boldsymbol{\kappa}^2)$ according to

$$\text{Im } A_{L \rightarrow L} \equiv \int \frac{d\boldsymbol{\kappa}^2}{\boldsymbol{\kappa}^2} \mathcal{F}(x_g, \boldsymbol{\kappa}) \cdot W_L(\boldsymbol{\kappa}^2);$$

$$\text{Im } A_{T \rightarrow T} \equiv \int \frac{d\boldsymbol{\kappa}^2}{\boldsymbol{\kappa}^2} \mathcal{F}(x_g, \boldsymbol{\kappa}) \cdot W_T(\boldsymbol{\kappa}^2).$$

Let us first understand qualitatively the properties of $W_i(\boldsymbol{\kappa}^2)$ in the case of large virtualities Q^2 . To the leading $\log Q^2$, these weight functions are

$$W_i(\boldsymbol{\kappa}^2) \propto \int_0^1 dz z(1-z) \int d^2 \boldsymbol{\kappa} \psi(z, \boldsymbol{\kappa}) \cdot \frac{2}{\varepsilon^2(\varepsilon^2 + \boldsymbol{\kappa}^2)}. \quad (2)$$

If we denote by $\langle \varepsilon^2 \rangle$ the typical values of $\varepsilon^2 = z(1-z)Q^2 + m_q^2$ that dominate the integral (2), then the weight factors $W_i(\boldsymbol{\kappa}^2)$ stay almost constant at $\boldsymbol{\kappa}^2 \ll \langle \varepsilon^2 \rangle$, and quickly decrease at $\boldsymbol{\kappa}^2 > \langle \varepsilon^2 \rangle$. They should have the form of a ‘‘smoothed step function,’’ and effectively cut off from above

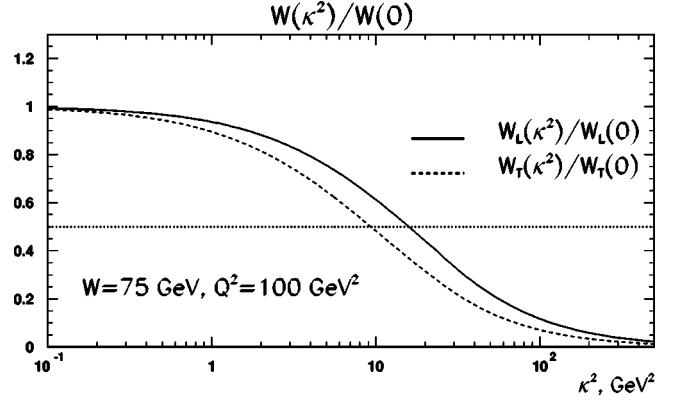


FIG. 1. The normalized weight functions $W_L(\boldsymbol{\kappa}^2)/W_L(0)$ and $W_T(\boldsymbol{\kappa}^2)/W_T(0)$ calculated at $Q^2 = 100 \text{ GeV}^2$. The $\boldsymbol{\kappa}^2$ values where W_L and W_T reach 1/2 are noticeably softer than $(Q^2 + m_\rho^2)/4$.

the $\boldsymbol{\kappa}^2$ region essential for the interaction. Therefore it is precisely $\langle \varepsilon^2 \rangle$ that settles the PQCD factorization scale.

In the case of heavy vector mesons, $z \approx 1/2$, and $\langle \varepsilon^2 \rangle \approx 1/4(Q^2 + m_q^2)$. However, for the ρ meson, the wave function is broad enough, so that typical values of $z(1-z)$ will be significantly less than 1/4, and the factorization scale will be noticeably softer than $(Q^2 + m_\rho^2)/4$.

We now check this expectation with the numerical analysis. Figure 1 shows the ratios $W_L(\boldsymbol{\kappa}^2)/W_L(0)$ and $W_T(\boldsymbol{\kappa}^2)/W_T(0)$ as a function of $\boldsymbol{\kappa}^2$ for $Q^2 = 100 \text{ GeV}^2$.

One sees that these ratios start decreasing at $\boldsymbol{\kappa}^2 \ll Q^2$ and reach 1/2 at $\boldsymbol{\kappa}^2 \approx 15 \text{ GeV}^2$ and $\boldsymbol{\kappa}^2 \approx 10 \text{ GeV}^2$, respectively. This shows that the above-mentioned effect of the broad wave function leads to significant softening of the relevant scale.

The exact factorization scales \bar{Q}_L^2 and \bar{Q}_T^2 can be defined in several ways. For example, one can take the $\boldsymbol{\kappa}^2$ points, where $W_i(\boldsymbol{\kappa}^2)$ reach $1/2W_i(0)$. Defined so, the longitudinal and transverse scales were found to be approximately equal to $1/6(Q^2 + 2.0 \text{ GeV}^2)$ and $1/11(Q^2 + 2.6 \text{ GeV}^2)$, respectively. The numbers 1/6 and 1/11 are very close to 0.15 and 0.07–0.1 obtained in [3].

The gluon density, however, has itself significant $\boldsymbol{\kappa}^2$ dependence [10]. Namely, in the region $\boldsymbol{\kappa}^2 \sim 1-10 \text{ GeV}^2$ and very small x_g ($x_g \leq 10^{-3}$) (which corresponds, at fixed $W = 75 \text{ GeV}$, to values of $Q^2 \leq 10 \text{ GeV}^2$), $\mathcal{F}(x_g, \boldsymbol{\kappa}^2)$ is a strongly rising function of $\boldsymbol{\kappa}^2$. At larger Q^2 , the effective x_g grows, and the unintegrated gluon density becomes flat. Finally, at large enough Q^2 (for $W = 75 \text{ GeV}$, this corresponds to $Q^2 \geq 100 \text{ GeV}^2$), the unintegrated gluon density decreases with $\boldsymbol{\kappa}^2$ growth in the region $\boldsymbol{\kappa}^2 \sim 1-10 \text{ GeV}^2$. Therefore the span of effectively contributive $\boldsymbol{\kappa}^2$ will extend to higher values of $\boldsymbol{\kappa}^2$ (at small Q^2) or reduce to smaller values of $\boldsymbol{\kappa}^2$ (at high Q^2). In order to take this into account, it is more useful to define the hard scales via the following implicit relations:

$$\frac{1}{W_i(0)} \int_0^\infty \frac{d\boldsymbol{\kappa}^2}{\boldsymbol{\kappa}^2} \mathcal{F}(x_g, \boldsymbol{\kappa}) W_i(\boldsymbol{\kappa}^2) \equiv G(x_g, \bar{Q}_i^2), \quad i = L, T. \quad (3)$$

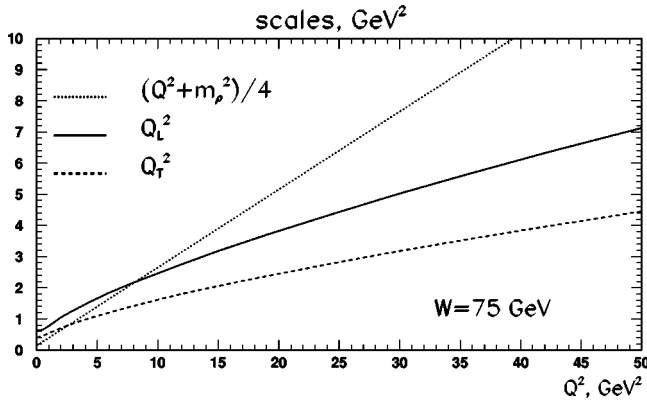


FIG. 2. The $Q^2 \rightarrow \bar{Q}_L^2$ and $Q^2 \rightarrow \bar{Q}_T^2$ mapping in the ρ production. The heavy meson analysis expectation $(Q^2 + m_\rho^2)/4$ is also shown.

Figure 2 shows the values of \bar{Q}_L^2 (solid line) and \bar{Q}_T^2 (dashed line), defined according to Eq. (3), as functions of Q^2 . These values start from 0.63 and 0.4 GeV^2 , respectively, in the photoproduction limit, and slowly grow with Q^2 rise. At $Q^2 = 27 \text{ GeV}^2$ (the highest Q^2 data point from H1 data on ρ production), these values are only around 4.5 and 3 GeV^2 , respectively. This confirms the conclusion of [3] that even at largest Q^2 where data are available, we still deal with a semi-perturbative situation. It is interesting to note that a better fit to these curves is given by a nonlinear, rather than a linear, approximation:

$$\bar{Q}_L^2 \approx 1.5 \cdot \bar{Q}_T^2 \approx 0.45 \cdot (Q^2 + 1.5)^{0.7}, \quad (4)$$

where all quantities are expressed in GeV^2 .

The same figure shows also, by dotted line, the expectation $(Q^2 + m_\rho^2)/4$ inspired by the heavy meson analysis. This expectation starts from 0.15 GeV^2 , which is noticeably smaller than $\bar{Q}_L^2(0)$ and $\bar{Q}_T^2(0)$, and rises with Q^2 significantly faster than \bar{Q}_L^2 and \bar{Q}_T^2 .

III. DISCUSSION

The quantitative understanding of the PQCD factorization scale in vector meson production allows one to address several phenomenological issues.

The Q^2 behavior of the ρ production cross section. The early data on ρ mesons were successfully parametrized (in the moderate and high- Q^2 region) by a simple law, $\sigma(\gamma^* p \rightarrow \rho p) \propto (Q^2 + m_\rho^2)^{-n}$, with $n = 2.32 \pm 0.10$ (ZEUS, [15]) or $n = 2.24 \pm 0.09$ (H1, [16]). However, further experiments in a much broader Q^2 region made it clear that such powerlike fits have very limited applicability domain. A natural question has been raised [9] as what would be the most insightful and physically motivated fit to the Q^2 behavior of $\sigma(\gamma^* p \rightarrow \rho p)$.

In most approaches to the exclusive ρ meson production, one has to deal with the gluon content of the proton, which contributes to the Q^2 dependence of the cross section. One might want to get rid of this rather “trivial” source of the Q^2 behavior and study the Q^2 properties of the underlying dy-

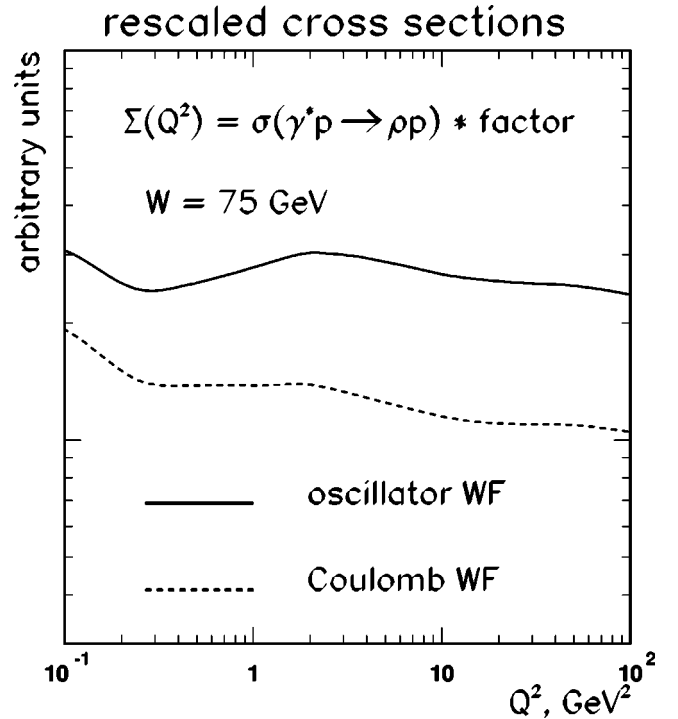


FIG. 3. The k_t -factorization predictions for the rescaled cross section $\Sigma(Q^2)$ for the ρ meson production.

namics. For this purpose, we suggest to consider rescaled cross section.

$$\Sigma(Q^2) = \sigma(\gamma^* p \rightarrow \rho p) \cdot \frac{(\bar{Q}^2)^3 \cdot b(Q^2)}{[G(x_g, \bar{Q}^2) \cdot \alpha_s(\bar{Q}^2)]^2}. \quad (5)$$

Here \bar{Q}^2 is familiar $1/4(Q^2 + m_V^2)$. Note that the gluon density is taken here at constant energy, so that $x_g = 0.41(Q^2 + m_\rho^2)/W^2$ also depends on Q^2 . This form of $\Sigma(Q^2)$ is motivated by the leading log Q^2 result (see, for example, [7]). If this result precisely reflects the real interaction, the rescaled cross section will be Q^2 -independent. Departure of the experimentally measured $\Sigma(Q^2)$ from the constant value will quantify how much the real situation differs from the leading log Q^2 result.

Figure 3 shows the k_t -factorization predictions for $\Sigma(Q^2)$ as function of Q^2 . One sees that although the factors in Eq. (5) have removed the strongest Q^2 -dependence from the cross section,² the result is still not constant. This is not surprising, since, as we showed in the previous section, the factorization scale in ρ production differs from $1/4(Q^2 + m_V^2)$. Indeed, when we considered $\Sigma_L(Q^2)$ and $\Sigma_T(Q^2)$, defined similarly to Eq. (5) but with replacements $\bar{Q}^2 \rightarrow \bar{Q}_L^2$ and $\bar{Q}^2 \rightarrow \bar{Q}_T^2$, respectively, we found almost constant values of $\Sigma_L(Q^2)$ and $\Sigma_T(Q^2)$.

²Recall that the ρ production cross section itself spans more than four orders of magnitude within the Q^2 interval shown.

In principle, all the quantities used in Eq. (5) are accessible in experiment. The only delicate issue will be the choice of the gluon density, especially at low Q^2 , since the DGLAP fits to conventional gluon density are available only for $Q^2 \geq 1 \text{ GeV}^2$ and can differ significantly from the k_t -factorization results, see [10]. It is still interesting to see how flat the experimental results on $\Sigma(Q^2)$ will be.

$\sigma(\gamma^{(*)}p \rightarrow \rho p)/\sigma_{\text{tot}}(\gamma^{(*)}p)$ problem. Another issue that demands the understanding of the hard scale in ρ production is a recent observation by ZEUS [9] that the measured value of the ratio

$$r_\rho = \frac{\sigma(\gamma^{(*)}p \rightarrow \rho p)(W^2, Q^2)}{\sigma_{\text{tot}}(\gamma^{(*)}p)(W^2, \bar{Q}^2)}, \quad (6)$$

is, within the errors, energy-independent. One expects from the Regge model, as well as from the PQCD approach, that $\sigma_{\text{tot}}(\gamma^{(*)}p)$ is linear, and $\sigma(\gamma^{(*)}p \rightarrow \rho p)$ is quadratic in the Pomeron exchange, therefore, both approaches predict this ratio to grow with energy rise. Thus energy independence of r_ρ appears to be at odds with theory.

The Pomeron intercept depends significantly on the hard scale involved in the interaction, see experimental data [15–18] and results of the phenomenological analysis of [10]. Therefore, when studying energy dependence of the ratio (6), one must make sure that the hard scales in both cross sections are equal.

In [9] these scales were identified with Q^2 for the total virtual photoabsorption cross section and with $\bar{Q}^2 = (Q^2 + m_\rho^2)/4$ for vector meson production. However, as we argue in this paper, the true scale of the ρ production can noticeably differ from $(Q^2 + m_\rho^2)/4$, especially at very small and very large Q^2 . This mismatch of the scales can be at least one of the sources of the discrepancy observed. Unfortunately, our numerical analysis showed that this effect was marginal and did not lead to resolution of the problem.

IV. CONCLUSIONS

In this work we investigated, at the quantitative level, the value of the PQCD factorization scale in the exclusive production of light vector mesons. The work was conducted in the k_t -factorization scheme, closely related to the familiar color dipole formalism, and was based on recent fits to the unintegrated gluon density obtained in [10]. The fact that we do not devise models for the gluon content but instead

heavily rely on the high-precision experimental data lends certain credence to the whole calculations.

We gave explicit results for PQCD factorization scales in longitudinal, \bar{Q}_L^2 , and transverse, \bar{Q}_T^2 , ρ production, and compared them with the widely used expression $(Q^2 + m_\rho^2)/4$. We found that: at smaller Q^2 ($Q^2 \lesssim 3-5 \text{ GeV}^2$) the DGLAP factorization scale is larger than $(Q^2 + m_\rho^2)/4$; at large enough Q^2 ($Q^2 \gtrsim 10 \text{ GeV}^2$ for the transverse amplitude and $Q^2 \gtrsim 20 \text{ GeV}^2$ for the longitudinal amplitude), the factorization scale is significantly smaller than $(Q^2 + m_\rho^2)/4$. This should be taken as a word of caution against an unwarranted application of the DGLAP approach to the problem of ρ meson production even at high Q^2 ; the overall Q^2 dependence of the PQCD factorization scale is significantly flatter than $(Q^2 + m_\rho^2)/4$. This is mostly due to the specific way the κ^2 behavior of the unintegrated gluon density changes, as the Q^2 increases (at fixed W); the PQCD factorization scale defined according to Eq. (3) is affected by the shape of unintegrated gluon density and is, therefore, energy dependent; and the presence of m_ρ in the often used scale $(Q^2 + m_\rho^2)/4$ is misleading, since the ρ meson mass has little relevance to the color dipole interaction with the target proton. Instead, Q^2 appears in combinations $Q^2 + M^2$ with $M^2 \approx 1.5-2.5 \text{ GeV}^2$.

These results allowed us to address the issue of Q^2 dependence of the ρ production cross section. Using scales \bar{Q}_L^2 and \bar{Q}_T^2 , we were able to factor out all sources of the Q^2 dependence of the cross section within the k_t -factorization approach. It would not be possible, if we used $(Q^2 + m_\rho^2)/4$ as a PQCD factorization scale, as illustrated by the rescaled cross section $\Sigma(Q^2)$, Fig. 3.

We also commented on a recent observation of energy independence of $r_\rho = \sigma(\gamma^{*}p \rightarrow \rho p)/\sigma_{\text{tot}}(\gamma^{*}p)$ ratio. We pointed out that the procedure used in experimental study of this ratio leads to a mismatch of the hard scales in $\sigma(\gamma^{*}p \rightarrow \rho p)$ and $\sigma_{\text{tot}}(\gamma^{*}p)$, which might be one of the causes of the observed discrepancy between the experiment and the theory expectations.

ACKNOWLEDGMENTS

I am thankful to Kolya Nikolaev for many valuable comments and to Igor Akushevich for his help in the early stage of the code development. I also wish to thank Professor J. Speth for hospitality at the Institut für Kernphysik, Forschungszentrum Jülich. The work was supported by INTAS grants 00-00679 and 00-00366, RFBR grant 02-02-17884, and grant “Universities of Russia” UR 02.01.005.

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