Time evolution via *S*-branes

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Using *S*(pacelike)-branes defined through rolling tachyon solutions, we show how the dynamical formation of D(irichlet)-branes and strings in tachyon condensation can be understood. Specifically, we present solutions of *S*-brane actions illustrating the classical confinement of electric and magnetic flux into fundamental strings and D-branes. The role of *S*-branes in string theory is further clarified and their Ramond-Ramond charges are discussed. In addition, by examining "boosted" *S*-branes, we find what appears to be a surprising dual *S*-brane description of strings and D-branes, which also indicates that the critical electric field can be considered as a self-dual point in string theory. We also introduce new tachyonic *S*-branes as Euclidean counterparts to non-Bogomol'nyi-Prasad-Sommerfield branes.

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I. INTRODUCTION

Tachyon condensation in open string theories has revealed new intriguing aspects of string theories and D-branes. One of the meritorious achievements in this area is that we can now describe D-branes as topological solitons in (effective) field theories of tachyons and string field theories. This approach to D-branes has also been extended to deal with the time dependent decay or creation of D-branes. In developing tools to deal with the complexities of time dependent systems, new ingredients called *S*(pacelike)-branes were introduced in Ref. [1]. Whereas ordinary D-branes are realized as timelike kinks and vortices of the tachyon field, spacelike defects can be defined as spacelike kinks and vortices in the background of a time dependent tachyon condensation process called rolling tachyons [2]. As defined *S*-branes are intrinsically related to and naturally arise in time dependent processes in string theory.[1]

In Ref. [3], some of the present authors demonstrated that *S*-branes can in fact describe the formation of topological defects in time dependent tachyon condensation. The key point was that while flat *S*-branes are defined as spacelike defects of a specific rolling tachyon solution, we can also introduce fluctuations into the rolling tachyon which will accordingly deform the *S*-branes. It was then found that the information from only the *S*-brane fluctuations is sufficient to describe the formation of individual fundamental strings as remnants of the original tachyon system. The advantage of the *S*-brane approach in describing tachyon remnant formation came from the fact that explicit knowledge of the full tachyon action was not necessary. This is a generalized correspondence between tachyon systems and Dirac-Born-Infeld (DBI) systems on the tachyon defects [12,13]. *S*-branes are universally governed by a Euclidean DBI effective action, independent of the specific details of the original tachyon systems, and with scalar excitations along the time direction. While many tachyonic Lagrangians have similar features and give rise to the same type of static solitons and rolling tachyon backgrounds, we must look for these solutions in each Lagrangian individually. Another advantage of the *S*-brane approach is then that an *S*-brane solution represents a class of solutions for many tachyonic Lagrangians; these solutions are classes in the sense that many different tachyonic Lagrangians give rise to the same type of *S*-brane solutions. So while in string theory the tachyon effective actions are obtained in various forms with different derivations, the *S*-brane approach gives a universal treatment. A third advantage is that it is easier to solve the equations of motion for the *S*-brane action than for arbitrary tachyon systems.

In this paper, after discussing *S*-branes and their role in time dependent physics in Sec. II, we will illustrate our ideas by presenting classical solutions of the *S*-brane actions, clarifying their role and obtaining their corresponding tachyon descriptions.[2] In Sec. III we recapitulate the solution [3] of the formation of confined electric fluxes which are fundamental strings. In addition we show how the *S*-brane solution is consistent with the tachyon picture of classical flux confinement. In Sec. IV new solutions representing the formation of (p,q) strings are presented and we relate these new solutions to an implementation of *S* duality for *S*-branes. The late time behavior of these *S*-brane solutions can be captured by simple linear solutions which we call "boosted" *S*-branes. These boosted *S*-branes are given corresponding explicit tachyon solutions and boundary state descriptions in Sec. V, and their consistency with the usual string and D-brane picture is checked. *T* duality in the time direction is found to interchange these two classes of D-brane solutions with the electric field above or below the critical value. In Sec. VI we examine the possibility that *S*-brane solutions may describe

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[1]See Refs. [4–10] for the development following Ref. [1]. Early work on tachyon condensation includes Ref. [11].

[2]We neglect closed string backreactions when describing the rolling tachyon.

D-brane scattering and Feynman diagrams for D-branes. We further find a generalized Ramond-Ramond (RR) charge conservation law for *S*- or D-branes. Section VII is devoted to conclusions and discussions.

It should be emphasized that although we are using the language of string theory, any theory with topological defects will have its own ‘‘*S*-branes’’ or spacelike defects. Some of these solutions should necessarily describe defect formation. It would be fascinating if our methods can be further applied to the formation of other topological defects and also provide dual descriptions of all kinds of defects and remnants.

In the paper we take $2\pi\alpha' = 1$ unless stated otherwise.

II. ROLES OF *S*-BRANES

The central idea we explore throughout this paper is how *S*-branes can be used to describe time dependent defect formation and tachyon condensation decay remnants. The detailed exploration of the classical solutions of *S*-brane actions will be provided in later sections, and we first concentrate on the general properties of *S*-branes, explaining their important roles in time-dependent tachyon condensation. Along the way we will see how *S*-branes and their classical solutions can be classified by the species of tachyon remnants, and discuss a new type of *S*-brane, which we name the tachyonic *S*-brane. We also derive *S*-brane actions which have a universal form, slightly generalizing the results in Ref. [3].

A. Remnant or defect formation

Assuming that the tachyon potential for a non-Bogomol’nyi-Prasad-Sommerfield (BPS) D-brane is minimized at some values for both $T>0$ and $T<0$, kink solutions can be approximately depicted by the $T=0$ loci. While the timelike kinks correspond to D-branes, the spacelike ones are *S*-branes. When *S*-branes were first introduced, they provided a fresh approach to the study of time dependent systems, but only fine tuned configurations were considered. Actually, as we will now demonstrate, *S*-branes appear ubiquitously during tachyon condensation. This is why it is worthwhile to define the *S*-brane action and to study its general solutions [3].

At late times of the tachyon condensation process, it is possible to describe D-brane remnants as kinks (or lumps) in the tachyon potential. In principle it should be possible to follow the time evolution of these $T=0$ regions. One might ask why we need to consider *S*-branes. The point is that, given a generic tachyon configuration, before the remnants are fully formed (before the tachyon profiles are localized), *S*-branes appear first in the time dependent formation of defects. These $T=0$ regions can ‘‘appear out of nowhere’’ at some time and are exactly *S*-branes. Only when the $T\simeq 0$ region becomes spatially localized has the *S*-brane metamorphosed or decayed into a D-brane (topological defect), see Fig. 1. In addition, even if there are no remnants, short-lived *S*-branes will appear as long as the energy is large enough to create local fluctuations over the top of the tachyon potential.

Furthermore, although it is suggested by its name and usually assumed that the *S*-branes are spacelike, the *S*-brane action admits timelike solutions which correspond to D-branes with a large electric field. We have seen such solutions in Ref. [3] and will present others below.

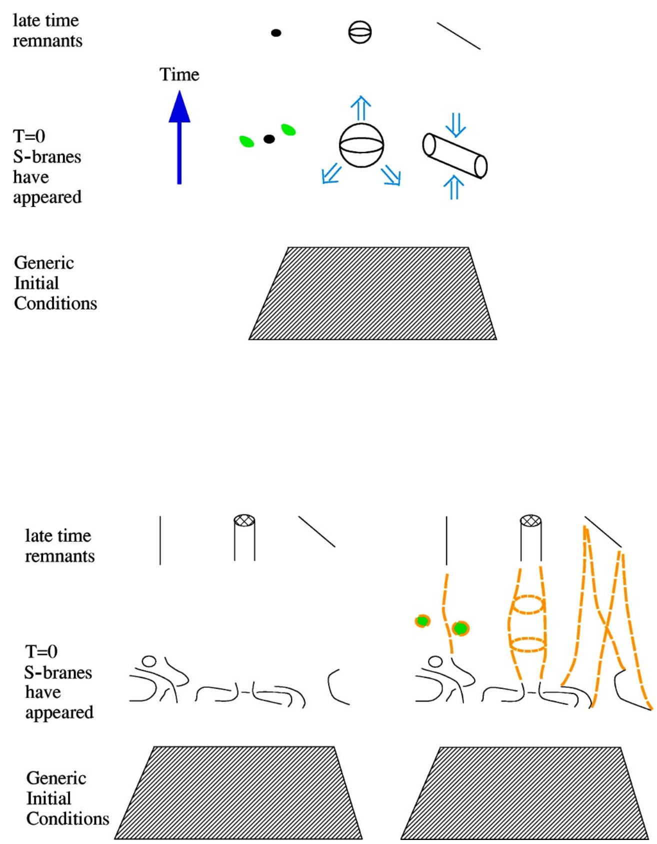

FIG. 1. The top figure is a series of snapshots of tachyon time evolution processes but since time is not explicit, the role of the *S*-brane is obscured. The bottom left figure is essentially just a redrawing of the top figure. The bottom right figure shows the entire dynamical evolution process with the *S*-branes outlined. The $T=0$ regions are drawn in as dashed lines. The main point is that at late times we have remnants with tachyon value zero and we can produce them from generic initial conditions. *S*-branes are how we ‘‘connect the lines’’ from the initial to final stage.

B. *S*-branes as classes of tachyon decay

In the case of tachyonic Lagrangians, it is possible to find kink solutions which represent lower dimensional excitations such as D-branes. These relations between unstable branes and ‘‘static’’ branes are also called the descent relations. A different question one can ask is how are the various objects in string theory related when we take into account time dependent processes? If we start off with a tachyonic system and end up with a stable system, then what is the time evolution process which connects these two systems? We propose that *S*-branes be used to classify the time evolution processes whenever there are remnants in the end.

We emphasize that there are differences between the *S*-branes of the non-BPS brane and the D-$\bar{D}$ system. It is clear that the *S*-branes share common properties but there should also be some differences due to the additional tachyon on the D-$\bar{D}$ pair. There are additional *S*-branes for

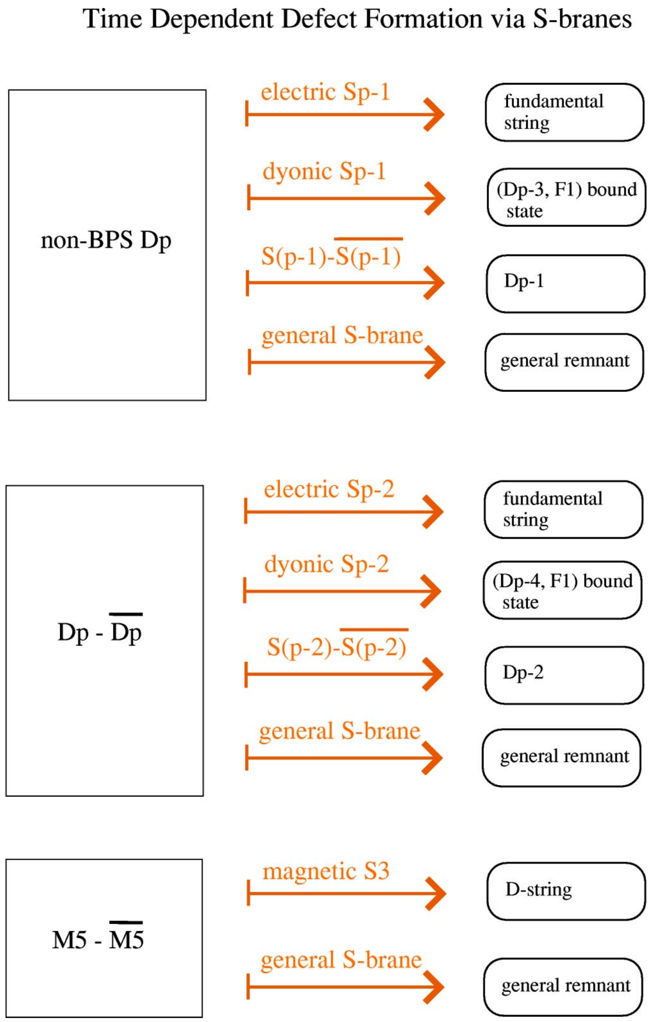

FIG. 2. Time evolution processes characterized by *S*-branes. The *S*-branes are the arrows. The upper three arrows starting from the non-BPS D*p*-brane will be treated in Secs. III, IV and VI, respectively. Although the *S*-branes from the non-BPS brane basically have counterparts in the D*p*-$\overline{\text{D}p}$, the arrows emanating from the D*p*-$\overline{\text{D}p}$ include processes previously unknown, especially the ones mediated by tachyonic *S*-branes. All arrows are commonly expected both in type IIA and IIB string theories. Finally, to understand the creation of D-strings, it is necessary to incorporate M-theory effects as indicated in the bottom figure and discussed in Sec. IV.

the D-$\overline{\text{D}}$ system that we call "tachyonic *S*-branes," which might be considered Euclidean counterparts of non-BPS branes in view of the correspondence that the original *S*-branes are Euclidean counterparts of BPS D-branes; the precise correspondence between tachyonic *S*-branes and Euclidean non-BPS branes is, however, not clear (see the next section for the precise definition of the tachyonic *S*-branes). Tachyonic *S*-branes should not be hard to differentiate from *S*-branes and describe essentially different time evolution processes. Some processes might be solutions of *S*-brane Lagrangians and some might be solutions of tachyonic *S*-brane Lagrangians. With this point in mind, we summarize the solutions discussed in this paper in Fig. 2.

In Ref. [1] *S*-branes represented a tachyon configuration rolling up and down the tachyon potential with the energy

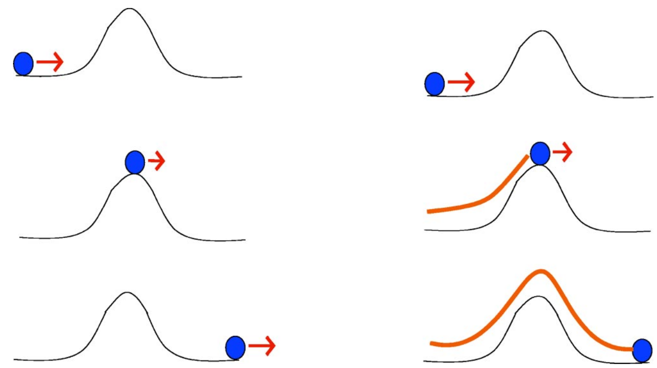

FIG. 3. Two different time evolution processes characterized by *S*-branes. The three pictures on the left characterize the rolling tachyon picture so the *S*-brane appears only when the tachyon crosses the top of the potential. The second three pictures give a schematic of remnant creation. We start off with some energy in the tachyon and perhaps in other fields. As the tachyon rolls, at some point it starts to create $T=0$ regions specified by the thick lines which eventually turn into remnants. At late times, the tachyon does not roll (no velocity arrow) as all the energy has been transferred into the remnant kink.

necessary to go up the potential remaining as some background contribution. This means at late times we have a time evolving system with energy stored in either radiation, the rolling tachyon or various other fields. In our case, however, long lived *S*-branes represent remnant formation and this difference implies that the process is not always time reversal invariant. As an example of the process we are considering, let us consider a finite energy configuration with the tachyon at large negative values. As the system evolves we climb up the tachyon potential, and at some point an *S*-brane shows up and eventually creates a remnant. The energy of the configuration can then be totally transferred to the remnant, so the *S*-brane shows how delocalized systems organize and transform energy into a remnant; in the end there might be no energy left to go into radiation, the rolling tachyon or anything else.[3] The *S*-brane schematically pulls the tachyon values over the potential and leaves a remnant solution in the process, see Figs. 3 and 4.

Reference [1] also discusses the width of an *S*-brane. In the context of tachyon condensation an analogous question is how easy is it to put one flat *S*-brane one after another in time. In general it is not clear if there is some limiting factor since it takes time for the tachyon to roll up and down the potential; however, it should not be impossible to have multiple *S*-branes. Any initial conditions forming the rolling tachyon can simply be repeated at some later time so this will roughly produce two separated rolling tachyon processes and two flat *S*-branes. It is the interactions between the initial conditions which will place a limit on how easy it is to

[3]In the argument here we compactify directions transverse to the resultant remnant in the world volume of the original unstable brane. This is necessary for the remnant to possess a finite tension. This observation is consistent with what has been studied in other literature [6,8,9].

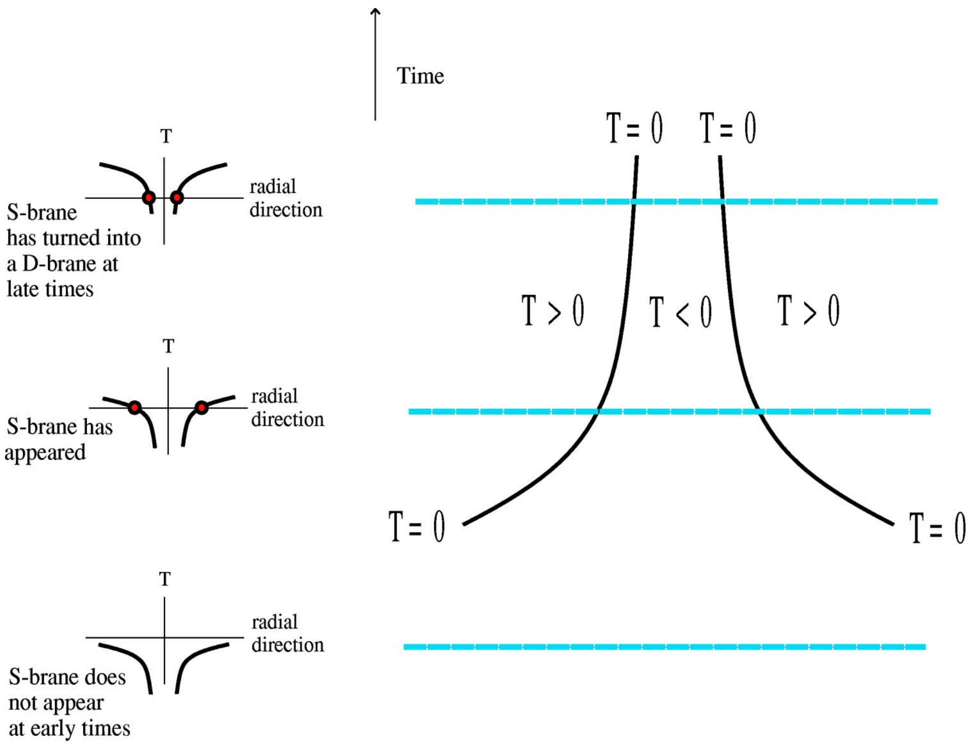

FIG. 4. The figure on the right is a schematic cross section of tachyon values on the non-BPS brane which gives rise to a decaying *S*-brane. To the left we have included snapshots of the tachyon values at specific times. At early times the tachyon configuration is changing but an *S*-brane has not appeared. The *S*-brane then appears, coming in from infinity, and then slows down to metamorphose into a D-brane. The tachyon configuration is not a kink or lump but more like an infinite well. Time dependent kinks do not necessarily leave spatial kink remnants. Related discussion can be found in Secs. V and VI.

produce multiple *S*-branes. This question could be explored further and it is related to coincident *S*-branes and their possible non-Abelian structure.

C. *S*-brane descent relations and new "tachyonic" *S*-branes

It has been argued that static tachyonic kink solutions on non-BPS branes correspond to codimension-one BPS branes, while vortex solutions on D-$\bar{D}$ pairs are codimension-two BPS branes. The relationship between these branes is summarized by the usual descent relations [14]. In analogy, Gutperle and Strominger [1] also defined *S*-branes as time dependent kinks (vortices) on non-BPS branes (D-$\bar{D}$ pairs), so it should be possible to extend the descent relations, shown in Fig. 5, to include both D-branes and *S*-branes. One may understand that the horizontal correspondence in the figure is just Euclideanization, or the change "timelike ↔ spacelike." For example, from this viewpoint the relation between the $S(p-2)$-brane and the non-BPS $\mathrm{D}(p-1)$-brane can be understood[4] as an arrow (1) in the extended descent relations. This arrow is how one can derive an *S*-brane action from the non-BPS D-brane action [3]. The $D(p-2)$ vortex solution on a $\mathrm{D}p$-$\bar{\mathrm{D}}p$ can be generalized to an *S*-brane counterpart. Later in this section we will derive the action of an *S*-brane spacetime vortex along the arrow (4).

First, starting at the top right of Fig. 5 we have an S-$\bar{S}$ pair. The figure also contains the tachyonic $S(p-1)$-brane. The tachyonic brane is naturally embedded into the extended descent relation since the space-time vortex [the arrow (4) in the figure] from D-$\bar{\mathrm{D}}$ to an $S(p-2)$ can be decomposed into two procedures: first construct a time-dependent kink (2) and then a space-dependent kink (3). The second procedure is almost the same as the arrow from the non-BPS $\mathrm{D}(p-1)$ to the BPS $\mathrm{D}(p-2)$.

To understand what a tachyonic *S*-brane is, let us first construct it. We begin with the Lagrangian of a $\mathrm{D}p$-$\bar{\mathrm{D}}p$ pair, choosing the Lagrangian of the boundary string field theory (BSFT) [15–17] since it is the best understood. The recent paper by Jones and Tye [18] proposed the action

$$S=-2T_{\mathrm{D9}}\int d^{10}x e^{-\pi|T|^2}\mathcal{F}(X+\sqrt{Y})\mathcal{F}(X-\sqrt{Y}), \quad (2.1)$$

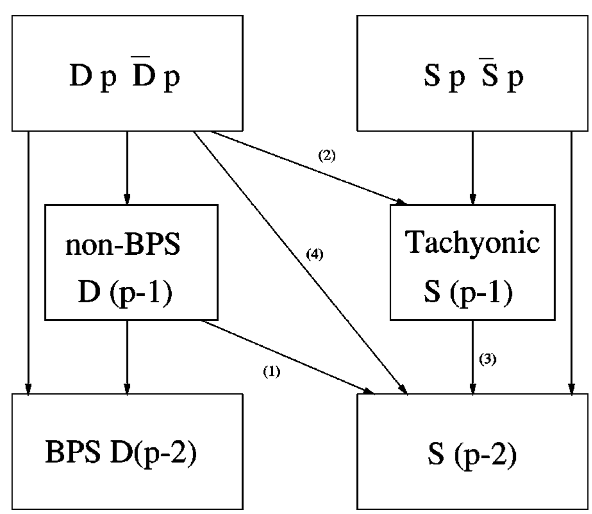

FIG. 5. The extended descent relation for tachyon condensations. We do not deal with the relation between type IIA and type IIB here.

[4]Note that the arrows in this figure are not the physical processes of formation which are depicted in Fig. 2. Here the arrows just represent construction of classical solutions from Lagrangians.

where we define $X\equiv\partial_\mu T\partial^\mu \overline{T}$ and $Y\equiv(\partial_\mu T)^2(\partial^\nu \overline{T})^2$, and for simplicity we choose $p=9$. We do not need detailed information of the kinetic function $\mathcal{F}$ here. This action is valid for linear tachyon profiles, but unfortunately a linear ansatz for time-dependent homogeneous solutions $T=T(x^0)$ leads to only trivial solutions (see Ref. [19]). Even though we exceed the validity of the action, let us proceed for the moment and examine the homogeneous tachyon solution. Noting that the D-$\overline{\text{D}}$ system reduces to the non-BPS brane system when we restrict the complex tachyon $T=T_1+iT_2$ to take only real value T_1, it is easy to see that the classical solution presented in Ref. [19],

$$T=T_{\rm cl}(x^0)=x^0+[\text{exponentially small terms for large } x^0], \tag{2.2}$$

is the tachyon solution on the D-$\overline{\text{D}}$ which we are looking for. The imaginary part T_2 of the complex tachyon appears in the Lagrangian only in squared form and so the equation of motion for T_2 has an overall factor T_2 or ∂T_2 and is trivially satisfied by $T_2=0$. However, the "tachyonic" fluctuation from T_2 leads to a new feature which we call the tachyonic *S*-brane. An effective tachyonic *S*-brane action is discussed in Appendix A.

Next, we consider arrow (4) in this section, which will provide another way to derive the *S*-brane action. This solution can be thought of as a combination of a time-dependent kink and the usual space-dependent kink along x^1. The solution of the BSFT action (2.1) is easily found

$$T=T_{\rm cl}(x^0)+iux^1 \tag{2.3}$$

where u goes to infinity by the usual BSFT argument for spatial kinks [16,17]. This classical solution has two zero modes in fluctuations since this "spacetime vortex" breaks two translation symmetries.

Following the analysis of Ref. [20] we construct an effective action of the spacetime vortex which we identify as an *S*-brane. The effective action of a D9-$\overline{\text{D}}$9 system takes the form

$$S=2T_{\rm D9}\int d^{10}x e^{-\pi|T|^2}\sqrt{\det(1+F)}f(X,Y) \tag{2.4}$$

where F is the diagonal linear combination of the two $U(1)$ gauge fields, $F=F_1+F_2$ and X,Y are now defined using the open string metric with respect for F

$$X\equiv G^{\mu\nu}\partial_\mu T\partial_\nu \overline{T},\quad Y\equiv|G^{\mu\nu}\partial_\mu T\partial_\nu T|^2. \tag{2.5}$$

This effective action is constrained by the usual assumption that the fields are slowly varying. The fluctuation fields which are zero modes (Nambu-Goldstone modes) are embedded in the action in a special manner since it is associated with the breaking of the translational symmetries. In fact, they appear as a kind of Lorentz transformation,

$$T=T_{\rm sol}(y_0,y_1),\quad y_0\equiv\frac{1}{\beta_0}[x_0-t_0(x_{\hat\mu})],$$

$$y_1\equiv\frac{1}{\beta_1}[x_1-t_1(x_{\hat\mu})], \tag{2.6}$$

$$x\to y=\Lambda x,\quad (\Lambda^t)G\Lambda=G, \tag{2.7}$$

where the open string metric is [we turn on only $F_{\hat\mu\hat\nu}$ ($\hat\mu,\hat\nu=2,\dots,9$)]

$$G^{\hat\mu\hat\nu}=\left(\frac{1}{1-F^2}\right)^{\hat\mu\hat\nu},\quad G^{00}=-1,\quad G^{11}=1,\quad G^{0\hat\mu}=G^{1\hat\nu}=0 \tag{2.8}$$

and the Lorentz transformation matrix Λ is

$$\Lambda=\left(\begin{array}{cc|c} 1/\beta_0 & 0 & -\partial_{\hat\mu}t_0/\beta_0 \\ 0 & 1/\beta_1 & -\partial_{\hat\mu}t_1/\beta_1 \\ \hline * & * & * \\ \vdots & \vdots & \vdots \end{array}\right). \tag{2.9}$$

Lorentz invariance (2.7) of the open string metric determines the beta factors

$$\beta_0=\sqrt{1-G^{\hat\mu\hat\nu}\partial_{\hat\mu}t_0\partial_{\hat\nu}t_0},\quad \beta_1=\sqrt{1+G^{\hat\mu\hat\nu}\partial_{\hat\mu}t_1\partial_{\hat\nu}t_1},$$
$$G^{\hat\mu\hat\nu}\partial_{\hat\mu}t_0\partial_{\hat\nu}t_1=0 \tag{2.10}$$

which can be substituted back into the action to give, after performing the integration over x^0 and x^1,

$$\begin{aligned} S&=S_0\int d^8x\beta_0\beta_1\sqrt{\det(\delta_{\hat\mu\hat\nu}+F_{\hat\mu\hat\nu})}\\ &=S_0\int d^8x\sqrt{\det(\delta_{\hat\mu\hat\nu}+F_{\hat\mu\hat\nu}-\partial_{\hat\mu}t_0\partial_{\hat\nu}t_0+\partial_{\hat\mu}t_1\partial_{\hat\nu}t_1)}. \end{aligned} \tag{2.11}$$

This is the effective action for the spacetime vortex, coinciding with the *S*-brane action which was derived in Ref. [3] if we set $t_1=0$. The new scalar field t_1 appears in the same way as how the usual D-brane action is generalized to the D-$\overline{\text{D}}$ pair. This action naturally leads to the following general form of the *Sp*-brane action in which the worldvolume embedding in the bulk spacetime (X^M with $M=0,1,\dots,9$) has not been gauge fixed

$$S=S_0\int d^{p+1}x\sqrt{\det(\partial_{\hat\mu}X_M\partial_{\hat\nu}X^M+F_{\hat\mu\hat\nu})}. \tag{2.12}$$

The field t_0 in Eq. (2.11) is identified with the embedding scalar X^0. Since in our derivation we did not refer to a specific tachyon effective action, the form of the *S*-brane action is universal in the slowly varying field approximation.[5]

[5]We expect that our *S*-brane action derived using a field theoretic approach is related to the long-distance *S*-brane effective field theory in Ref. [10].

III. STRINGS FROM S-BRANES

S-brane solutions describing a flux tube confining into a fundamental string have been previously discussed in Ref. [3]. In this section we reexamine the solution from a spacetime perspective which will be helpful in finding other *S*-brane solutions in the next section. Also, by directly analyzing the tachyon system, we find further evidence that the *S*-brane solution should be regarded as a fundamental string.

A. Solution of F-string formation

Let us review the electric $S3$-brane spike solution of Ref. [3].[6] The *S*-brane actions of Eq. (2.11) were derived in a certain gauge in which the time direction was treated as a scalar field X^0. In the following sections we will discuss *S*-brane solutions with nontrivial time dependence, so we take the following gauge choice which is preferable in the spacetime point of view

$$\begin{aligned} X^0 &= t \\ X^1 &= r(t)\cos\theta \\ X^2 &= r(t)\sin\theta\cos\phi \\ X^3 &= r(t)\sin\theta\sin\phi \\ X^4 &= \chi \\ F_{t\chi} &= E(t) \end{aligned} \tag{3.1}$$

$$ds^2 = (-1+\dot{r}^2)dt^2 + d\chi^2 + r^2(t)[d\theta^2 + \sin^2\theta d\phi^2], \tag{3.2}$$

where we parametrize the worldvolume of the $S3$-brane by (t,θ,ϕ,χ). At any given moment, the *S*-brane worldvolume is a cylinder, $R\times S^2$. The open string metric and its inverse are

$$(g+F)_{ab} = \begin{pmatrix} -1+\dot{r}^2 & E(t) & 0 & 0 \\ -E(t) & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}, \quad (\text{rows/columns: } t, \chi, \theta, \phi) \tag{3.3}$$

$$(g+F)^{ab} = \frac{1}{-1+\dot{r}^2+E^2}\begin{pmatrix} 1 & -E(t) \\ E(t) & -1+\dot{r}^2 \end{pmatrix}. \quad (\text{rows/columns: } t, \chi) \tag{3.4}$$

The Lagrangian for this *S*-brane is (up to a normalization constant for the *S*-brane tension)

$$\sqrt{\det(g+F)} = r^2\sin\theta\sqrt{-1+\dot{r}^2+E^2} \tag{3.5}$$

and the equation of motions for the embedding are

$$\partial_a[\sqrt{\det(g+F)}(g+F)^{ab}\partial_b X^M] = 0, \tag{3.6}$$

where $M=0,\ldots,4$. There are only two distinct equations of motion for this system (the gauge field equations of motion can also be checked), the first of which is

$$\partial_a[r^2\sin\theta\sqrt{-1+\dot{r}^2+E^2}(g+F)^{ab}\partial_b t] = 0 \tag{3.7}$$

while the second equation of motion is

$$\partial_a[r^2\sin\theta\sqrt{-1+\dot{r}^2+E^2}(g+F)^{ab}\partial_b(r\cos\theta)] = 0. \tag{3.8}$$

We use the first equation of motion to simplify the derivative term in the second equation of motion and then rearrange terms slightly, to obtain

$$\partial_t\left(\frac{r^2}{\sqrt{-1+\dot{r}^2+E^2}}\right) = 0, \quad r\ddot{r} + 2(1-\dot{r}^2-E^2) = 0. \tag{3.9}$$

Finally, substituting the second equation into the first, we get the differential equation for the radius

$$\partial_t\left(\frac{r^{3/2}}{\sqrt{\ddot{r}}}\right) = 0 \quad \Leftrightarrow \quad \ddot{r} = Ar^3 \tag{3.10}$$

which has a solution describing the confinement of electric flux

$$r = \frac{c}{t}, \quad E = 1. \tag{3.11}$$

The electric field is always constant and takes the critical value, while the radius of this flux tube shrinks to zero at $t=\infty$. The electric field is necessarily constant since there are no magnetic fields; a changing electric field would necessarily also produce a magnetic field. Although this solution only exists for $t>0$, this does not mean that the dynamics on the non-BPS mother brane is trivial for $t<0$. Before $t=0$ it is still possible to have flux on the non-BPS mother brane and yet no $T=0$ regions. The key point is that the *S*-brane is only defined where the tachyon value is zero and so captures partial knowledge of the full tachyon configuration and flux. Yet, at the same time there is no violation of fundamental string charge from the *S*-brane viewpoint.[7] This *S*-brane comes in from spatial infinity and brings in charge through the gauge fields on its worldvolume. For charge conservation we do not have to have time reversal *S*-brane solutions which would correspond to including a mirror copy of the above solution describing an expanding worldvolume. We point out, however, that the expanding string solution is interesting in its own right and is possibly related to instabilities due to

[6] Reference [3] discusses *Sp*-branes with $p\geq 3$, but in this section we consider the $p=3$ case in preparation of Sec. IV.

[7] For this solution (3.11) the total fundamental string number is $4\pi c$. See Eq. (4.41).

critical electric fields and possibly the Hagedorn temperature. Further discussion on why this solution represents a fundamental string at late times is given in Ref. [3].

These spike solutions correspond to inhomogeneous tachyon configurations which spontaneously localize into lower dimensional systems. An example of such a solution was found by Sen in Ref. [6].

B. Discussion on confinement

In Ref. [3] and in the previous section, we have seen *S*-brane solutions describing the decay of an unstable D-brane into fundamental strings. A peculiar feature of these solutions is that eventually the electric flux becomes concentrated around the *S*-brane remnant where $T=0$. Is this a generic phenomenon corresponding to the confinement[8] of fundamental strings? In this section, we will discuss how the *S*-brane configuration is related to confinement in a tachyon system by showing that it is the lowest energy configuration for fixed electric flux. Furthermore, the magnetic field is also shown to be classically confined, which is consistent with the *S*-brane solution of the D-string formation presented in Sec. IV.

The main idea is that as an unstable D-brane decays, the tachyon condenses $T\to\infty$ almost everywhere except at the location of the *S*-brane remnant where $T=0$. We wish to show that the electric flux will concentrate around the region $T=0$.

Take an unstable D2-brane for simplicity. To begin, let us first consider homogeneous configurations with electric field $F_{01}=E$. The Lagrangian density is of the form

$$\mathcal{L}=-\sqrt{1-E^2}\,\tilde{\mathcal{L}}(T,z), \tag{3.12}$$

where

$$z=-\frac{\dot{T}^2}{1-E^2},\qquad E=\dot{A}, \tag{3.13}$$

and this Lagrangian is valid for $0\leqslant E^2<1$. The conjugate variables of T and A are

$$P=\frac{\partial\mathcal{L}}{\partial\dot{T}}=\frac{1}{\sqrt{1-E^2}}\frac{\partial\tilde{\mathcal{L}}}{\partial z}\dot{T}, \tag{3.14}$$

$$D=\frac{\partial\mathcal{L}}{\partial E}=\frac{E}{\sqrt{1-E^2}}\left(\tilde{\mathcal{L}}-2z\frac{\partial\tilde{\mathcal{L}}}{\partial z}\right) \tag{3.15}$$

so the Hamiltonian density is

$$\mathcal{H}=P\dot{T}+DE-\mathcal{L}=\frac{1}{\sqrt{1-E^2}}\left(\tilde{\mathcal{L}}-2z\frac{\partial\tilde{\mathcal{L}}}{\partial z}\right). \tag{3.16}$$

As long as $E\neq0$, we have the simpler expression [24]

$$\mathcal{H}=\frac{D}{E}. \tag{3.17}$$

Now consider those configurations which can be approximated by a homogeneous region for $|x_2|<l/2$, and a different homogeneous region when $|x_2|>l/2$. For our purposes the two regions will correspond to the *S*-brane region $T=0$, and the tachyon condensation region $T\to\infty$. When the D2-brane decays, some energy will be dissipated or radiated away but the electric flux

$$\Phi=\int dx^2 D \tag{3.18}$$

will be preserved. The final state of the process should be the most energy-efficient configuration for a given flux.

According to Eq. (3.16), the energy in the region of tachyon condensation can be arbitrarily close to zero. As an example, for the effective theory with $\tilde{\mathcal{L}}=V(T)f(z)$, where $V(T)\to0$ as $T\to\infty$, we can set $T\to\infty$ and $\dot{T}\to0$ such that $\mathcal{H}=0$. It follows from Eq. (3.15) that $D=0$ in the condensate region as long as $E<1$. Although there is electric field everywhere on the non-BPS brane, the flux is only non-zero in the *S*-brane region

$$lD=\Phi, \tag{3.19}$$

where D is the electric flux density for $|x_2|<l/2$. The total energy is

$$H=l\mathcal{H}=l\frac{D}{E}=\frac{\Phi}{E}, \tag{3.20}$$

where we used Eq. (3.17). Since Φ is a given fixed number, the energy H is minimized by maximizing E. We conclude that the minimal energy state has

$$E\to1 \tag{3.21}$$

around the *S*-brane, and so the energy is from pure flux $H=\Phi$, that is, the total energy is the same as the energy due to the tension of the fundamental strings. Finally, due to Eq. (3.15), in the limit where the electric field goes to the critical value, $D\to\infty$, and so the width of the *S*-brane region with nonzero electric flux shrinks to zero

$$l=\frac{\Phi}{D}\to0. \tag{3.22}$$

We have therefore shown that the electric flux is confined to the infinitesimal region around $T=0$.

We hope that the analysis above captures the physical reason for confinement in the low energy limit and with the present result one can show that the confined flux behaves as a fundamental string governed by a Nambu-Goto action following the argument given in Refs. [23,24]. In the above discussion, however, we ignored the transition interpolating the two homogeneous regions. When the transition region is

[8]See Ref. [21] for a discussion on the dielectric effect on classical confinement of fluxes, and also Refs. [22,23] for the confinement on branes.

taken into account, it might happen that the confinement profile has an optimal width at some characteristic scale.

Is there confinement for the magnetic flux as well? Since *S*-duality interchanges fundamental strings with D-strings, we expect the answer to be yes. We will study the consequences of *S*-duality for *S*-branes in the next section, while here we will continue with a direct analysis of the tachyon system. It is well known that a magnetic field on a BPS Dp-brane gives a density of lower-dimensional BPS $D(p-2)$-branes on the mother D-brane. Naively, however, a magnetic field on the non-BPS brane does not give any lower-dimensional BPS D-brane charge. The effect of the magnetic field appears only on the tachyon defects. For example, on a non-BPS D3-brane, a tachyon kink is equivalent with a BPS D2-brane. Suppose that we have a magnetic field on the original non-BPS brane along the kink. Then this induces BPS D0-brane charge only on the D2-brane, while apart from the kink no charge is induced though the magnetic field is present all over the non-BPS D3-brane worldvolume.

Keeping the above charge conservation in mind, let us try the same confinement argument to tackle this problem. The analogue of Eq. (3.12) is

$$\mathcal{L}=-\sqrt{1+B^2}\tilde{\mathcal{L}}(T,z), \tag{3.23}$$

where $z=-\frac{1}{2}\dot{T}^2$, and the analogue of Eq. (3.16) is

$$\mathcal{H}=\sqrt{1+B^2}\left(\tilde{\mathcal{L}}-2z\frac{\partial\tilde{\mathcal{L}}}{\partial z}\right). \tag{3.24}$$

As in the case of electric flux, we consider a homogeneous *S*-brane region[9] of width l and a tachyon condensation region. Let the magnetic fields in the two regions be B_0 and B_1. The energy in the condensed region can be minimized to zero by assigning $T\to\infty$ and $\dot{T}=0$. The total energy is

$$H=l\mathcal{H}=Cl\sqrt{1+B_0^2}, \tag{3.25}$$

where C is a constant independent of B_0 and l. This energy H is to be minimized with the constraint that the total flux only on the *S*-brane region is conserved (or to assume that $B_1=0$), that is

$$\Phi_0=lB_0=\text{fixed}. \tag{3.26}$$

Using the same arguments as before, we see that H is minimized for $l=0$ (and also $B_0\to\infty$), which shows the confinement of the lower dimensional RR charge.

We will see in Sec. IV that in fact one can construct an $S3$-brane spike solution which represents the formation of (p,q) strings. The argument for confinement of the electric and magnetic fields we have presented here is therefore consistent with our interpretation of the spike solutions.

[9]Although a homogeneous tachyon profile $T=0$ will not help to give the lower dimensional RR charge because the RR coupling on the non-BPS brane is proportional to $dT\wedge F$ while dT vanishes, we believe that the argument here captures an important feature of confinement.

IV. D-BRANES FROM *S*-BRANES

In the previous section, we reviewed the formation of fundamental strings from *S*-branes and showed how confinement of the electric flux can be a strong coupling but classical process. We found also that magnetic flux on a non-BPS brane is confined, which was expected due to the electric-magnetic duality in string theory. Confinement of magnetic fields should occur in any theory with electric-magnetic duality with confined electric flux bundles. In string theory the electric fluxes act as fundamental strings while confined magnetic fluxes act as branes; D-strings will be the focus of our attention. In this section we show how an $S3$-brane can realize the dynamical formation of (p,q) string bound states and D-strings, and so in a similar vein this will demonstrate that magnetic fields also confine. Magnetic fields can have an effect on tachyon dynamics.

Another motivation for searching for these solutions is the fact that, as opposed to fundamental strings, it is already known that D-branes can be described in the context of tachyon condensation. If we can discuss D-branes formation using *S*-branes then the related tachyon solutions should be easier to obtain. (An understanding of tachyon solutions would also help to explain how to construct closed strings from an open string picture.) A schematic cross section of expected tachyon values is shown in Fig. 4. From this illustration we see that while the *S*-brane region $(T=0)$ seems to appear "out of nowhere" and therefore seems to violate causality, from the tachyon picture there is in fact no difficulty. Before the *S*-brane appears, the tachyon field is simply evolving with no $T=0$ regions. Also at very early times, the entire spacetime is filled with only one of the vacua and so it is impossible to consider stable lower dimensional defects. When the tachyon has evolved closer to the second vacuum at late times, it is possible to interpret the $T=0$ regions as physical objects. By the time we can interpret the *S*-brane as a standard localized defect, it has already slowed down to less than the speed of light.

A. Tachyon solutions with homogeneous electric/magnetic fields

Before turning to the formation of (p,q) strings, we first consider homogeneous tachyon solutions with magnetic fields in analogy to the electric case in Ref. [24]. To better understand the tachyon condensate, it has been proposed [24] that in the effective action description of non-BPS branes

$$L=V(T)\sqrt{-\det(\eta+F)}\mathcal{F}(z), \tag{4.1}$$

$$z\equiv[(\eta+F)^{-1}]^{\mu\nu}\partial_\mu T\partial_\nu T=[(\eta-F\eta^{-1}F)^{-1}]^{\mu\nu}\partial_\mu T\partial_\nu T, \tag{4.2}$$

not only does the potential go to zero but that the kinetic energy contribution of the tachyon also vanishes

$$\mathcal{F}(z)=0 \quad\Leftrightarrow\quad z=-1 \tag{4.3}$$

after tachyon condensation. For uniform electric fields and tachyon fields this leads to a constraint

$$\dot{T}^2+E^2=1 \tag{4.4}$$

which governs the tachyon system near the bottom of the tachyon potential.[10] One motivation for searching for such a constraint is that it should help to describe confinement of the electric flux on a non-BPS brane, and it was shown that this constraint leads to a Carrollian limit for the propagating degrees of freedom on the brane. The effect of the Carrollian limit is to make the condensate a fluid of electric strings.

It is straightforward to extend the above analysis to include magnetic fields as well as electric fields. For simplicity we explicitly work out the $2+1$ dimensional case but all other cases can be treated in the same manner. Similar discussion has also recently appeared in Ref. [25].

When the fields are all spatially homogeneous the open string metric is

$$(g+F)_{ab}=\begin{array}{c} \begin{matrix} t & \quad x & \quad y \end{matrix} \\ \begin{pmatrix} -1 & E_x & E_y \\ -E_x & 1 & B \\ -E_y & -B & 1 \end{pmatrix} \end{array} \tag{4.5}$$

and to calculate the constraint we only need the G^{tt} component of the inverse of this matrix. A simple calculation shows that the constraint $z=-1$ becomes

$$\dot{T}^2+\frac{E^2}{1+B^2}=1. \tag{4.6}$$

There is no obvious duality between electric and magnetic fields since the tachyon scalar field breaks the world volume Lorentz invariance. The effect of the magnetic field is to increase the critical electric field, and if we take $\dot{T}=0$ then we reduce to the simple Lorentz invariant condition

$$E^2-B^2=1. \tag{4.7}$$

The role played by electric and magnetic fields is interesting and we make the following observations. First, a critical electric field will stop the tachyon from rolling near the end of the tachyon condensation process. Second, it has been shown that a D-$\overline{\text{D}}$ pair with critical electric field is supersymmetric [26]. Even though these results were derived in different contexts, there is an overlap in the way a critical electric field on branes removes tachyon dynamics and one wonders if there are further connections. For example, perhaps the reason why the tachyon ceases to roll in the presence of the electric field is also due to supersymmetry. In general we should be able to see regions of supersymmetry develop during the tachyon condensation process, where $\dot{T}=0$ and these regions could have interpretations as various lower dimensional supersymmetric objects. We further point out the existence of supersymmetric D-$\overline{\text{D}}$ configurations which are distinct [27] from the critical electric field case. These solutions should also appear as end products of tachyon condensation and be related to different constraints on the tachyon Lagrangian.

As we have just observed in $2+1$ dimensions, if there are no electric fields, then there is apparently no effect due to the magnetic field near the tachyon minimum. For higher dimensions, it is clear that if we follow similar steps, the homogeneous magnetic field by itself does not effect tachyon dynamics. One way to understand why the magnetic field does not change the rolling tachyon condition is that a constant magnetic field on a non-BPS *Dp*-brane can be understood as a bound state of a non-BPS *Dp*-brane and non-BPS D$(p-2)$-branes. Both of these have a rolling tachyon ($\dot{T}=1$), so the resultant bound state also has the rolling tachyon. Constant magnetic fields in this situation are not capable of generating stable lower dimensional objects. On the other hand, more complicated configurations with magnetic fields can create lower dimensional branes as we will see in the next section.

Finally, let us obtain the results of Eq. (4.3) from the worldsheet point of view. An open string on the D-brane has opposite charges at its end points. In a constant electric field background, the charges are pulled in opposite directions, with the electrostatic force in competition with the tension. When we stretch a string in an electric field which is strong enough ($E=\pm 1$), the increase in energy due to tension is compensated by the decrease in electric potential energy. The strings can have infinite length with vanishing energy. It appears as if the strings have no tension, resembling a collection of particles or dust. We propose to interpret this situation as tachyon condensation, or the point at which the D-brane vanishes.

Consider an open string with the worldsheet action

$$\begin{aligned} S&=\int d^2\sigma\left[\frac{1}{2}(\dot{X}^2-X'^{\,2}+F_{\mu\nu}\dot{X}^\mu X'^{\,\nu})+X'^{\,\mu}\partial_\mu\Phi(X)\right]\\ &=\int d^2\sigma\frac{1}{2}(\dot{X}^2-X'^{\,2})+\int d\tau\left(-\frac{1}{2}F_{\mu\nu}X^\mu\dot{X}^\nu+\Phi(X)\right). \end{aligned} \tag{4.8}$$

The spacetime momentum densities are

$$P_\mu=\dot{X}_\mu+F_{\mu\nu}X'^{\,\nu}. \tag{4.9}$$

The equation of motion is

$$\ddot{X}^\mu-X''^{\,\mu}=0, \tag{4.10}$$

and the boundary condition is

$$X'_\mu+F_{\mu\nu}\dot{X}^\nu=\partial_\mu\Phi(X), \tag{4.11}$$

at the string end points $\sigma=0,\pi$.

[10] In Ref. [24], this condition $z=-1$ comes from requiring that D and H be preserved while $V(T)\to 0$ for a homogeneous background.

We do not consider oscillation modes, so we impose the above boundary condition on the whole string. From Eqs. (4.9) and (4.11), we obtain the relation

$$(\delta^{\mu}_{\nu} - F^{\mu\kappa}F_{\kappa\nu})X'^{\nu} = -F^{\mu\nu}P_{\nu} + \partial^{\mu}\Phi(X). \tag{4.12}$$

From this equation we see that there are solutions with arbitrarily large X' and $P_{\mu}=0$ (that is, arbitrarily long strings at no cost in energy or momentum) if either

$$\det(1-F^2) = \det(1+F)\det(1-F) = [\det(1+F)]^2 = 0, \tag{4.13}$$

or

$$\partial_{\mu}\Phi = \infty. \tag{4.14}$$

The first condition (4.13) agrees with (4.6) when $\dot{T}=0$. The second condition (4.14) agrees with the final state of the rolling tachyon solution of Sen

$$\Phi \propto e^{X^0}. \tag{4.15}$$

It can be related to the desired condition for T via a change of variable such as

$$\Phi = \frac{T}{\sqrt{1+z}}, \tag{4.16}$$

where z is defined in Eq. (4.2). The condition (4.14) is now

$$z = -1. \tag{4.17}$$

B. $S3$-branes with electric and magnetic fields

Let us proceed to construct a solution of the $S3$-brane action which represents a formation of a (p,q) string bound state. The ansatz is identical to the one in the previous section, Eq. (3.1), except that we also include an additional magnetic field

$$\begin{aligned} X^0 &= t \\ X^1 &= r(t)\cos\theta \\ X^2 &= r(t)\sin\theta\cos\phi \\ X^3 &= r(t)\sin\theta\sin\phi \\ X^4 &= \chi \\ F_{t\chi} &= E(t) \\ F_{\theta\phi} &= b\sin\theta. \end{aligned} \tag{4.18}$$

The open string metric and its inverse are just direct products of the example we gave before and

$$(g+F)_{ab} = \begin{pmatrix} r^2\sin^2\theta & b\sin\theta \\ -b\sin\theta & r^2 \end{pmatrix} \quad (\text{rows/columns } \theta, \phi)$$

$$(g+F)^{ab} = \frac{1}{r^4\sin^2\theta\left(1+\dfrac{b^2}{r^4}\right)} \begin{pmatrix} r^2\sin^2\theta & -b\sin\theta \\ b\sin\theta & r^2 \end{pmatrix} \quad (\text{rows/columns } \theta, \phi) \tag{4.19}$$

so the action is proportional to

$$\sqrt{\det(g+F)} = r^2\sin\theta\sqrt{(-1+\dot{r}^2+E^2)\left(1+\frac{b^2}{r^4}\right)}. \tag{4.20}$$

We first examine the equation of motion of the embedding coordinate

$$\partial_t\left[r^2\sin\theta\sqrt{(-1+\dot{r}^2+E^2)\left(1+\frac{b^2}{r^4}\right)}(g+F)^{tt}\partial_t t\right] = 0 \tag{4.21}$$

and try a solution of the form

$$r = \frac{c_{\rm d}}{t}, \qquad E = \text{const}, \tag{4.22}$$

where $c_{\rm d}$ is a constant parameter. This ansatz gives a solution as long as we satisfy the relation

$$E^2 - \frac{b^2}{c_{\rm d}^2} = 1 \tag{4.23}$$

which is consistent with the constraint in Eq. (4.3) since on the S-brane world volume $\dot{T}=0$. It is straightforward to check that the other equations of motion such as

$$\partial_a\left[r^2\sin\theta\sqrt{(-1+\dot{r}^2+E^2)\left(1+\frac{b^2}{r^4}\right)} \times (g+F)^{ab}\partial_b(r\cos\theta)\right] = 0 \tag{4.24}$$

and

$$\partial_a\left[r^2\sin\theta\sqrt{(-1+\dot{r}^2+E^2)\left(1+\frac{b^2}{r^4}\right)}(g+F)^{ab}\right] = 0 \tag{4.25}$$

are also satisfied. The field strength $F_{\theta\phi}$ generates a magnetic field along χ and parallel to the electric field. This S-brane is an electric-magnetic flux tube confining into a $1+1$ dimensional remnant. At late times this S-brane becomes a (p,q) string bound state. The existence of these additional solutions should be expected due to S duality on the $S3$-brane as we will explain in the following section.

We note that these solutions have real $F_{\theta\phi}$ as long as the electric field is greater than the critical value due to Eq. (4.23). Although the appearance of large electric fields is unusual on D-branes, they appear quite naturally on S-branes and large electric fields do not lead to imaginary S-brane actions. We will see in the following section how large electric fields show up on S-branes by examining the tachyon solutions on the non-BPS mother branes.

C. S duality for $S3$-branes

For the purposes of this section, the following parametrization:

$$\begin{aligned} X^0 &= X^0(x^1,x^2,x^3)\\ X^1 &= x^1\\ X^2 &= x^2\\ X^3 &= x^3\\ X^4 &= \chi\\ F_{a\chi} &= \partial_b A_\chi(x^1,x^2,x^3)\\ F_{ab} &= F_{ab}(x^1,x^2,x^3) \end{aligned} \tag{4.26}$$

turns out to be useful to see the duality transformations, where $a,b=1,2,3$. The world volume of the $S3$-brane is now parametrized by (x^1,x^2,x^3,χ) as in Ref. [3]. In the above parametrization we have assumed that all the fields are independent of χ just like in our explicit S-brane solutions. We follow Ref. [28] in deriving the extended duality symmetry. This will help clarify how the (p,q) string formation solutions are related to the F-string formation solution in Sec. III, and suggests other "non-BPS" throat-type solutions.

The $S3$-brane Lagrangian in this coordinate choice is written as [3]

$$\begin{aligned} L &= \sqrt{\det(\delta_{ij}-\partial_i X^0 \partial_j X^0 + F_{ij})} \qquad (4.27)\\ &= [1-(\partial_a X^0)^2+(F_{a\chi})^2+(F_{ab})^2/4\\ &\quad +(\partial_a X^0 F_{a\chi})^2-(\partial_a X^0)^2(F_{b\chi})^2\\ &\quad -(\epsilon^{abc}F_{bc}\partial_a X^0)^2/4\\ &\quad +(\epsilon^{abc}F_{bc}F_{a\chi})^2/4]^{1/2}. \end{aligned}$$

We omit the overall constant factor S_0 in the S-brane Lagrangian. We next introduce the Lagrange multiplier field ϕ_B for the Bianchi identity of F_{ab} as

$$\Delta L = (\phi_B/2)[\epsilon_{abc}\partial_a F_{bc}]. \tag{4.28}$$

With this multiplier term we can regard F_{ab} as fundamental fields and integrate out the field strength F_{ab}. The final form of the Lagrangian is simply

$$L+\Delta L = \sqrt{\det(\eta^{RS}+\nabla\Phi^R\cdot\nabla\Phi^S)} \tag{4.29}$$

where $\Phi^R=(X^0,\phi_B,A_\chi)$ and the metric in the virtual transverse space parametrized by Φ^R is $\eta^{RS}=\mathrm{diag}(-1,-1,1)$. The Lagrangian shows that the whole duality symmetry is $SO(2,1)$. It is interesting that the subgroup of the duality symmetry which rotates the electric and magnetic fields A_χ and ϕ_B, in other words $F_{a\chi}$ and $\epsilon_{abc}F_{bc}$ and so this should be S duality, is in this case $SO(1,1)$. This duality is more like a Lorentz boost between electric and magnetic fields than the usual duality rotations. We will explain how to obtain the more usual duality rotations in the next section.

If X^0 and A_χ are turned on, the duality group becomes $SO(1,1)$ whose fixed point is the spike solution

$$X^0 = A_\chi = \frac{c}{r} \tag{4.30}$$

which represents the formation of a fundamental string. If we also turn on ϕ_B, we obtain the spike solution representing the formation of a (p,q) string at late time

$$X^0 = \frac{A_\chi}{\cosh\alpha} = \frac{\phi_B}{\sinh\alpha} = \frac{c_{\rm d}}{r} \tag{4.31}$$

which we provided in the previous section from an alternate viewpoint, Eq. (4.7). The relationship between these two parametrizations is

$$E = \cosh\alpha, \quad b = c_{\rm d}\sinh\alpha. \tag{4.32}$$

The fundamental string charge and the D-string charge are $4\pi c_{\rm d}\cosh\alpha$ and $4\pi c_{\rm d}\sinh\alpha$, respectively.

The duality group $SO(1,1)$ above is only a subgroup of the full S-duality symmetry group $SL(2)$. [It becomes $SL(2,Z)$ upon charge quantization.] Here we started off with an S-brane solution which decayed into fundamental strings $(n,0)$. The $SO(1,1)$ symmetry connects it to (p,q) strings with $p>q$, but we are still missing all other (p,q) strings with $p<q$. We will discuss how to obtain these other cases in the next section.

D. Magnetic S-branes from M-theory

Since D-branes can be realized as defects on the non-BPS brane worldvolume, one is tempted to try to find the $S3$-brane spike solution decaying into just D-strings. However, the condition in Eq. (4.23) implies that if there are no fundamental strings $(E=0)$, there is no solution with real magnetic fields; one can prove this from the equations of motion, Eq. (4.21), and by assuming only rotational symmetry. In the context of tachyon condensation of non-BPS branes, the magnetic field on any S-brane induced from the field strength on the corresponding non-BPS brane should be real. Magnetic solutions do exist, however, if we allow for imaginary field strengths. It is possible to investigate the implications for allowing imaginary field strength solutions. Imaginary field strengths have been noted to potentially arise in time dependent systems [29] and it remains to be seen whether they will play a physical role in a theory.

However, instead of introducing imaginary field strengths we will find a way to mimic their behavior with real magnetic fields and so avoid the constraint of Eq. (4.23). The key point will be to consider M-theory effects by dualizing the scalar field from the M-theory circle to a gauge field. This dualized gauge field will not be induced from the non-BPS brane but from M-theory.

Earlier, in Sec. II, we discussed a generalized S-brane action for spacetime vortices in Eq. (2.11). The main difference was that this generalized action included fluctuations of a transverse scalar along a spatial direction. Up to now we have not used this scalar; however, we will now use this to solve the riddle of D-string generation from an $S3$-brane. The idea is to consider M-theory compactified on two circles, one the M-theory circle which reduces us to type IIA and another to take us from type IIA to type IIB string theory.

In this case we can begin with an M5-$\overline{\text{M5}}$ pair and look for a codimension-three-generalized vortex solution representing a spacelike M2-brane. This should be present just by generalizing the argument in Ref. [22] where an M2-brane is realized as a topological soliton in M5-$\overline{\text{M5}}$. The spacelike M2-brane Lagrangian of this spacetime vortex is

$$L=\sqrt{\det(\delta_{ij}-\partial_i X^0\partial_j X^0+\partial_i X^4\partial_j X^4+\partial_i X^{10}\partial_j X^{10})},\quad i,j=1,2,3 \tag{4.33}$$

where the spatial transverse direction is along the M-theory circle X^{10}. Let us dualize the scalar X^{10} into a gauge field with field strength $\tilde{F}$. We perform the dualization by adding the Lagrange multiplier term

$$\Delta L=\frac{1}{2}X^{10}\epsilon_{ijk}\partial_i\tilde{F}_{jk} \tag{4.34}$$

and then integrating out X^{10}. The final form of the Lagrangian is

$$L+\Delta L=\sqrt{\det(\delta_{ij}-\partial_i X^0\partial_j X^0+\partial_i X^4\partial_j X^4+i\tilde{F}_{ij})} \tag{4.35}$$

where the factor of "i" now accompanies the dual field strength! This factor does not need to be added into the Lagrange multiplier term but instead is a direct consequence of the Euclidean nature of the S-brane action. If the scalar X^4 is trivial as in the present situation $X^4=\chi$ of Secs. III and IV, this $S2$-brane action in type IIA can be regarded as an $S3$-brane action in type IIB theory. In this action we can now solve for a purely magnetic $S3$-brane solution as in Sec. IV B but now with real field strength. We emphasize that the field strength is real and the factor of "i" does not effect the Hermiticity of the action. One might ask if the $S3$-brane can be constructed directly from an unstable 4-brane object. It is possible that this $S3$-brane construction can be studied on the S-dual of the non-BPS D4-brane which has also been called an NS4-brane [30].

This $S3$-brane decays into a one dimensional remnant with magnetic field, so our expectation should be that this is a D-string. Let us obtain the explicit solution and see how the D-string tension is reproduced. We begin with the action for this magnetic $S3$-brane written in the spacetime point of view with the parametrization (4.18),

$$L=r^2\sin\theta\sqrt{-1+\dot{r}^2+B^2-B^2\dot{r}^2},\quad B\equiv\frac{\tilde{F}_{\theta\phi}}{r^2\sin\theta}. \tag{4.36}$$

The solution for this decaying $S3$-brane is

$$r=\frac{c_{\rm m}}{t},\quad B=\frac{c_{\rm m}}{r^2}, \tag{4.37}$$

where we take $c_{\rm m}$ to be positive. We calculate the conjugate momenta and Hamiltonian

$$P_r=r^2\sin\theta\frac{\dot{r}[1-B^2]}{\sqrt{-1+\dot{r}^2+B^2-B^2\dot{r}^2}}, \tag{4.38}$$

$$H\equiv\int d\chi d\theta d\phi[P_r\dot{r}-L]$$
$$=\int d\chi d\theta d\phi r^2\sin\theta\frac{-1+B^2}{\sqrt{-1+\dot{r}^2+B^2-B^2\dot{r}^2}}. \tag{4.39}$$

At late times $\dot{r}=0$ and B is large so in this limit the Hamiltonian has the simple form

$$H=\int d\chi\int_{S^2}d\theta d\phi Br^2\sin\theta=4\pi c_{\rm m}\int d\chi. \tag{4.40}$$

At this stage we recall that in the above analysis we omitted the overall factor of the $S3$-brane tension, and also that the parameter $c_{\rm m}$ should be subject to the Dirac quantization condition. It is naturally expected that the $S3$-brane tension is given by the D3-brane tension, $T_{\rm D3}=1/2\pi g_s$ in our convention $2\pi\alpha'=1$. Now what about the Dirac quantization condition? Let us compare this magnetic S-brane with the electric case in the previous section. A straightforward calculation shows that the energy there is given by the same expression

$$H_{\text{electric S3}}=4\pi c\int d\chi \tag{4.41}$$

where again the overall tension $T_{\rm D3}$ is omitted, and c is the parameter appearing in the solution (3.11). Now the Dirac quantization condition is

$$4\pi c\cdot 4\pi c_{\rm m}=\frac{2\pi n}{T_{\rm D3}} \tag{4.42}$$

where n is an integer and the factors of 4π come from integrating over the S^2 angular directions of the S-brane worldvolume. The factor T_{D3} appears here since this factor appears in the action and so the right hand side is proportional to the string coupling constant g_s.

Let us see how this condition works. What we are doing is a generalization of Ref. [31]. Suppose that Eq. (4.41) gives the correct tension of a fundamental string,

$$4\pi c T_{D3} = T_{F1} \tag{4.43}$$

which is 1 in our convention. This equation together with the condition (4.42) provides the correct tension of a D-string,

$$4\pi c_m T_{D3} = \frac{n}{g_s} = n T_{D1}. \tag{4.44}$$

Here n should be a positive integer since the left-hand side is positive. We have shown that the remnant, represented by the magnetic $S3$-brane solution, has the tension of a D-string, which supports our claim that the remnant is a D-string. A boundary state discussion of this claim is also presented in Sec. V F.

It is interesting to relate the above dualization procedure to a discussion of S duality. In fact Ref. [32] discussed S duality for D3-branes and used a Euclideanized version of the D3-brane action for simplicity, which from our viewpoint is an $S3$-brane action. As compared to our dualization procedure, Eq. (4.28), in the dualization process of Ref. [32] the Lagrange multiplier field ϕ_B enforcing the Gauss condition came with a factor of "i." The factor of "i" was argued to arise from the Euclidean nature of the brane worldvolume. The effect of this alternate dualization procedure with an explicit factor of "i" is that we reproduce the action in Eq. (4.35). Therefore this alternate dualization procedure is equivalent to field strengths coming from the M-theory circle and not from the non-BPS brane.

Finally, for this case the duality group discussed in Sec. IV C becomes $SO(1,2)$ acting on (X^0,ϕ_B,A_χ), due to the "i" factor. The electric-magnetic duality is now the more standard $SO(2)$ duality rotation, which is consistent with the interpretation that this $S3$-brane decays into a D-string. Interestingly, for the solution with the factor of "i," we can ignore the χ direction and regard the solution as an $S2$-brane instead of the $S3$-brane. This solution represents the formation of a D0-brane from the $S2$-brane in type IIA theory. The magnetic field was originally the scalar field for the M-theory circle, thus this solution in the M-theory side represents a lightlike particle emission process from the spacelike M2-brane.

V. STRINGS AND D-BRANES AS BOOSTED S-BRANES

A succinct summary of our characterization of S-branes so far is that there are ways to follow the time dependent defect formation process. In this section we further discuss the (p,q) strings of the previous sections. We will find how certain "boosted" $S1$-branes apparently become ordinary D-branes and fundamental strings moving in the bulk. In fact, these boosted $S1$-branes extract late time information of the remnant formation solutions which we studied before. We start by presenting solutions of $S1$-brane actions and discussing their properties. The corresponding tachyon solutions are then presented, and it is shown how in a certain limit these solutions apparently become $(p,1)$ strings. Boundary states for the boosted S-branes are also constructed, and we show that they become boundary states of $(p,1)$ strings in the limit relevant to the S-branes discussed in the previous sections. Finally, we examine the boundary state of the magnetic $S3$-brane in Sec. IV D, and show that at late times this solution produces a D-string boundary state consistently.

A. Boosted S1-branes

The $S3$-branes of Secs. III and IV are eventually confined into $(1+1)$-dimensional remnants so it should be interesting to analyze $S1$-branes directly. Since the "static" $S1$-branes are spacelike in the target space we will have to "boost" them to become timelike in the target space. These boosted $S1$-branes are expected to be almost the same as the spike solutions in Secs. III and IV at late times, except that the boosted $S1$-branes have at least one D1-brane charge. The general $S1$-brane action is

$$S = \int d^2x \sqrt{\det(\delta_{ij} - \partial_i X^0 \partial_j X^0 + F_{ij})} \tag{5.1}$$

where the Euclidean worldvolume is parametrized by x^i with $i=1,\chi$. First let us consider a solution relevant for the fundamental string formation in Sec. III. As in the previous solutions, we turn on only A_χ among the gauge fields and assume $\partial_\chi = 0$, so the action simplifies to

$$S = \int d^2x \sqrt{1 - (\partial_1 X^0)^2 + (\partial_1 A_\chi)^2}. \tag{5.2}$$

When the BPS-like relation $X^0 = \pm A_\chi$ holds, the equations of motion become linear:

$$\partial_1 \partial_1 X^0 = 0. \tag{5.3}$$

This holds for any Sp-brane if the above ansatz is applied, and the spike solution of Sec. III and Ref. [3] was of this type. In the present case $p=1$, the solutions are simple

$$X^0 = c x^1, \quad F_{1\chi} = c \tag{5.4}$$

where the parameter c describes the velocity in the target space

$$\frac{\partial x^1}{\partial X^0} = 1/c. \tag{5.5}$$

Due to the presence of the field strength $F_{1\chi}$ on the S-brane, the resultant configuration can be timelike, $c>1$. The configuration is a one dimensional object moving in the target space with speed $1/c$ along the x^1 direction. If $c>1$, this object moves slower than the speed of light and apparently becomes a physically meaningful moving 1-brane. The induced electric field on the 1-brane is

$$F_{0\chi}=\frac{\partial x^1}{\partial X^0}F_{1\chi}=1 \tag{5.6}$$

which is the critical value. If one tries to use a usual DBI analysis for this moving 1-brane by assuming that this 1-brane is a D1-brane, the DBI action becomes imaginary. So although this seems to be similar to a normal bound state of strings and branes, this configuration seems to only have an *S*-brane description using the *S*-brane action.

We can generalize this solution so that it deviates from the BPS-like relation. A simple calculation shows that a generalized solution is

$$\partial_1 X^0=\frac{c_1}{\sqrt{1-c_2^2+c_1^2}}, \quad F_{1\chi}=\frac{c_2}{\sqrt{1-c_2^2+c_1^2}}. \tag{5.7}$$

In this case the induced electric field takes on arbitrary values

$$F_{0\chi}=\frac{c_2}{c_1}, \tag{5.8}$$

although we still have the restriction on the parameters c_1 and c_2

$$1-c_2^2+c_1^2\geqslant 0 \tag{5.9}$$

coming from the reality condition for the $S1$-brane action. The velocity of the moving D1-brane has a lower bound related to the field strength $F_{1\chi}$. Expressing c_2 in terms of $F_{1\chi}$ and c_1 as

$$c_2=\frac{F_{1\chi}}{\sqrt{1+(F_{1\chi})^2}}\sqrt{1+c_1^2}, \tag{5.10}$$

it is not difficult to see that

$$\left|\frac{\partial x^1}{\partial x^0}\right|=\frac{\sqrt{1+c_1^2}}{c_1}\frac{1}{\sqrt{1+(F_{1\chi})^2}}\geqslant\frac{1}{\sqrt{1+(F_{1\chi})^2}}. \tag{5.11}$$

Setting $c_1=c_2=c$ brings us back to the BPS solution (5.4).

These solutions include ones which describe static configurations in the bulk. Setting the velocity to zero in Eq. (5.7), we get the relationship $c_2^2=1+c_1^2$ and in this case the induced electric field can be larger than the critical value

$$F_{0\chi}=\frac{\sqrt{1+c_1^2}}{c_1}\geqslant 1. \tag{5.12}$$

Again, we see that this static one-dimensional object exceeds the validity of the usual DBI action, unless $c_1=\infty$. In the limit $c_1=\infty$ the configuration is static and has a critical electric field so this configuration can also be described by the usual D1-brane action. However, this limit is rather singular and it apparently represents an $(n,1)$ string with $n\to\infty$. We identify this as an infinite number of fundamental strings where the D1-brane effect has disappeared [33]. On the other hand, the limit $c_1\sim c_2=\infty$ is just like the late time behavior of the spike solution found in Sec. III and Ref. [3] so here we see a nice agreement between these two *S*-brane solutions.

B. Tachyon condensation representation

Our general *S*-brane analysis is based on the belief that any solution of the *S*-brane action has a corresponding tachyon solution on an unstable brane. The solution given in the previous section should hence have a tachyon description. Since the solution is just a boosted *S*-brane, it is natural to expect that the corresponding tachyon solution can be generated by the worldvolume boost from the homogeneous rolling tachyon solution. In this case, one has to perform a Lorentz boost respecting the open string metric. Let us see this in more detail.

We start with the following general Lagrangian for a non-BPS D2-brane,

$$L=-V(T)\sqrt{-\det(\eta+F)}\mathcal{F}(G^{\mu\nu}\partial_\mu T\partial_\nu T), \tag{5.13}$$

where $\mathcal{F}$ is a function defining the kinetic energy structure of the tachyon, and $G^{\mu\nu}$ is the open string metric. This action is the general form for the linear tachyon profiles. Almost all the Lagrangians which have been investigated so far, such as Sen's rolling Lagrangian [2,34], BSFT [15–17], and the Minahan-Zwiebach model [35], are included in this general form. Let us examine the tachyon field which depends only on x^0 and x^1. If one chooses a gauge $A_\chi=0$ and turns on only A_1, then the gauge field equations of motion are satisfied trivially for the constant gauge field strength $F_{1\chi}$. Then the problem reduces to the situation where we have to solve only the tachyon equation of motion under the background of the field strength which appears only in the open string metric. In our case the explicit form of the inverse open string metric is

$$G^{\mu\nu}=\mathrm{diag}\left(-1,\frac{1}{1+(F_{1\chi})^2},\frac{1}{1+(F_{1\chi})^2}\right) \tag{5.14}$$

where $\mu=0,1,\chi$. The metric in the x^0-x^1 spacetime is

$$G_{\mu\nu}=\mathrm{diag}[-1,1+(F_{1\chi})^2]. \tag{5.15}$$

The simplest solution is a homogeneous solution, $\partial_1 T=\partial_\chi T=0$. Since in this case we turned on only the magnetic field, we have that $G_{00}=-1$, and so this solution is just the same as the one with vanishing field strength. One can integrate the equations of motion for T and then obtain a solution[11] $T=T_{\rm cl}(x^0)$. Without loss of generality, we may assume that the tachyon passes the top of its potential at $x^0=0$, i.e. the equation $T_{\rm cl}(x^0)=0$ is solved by $x^0=0$.

We next perform a Lorentz boost in the 01 spacetime directions which preserves the open string metric. For this purpose we define a rescaled coordinate $\tilde{x}^1\equiv\sqrt{G_{11}}x^1$. In

[11] At this stage we exceed the validity of the BSFT tachyon action (5.13) since the solution is not linear in x^0 [19].

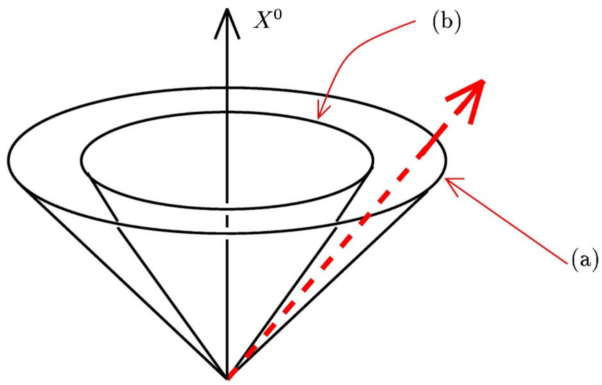

FIG. 6. The light cone structure on the non-BPS D2-brane worldvolume. The closed string light cone (a) is always located outside the open string light cone (b). The dashed line denotes the motion of the boosted *S*-brane which is both timelike with respect to the closed string light cone and spacelike with respect to the open string light cone.

these rescaled coordinates the metric becomes $\tilde{G}_{\mu\nu} = \mathrm{diag}(-1,1)$ and the Lorentz boost takes the usual form

$$\begin{pmatrix} x^0 \\ \tilde{x}^1 \end{pmatrix} \rightarrow \begin{pmatrix} x^{0\prime} \\ \tilde{x}^{1\prime} \end{pmatrix} = \begin{pmatrix} \cosh\gamma & \sinh\gamma \\ \sinh\gamma & \cosh\gamma \end{pmatrix} \begin{pmatrix} x^0 \\ \tilde{x}^1 \end{pmatrix}. \tag{5.16}$$

The line where the original defect is located, $x^0=0$, is boosted to a tilted line

$$x^0 + \tanh\gamma\sqrt{G_{11}}x^1 = 0 \tag{5.17}$$

so the defect is now moving along the x^1 direction with velocity

$$\frac{\partial x^1}{\partial x^0} = \frac{-1}{\sqrt{G_{11}}\tanh\gamma}. \tag{5.18}$$

The important point here is that the absolute value of this velocity can be made less than unity. By definition $|\tanh\gamma| \leqslant 1$, so if the field strength vanishes, the velocity of the configuration is greater than that of light; the worldvolume of the defect is still spacelike. If we turn on a constant field strength, then a large boost will make the defect timelike. This property is a direct result of the fact that the open string light cone lies inside the closed string light cone [36]. Because of this fact one may obtain timelike D-branes from spacelike-branes (see Fig. 6).

The lower bound for the velocity of the moving D-brane (5.11) should be seen also in this tachyon solution. In fact, it is given by

$$\left|\frac{\partial x^1}{\partial x^0}\right| \geqslant |1/\sqrt{G_{11}}| = \frac{1}{\sqrt{1+(F_{1\chi})^2}}, \tag{5.19}$$

which coincides with Eq. (5.11). For the *S*-brane, the limit $F_{1\chi}\rightarrow\infty$ makes the *S*-brane worldvolume static. Let us study what happens to the tachyon solution in this limit. The boosted tachyon configuration is

$$T = T_{\mathrm{cl}}[(\cosh\gamma)x^0 + (\sinh\gamma)\sqrt{G_{11}}x^1]. \tag{5.20}$$

The original solution $T_{\mathrm{cl}}(x^0)$ has the rolling tachyon behavior for large x^0, $T_{\mathrm{cl}} \sim x^0$. Therefore in the limit $F_{1\chi}\rightarrow\infty$, this boosted tachyon solution becomes

$$T \sim ux^1, \quad u \equiv F_{1\chi}\sinh\gamma \rightarrow \infty \tag{5.21}$$

and this linear dependence on x^1 coincides with the familiar static D-string kink solution. The coefficient of the linear term diverges, which is also consistent with the BSFT renormalization argument for D-brane kink solutions [16,17].

It is clear that the moving 1-brane has a unit D-string charge. Taking into account that the integration surface enclosing the defect in the original non-BPS brane worldvolume is not necessarily timelike, the *S*-brane charge is just the same as the D-brane charge [1]. So, if the *S*-brane worldvolume is deformed to be timelike, it should give an ordinary D-brane charge. This can be easily seen from the RR-tachyon coupling in the non-BPS brane [17],

$$\int C \wedge dT e^{-T^2}. \tag{5.22}$$

Here dT can be evaluated as

$$dT = \frac{\partial T_{\mathrm{cl}}(x^{0\prime})}{\partial x^{0\prime}} d(x^{0\prime}). \tag{5.23}$$

Therefore if the boosted line $x^{0\prime}=0$ becomes timelike, the usual D-brane charge is generated in which the RR source is distributed on a hypersurface timelike in the bulk closed string metric.

Here we stress that the boosted tachyon configuration has the usual D-string charge, so the configuration should represent an $(n,1)$ string with $n\rightarrow\infty$, as seen in Sec. V A. Then, how is the fundamental string charge n seen in the tachyon description? The answer is that the fundamental string charge is expected to be realized only in the induced electric field, not in the tachyon field. In fact, if we recall the noncommutative soliton representing a fundamental string [37], there the tachyon sits at the bottom of the potential from the first place. In the present case using Eq. (5.20), it is easy to evaluate the induced electric field

$$F_{0\chi} = \frac{\partial x^1}{\partial x^0}F_{1\chi} = -\frac{F_{1\chi}}{\sqrt{G_{11}}\tanh\gamma} \overset{F_{1\chi}\rightarrow\infty}{\mapsto} -\coth\gamma \tag{5.24}$$

and we find that this agrees with the $S1$-brane analysis Eq. (5.12). So in the limit $\gamma\rightarrow\infty$ we have a critical electric field $F_{0\chi}=-1$.

In addition to the charges of the defects, their energy is another important physical quantity to study. Though one expects that the energy of the boosted *S*-brane should depend on the tension of the *S*-branes whose precise value is unknown, we may proceed by using the explicit expression of the corresponding tachyon solution. The detailed analysis is presented in Appendix B.

Although we have just seen how the $S1$-brane seems to describe $(n,1)$ string bound states, one might question the validity of the solutions since the S-brane solutions allow for faster than light travel. Let us examine the tachyon configurations to see how this occurs. As discussed around Eq. (5.21) a static brane has zero width while all moving configurations acquire a finite width. When the width is small relative to the background it is easy to say that there is a lump which is actually moving, and in such cases the lump is moving slower than the speed of light. If we speed up the configuration, its width increases and the lump in the tachyon field becomes hard to separate from the background. In such cases it is difficult to say if the lump is moving and instead we should describe the configuration as a collective motion of the tachyon field which just resembles a lump moving. When the S-branes move faster than light, the configurations do not have good interpretations in terms of lumps or branes in motion and so it is okay if the configuration "moves" at a speed greater than light.

C. Boundary state and fundamental string charge

In the previous section it was shown that the boosted S-brane carries D-string charge and the tachyon configuration had the usual D-string form. However, since an electric field is induced on this D-string as shown in Eq. (5.8), the 1-brane is expected to be an $(n,1)$ string which also possesses fundamental string charge. The easiest way to see if this object carries such a charge is to study its boundary state, especially its coupling to the bulk Neveu-Schwarz–Neveu-Schwarz (NS-NS) gauge field. In this section we explicitly construct a boundary state for the boosted S-branes of Eq. (5.7).

According to Gutperle and Strominger [1], the boundary state for an Sp-brane[12] satisfies the following boundary conditions:

$$(\alpha_n^\mu+\mathcal{O}^\mu{}_\nu\tilde{\alpha}^\nu_{-n})|B,\eta\rangle=0 \tag{5.25}$$

(and similar expressions for the worldsheet fermions). The orthogonal matrix $\mathcal{O}$ is given by

$$\mathcal{O}^\mu{}_\nu=\mathrm{diag}(-1,1,\dots,1,-1,\dots,-1) \tag{5.26}$$

where we have $p+1$ entries giving $+1$, specifying the Neumann directions. For spacelike branes the first entry $\mathcal{O}^0{}_0$ is negative due to the Dirichlet boundary condition for the time direction.

We now proceed to find the boundary state for the boosted $S1$-brane. Since our solution has constant field strength and constant velocity, it is expected that only the orthogonal matrix $\mathcal{O}$ will be modified.[13] We work out the bosonic string case for simplicity. The worldsheet boundary coupling in the string sigma model should be

$$\oint d\tau\left(F_{1\chi}X^1\frac{\partial}{\partial\tau}X^\chi+VX^1\frac{\partial}{\partial\sigma}X^0\right) \tag{5.27}$$

where V is the inverse of the velocity of the moving D1-brane, while we normalize the bulk action as

$$\frac{1}{2}\int d\sigma d\tau\partial_a X^\mu\partial_b X^\nu\eta^{ab}\eta_{\mu\nu} \tag{5.28}$$

with the oscillator expansion

$$X^\mu=x^\mu+p^\mu\tau\frac{i}{2}+\sum_{n\neq 0}\frac{1}{n}(\alpha_n e^{in(\sigma+\tau)}+\tilde{\alpha}_n e^{in(\sigma-\tau)}). \tag{5.29}$$

The variation of the action gives the boundary conditions at $\sigma=0,\pi$ as

$$\begin{aligned}&\partial_\sigma X^1-F_{1\chi}\partial_\tau X^\chi-V\partial_\sigma X^0=0,\\&\partial_\sigma X^\chi+F_{1\chi}\partial_\tau X^1=0,\\&\partial_\tau X^0-V\partial_\tau X^1=0.\end{aligned} \tag{5.30}$$

The last condition is due to the original Dirichlet boundary condition for the time direction X^0. Substituting

$$\partial_\sigma X^\mu|_{\sigma=0}=-\frac{1}{2}\sum_n(\alpha_n^\mu+\tilde{\alpha}^\mu_{-n}),\quad \partial_\tau X^\mu|_{\sigma=0}=-\frac{1}{2}\sum_n(\alpha_n^\mu-\tilde{\alpha}^\mu_{-n}) \tag{5.31}$$

into the above boundary conditions (5.30), we obtain

$$\begin{aligned}&\alpha_n^0-\tilde{\alpha}^0_{-n}-V(\alpha_n^1-\tilde{\alpha}^1_{-n})=0,\\&\alpha_n^1+\tilde{\alpha}^1_{-n}-F_{1\chi}(\alpha_n^\chi-\tilde{\alpha}^\chi_{-n})-V(\alpha_n^0+\tilde{\alpha}^0_{-n})=0,\\&\alpha_n^\chi+\tilde{\alpha}^\chi_{-n}+F_{1\chi}(\alpha_n^1-\tilde{\alpha}^1_{-n})=0.\end{aligned} \tag{5.32}$$

Solving these equations, we obtain a new orthogonal matrix specifying the boundary condition

$$\mathcal{O}^\mu{}_\nu=\frac{1}{1+F_{1\chi}^2-V^2}\times\begin{pmatrix}-(1+F_{1\chi}^2+V^2) & 2V & 2F_{1\chi}V\\ -2V & 1-F_{1\chi}^2+V^2 & 2F_{1\chi}\\ 2F_{1\chi}V & -2F_{1\chi} & 1-F_{1\chi}^2-V^2\end{pmatrix}, \tag{5.33}$$

where $\mu,\nu=0,1,\chi$. It should be noted here that off-diagonal entries appear in $\tilde{\mathcal{O}}$, and these are responsible for the fundamental string charge. There is now a non-vanishing overlap

[12]In the following, we identify our flat S-brane in the rolling tachyon context with the SD-brane which is defined to be a brane on which open strings can end with Dirichlet boundary conditions along time.

[13]Also the normalization of the boundary state, which is usually identified with the DBI Lagrangian, will be modified but in this paper we will not consider this point.

of the boundary state with a NS-NS B field state $|B^{\rm NSNS}_{\mu\nu}\rangle$. This represents a source for the B-field

$$\langle B|B^{\rm NSNS}_{0\chi}\rangle \propto -\tilde{\mathcal{O}}_{0\chi}+\tilde{\mathcal{O}}_{\chi 0}=\tilde{\mathcal{O}}^0{}_\chi+\tilde{\mathcal{O}}^\chi{}_0\neq 0. \qquad (5.34)$$

Here we lowered the indices by $\eta_{\mu\nu}$ which appears in the oscillator commutation relations. This shows that the moving D-string carries fundamental string charge and becomes a source for the target space NS-NS B field.

To gain a better understanding of this source, such as the amount of charge n it has, let us study the structure of the orthogonal matrix $\mathcal{O}$ in more detail. We started from an $S1$-brane boundary state (5.26) which has a Dirichlet boundary condition along time and then boosted it to obtain the matrix in Eq. (5.33). This can be compared with the ordinary $(n,1)$ string boundary state constructed in Ref. [38] which is obtained from the boundary state of a *D1-brane* by introducing the boundary coupling[14]

$$\oint d\tau\left(EX^0\frac{\partial}{\partial\tau}X^\chi - vX^0\frac{\partial}{\partial\sigma}X^1\right). \qquad (5.35)$$

The orthogonal matrix obtained in Ref. [38] was

$$\tilde{\mathcal{O}}^\mu{}_\nu=\frac{1}{1-E^2-v^2}\times\begin{pmatrix}1+E^2+v^2 & -2v & 2E\\ 2v & -1+E^2-v^2 & 2vE\\ 2E & -2vE & 1+E^2-v^2\end{pmatrix} \qquad (5.36)$$

and the associated boundary state describes an $(n,1)$ string moving with the speed v along the x^1 direction. The charge n is given by the electric flux on the worldvolume theory,

$$n=\frac{E}{\sqrt{1-E^2-v^2}}. \qquad (5.37)$$

Remarkably, the matrix (5.33) is identical with (5.36) under the relation

$$V=\frac{1}{v},\quad F_{1\chi}=\frac{E}{v}. \qquad (5.38)$$

This is indeed what we expected since the first equation is just $v=\partial x^1/\partial x^0=1/V$ and the second equation is just the change of the coordinates for $E=F_{0\chi}$ which we have found in previous sections. This suggests that the boosted S-brane boundary state (5.33) describes a moving $(n,1)$ string, but in a strict sense this is not the case. Let us compare the regions of parameter space where the actions are valid. The description (5.33) is valid if the S-brane Lagrangian is real,

$$1+F_{1\chi}^2-V^2\geqslant 0. \qquad (5.39)$$

Substituting the identification (5.38) into the above inequality, we find

$$1-E^2-v^2\leqslant 0, \qquad (5.40)$$

which is the region where the description (5.36) is invalid since the D1-brane Lagrangian becomes imaginary. Therefore, although the boundary states have the same structure, their valid regions of parameter space are different. The two descriptions overlap only in the case of vanishing Lagrangians where the fundamental string charge n goes to infinity. This means that the fundamental string (limit) can be described by both the boosted $S1$-brane and the D1-brane.

In the static case we can see this correspondence more directly. In the S-brane boundary conditions (5.30), we take the limit

$$E=\frac{F_{1\chi}}{V}\to 1,\quad v=\frac{1}{V}\to 0 \qquad (5.41)$$

which is expected to give static fundamental strings. Then Eq. (5.30) reduces to

$$\partial_\tau X^1=0,\quad \partial_\tau X^\chi+\partial_\sigma X^0=0. \qquad (5.42)$$

The first equation tells us that the object has Dirichlet boundary condition along x^1 and so it has worldvolume along x^0 and x^χ, while the second equation is the $|E|=1$ limit of the mixed boundary condition on a D-string,

$$F_{0\chi}\partial_\tau X^\chi+\partial_\sigma X^0=0. \qquad (5.43)$$

So this is precisely the fundamental string limit.

D. *S*-brane description and *T* duality

At this stage it is very natural to ask, ''What is the boosted S-brane without taking the fundamental string limit (= vanishing Lagrangian limit)?'' To approach a possible answer to this question, let us observe what happens to the orthogonal matrix in the boundary state. For simplicity we examine the static case. The boundary state of a static $(n,1)$ string presented in Ref. [38] is defined through its orthogonal matrix

$$\tilde{\mathcal{O}}^\mu{}_\nu(E)=\frac{1}{1-E^2}\begin{pmatrix}1+E^2 & 2E\\ 2E & 1+E^2\end{pmatrix}, \qquad (5.44)$$

where $\mu,\nu=0,\chi$. Here of course E should be less than or equal to 1. On the other hand, the boosted $S1$-brane with the static limit $V=\infty$ is also described by the above matrix with $E\geqslant 1$. To relate these two descriptions, we see that if we perform the transformation

$$E\to\tilde{E}=1/E, \qquad (5.45)$$

then the matrix $\tilde{\mathcal{O}}$ transforms as

$$\tilde{\mathcal{O}}(\tilde{E})=-\tilde{\mathcal{O}}(E). \qquad (5.46)$$

[14]Here we changed the notation from Ref. [38] as $\sigma\leftrightarrow\tau$ and $(0,1,2)\to(0,\chi,1)$ to fit our computation, and to avoid confusion we used $-v$ instead of the V used in Ref. [38].

Interestingly, this means that the case with electric field E larger than 1 is related to an E smaller than 1 only by a sign change of $\tilde{\mathcal{O}}$. The change in the sign of $\tilde{\mathcal{O}}$ is equivalent to the replacement $\tilde{\alpha}\to-\tilde{\alpha}$ which is a T duality along x^0 and χ directions, see Eq. (5.25).

So what we have found here is that the description of E larger than 1 can be obtained by T duality along x^0 and χ. Let us discuss the meaning of this duality more. Before examining our present case, it is instructive to remember the ordinary T duality along spatial directions for D-branes. Let us consider a bound state of n D0-branes and m D2-branes. The D2-brane worldvolume is extended along x^1 and x^2. The density of the D0-branes per unit area on the worldvolume of a single D2-brane is just the magnetic field induced on the D2-brane, $F_{12}=n/m$. The open string boundary condition becomes a mixed boundary condition. Now let us take a T duality along x^1 and x^2. First, T dualizing along x^1 transforms this D2-D0 bound state to a D1-brane winding the 12 torus n times along x^1 and m times along x^2. Second, take the T duality along x^2. We then get a bound state of n D2-branes and m D0-branes, giving an induced magnetic field $\tilde{F}_{12}=m/n=(F_{12})^{-1}$. This shows that the inversion of the magnetic field can be understood as T duality.

Let us apply this well-known idea to our case, and see what happens to a $(n,1)$ string when we T-dualize along x^0 and χ. Consider a static $(n,1)$ string stretched along the χ direction. The induced electric field $E=F_{0\chi}<1$ parametrizes the number of bound fundamental strings. First let us take a T-duality along χ. The resultant configuration is a D0-brane moving at the speed E which does not exceed the speed of light. This moving D0-brane can be thought of as a "winding" D0-brane, that is, a D0-brane winding $1/E$ times along x^0 and 1 time along χ. The winding along χ should be thought of as an $S0$-brane since the worldvolume is only along this spatial direction. Now take a second T duality along x^0. The former $1/E$ D0-brane becomes $1/E$ $S(-1)$-branes, while the latter $S0$-brane becomes a single D1-brane. Therefore, after the T dualities, we have a bound state of a single D1-brane and $1/E$ $S(-1)$-branes. This statement is very plausible in view of how we derived the boosted S-brane: there we considered an $S1$-brane with magnetic field $F_{1\chi}$, which is exactly a bound state of an $S1$-brane and $S(-1)$-branes. If we consider now the boosted S-brane so the $S1$-brane is timelike, i.e. a D1-brane, the resultant object should be a bound state of a D1-brane and $S(-1)$-branes.

Since the S-brane description in the previous sections is valid for $E\geqslant 1$, the case $E=1$ is the only overlapping region and has two equivalent descriptions. However, the above observation leads us to an intriguing conjecture: Any $(n,1)$ string can be thought of as a bound state of a D1-brane and E $S(-1)$-branes with $E<1$. Here we do not specify how the latter bound state should be described but there might be some advantages in treating the $(n,1)$ strings from the S-brane point of view. To illustrate this point, consider the RR coupling on a Dp-brane

$$\int C^{(p+1)}+F\wedge C^{(p-1)}+\cdots. \tag{5.47}$$

Let us turn on a constant electric field E_{01}. Usually this is said to turn the Dp-brane into an (F, Dp) bound state, but what does the above RR coupling tell us? The second term gives

$$E_{01}\int C^{(p-1)}_{23\cdots p}. \tag{5.48}$$

This is a source term for the RR $(p-1)$-form with spatial indices, or in other words for an $S(p-2)$-brane. This suggests that the fundamental strings can be thought of as smeared S-branes, at least in the worldvolume of other mother D-branes in which the fundamental strings are bound.

E. Relation between *S*- and D-brane descriptions

In the above we have learned that while D-branes with small electric fields are described by D-brane actions, D-branes with large electric fields are described by S-brane actions. Following the previous section, here we further explore the T duality which interchanges these two classes of configurations.

For simplicity, only the electric field in the χ direction is turned on. The Lagrangian, electric flux density and the Hamiltonian for the D-brane are given by

$$L=-\sqrt{1-E^2},\quad D=\frac{E}{\sqrt{1-E^2}},\quad H=\frac{1}{\sqrt{1-E^2}}, \tag{5.49}$$

and those for the S-brane are

$$L=\sqrt{-1+E^2},\quad D=\frac{E}{\sqrt{-1+E^2}},\quad H=\frac{1}{\sqrt{-1+E^2}}. \tag{5.50}$$

The range of electric fields valid for the D-brane description is $E^2<1$, which is mapped to the range of validity $E^2>1$ for the S-brane description by the T-duality along the time direction

$$E\to\frac{1}{E} \tag{5.51}$$

considered in the previous section. From the expressions above, we find that this map induces the interchange of D and H, or equivalently the interchange of the fundamental string charge and the energy. Recall that ordinary T-duality interchanges winding modes with Kaluza-Klein modes. Since the total string number can be thought of as the "winding number," and the energy as the "momentum" in the time direction, roughly speaking, the interchange of D and H is what one would expect for the T duality in the time direction.

F. Boosted *S*3-brane as a D-string

Earlier in this section we saw how the late time part of the solution of Sec. III can be realized as a boosted $S1$-brane. We may expect that in the same manner the late time configuration of the spike solution of Sec. IV D can also be obtained as a boosted *S*-brane. Here we will present a boosted solution of an *S*3-brane action with magnetic fields,[15] and show that actually the boundary state of the boosted *S*3-brane reduces to that of a static D-string.

As explained in Sec. IV D we may consider field strengths on the *S*3-brane arising from the excitations of a scalar field along the M-theory circle. If we assume that the fields in Eq. (4.33) are independent of x^2, x^3 as well as x^4, we obtain for vanishing A_4 $(=X^4)$

$$L=\sqrt{1-(\partial_1 X^0)^2+(\partial_1 X^{10})^2}. \tag{5.52}$$

The field X^{10} is related to the original field strength $B_1 \equiv \tilde{F}_{23}$ through the Legendre transformation,

$$\frac{\delta}{\delta B_1}\left[\sqrt{1-(\partial_1 X^0)^2-B_1^2+(B_1\partial_1 X^0)^2}-B_1\partial_1 X^{10}\right]=0 \tag{5.53}$$

where the factor of *i* has been included as discussed earlier. This is rewritten as

$$\partial_1 X^{10}=-B_1\sqrt{\frac{1-(\partial_1 X^0)^2}{1-B_1^2}} \tag{5.54}$$

so that the *S*3-brane can become a timelike object, $|\partial_1 X^0|>1$.

The Lagrangian (5.52) has the same form as Eq. (5.2), as it should due to *S* duality. There exists a general solution similar to Eq. (5.7),

$$\partial_1 X^0=\frac{c_1}{\sqrt{1-c_2^2+c_1^2}},\quad \partial_1 X^{10}=\frac{c_2}{\sqrt{1-c_2^2+c_1^2}}. \tag{5.55}$$

Let us take the BPS limit $c_1=c_2$ and furthermore the static limit $c_1\to\infty$. This is expected to be a D-string since this limit provides the late time behavior of the spike solution in Sec. IV D. To check this, let us again look at the worldsheet boundary condition of an attached fundamental string. The appropriate inclusion of the boundary coupling leads to[16]

$$\partial_\sigma X^2-i\tilde{F}_{23}\partial_\tau X^3=0,\quad \partial_\sigma X^3+i\tilde{F}_{23}\partial_\tau X^2=0, \tag{5.56}$$

$$\partial_\tau X^0-V\partial_\tau X^1=0,\quad V\partial_\sigma X^0-\partial_\sigma X^1=0, \tag{5.57}$$

where *V* is defined to be the value of $\partial_1 X^0$ in the solution as before. In the static limit, $V\to\infty$ and $\tilde{F}_{23}\to\infty$, the above boundary conditions reduce to

$$\partial_\tau X^3=\partial_\tau X^2=\partial_\tau X^1=0,\ \ \partial_\sigma X^0=0. \tag{5.58}$$

Remembering that we have a Neumann boundary condition for x^4, this is precisely a boundary condition for a D-string extended along x^4.

This analysis provides more evidence for the claim that the late time remnant of the solution in Sec. IV D is just a D-string. Here we demonstrated that D-strings can be described by an *S*3-brane, suggesting another interesting duality.

VI. *S*-BRANE AND D-BRANE INTERACTIONS

In this section we discuss how the formation of a codimension-one D-brane can be understood using an *S*-brane description of brane creation. In comparison, the solutions in Sec. IV B describe the formation of a (p,q) string from an *S*3-brane which is defined to be a spacelike defect on a non-BPS D4-brane. On the non-BPS D4-brane, these *S*-brane solutions are therefore describing the formation of codimension-three defects. However, the simplest case should be the formation of a codimension-one D-brane, which has been studied in some literature [6,39,8,40,9].

Here we make a preliminary discussion of the interesting role which *S*-branes play in RR charge conservation. Our main point is that in order to create charged defects we must also have charged *S*-branes whose time dependent charge represents specific inflow and outflow of charge into the system. In a time evolution transition, for example, we will discuss how RR charge can be thought to be "added" by the *S*-brane

$$A\ \ (\text{with charge } q_1)\ \overset{S\text{-brane ``charge'' } q_2}{\longmapsto}\ B\ \ (\text{with charge } q_1+q_2). \tag{6.1}$$

An interesting candidate process to examine is the time dependent formation of a kink, see also Refs. [6,39,8,40,9]. For simplicity consider a kink D0-brane on a non-BPS D1-brane system. The kink solutions for a D0-brane and the anti-kink solution for a $\overline{\text{D0}}$-brane are schematically

$$\begin{aligned} T_D(x)&>0 \quad \text{for } x>0\\ &<0 \quad \text{for } x<0,\\ T_{\bar D}(x)&<0 \quad \text{for } x>0\\ &>0 \quad \text{for } x<0. \end{aligned} \tag{6.2}$$

Consider now a transition from kink to anti-kink. This is a configuration where the absolute values of the tachyon field decrease and then increase again. The crucial point is that there should be a transition in the entire tachyon profile as it goes through zero. The time evolution of the configuration should roughly pass through

$$T(x)=0 \quad \forall x \tag{6.3}$$

which is flat. Since the *S*-brane always appears in such a transition, we attempt to ascribe the change in charge as being due to the *S*-brane. Although from the point of view of

[15] Though so far in this section we have used *S*1-branes, in this section we need magnetic fields and so use an *S*3-brane instead.

[16] Although there appears an "*i*" in this expression, this might be absorbed into the redefinition of the worldsheet variables.

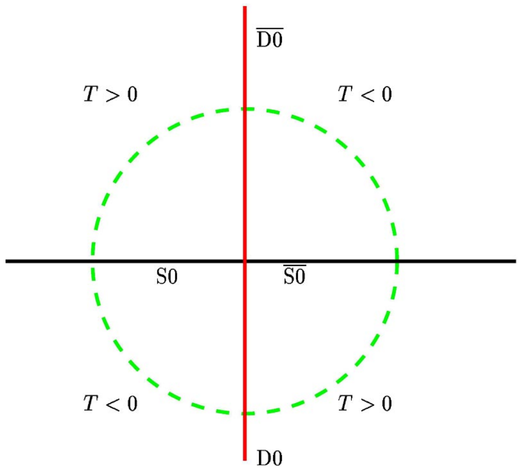

FIG. 7. Formation of an anti-kink using a kink and *S*-branes.

the effective theory the *S*-brane is a very non-localized instantaneous charged object, the complete tachyon profile paints a more standard picture which shows that the transition is not instantaneous. We will see, however, the consistency and simplicity of the *S*-brane picture.

To go from kink to anti-kink, the *S*-brane must have charge two, one to annihilate with the $\overline{\text{D0}}$-brane and one to create the D0-brane. The fact that a flat *S*-brane describes such a process is very surprising as it is so simple and is different from our other *S*-brane solutions. Also as discussed in Ref. [40], many branes and anti-branes can be essentially created from a flat $T=0$ initial condition. It seems then that a flat charge one *S*-brane can either destroy a D0-brane, or destroy a D0-brane and also create equal numbers of branes and anti-branes. If this statement were true it would greatly reduce the usefulness of *S*-branes since each *S*-brane would represent an infinite number of qualitatively different processes. Fortunately, we shall see by considering things more carefully that this is not the case and our consideration here was too naive. In fact we can consistently conserve RR charge in the tachyon condensation process by properly accounting for the *S*-branes.

Figure 7 illustrates the time dependent kink formation process and represents the entire non-BPS D1-brane worldvolume with the vertical and horizontal directions corresponding to time and space, respectively. The horizontal line $t=0$ indicates the location of the $S0$-brane, the upper half vertical line is a $\overline{\text{D0}}$-brane and the lower half vertical line is a D0-brane. For $t<0$, $T(x)>0$ for $x>0$ while $T(x)<0$ for $x<0$. For $t>0$, $T(x)<0$ for $x>0$ while $T(x)>0$ for $x<0$.

Although the horizontal line marks the $T=0$ region, it actually consists of an $S0$-brane and an $\overline{S0}$-brane. The $S0$-brane is located at $x<0$, $t=0$ while the $\overline{S0}$-brane is at $x>0$, $t=0$. This is clear if we look at the tachyon configuration at $t=0$ since $\dot{T}<0$ for $x>0$ while $\dot{T}>0$ for $x<0$. This pair of *S*-branes seems to be necessary to create a D0-brane on a non-BPS D1-brane.

Now we can define our charge conservation rule. If we just consider the D0-brane and the $\overline{\text{D0}}$-brane, charge is not conserved. To conserve charge we must include the *S*-brane charge and so propose the following conservation law. For any closed curve, for example the dashed circle in the figure, count the number of D-branes and *S*-branes which flow into the curve in such a way that a D-brane (anti-*S*-brane) contributes a charge $+1$ while an anti-D-brane (S-brane) counts as a -1. Naturally, a single stationary D0-brane conserves charge as does a single flat $S0$-brane (which is consistent with the charge conservation of the known flat *S*-branes of the rolling tachyon.) In the above figure the net change inflow is zero, $+2-2=0$.

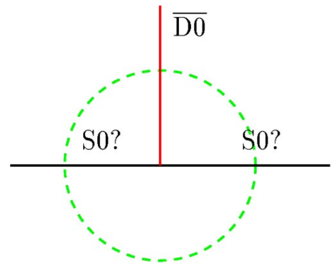

FIG. 8. Creating a $\overline{\text{D0}}$-brane does not conserve charge.

The verification of this conservation law is straightforward. Draw an arbitrary simple closed curve over the spacetime plot of any tachyon configuration and parametrize the curve by l, so the values of the tachyon are $T=T(l)$, $0\leq l \leq 2\pi$. The zeros of the tachyon configuration are located at $l=l_i$ where $i=1,2,\ldots,2n$. Now the important point is that we take the tachyon field to be a single valued function over the worldvolume $T(l=0)=T(l=2\pi)$, so integrating the derivative $\partial T/\partial l$ over the curve we get

$$\sum_i \operatorname{sgn}\left[\frac{\partial T}{\partial l}\bigg|_{l=l_i}\right]=0. \qquad (6.4)$$

The locations l_i with $\operatorname{sgn}[\partial T/\partial l|_{l=l_i}]=+1$ are physically interpreted as intersections of the circle with either a D0-brane or $\overline{S0}$-brane, depending on how fast the tachyon field zeros are moving. This proves our conservation law and clearly shows that *S*-branes play an essential role in charge conservation.[17]

Consider next a similar case where the entire tachyon configuration is situated at $T=0$. We are tempted to imagine the formation of a net kink or anti-kink by tiny perturbations as shown in Fig. 8, and this fact gives some support to our previous statement that a flat *S*-brane is a good candidate to describe the transition. Unfortunately this observation is in direct contradiction to our charge conservation law. How do we resolve charge conservation with our above observation? One way is to place the *S*-brane at past infinity by reparametrizing time, see Fig. 9. The *S*-brane can never be enclosed by any finite closed curve, so charge is conserved. Putting

[17]More precisely, the "location" l_i does not specify the location of the branes but gives the maximum of the RR charge density. The RR charge density is given by $\sim e^{-T^2}dT$, and the integration over $T\in[-\infty,\infty]$ gives a unit RR charge. In the following the location should be understood in this sense of the maximum charge density.

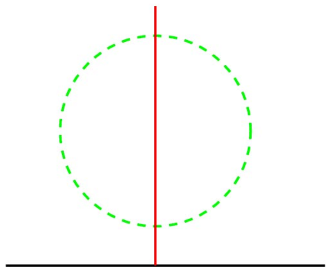

FIG. 9. Putting the *S*-brane at past infinity will ensure charge conservation.

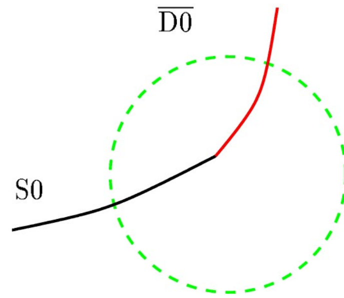

FIG. 10. An $S0$-brane is changing into a $\overline{\text{D0}}$-brane.

the *S*-brane at past infinity was also discussed in Refs. [7,8] as a "half *S*-brane," where the tachyon was taken to be $T(t)=e^{\lambda t}$. This tachyon configuration is just like a flat *S*-branes in our sense at early times and then dissipates into the vacuum at late times. [To go from the D0-brane to the $\overline{\text{D0}}$-brane we would need something like $T(t,x)=x\sinh(\lambda t)$.] We may also think of the situation illustrated in Fig. 10 in which an $S0$-brane turns into a $\overline{\text{D0}}$-brane so charge is again conserved. Although charge conservation cannot solely determine the possible dynamics, it clearly does limit the dynamical processes.

It should be remembered that we can produce chargeless remnants.[18] The fundamental string formation process studied in Sec. III provides an example. There the net (S-)brane RR charge disappeared due to the shrinking worldvolume. Of course if we took the branes to have zero charge then charge conservation would play no role. However, as long as we treat topological defects with topological charges, the same argument should apply.

Our discussion on charge conservation for codimension-one kinks of a real tachyon can be generalized to codimension two vortices of complex tachyons, which exist on the worldvolume of a D-$\overline{\text{D}}$ pair. Therefore in analogy to Eq. (6.4), the number of vortices and anti-vortices intersecting a sphere should be equal.

Seeing how *S*-branes and D-branes interact, we are reminded of string networks. Also, one could attempt to interpret the process in Fig. 7 as two copies of the process in Fig. 10. Solutions of Fig. 10 are not solutions of the *S*-brane action, but could be solutions of an S-$\overline{S}$ pair.

VII. CONCLUSIONS AND DISCUSSIONS

In this paper, we have explained how *S*-branes play a role in time evolution in string theory, especially in the D-brane/F-string formation during tachyon condensation. In general we have classified *S*-brane solutions according to their remnants as in Fig. 2. Although there are some "expected" solutions which we have not yet obtained, the arrows in Fig. 2 typically show how *S*-branes work in regards to time evolution of string theory processes. Although our *S*-brane is defined through the rolling tachyon on non-BPS D-branes, we may expect that this scenario of D-brane/F-string formation via *S*-branes is more general and may be applied to other situations of brane creation in string theory and also to brane cosmology [41]. Possibly we may even apply these *S*-brane methods to understand defect formation in non-stringy systems with topological defects, such as the standard model, since it has recently been reported that the generic features of D-branes can be reconstructed in the context of usual field theories [42].

To illustrate the roles of the *S*-branes, we presented several classical solutions of *S*-brane actions, including electric $S3$-brane spike solutions (Sec. III and Ref. [3]) which described fundamental string formation, electric-magnetic $S3$-brane spike solutions in Sec. IV which produced (p,q) strings and D-strings, and "boosted" *S*-branes which are flat and timelike branes capturing the late time configuration of the spike solutions. By directly analyzing the non-BPS tachyon system in Sec. III B, the confinement of electric flux was shown to minimize the energy of the corresponding tachyon system, and this result agrees with our interpretation of the electric spike solution. *S*-duality on the $S3$-brane was studied in Sec. IV C, which turned out to be consistent with the rolling tachyon with electric and magnetic fields obtained in Sec. IV A. By taking into account M-theory effects, we found out how to produce D-strings from an $S3$-brane. The existence of these solutions therefore demonstrates that *S*-duality could in fact be used in a new way to constrain remnant formation. Our resolution of the imaginary field strength on the $S3$-brane worldvolume is potentially relevant in other cases [29]. The boosted *S*-brane was introduced and we provided their corresponding tachyon configurations in Sec. V B. We also obtained the boosted *S*-brane boundary state which clarified that the boosted *S*-brane is T dual in the time direction to (p,q) strings. In our analysis the fundamental string limit of (p,q) strings can be described by both D-branes and *S*-branes so the critical electric field $E=1$ is likely a self-dual point between these two descriptions.

We now turn to detailed comments on some results we obtained in this paper. Although the late time configuration of the spike solution in Sec. III is given by the boosted *S*-brane in Sec. V, we have not found explicit tachyon solutions corresponding to the spike solutions of Sec. III and Sec. IV. The results of Ref. [13], which discussed tachyon

[18]Many field theories have solitons and so we believe that Spacelike solitons (branes) should also exist in these theories. For example in scalar ϕ^4 theory, it might be possible for *S*-branes to describe the formation of the kink solution. In this case since the kink solution has Z_2 topological charge which should be conserved, the process illustrated in Fig. 8 still does not exist.

spike configurations of D-branes (the brane/F-string ending on branes), might be useful in the construction of tachyon configurations for S-branes. It might be possible to generalize the recent result in Ref. [43] on the correspondence between the tachyon system and DBI on their defects, to our S-brane situations. It is inevitable, however, that the tachyon solutions will be approximate since the precise Lagrangian in string theory is still missing. Also, while work has been done to check various static properties of tachyon actions, their time dependent properties are not as well understood.

We also point out various other solutions and generalizations. Another type of solution to look for on the S-brane worldvolumes we have discussed, is to have the electric field and magnetic fields in different directions. One example is to have the electric field along the χ direction and to have the magnetic field along one of the angular directions, let us say ϕ. A similar static case has been discussed in Refs. [26,44]. Also, in the solution of Sec. III, it is possible to take a T duality along the χ direction. This simply turns $F_{t\chi}$ into the velocity along that direction, so the criticality of the original electric field will result in the S-brane worldvolume moving at the speed of light. This is a null geodesic, and looks like an emission process of a D-brane. Another interesting generalization is to have multiple spikes. This is possible because the bion spike solutions in Refs. [31,28] decouple from each other and so do the multiple S-brane spikes. These solutions are similar to the above emission processes. In this case we observe many D-branes and strings coming in from past infinity and scattering to various directions in the target space. However, we would like to state that such a configuration is odd since although the Hamiltonian is simply the sum of spikes, and hence gives seemingly independent worldvolumes, we see that the worldvolumes also apparently intersect for some time.

The analysis in Sec. IV C also implies that there are also throat solutions in S-brane systems as in the D-brane cases. In the ordinary D-brane case the throat solutions are relevant for the brane and anti-brane annihilation process [31,45,5]. It would be very interesting if the role of these S-brane throat solutions (the throat is along time direction X^0 in the S-brane case) is clarified. In fact this question is related to the possible non-Abelian structure of S-branes which should be not just the result of a non-Abelian structure of the original non-BPS D-branes but is more intrinsic to time evolution and tachyon condensation on a single non-BPS D-brane. Since the throat can also carry an electric charge, it is possible that these throat solutions are involved with the mechanism of electric flux confinement.

Finally, the various S-brane solutions we have found are reminiscent of interactions between branes and strings, and the interpretation that particular S-brane solutions can be thought of as Feynman diagrams was pursued partly in Sec. VI and Ref. [5]. In Sec. VI the creation of codimension-one D-branes was qualitatively discussed from the viewpoint of charge conservation. We believe that this creation process can be described by some classical solution of the (tachyonic) S-brane action which might be the action of an S-$\bar{S}$ pair. However, here we outline another possible way to describe this D-brane creation process. The tachyon configuration of Fig. 7 has an interesting property. The rolling tachyon energy at $x\neq 0$ is nonzero while at $x=0$ the energy is equal to the non-BPS brane since $T=\dot{T}=0$ there. The tension of the $S0$-brane depends on the rolling tachyon energy $\mathcal{E}(x)$, since in the derivation of the S-brane action, the tension of the S-brane is just the value of the non-BPS brane Lagrangian integrated over x^0 with substitution of the classical solution $T(x^0)$ which is dependent on $\mathcal{E}$. Hence it may be possible to regard $\mathcal{E}(x)$ (or equivalently, the tension of the S-brane) as another dynamical variable that the S-brane system has. At values of x with $\mathcal{E}(x)=\mathcal{T}_{\text{non-BPSD1}}$, a $\overline{\text{D0}}$-brane is created as in Fig. 7. If we may introduce a term like $(\partial\mathcal{E})$ in the S-brane Lagrangian, it may fix the spatial dependence of the S-brane tension via equations of motion for $\mathcal{E}$ and so govern the D-brane creation process. However, since $\mathcal{E}(x)$ is not a localized mode on the S-brane but defined through the integration over all the x^0 region, it might be difficult to proceed along this direction to generalize the S-brane Lagrangian.

If a configuration like Fig. 7 is explicitly constructed, however, it should provide an interesting procedure to compute Feynman diagrams for D-brane scattering. It is possible that physical quantities associated with the scattering process are directly related to S-brane actions and their solutions. Understanding these S-brane systems might provide a theory of interacting D-branes and strings in a general context and an alternative to matrix theory.

By analyzing boundary states with electric fields and an inhomogeneous tachyon background, the authors of Ref. [46] have also recently discussed solutions which can dynamically produce fundamental strings. It would be interesting to further explore the relationship between their boundary state analysis and S-brane solutions. We leave these issues to future investigation.

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APPENDIX A: TACHYONIC *S*-BRANE ACTION

In this appendix, we explicitly demonstrate how the tachyonic S-branes considered in Sec. II C appear in the tachyon condensation of D-brane anti-D-brane. In Fig. 4 this is the arrow (2). Since arrow (1) has already been discussed in Ref. [3], while arrow (3) is just the same as the usual D-brane descent relation and arrow (4) was realized in Sec. II C, the derivation of arrow (2) completes the explanation of the generalized descent relations of Fig. 4.

To derive the effective action of the "tachyonic *S*-brane" by using the fluctuation analysis of the time dependent kink as performed in Ref. [3], we return to the Lagrangian of the D-$\overline{\mathrm{D}}$ pair in Eq. (2.1) and the solution representing the tachyonic *S*-brane in Eq. (2.2). A direct analysis of this fluctuation mode is difficult due to the complexity of the Lagrangian.

The easiest way to proceed is to simplify the situation and truncate the derivatives of the Lagrangian at fourth order

$$S=2T_{\mathrm{D9}}\int d^{10}x e^{-|T|^2}[1+|\partial_\mu T|^2+p_1(|\partial_\mu T|^2)^2 + p_2(\partial_\mu T)^2(\partial_\nu \overline{T})^2], \tag{A1}$$

where $T\equiv T_1+iT_2$ and p_1,p_2 are numerical constants. We must keep at least fourth order derivative terms since if we only keep the usual canonical kinetic energy there is no tachyon solution linear in time, and unless the solution is linear it is again technically difficult to perform a fluctuation analysis. The equation of motion for a homogeneous time dependent tachyon is

$$-T_1(1+\dot{T}_1-3p\dot{T}_1^4)+\ddot{T}_1(1-6p\dot{T}_1^2)=0, \tag{A2}$$

where $p\equiv p_1+p_2$, and we have set $T_2=0$. Therefore the linear solution

$$T_{\mathrm{cl}}=ax^0, \tag{A3}$$

exits for $a=\sqrt{(1+\sqrt{1+12p})/6p}$.

It is actually strange that we have a completely linear solution in spite of the presence of the tachyon potential. The higher-order kinetic term makes this situation possible. The general solution does not exhibit the rolling tachyon behavior at late time, since this model is just a generalization of the Minahan-Zwiebach model which does not possess the rolling tachyon behavior. The general solution reaches the true vacuum $T=\infty$ in finite time. But if we tune the initial condition then we have the completely linear solution for the rolling tachyon. The strangeness of this solution is also apparent in that its energy vanishes

$$\mathcal{E}=\int e^{-|T|^2}(1+\dot{T}_1-3p\dot{T}_1^4)=0. \tag{A4}$$

For the meantime we treat this special solution as just an illustration of the new descent relations.

1. Fluctuation spectrum

Let us consider the following fluctuation:

$$T=T_{\mathrm{cl}}(x^0)+t_1(x^\mu)+it_2(x^\mu), \tag{A5}$$

where $\mu=0,1,\ldots,9$. Substituting this into the action and collecting terms quadratic in the fluctuation fields, we obtain the fluctuation action

$$S_{\mathrm{fluc}}=2T_{\mathrm{D9}}\int d^{10}x e^{-T_{\mathrm{cl}}^2}\bigg[\left(\frac{4}{3}-\frac{2}{3}a^2\right)[(2a^2x_0^2-1)t_1^2-t_2^2] - \frac{8-4a^2}{3}x_0t_1\dot{t}_1+\frac{a^2-2}{3a^2}(\partial_\mu t_1)^2+\frac{4(1+a^2)}{3a^2}\dot{t}_1^2 + (1-2a^2p_1+2a^2p_2)(\partial_\mu t_2)^2+4p_2a^2\dot{t}_2^2\bigg]. \tag{A6}$$

The two fluctuation modes are completely decoupled from each other. Integrating by parts, we find that

$$S_1=2T_{\mathrm{D9}}\int d^{10}x e^{-T_{\mathrm{cl}}^2}\left[\frac{a^2-2}{3a^2}(\partial_\mu t_1)^2+\frac{4(1+a^2)}{3a^2}\dot{t}_1^2\right], \tag{A7}$$

$$S_2=2T_{\mathrm{D9}}\int d^{10}x e^{-T_{\mathrm{cl}}^2}\bigg[\left(-\frac{4}{3}+\frac{2}{3}a^2\right)t_2^2 + (1-2a^2p_1+2a^2p_2)(\partial_\mu t_2)^2+4p_2a^2\dot{t}_2^2\bigg]. \tag{A8}$$

To see the physical meaning of these fluctuations, we redefine the fields as $\hat{t}_{1,2}=e^{-(ax^0)^2/2}t_{1,2}$ so the newly defined fields $\hat{t}_{1,2}$ have canonical kinetic terms. Then we can decompose the fields $\hat{t}_{1,2}(x^\mu)$ into the eigenfunctions of the harmonic potential along x^0, as performed in Ref. [3]. We may determine the "mass" spectra for these fluctuations as the eigenvalues of the Laplacian, ∂_i^2, for the spatial directions.

The t_1 fluctuation contains a zero mode which is the Nambu-Goldstone mode associated with the symmetry breaking of the translation along x^0 by the presence of the kink solution. The "mass" tower of t_1 is obtained as

$$m^2=\frac{8a^2(1+a^2)}{a^2-2}n, \quad n=0,1,\ldots . \tag{A9}$$

The constant a should be less than $\sqrt{2}$ to keep the coefficient of the term $(\partial_\mu t_1)^2$ negative.

Next, we use the field redefinitions to rewrite the action S_2 as

$$S_2=2T_{\mathrm{D9}}\int d^{10}x\bigg[(1-2a^2p_1+2a^2p_2)(\partial_i\hat{t}_2)^2+\frac{2-a^2}{3a^2}\hat{t}_2^2 + \frac{a^2(2-a^2)}{3a^2}x_0^2\hat{t}_2^2+(a^2-2)\hat{t}_2^2\bigg]. \tag{A10}$$

From this expression it is easy to extract the mass spectrum

$$m^2=\frac{2-a^2}{3a^2(1-2a^2p_1+2a^2p_2)}\bigg[a\sqrt{\frac{2-a^2}{3}}(2n+1) + a^2-2\bigg]. \tag{A11}$$

Here again $1-2a^2p_1+2a^2p_2>0$ should be satisfied so that the fluctuation Lagrangian is positive definite. The lowest mode becomes tachyonic, and this tachyonic mode is associated with the instability of the time-dependent kink solution.

2. Effective action

The lowest modes in the fluctuations $\hat{t}_{1,2}$ are Gaussian, and if one expresses these in term of the original fluctuation then they are actually constant, independent of x^0. Using this property, we can calculate the effective action for the tachyonic *S*-brane. By substitution of the fluctuation into the original D-$\overline{\text{D}}$ action, we have

$$\begin{aligned} S=&2T_{\text{D9}}\int dx_0 e^{-(ax_0+t_2)^2}\int d^9x\{1-a^2+(\partial_i t_1)^2+(\partial_i t_2)^2\\ &+p_1[(a^4-2a^2(\partial_i t_1)^2-2a^2(\partial_i t_2)^2]\\ &+p_2[a^4-2a^2(\partial_i t_1)^2+2a^2(\partial_i t_2)^2]\}\\ =&2T_{\text{D9}}\frac{\sqrt{\pi}}{a}\int d^9x e^{-t_2^2}\bigg[\frac{2}{3}(2-a^2)-\frac{2-a^2}{a^2}(\partial_i t_1)^2\\ &+(1-2a^2p_1+2a^2p_2)(\partial_i t_2)^2+[(\partial t)^4\text{term}]\bigg]. \end{aligned} \tag{A12}$$

(In the last line we have performed the integration over x^0.) This is the tachyonic *S*-brane effective action, which resembles a Minahan-Zwiebach model [35]. The differences between them are as follows: (1) The sign of $(\partial_i t_1)^2$ term is negative, indicating that this mode represents the translation along the time direction. (2) The worldvolume metric defining this theory is Euclidean. These two properties are shared with the *S*-brane action obtained in our previous paper.

Although we have adopted a derivative truncation as the starting point (A1) and also a special solution (A3), we believe that this effective action (A12) may capture essential features of the tachyonic *S*-branes.

APPENDIX B: EVALUATION OF THE TACHYON ENERGY OF THE BOOSTED *S*-BRANE

Though the energy of the (deformed) *S*-brane configurations has been studied in Sec. III, Sec. IV, and Ref. [3], the overall normalization of the *S*-brane action has not been specified there. This can be fixed in principle in the derivation of the *S*-brane actions in Sec. II C and Ref. [3]. It is clear that the factor S_0 in Ref. [3], which is an "*S*-brane tension," can be computed by substituting the rolling tachyon solution into the original tachyon action. This tension S_0 is, therefore, not fixed since it is dependent on the rolling tachyon energy $\mathcal{E}$. This situation is different from the case of static tachyon defects of D-branes where the tension is fixed completely.

Let us evaluate S_0 using the BSFT Lagrangian as a starting point.[19] The BSFT action of a non-BPS D2-brane is[20]

$$S_{\text{nonBPS}}=-\mathcal{T}_2\int d^3x e^{-\pi T^2}\sqrt{-\det(\eta+F)}\mathcal{F}(z), \tag{B1}$$

where the worldvolume coordinates are x^0,x^1,χ and

$$z\equiv G^{\mu\nu}\partial_\mu T\partial_\nu T. \tag{B2}$$

We are working in the units $2\pi\alpha'=1$, and $G_{\mu\nu}$ is the open string metric. The function $\mathcal{F}$ is defined by BSFT and its explicit form is given in Refs. [15–17], for example. The properties of this $\mathcal{F}$,

$$\mathcal{F}(z)\sim-\frac{1}{2}\frac{1}{z+1}\quad(z\sim-1) \tag{B3}$$

will turn out to be important later.

For vanishing field strength the homogeneous rolling tachyon solution $T=T_{\text{cl}}(x^0)$ presented in Ref. [19] has an asymptotic expansion for large x^0

$$T_{\text{cl}}(x^0)=x^0+\epsilon(x^0)+\text{higher}, \tag{B4}$$

where

$$\dot{\epsilon}(x^0)=\sqrt{\frac{\mathcal{T}_2}{4\mathcal{E}}}\exp\left[-\frac{\pi}{2}(x^0)^2\right]. \tag{B5}$$

Here $\mathcal{E}$, the energy density of the above homogeneous rolling tachyon solution, is defined by the following Hamiltonian density formula:

$$H=\mathcal{T}_2 e^{-\pi T_{\text{cl}}{}^2}\sqrt{-\det(\eta+F)}\left[\mathcal{F}(z)-\dot{T}\frac{\delta z}{\delta\dot{T}}\frac{\delta\mathcal{F}(z)}{\delta z}\right]. \tag{B6}$$

Note that $T_{\text{cl}}(x^0)$ is a function dependent on the integration constant $\mathcal{E}$ implicitly. The *S*-brane tension S_0 is just the value of the action (B1) into which the solution T_{cl} is substituted (while the integration over the spatial worldvolume is left unperformed, to give the worldvolume of the *S*-brane). Although the complexity of the function $\mathcal{F}(z)$ obstructs the analytic evaluation of the action, we can read off the integrand in the asymptotic region $x^0\sim\infty$. Noting that z approaches -1 in this limit

$$z\sim-1-\sqrt{\frac{\mathcal{T}_2}{\mathcal{E}}}\exp\left[-\frac{\pi}{2}(x^0)^2\right], \tag{B7}$$

we obtain

[19] So far, among many tachyonic Lagrangians, only the BSFT Lagrangians reproduce the D-brane tensions correctly and consistently.

[20] We have rescaled the tachyon from that of Ref. [19] as $T\to T/\sqrt{4\pi}$.

$$L_{\rm nonBPS}\sim -\mathcal{T}_2 e^{-\pi(x^0)^2}\left(-\frac{1}{2}\right)\left[-\sqrt{\frac{\mathcal{T}_2}{\mathcal{E}}}\exp\left[-\frac{\pi}{2}(x^0)^2\right]\right]^{-1}$$
$$=-\frac{\sqrt{\mathcal{E}\mathcal{T}_2}}{2}e^{-\pi(x^0)^2/2}. \qquad \text{(B8)}$$

This means that the value of S_0, which is given by the integral of $L_{\rm nonBPS}$ over x^0, is in fact finite and may be approximated as

$$S_0\sim -\frac{\sqrt{\mathcal{E}\mathcal{T}_2}}{2}\int_{-\infty}^{\infty}dx^0 e^{-\pi(x^0)^2/2}=-\sqrt{\frac{\mathcal{E}\mathcal{T}_2}{2}}. \qquad \text{(B9)}$$

Let us move on to the evaluation of the energy of the boosted *S*-brane which is a timelike object. It is straightforward to show that the rolling tachyon solution in the presence of a constant magnetic field is also a solution of the non-BPS D2-brane system (B1),

$$T=T_{\rm cl}(x^0),\quad F_{1\chi}=\text{const}. \qquad \text{(B10)}$$

Basically we can turn on the constant field strength transverse to the *S*-brane freely. Next, consider the boosted solution

$$T=T_{\rm cl}(x^{0\prime}),\quad F_{1\chi}=\text{const} \qquad \text{(B11)}$$

where

$$x^{0\prime}\equiv x^0\cosh\gamma+x^1\sqrt{G_{11}}\sinh\gamma. \qquad \text{(B12)}$$

Here the open string metric is

$$G_{\mu\nu}=\text{diag}(-1,1+F_{1\chi}^2,1+F_{1\chi}^2). \qquad \text{(B13)}$$

One can show that Eq. (B11) is again a solution[21] of the non-BPS D2-brane system (B1). In the limit

$$F_{1\chi}\to\infty \qquad \text{(B14)}$$

the *S*-brane becomes timelike and in this case the tachyon configuration is approximately

$$T\sim(\sqrt{G_{11}}\sinh\gamma)x^1, \qquad \text{(B15)}$$

which resembles the usual D-string kink solution. This suggests that the energy is localized at $x^1=0$.

We keep this in mind and proceed to carefully evaluate the Hamiltonian at $x^0=0$ for simplicity. The asymptotic expansion of $T_{\rm cl}$ at $x^0=0$ is

$$T=\sinh\gamma\sqrt{G_{11}}x^1+\sqrt{\frac{\mathcal{T}_2}{4\mathcal{E}}}\exp\left[-\frac{\pi}{2}(\sinh\gamma\sqrt{G_{11}}x^1)^2\right]$$
$$+\text{higher} \qquad \text{(B16)}$$

[21]The nontrivial check is on the equations of motion for the gauge fields. The tachyon equation of motion is trivially satisfied since we made a boost respecting the open string metric.

and this approximation is very good for nonzero x^1 and large $F_{1\chi}$. For this solution the argument z is

$$z=[-\dot{T}^2+G^{11}(\partial_1 T)^2]=\cdots=-(T'_{\rm cl})^2 \qquad \text{(B17)}$$

where the prime denotes a derivative with respect to the argument of the function $T_{\rm cl}$, i.e. in the above

$$T'_{\rm cl}\equiv\left[\frac{\delta T_{\rm cl}(a)}{\delta a}\right]_{a=\cosh\gamma x^0+\sinh\gamma\sqrt{G_{11}}x^1}. \qquad \text{(B18)}$$

Since $T'_{\rm cl}$ approaches 1, z approaches -1 everywhere except $x^1=0$ in the limit $F_{1\chi}\to\infty$. This means that in the evaluation of the energy $\delta\mathcal{F}/\delta z$ [the second term in Eq. (B6)] is much larger than $\mathcal{F}$ [the first term in Eq. (B6)] due to the expansion (B3), so the Hamiltonian at $x^0=0$ is given by

$$H=\mathcal{T}_2\exp[-\pi(\sinh\gamma\sqrt{G_{11}}x^1)^2]\sqrt{-\det(\eta+F)}2(\dot{T})^2\frac{\delta\mathcal{F}(z)}{\delta z}$$
$$=\mathcal{T}_2\exp[-\pi(\sinh\gamma\sqrt{G_{11}}x^1)^2]\sqrt{1+F_{1\chi}^2}2(\cosh^2\gamma)$$
$$\times(T'_{\rm cl})^2\frac{\delta\mathcal{F}(z)}{\delta z}$$
$$=\mathcal{T}_2\exp[-\pi(\sinh\gamma\sqrt{G_{11}}x^1)^2]\sqrt{1+F_{1\chi}^2}2(\cosh^2\gamma)$$
$$\times\frac{1}{\frac{2\mathcal{T}_2}{\mathcal{E}}\exp[-\pi(\sinh\gamma\sqrt{G_{11}}x^1)^2]}$$
$$=\mathcal{E}\sqrt{1+F_{1\chi}^2}\cosh^2\gamma. \qquad \text{(B19)}$$

This is independent of x^1, and we have shown that the background rolling tachyon energy is still present everywhere, even in the limit $F_{1\chi}\to\infty$. (The above result is consistent with the original rolling tachyon with $F_{1\chi}=0$ and $\gamma=0$, since this should give the energy $\mathcal{E}$.)

Let us consider higher order terms in the Hamiltonian to see the localization of the energy which should correspond to the energy of the boosted *S*-brane. In the limit (B14), it turns out that the next-to-leading order term coming from the expansion of the potential term $e^{-\pi T^2}$ can be ignored. First, we expand the function z for large x^1 at $x^0=0$ as

$$z=-1-\sqrt{\frac{\mathcal{T}_2}{\mathcal{E}}}\exp\left[-\frac{\pi}{2}(\sinh\gamma\sqrt{G_{11}}x^1)^2\right]$$
$$-\frac{\mathcal{T}_2}{\mathcal{E}}\exp[-\pi(\sinh\gamma\sqrt{G_{11}}x^1)^2]+\text{higher}. \qquad \text{(B20)}$$

Then the Hamiltonian is evaluated to the next-to-leading order as

$$H=\mathcal{E}\sqrt{1+F_{1\chi}^2}\cosh^2\gamma\left[1-\frac{1}{2}\sqrt{\frac{\mathcal{T}_2}{\mathcal{E}}}\right.$$
$$\left.\times\exp\left[-\frac{\pi}{2}(\sinh\gamma\sqrt{G_{11}}x^1)^2\right]\right]+\frac{1}{2}\sqrt{\mathcal{E}T_2}\sqrt{1+F_{1\chi}^2}$$
$$\times\exp\left[-\frac{\pi}{2}(\sinh\gamma\sqrt{G_{11}}x^1)^2\right]+\text{higher.} \tag{B21}$$

Here the second term in the first line is from the higher order evaluation of $\delta\mathcal{F}/\delta z$ in Eq. (B6), while the second line comes from evaluation of the $\mathcal{F}(z)$ term in the Hamiltonian (B6). Interestingly, though these two exponential terms become infinitely small in the limit $F_{1\chi}\to\infty$, they are combined and approach a δ function whose coefficient is finite. More precisely, the above expression is arranged in this limit as

$$H=\mathcal{E}\sqrt{1+F_{1\chi}^2}\cosh^2\gamma-\sqrt{\frac{\mathcal{E}\mathcal{T}_2}{2}}|\sinh\gamma|\,\delta(x^1). \tag{B22}$$

So, in addition to the homogeneous energy of the background rolling tachyon, we have a localized energy with a finite coefficient. This second term should be identified with the energy of the boosted $S1$-brane.

We now show that the localized energy contribution we just calculated agrees with the Hamiltonian of the $S1$-brane action. The action of a static $S1$-brane located at $x^1=0$ is

$$S_{S1}=S_0\int dx^0dx^1d\chi\,\delta(x^1)\sqrt{E^2-1}. \tag{B23}$$

Using this action, one finds that the $S1$-brane Hamiltonian density is

$$H_{S1}=S_0\frac{1}{\sqrt{E^2-1}}\delta(x^1). \tag{B24}$$

Now this electric field E is the induced electric field as seen in Eq. (5.24). After taking the limit $F_{1\chi}\to\infty$, we have $E=-\coth\gamma$. Substituting this into the S-brane Hamiltonian (B24), we obtain

$$H_{S1}=S_0|\sinh\gamma|\,\delta(x^1). \tag{B25}$$

Remarkably this agrees with the finite energy contribution in Eq. (B22) using the S-brane tension of Eq. (B9).

Lastly we provide a comment on this localized energy. In the final expression (B22), the S-brane contribution was found to be negative. This suggests that the S-brane has a negative energy, which agrees with the result of the boundary state analysis in which the time-time component of the boosted S-brane boundary state is given by a negative value as opposed to the usual boundary states for $(p,1)$ strings. In this appendix we have shown why this does not result in any of the usual problems. While the contribution of the S-brane is negative, there is an additional leading order energy contribution in Eq. (B22) which is due to the energy of the background rolling tachyon, and so the total energy is still positive.

The picture is reminiscent of anti-particles in the "Dirac sea." The boosted S-brane is like something existing in a cloud of fundamental strings. Since our non-BPS D-brane formulation did not take care of the radiation of the fundamental strings, it keeps the energy and effect of all these strings which are supposed to radiate away. (One of the effects of this cloud of fundamental strings might possibly be to make the S-brane energy negative.) Actually, the string cloud will dissipate, and the S-brane with strings attached to it will become a D-brane with strings attached to it. (Here we have to distinguish the strings on the non-BPS D-brane which will decay away, from strings stuck to the S-brane.) As a final remark, the energy of the background rolling tachyon in Eq. (B19) diverges in the limit $F_{1\chi}\to\infty$. The validity of some of the calculations are not so rigorous due to this singular limit. Although the boosted S-brane is expected to capture the late time behavior of the spike solution in Sec. III, apparently this divergence comes from the fact that we have not taken into account the curved worldvolume of the S-brane in the spike solution where $F_{1\chi}$ is divergent only at $r=0$. In this sense the correspondence between the spike solution and the boosted S-brane is not exact.

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