Becchi-Rouet-Stora symmetry restoration of chiral Abelian Higgs-Kibble theory in dimensional renormalization with a nonanticommuting γ_5

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The one-loop renormalization of the Abelian Higgs-Kibble model in a general 't Hooft gauge and with chiral fermions is fully worked out within dimensional renormalization scheme with a nonanticommuting γ_5. The anomalous terms introduced in the Slavnov-Taylor identities by the minimal subtraction algorithm are calculated and the asymmetric counterterms needed to restore the Becchi-Rouet-Stora symmetry, if the anomaly cancellation conditions are met, are computed. The computations draw heavily from regularized action principles and algebraic renormalization theory.

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I. INTRODUCTION

Because of the availability of high precision tests of the standard model in particle accelerators, it is mandatory to compute higher order quantum corrections and therefore to investigate thoroughly consistent and systematic renormalization schemes. Dimensional regularization [1,2] is the standard regularization method applied to particle physics. Both its axiomatic and properties were rigorously established long ago [3–5]. Its success as a practical regularization method stems from the fact that in vectorlike nonsupersymmetric gauge theories preserves enough properties so that the minimal subtraction (MS) scheme [6] leads to a renormalized gauge invariant theory [2]. But the electroweak interactions of the standard model are chiral and, unfortunately, in dimensional renormalization the algebraic properties of γ_5 cannot be maintained consistently as we move away from four dimensions [7]. Thus, in the so called "naive" prescription of dimensional renormalization (NDR), commonly used in multiloop computations in the standard model, one assumes $[\gamma_5,\gamma_\mu]=0$ and the cyclicity of the trace [8]. These assumptions have the consequence [3,5,7,9] that $\mathrm{Tr}[\gamma_{\mu_1}\cdots\gamma_{\mu_4}\gamma_5]$ is identically 0 in the dimensionally regularized theory ($d\neq 4$), which is incompatible with the property in four dimensions $\mathrm{Tr}[\gamma^{\mu_1}\cdots\gamma^{\mu_4}\gamma_5]=i\,\mathrm{Tr}\mathbb{I}\epsilon_{\mu_1\ldots\mu_4}$ in theories where a not null tensor ϵ is needed.

Therefore it seems unavoidable to find ambiguities as we use NDR to carry out computations in any chiral theory with fermionic loops with an odd number of γ_5's. However, it is commonly claimed that with usual manipulations the resulting ambiguities are proportional to the coefficient of the (chiral gauge) anomaly—null in the standard model—and the "naive" prescription is then thought to be safe [10]. Calculations to low orders in perturbation theory support this idea, but there is not a rigorous proof of it valid to all orders. Higher order computations within the SM have already reached a point where the possible inconsistencies of NDR cannot be simply forgotten. To avoid these algebraic inconsistencies it has been suggested not to assume the cyclicity of the trace [10,11], but in this case extra checks or proofs are needed. The only dimensional regularization scheme [sometimes called the Breitenlohner–Maison–'t Hooft–Veltman (BMHV) scheme] which is known rigorously [7] to be consistent in presence of γ_5 is the original one devised by 't Hooft and Veltman [2] and later systematized by Breitenlohner and Maison [3] following the definition of γ_5 in Ref. [12]. A similar scheme is considered in Ref. [13] and an extension has been recently proposed in Ref. [14].

In BMHV scheme, besides the "d-dimensional" metrics $g_{\mu\nu}$, a new one is introduced [3] $\hat{g}_{\mu\nu}$, which can be considered as a "$(d-4)$-dimensional covariant." Defining $\overline{g}^{\mu\nu}=g^{\mu\nu}-\hat{g}^{\mu\nu}$, $\overline{g}^{\mu\nu}$ can be thought of as a projector over the "four-dimensional space" and $\hat{g}^{\mu\nu}$ as a projector over the "$(d-4)$-dimensional" one. Moreover the ϵ tensor is considered to be a "four-dimensional covariant" object, because it is assumed to satisfy $\epsilon_{\mu_1\ldots\mu_4}\epsilon_{\nu_1\ldots\nu_4}=-\sum_{\pi\in S_4}\mathrm{sgn}\,\pi\prod_{i=1}^4\overline{g}_{\mu_i\nu_{\pi(i)}}$. With the definition $\gamma_5=(i/4!)\epsilon_{\mu_1\cdots\mu_4}\gamma^{\mu_1}\cdots\gamma^{\mu_4}$ and the *assumption of cyclicity* of the symbol Tr, it can be proved algebraically [3]:

$$\mathrm{Tr}[\gamma^{\mu_1}\cdots\gamma^{\mu_4}\gamma_5]=\mathrm{Tr}[\overline{\gamma}^{\mu_1}\cdots\overline{\gamma}^{\mu_4}\gamma_5]=i\,\mathrm{Tr}\mathbb{I}\epsilon_{\mu_1\cdots\mu_4};$$
$$\{\gamma_5,\gamma^\mu\}=\{\gamma_5,\hat{\gamma}^\mu\}=2\gamma_5\hat{\gamma}^\mu=2\hat{\gamma}^\mu\gamma_5,$$
$$\{\gamma_5,\overline{\gamma}^\mu\}=[\gamma_5,\hat{\gamma}^\mu]=0, \tag{1}$$

which is the same algebra of symbols than of the original one of 't Hooft and Veltman [2,5]. Notice that γ_5 no longer anticommutes with γ_μ as it does in NDR.

The four-dimensional projection of the minimal subtraction (MS) of the singular part of the dimensionally regularized Feynman diagrams, their divergences having been also minimally subtracted in advance, leads to a renormalized quantum field theory [3,4] that satisfies Hepp axioms [15] of renormalization theory, field equations, the action principles, and Zimmerman-Bonneau identities [3,4,16]. The reader can find a review in Ref. [17].

Not many computations have been done in the BMHV scheme. Most of them involve theories without a chiral gauge symmetry [18], where there is no room for the introduction of spurious anomalies but subtleties related with eva-

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nescent operators [5] appear, or quantities related with axial currents [12,19], where it gives correctly the essential axial current anomalies. Several practical computations, taking care of the fulfillment of Slavnov-Taylor identities (STIs), but involving only some restricted set of diagrams, have been done in the case of the standard model [20,21,22] or in supersymmetric QED [23]. In Ref. [22], the authors report a relevant finite difference between the results of NDR and BMHV in two-loop diagrams of the standard model containing triangle subdiagrams.

In the BMHV scheme the regularization breaks the gauge symmetry, and the MS procedure gives Green's functions which does not fulfill the STIs, essential ingredients to guarantee unitarity and gauge independence. The axioms of local relativistic quantum field theories allows for local ambiguities [15,24] to be removed by imposing renormalization conditions. The quantum action principle [25] tells us that any symmetry breaking term generated in the renormalization process is local at the lowest nonvanishing order, and the question here is whether it is possible to remove the breaking terms of the STIs by adding to the classical action appropriate local finite counterterms. (The removable breaking terms are called *spurious anomalies* [9].) Algebraic renormalization theory [26,27,28] establish elegantly how and when this process can be accomplished: when there are no obstructions in the form of (*essential*) anomalies. This imposes anomaly cancellation conditions on the field representations. These anomalies belong to the cohomology of the Slavnov-Taylor operator that governs the chiral gauge symmetry at the quantum level. On the other hand, nonphysical or spurious anomalies correspond to trivial objects in the cohomology.

Algebraic renormalization has been recently applied to theoretical studies of the standard model [29] and important progress towards practical uses of noninvariant regularization schemes [30,31] have been achieved. In Ref. [17] a systematic computation of the finite one-loop counterterms needed to restore the STI of non-Abelian gauge theories without scalar fields in BMHV dimensional renormalization was done. The modified action which gives gauge invariant results in MS of the BMHV scheme was explicitly given. Especially use of the rigorous identity (see Ref. [17] for notations and a deduction based on the action principles of dimensional regularization [3])

$$\begin{aligned}\mathcal{S}_d(\Gamma_{\mathrm{DR}}) &\equiv \int d^dx(s_d\varphi)\frac{\delta\Gamma_{\mathrm{DR}}}{\delta\varphi}+\frac{\delta\Gamma_{\mathrm{DR}}}{\delta K_\Phi}\frac{\delta\Gamma_{\mathrm{DR}}}{\delta\Phi}\\ &= s_dS_0\cdot\Gamma_{\mathrm{DR}}+s_dS_{ct}^{(n)}\cdot\Gamma_{\mathrm{DR}}\\ &\quad+\int d^dx\left[\frac{\delta S_{ct}^{(n)}}{\delta K_\Phi(x)}\cdot\Gamma_{\mathrm{DR}}\right]\frac{\delta\Gamma_{\mathrm{DR}}}{\delta\Phi(x)} \qquad (2)\end{aligned}$$

was made, which allowed at one loop level for a direct computation of the breaking in terms of finite parts of diagrams with an insertion of an evanescent operator, thus avoiding the evaluation of the left-hand side (LHS) of the STIs.

Multiloop extension of these techniques and the explicit form of the two-loop modified action for the standard model which would give Becchi-Rouet-Stora (BRS) invariant results with a MS procedure would be valuable. For reasons of simplicity and as a previous step, the present paper is devoted to a simpler model, the Abelian Higgs-Kibble model with chiral fermions [32,33,26,30] in a general gauge of the 't Hooft class. Compared with the models of Ref. [17], this model presents the new feature of spontaneous symmetry breaking, which makes more involved the restoration process of the (hidden) BRS symmetry. Moreover, it is free of IR problems and it is very easy to write in an exhaustive manner the list of the monomials needed, to all orders, in the restoration procedure of its BRS symmetry. This makes the model a perfect training ground for future work on the standard model and two-loop computations. Since algebraic renormalization is not well known to practitioners, we will show in detail each step of the algorithm.

The layout of this paper is as follows. In Sec. II we give the classical action, fields, and symmetry of the model. Section III is devoted to a quick reminder of the general theory of algebraic renormalization, suited to this model and which is independent of the order of the renormalization procedure. In Sec. IV the BMHV dimensional regularization of the model is presented. In Sec. V we use the techniques of Ref. [17] to compute the breaking at one loop. Of course, the correct form of the anomaly is thus obtained and in Sec. VI, following the lines of Sec. III, the finite counterterms needed to restore the BRS symmetry in the anomaly free case are finally computed. In Appendix A the explicit matrix form of the linearized Slavnov-Taylor operator, needed to do the computation, is given. Appendix B is devoted to a pedagogical explicit computation of the BRS cohomology of order one. In Appendix C the results for each diagram contributing to the one loop breaking are presented in full detail.

II. CLASSICAL ACTION

The *CP* symmetric four-dimensional classical action is the following:

$$\begin{aligned}S_{\mathrm{inv}}=\int d^4x\Big\{&-\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}+(D_\mu\phi^\dagger)(D^\mu\phi)+\mu^2\phi^\dagger\phi\\ &-\lambda(\phi^\dagger\phi)^2+\sum_{k\in I\cup J}\overline{\psi}_k[i\partial\!\!\!/+A\!\!\!/(e_{Lk}P_L+e_{Rk}P_R)]\psi_k\\ &-\sum_{i\in I}(\sqrt{2}f_i\phi\overline{\psi}_iP_R\psi_i+\sqrt{2}f_i\phi^\dagger\overline{\psi}_iP_L\psi_i)\\ &-\sum_{j\in J}(\sqrt{2}f_j\phi\overline{\psi}_jP_L\psi_j+\sqrt{2}f_j\phi^\dagger\overline{\psi}_jP_R\psi_j)\Big\}. \qquad (3)\end{aligned}$$

Here A_μ is an Abelian gauge field, ϕ a scalar complex field, ψ_i and ψ_j are families of Dirac fermion fields $F_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu$ and $D_\mu\phi\equiv(\partial_\mu-iA_\mu)\phi$. This action is invariant also under the Abelian gauge transformations $\delta A_\mu=\partial_\mu\omega$, $\delta\phi=i\omega\phi$, $\delta\psi_k=i\omega(e_{Lk}P_L+e_{Rk}P_R)\psi_k$ provided that the

TABLE I. Ghost number, dimension, conmmutativity, and *CP* transformation of fields, coordinates, and the BRS operator. In the third row, $+1$ (-1) means that the symbol commutes (anticommutes) and in last one, $D\equiv\gamma^0 C$, where C is the usual conjugation matrix. Note that the chosen *CP* properties of the ghosts make the BRS operator *CP* invariant.

	s	x_μ	$\phi_{1(2)}$	A_μ	ψ	c	$\bar{c}$	B	$K_{\phi_{1(2)}}$	K_ψ
Gh. No.	1	0	0	0	0	1	-1	0	-1	-1
Dimen.	0	-1	1	1	3/2	0	2	2	3	5/2
Comm.	-1	$+1$	$+1$	$+1$	-1	-1	-1	$+1$	-1	$+1$
CP	s	x^μ	$(-)\phi_{1(2)}$	$-A^\mu$	$D\bar{\psi}^t$	$-c$	$-\bar{c}$	$-B$	$(-)K_{\phi_{1(2)}}$	$-K^t_{\bar{\psi}}D^{-1}$

fermion charges satisfy[1]

$$e_{Li}=e_{Ri}+1 \quad \text{if } i\in I,$$
$$e_{Lj}=e_{Rj}-1 \quad \text{if } j\in J. \tag{4}$$

We shall consider spontaneous symmetry breaking due to a nonvanishing vacuum expectation value (VEV) of the real component of ϕ. Let us now write the classical BRS invariant action

$$S_{\rm cl}=S_{\rm inv}+S_{\rm GF}+S_{\rm ext}, \tag{5}$$

where

$$S_{\rm inv}=\int d^4x\Big\{-\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}+[(D_\mu\phi^+)(D^\mu\phi)+\mu^2\phi^+\phi$$
$$-\lambda(\phi^+\phi)^2]_{\phi=(1/\sqrt{2})(\phi_1+v+i\phi_2)}$$
$$+\bar{\psi}\{i\partial\!\!\!/+A\!\!\!/[(\theta+r)P_L+\theta P_R]\}\psi$$
$$-f[(v+\phi_1)\bar{\psi}\psi+ir\phi_2\bar{\psi}\gamma_5\psi]\Big\}$$

$$S_{\rm GF}=\int d^4x s\left[\frac{1}{2}\xi\bar{c}B+\bar{c}\Sigma\right]=\int d^4x\frac{\xi}{2}B^2+B(\partial_\mu A^\mu+\rho\phi_2)$$
$$-\bar{c}[\partial^\mu\partial_\mu+\rho(\phi_1+v)]c,$$

$$S_{\rm ext}=\int d^4x K_{\phi_1}s\phi_1+K_{\phi_2}s\phi_2+K_\psi s\psi+s\bar{\psi}K_{\bar{\psi}}. \tag{6}$$

In this action, v is a parameter with dimension of mass, ϕ_1 and ϕ_2 are real scalar fields, c is the ghost field, $\bar{c}$ the antighost field, B is the Lautrup-Nakanishi field, K_Φ are the external fields coupled to the corresponding BRS variations,[2] and $\Sigma=\partial_\mu A^\mu+\rho\phi_2$ is the most general gauge-fixing functional which is linear, preserves *CP* symmetry, has the appropriate ghost number, and is consistent with power counting (see Table I). ξ and ρ are the gauge parameters. We have omitted the index labeling the fermion families, a notation that we shall keep unless otherwise stated. We have set $e_{Rk}\equiv\theta_k$ and $e_{Lk}\equiv\theta_k+r_k$. Therefore, $\{r_k\}_{k\in I\cup J}$ are not free parameters but convenient shorthand for $+1$ if $k\in I$ and -1 if $k\in J$.

The BRS transformation is

$$s\phi_1=-\phi_2 c,\quad s\phi_2=(v+\phi_1)c,$$
$$s\psi=ic[(\theta+r)P_L+\theta P_R]\psi,$$
$$s\bar{\psi}=i\bar{\psi}[(\theta+r)P_R+\theta P_L]c,$$
$$sA_\mu=\partial_\mu c,\quad sc=0,\quad s\bar{c}=B,$$
$$sB=sK_{\phi_i}=sK_\psi=sK_{\bar{\psi}}=0, \tag{7}$$

which leaves $S_{\rm cl}$ invariant due to the anticommutativity of γ_5 in four dimensions.

If $\mu^2=\lambda v^2$ there are no linear terms in the classical action (5). This is equivalent to the renormalization condition which sets to zero the VEV of the field ϕ_1. The quadratic terms in Eq. (5) give the propagators, which are shown in Fig. 1 for $\mu^2=\lambda v^2$. Note that the propagators with B fields only contribute to the one particle irreducible (1PI) Feynman diagrams at the tree level and that no special value will be chosen for the ρ parameter; i.e., there will be $A\phi_2$ mixing at the tree level and beyond in the loop expansion. Notice also that although the theory is Abelian, the ghost fields are not free: they interact with ϕ_1 through the gauge-fixing part of the action. This makes the BRS-Tyutin (BRST) formalism very convenient to use.

III. ALGEBRAIC RENORMALIZATION

The classical action (5) is a solution, over the space of *CP*-invariant integrated polynomials of ghost number 0 and mass dimension 4, to the Slavnov-Taylor identity (STI)

[1] If the field redefinitions $A_\mu\to 2gA_\mu$, $\omega\to 2g\omega$ are done the model coincides with the model of Sec. IV A of Ref. [33], with ψ (ψ') being our fermions of type $r=+1$ $(r=-1)$ and their charge f our $(2e_R+1)g/2$ $[(2e_R-1)g/2]$.

[2] "Antifields" or sources to the trivial BRS variations of A_μ and c could also have easily been introduced. This would make more clear the study and interpretation of the cohomology of $\tilde{b}$ at order 0, but they are neither relevant nor necessary for the practical purposes we pursue in this paper.

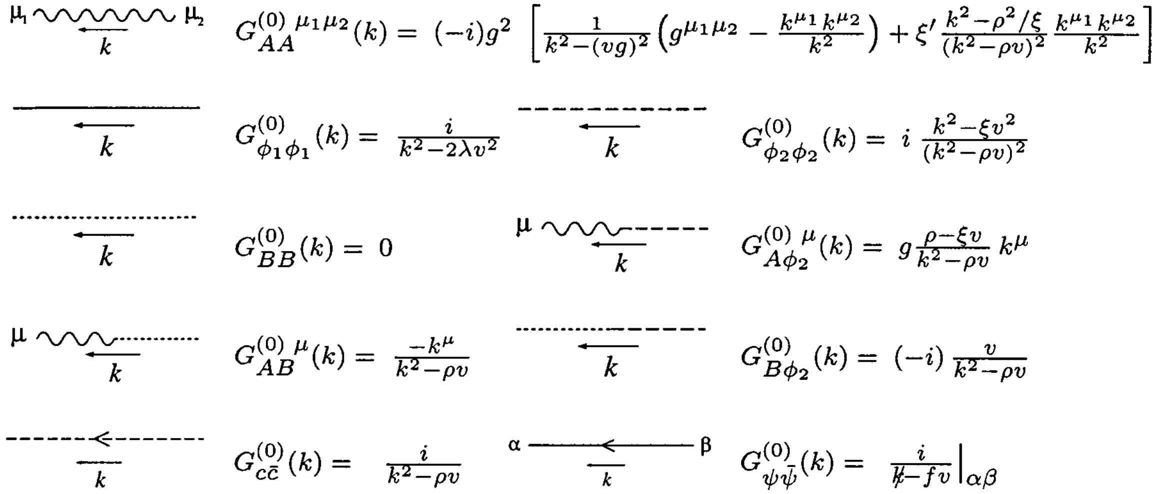


FIG. 1. Free field propagators if $\mu^2=\lambda v^2$. For convenience, the following abbreviation has been defined: $\xi'=\xi/g^2$. Momentum k flows from the second to the first field of Green functions. μ, μ_1, and μ_2 are Lorentz indices. α and β are the indices of Dirac matrices and will be ommitted.

$$\int d^4x\left\{(\partial_\mu c)\frac{\delta S_{\rm cl}}{\delta A_\mu}+B\frac{\delta S_{\rm cl}}{\delta\bar{c}}+\frac{\delta S_{\rm cl}}{\delta K_{\phi_1}}\frac{\delta S_{\rm cl}}{\delta\phi_1}+\frac{\delta S_{\rm cl}}{\delta K_{\phi_2}}\frac{\delta S_{\rm cl}}{\delta\phi_2}+\frac{\delta S_{\rm cl}}{\delta K_\psi}\frac{\delta S_{\rm cl}}{\delta\psi}+\frac{\delta S_{\rm cl}}{\delta K_{\bar\psi}}\frac{\delta S_{\rm cl}}{\delta\bar\psi}\right\}=0, \tag{8}$$

which rules the BRS invariance of the theory, and to the gauge-fixing equation

$$\frac{\delta S_{\rm cl}}{\delta B}=\xi B+\partial_\mu A^\mu+\rho\phi_2, \tag{9}$$

which is the equation of motion of the Lagrange multiplier field B.

Any functional $\mathcal{F}$ satisfying both equations, also satisfies the ghost equation

$$\frac{\delta\mathcal{F}}{\delta\bar{c}}+\Box c+\rho\frac{\delta\mathcal{F}}{\delta K_{\phi_2}}=0. \tag{10}$$

After renormalization of the perturbative expansion, the 1PI generating functional Γ will be required to be, in the sense of a formal series of functionals in $\hbar$, a deformation of $S_{\rm cl}$ constrained by the same equations. If the renormalization procedure respects both the gauge-fixing and ghost equations, Γ will have the form

$$\begin{aligned}\Gamma[\phi_1,\phi_2,A_\mu,\psi,\bar\psi,c,\bar c,B,K_{\phi_1},K_{\phi_2},K_\psi,K_{\bar\psi}]&=\int d^4x\,\frac{\xi}{2}B^2+B(\partial_\mu A^\mu+\rho\phi_2)-\bar c\Box c\\&\quad+\tilde\Gamma[\phi_1,\phi_2,A_\mu,\psi,\bar\psi,c,K_{\phi_1},\tilde K_{\phi_2}\\&\quad\equiv K_{\phi_2}-\rho\bar c,K_\psi,K_{\bar\psi}],\end{aligned} \tag{11}$$

so that the left hand side of the STI will read

$$\tilde{\mathcal{S}}(\tilde\Gamma)\equiv\int d^4x\left\{(\partial_\mu c)\frac{\delta\tilde\Gamma}{\delta A_\mu}+\frac{\delta\tilde\Gamma}{\delta K_{\phi_1}}\frac{\delta\tilde\Gamma}{\delta\phi_1}+\frac{\delta\tilde\Gamma}{\delta\tilde K_{\phi_2}}\frac{\delta\tilde\Gamma}{\delta\phi_2}+\frac{\delta\tilde\Gamma}{\delta K_\psi}\frac{\delta\tilde\Gamma}{\delta\psi}+\frac{\delta\tilde\Gamma}{\delta K_{\bar\psi}}\frac{\delta\tilde\Gamma}{\delta\bar\psi}\right\}. \tag{12}$$

From now, a tilde will indicate dependence on the same fields as $\tilde\Gamma$ does.

Let us introduce the linearized Slavnov-Taylor operator

$$\begin{aligned}\tilde{\mathcal{S}}_{\tilde{\mathcal{F}}}\equiv\int d^4x\Bigg\{&(\partial_\mu c)\frac{\delta}{\delta A_\mu}+\frac{\delta\tilde{\mathcal{F}}}{\delta\phi_1}\frac{\delta}{\delta K_{\phi_1}}+\frac{\delta\tilde{\mathcal{F}}}{\delta K_{\phi_1}}\frac{\delta}{\delta\phi_1}\\&+\frac{\delta\tilde{\mathcal{F}}}{\delta\phi_2}\frac{\delta}{\delta\tilde K_{\phi_2}}+\frac{\delta\tilde{\mathcal{F}}}{\delta\tilde K_{\phi_2}}\frac{\delta}{\delta\phi_2}+\frac{\delta\tilde{\mathcal{F}}}{\delta\psi}\frac{\delta}{\delta K_\psi}+\frac{\delta\tilde{\mathcal{F}}}{\delta K_\psi}\frac{\delta}{\delta\psi}\\&+\frac{\delta\tilde{\mathcal{F}}}{\delta\bar\psi}\frac{\delta}{\delta K_{\bar\psi}}+\frac{\delta\tilde{\mathcal{F}}}{\delta K_{\bar\psi}}\frac{\delta}{\delta\bar\psi}\Bigg\},\end{aligned} \tag{13}$$

which has the nilpotency properties

$$\begin{aligned}&\tilde{\mathcal{S}}_{\tilde{\mathcal{F}}}\tilde{\mathcal{S}}(\tilde{\mathcal{F}})=0,\ \ \forall\tilde{\mathcal{F}},\\&\tilde{\mathcal{S}}_{\tilde{\mathcal{F}}}\tilde{\mathcal{S}}_{\tilde{\mathcal{F}}}=0,\ \ \text{if}\ \ \tilde{\mathcal{S}}(\tilde{\mathcal{F}})=0.\end{aligned} \tag{14}$$

For $\tilde{\mathcal{F}}$ equal to the classical action $\tilde S_{\rm cl}$ we have the important linear operator

$$\tilde b\equiv\tilde{\mathcal{S}}_{\tilde S_{\rm cl}}. \tag{15}$$

The local part of maximal dimension four of $\tilde\Gamma$ at a determined order in the perturbative expansion and prior to imposition of the STI is an arbitrary linear combination of the following basis of the space $\tilde{\mathcal{V}}_0$ of the integrated Lorentz scalar *CP*-invariant polynomials in the fields and their derivatives with maximal canonical dimension 4 and ghost number 0 (note that we choose the same first twenty monomials as in Ref. [26]):

$$\tilde{e}_1 \equiv \int \phi_1, \;\; \tilde{e}_2 \equiv \int {\phi_1}^2, \;\; \tilde{e}_3 \equiv \int {\phi_2}^2,$$

$$\tilde{e}_4 \equiv \int {\phi_1}^3, \;\; \tilde{e}_5 \equiv \int \phi_1 {\phi_2}^2, \;\; \tilde{e}_6 \equiv \int {\phi_1}^4,$$

$$\tilde{e}_7 \equiv \int \phi_2^4, \;\; \tilde{e}_8 \equiv \int {\phi_1}^2 {\phi_2}^2, \;\; \tilde{e}_9 \equiv \int (\partial_\mu \phi_1)(\partial^\mu \phi_1),$$

$$\tilde{e}_{10} \equiv \int (\partial_\mu \phi_2)(\partial^\mu \phi_2), \;\; \tilde{e}_{11} \equiv \int \phi_2(\partial_\mu A^\mu),$$

$$\tilde{e}_{12} \equiv \int A_\mu \phi_1 (\partial^\mu \phi_2),$$

$$\tilde{e}_{13} \equiv \int A_\mu \phi_2 (\partial^\mu \phi_1), \;\; \tilde{e}_{14} \equiv \int A_\mu A^\mu, \;\; \tilde{e}_{15} \equiv \int A_\mu A^\mu \phi_1,$$

$$\tilde{e}_{16} \equiv \int A_\mu A^\mu {\phi_1}^2, \;\; \tilde{e}_{17} \equiv \int A_\mu A^\mu {\phi_2}^2, \;\; \tilde{e}_{18} \equiv \int (\partial_\mu A^\mu)^2,$$

$$\tilde{e}_{19} \equiv \int (\partial_\mu A_\nu - \partial_\nu A_\mu)^2, \;\; \tilde{e}_{20} \equiv \int (A_\mu A^\mu)^2,$$

$$\tilde{e}_{21} \equiv \int K_{\phi_1} \phi_2 c, \quad \tilde{e}_{22} \equiv \int \tilde{K}_{\phi_2} c, \;\; \tilde{e}_{23} \equiv \int \tilde{K}_{\phi_2} \phi_1 c,$$

$$\tilde{e}_{24} \equiv \int \bar{\psi}\psi,$$

$$\tilde{e}_{25} \equiv \int \bar{\psi} i \not{\partial} P_L \psi, \;\; \tilde{e}_{26} \equiv \int \bar{\psi} i \not{\partial} P_R \psi,$$

$$\tilde{e}_{27} \equiv \int \bar{\psi} \not{A} P_L \psi, \;\; \tilde{e}_{28} \equiv \int \bar{\psi} \not{A} P_R \psi,$$

$$\tilde{e}_{29} \equiv \int \phi_1 \bar{\psi}\psi = \phi_1 \bar{\psi} P_R \psi + \phi_1 \bar{\psi} P_L \psi,$$

$$\tilde{e}_{30} \equiv \int \phi_2 \bar{\psi} \gamma_5 \psi = \phi_2 \bar{\psi} P_R \psi - \phi_2 \bar{\psi} P_L \psi,$$

$$\tilde{e}_{31} \equiv \int (K_\psi P_L \psi - \bar{\psi} P_R K_{\bar{\psi}}) c,$$

$$\tilde{e}_{32} \equiv \int (K_\psi P_R \psi - \bar{\psi} P_L K_{\bar{\psi}}) c. \tag{16}$$

This means that we have the freedom to add to the starting action any actionlike term of the form $\tilde{X} = \Sigma_{i=1}^{32} \tilde{x}^i \tilde{e}_i$, each $\tilde{x}^i$ being a formal series in $\hbar$ of order $O(\hbar)$.

In a noninvariant renormalization scheme, the renormalized STI will have a breaking

$$\tilde{\mathcal{S}}(\tilde{\Gamma}) = \tilde{\Delta} \cdot \tilde{\Gamma}, \tag{17}$$

the right-hand side (RHS) of last equation being the insertion of a *CP*-invariant integrated local operator of maximal dimension 4 and ghost number 1.

Now, in accordance with the algebraic theory of renormalization and supposing that the breaking vanish at lower orders of the perturbative expansion, the insertion at order n is simply a local integrated polynomial $\tilde{\Delta}$. $\tilde{\Gamma} = \hbar^n \tilde{\Delta}^{(n)} + O(\hbar^{n+1})$ which can be decomposed as a linear combination of the following basis of the space $\tilde{\mathcal{V}}_1$ of *CP* invariant actionlike polynomials of maximal dimension 4 and ghost number 1 (again, we choose a basis following the strategy in Ref. [26]):

$$\tilde{u}_1 \equiv \int \phi_2 c, \;\; \tilde{u}_2 \equiv \int \phi_1 \phi_2 c, \;\; \tilde{u}_3 \equiv \int {\phi_2}^3 c,$$

$$\tilde{u}_4 \equiv \int {\phi_1}^2 \phi_2 c, \;\; \tilde{u}_5 \equiv \int (\Box \phi_2) c, \;\; \tilde{u}_6 \equiv \int {\phi_1}^3 \phi_2 c,$$

$$\tilde{u}_7 \equiv \int \phi_1 {\phi_2}^3 c, \;\; \tilde{u}_8 \equiv \int \phi_2 (\Box \phi_1) c, \;\; \tilde{u}_9 \equiv \int \phi_1 (\Box \phi_2) c,$$

$$\tilde{u}_{10} \equiv \int (\partial_\mu \phi_1)(\partial^\mu \phi_2) c, \;\; \tilde{u}_{11} \equiv \int (\partial_\mu A^\mu) c,$$

$$\tilde{u}_{12} \equiv \int A_\mu (\partial^\mu \phi_1) c,$$

$$\tilde{u}_{13} \equiv \int (\partial_\mu A^\mu) \phi_1 c, \;\; \tilde{u}_{14} \equiv \int (\partial_\mu A^\mu) {\phi_1}^2 c,$$

$$\tilde{u}_{15} \equiv \int A_\mu \phi_1 (\partial^\mu \phi_1) c,$$

$$\tilde{u}_{16} \equiv \int (\partial_\mu A^\mu) {\phi_2}^2 c, \;\; \tilde{u}_{17} \equiv \int A_\mu \phi_2 (\partial^\mu \phi_2) c,$$

$$\tilde{u}_{18} \equiv \int \Box (\partial_\mu A^\mu) c,$$

$$\tilde{u}_{19} \equiv \int A_\mu A_\nu (\partial^\mu A^\nu) c, \;\; \tilde{u}_{20} \equiv \int A_\mu A^\nu \phi_2 c,$$

$$\tilde{u}_{21} \equiv \int A_\mu A^\mu \phi_1 \phi_2 c,$$

$$\tilde{u}_{22} \equiv \int A_\mu A^\mu (\partial^\nu A^\nu) c,$$

$$\tilde{u}_{23} \equiv \int \bar{\psi} \gamma_5 \psi c, \;\; \tilde{u}_{24} \equiv \int \partial_\mu (\bar{\psi} \gamma^\mu P_L \psi) c,$$

$$\tilde{u}_{25} \equiv \int \partial_\mu (\overline{\psi}\gamma^\mu P_R \psi) c,$$

$$\tilde{u}_{26} \equiv \int \phi_2 \overline{\psi}\psi c, \quad \tilde{u}_{27} \equiv \int \phi_1 \overline{\psi}\gamma_5 \psi c,$$

$$\tilde{u}_{28} \equiv \int \varepsilon_{\mu_1\mu_2\mu_3\mu_4} (\partial^{\mu_1} A^{\mu_2})(\partial^{\mu_3} A^{\mu_4}) c, \tag{18}$$

that is, $\tilde{\Delta}^{(n)} = \Sigma_{j=1}^{28} \tilde{\Delta}^{j(n)} \tilde{u}_j$. Notice that, incidentally, external fields do not appear in the basis due to power counting and the property of the Abelian ghosts $cc=0$.

The projection of the breaking over the direction of the last element of the basis constitutes the anomaly of the theory, for it can be shown that the linear system $\tilde{b}\tilde{X}^{(n)} = \tilde{\Delta}^{(n)}$ has an (underdetermined) solution if and only if the coefficient $\tilde{\Delta}^{28(n)}$ vanishes. This can be established by using cohomological methods: the first expression in Eq. (14) leads to $\tilde{b}\tilde{\Delta}=0$, which is the famous consistency condition, and, the second expression in Eq. (14) implies $\tilde{b}^2=0$; hence, $\tilde{\Delta} \equiv \text{anomaly} + \tilde{b}\tilde{X}$, the anomaly belonging to the ghost number one nontrivial cohomology space of the $\tilde{b}$ operator. See Appendix B for a proof of this statement by explicit computation.

Now, let us define the matrix elements of the operator $\tilde{b}$ restricted to its action from $\tilde{\mathcal{V}}_0$ to $\tilde{\mathcal{V}}_1$ in the following manner $\tilde{b}\tilde{e}_i \equiv \tilde{b}_{0\,i}{}^j u_j$ [explicit values of these matrix elements for $S_{\rm cl}$ in Eq. (5) are given in Appendix A]. Then, the linear system

$$\sum_{j=1}^{27} \tilde{b}_{0\ i}{}^{j} \tilde{x}^{i(n)} = \tilde{\Delta}^{j(n)}, \quad i=1,\ldots,32 \tag{19}$$

always has a solution up to an $O(\hbar^n)$ arbitrary linear combination of $\tilde{b}$ invariants. These $\tilde{b}$ invariants are any basis of the kernel $\tilde{\mathcal{K}}_0$ of the restricted linear operator $\tilde{b}_0 \equiv \tilde{b}$: $\tilde{\mathcal{V}}_0 \to \tilde{\mathcal{V}}_1$. We can choose for example the following basis of $\tilde{\mathcal{K}}_0$ for all values of θ:

$$\tilde{\mathcal{I}}_1 = -\frac{1}{4}\tilde{e}_{19} = -\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}, \tag{20}$$

$$\tilde{\mathcal{I}}_2 = v\tilde{e}_1 + \frac{\tilde{e}_2+\tilde{e}_3}{2} = \int \phi^\dagger \phi,$$

$$\tilde{\mathcal{I}}_3 = v^3\tilde{e}_1 + \frac{3v^2}{2}\tilde{e}_2 + \frac{v^2}{2}\tilde{e}_3 + v(\tilde{e}_4+\tilde{e}_5) + \frac{\tilde{e}_6+\tilde{e}_7}{4} + \frac{\tilde{e}_8}{2} = \int (\phi^\dagger\phi)^2,$$

$$\tilde{\mathcal{I}}_4 = v\tilde{e}_{24} + \tilde{e}_{29} + ir\tilde{e}_{30} = \int v\overline{\psi}\psi + \phi_1\overline{\psi}\psi + ir\phi_2\overline{\psi}\gamma_5\psi,$$

$$\tilde{\mathcal{I}}_5 = (\mu^2 - 3\lambda v^2)\tilde{e}_1 - 3\lambda v\tilde{e}_2 - \lambda v\tilde{e}_3 - \lambda v\tilde{e}_3 - \lambda(\tilde{e}_4+\tilde{e}_5) + \tilde{e}_{11} + v\tilde{e}_{14} + \tilde{e}_{15} + \tilde{e}_{22} - f\tilde{e}_{24} = \tilde{b}\int K_{\phi_1},$$

$$\tilde{\mathcal{I}}_6 = (\mu^2 - \lambda v^2)v\tilde{e}_1 + (\mu^2 - 3\lambda v^2)\tilde{e}_2 - 3\lambda v\tilde{e}_4 - \lambda v\tilde{e}_5 - \lambda(\tilde{e}_6+\tilde{e}_8) + \tilde{e}_9 - \tilde{e}_{12} + \tilde{e}_{13} + v\tilde{e}_{15} + \tilde{e}_{16} + \tilde{e}_{21} + \tilde{e}_{23} - f\tilde{e}_{29} = \tilde{b}\int K_{\phi_1}\phi_1,$$

$$\tilde{\mathcal{I}}_7 = (\mu^2 - \lambda v^2)\tilde{e}_3 - 2\lambda v\tilde{e}_5 - \lambda(\tilde{e}_7+\tilde{e}_8) + \tilde{e}_{10} + v\tilde{e}_{11} - \tilde{e}_{12} + \tilde{e}_{13} + \tilde{e}_{17} - \tilde{e}_{21} - v\tilde{e}_{22} - \tilde{e}_{23} - irf\tilde{e}_{30} = \tilde{b}\int K_{\phi_2}\phi_2,$$

$$\tilde{\mathcal{I}}_8 = -2\tilde{e}_{25} - 2(r+\theta)\tilde{e}_{27} + vf\tilde{e}_{24} + f\tilde{e}_{29} + irf\tilde{e}_{30} = \tilde{b}\int (K_\psi P_L\psi + \overline{\psi}P_R K_{\overline{\psi}}),$$

$$\tilde{\mathcal{I}}_9 = -2\tilde{e}_{26} - 2\theta\tilde{e}_{28} + vf\tilde{e}_{24} + f\tilde{e}_{29} + irf\tilde{e}_{30} = \tilde{b}\int (K_\psi P_R\psi + \overline{\psi}P_L K_{\overline{\psi}}),$$

$$\tilde{\mathcal{I}}_{10} = \theta(\tilde{e}_9 + \tilde{e}_{10} + v\tilde{e}_{11} - \tilde{e}_{12} + \tilde{e}_{13} + \tilde{e}_{21} - v\tilde{e}_{22} - \tilde{e}_{23}) - r\tilde{e}_{26} - ir\theta\tilde{e}_{32},$$

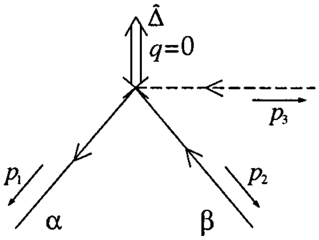
$$\tilde{\mathcal{I}}_{11} = (\theta+r)(\tilde{e}_9 + \tilde{e}_{10} + v\tilde{e}_{11} - \tilde{e}_{12} + \tilde{e}_{13} + \tilde{e}_{21} - v\tilde{e}_{22} - \tilde{e}_{23}) + r\tilde{e}_{25} + ir(\theta+r)\tilde{e}_{31},$$

which is fixed by choosing an appropriate set of normalization conditions.

Notice that the rank of $\tilde{b}_0$ is dim of $\tilde{\mathcal{V}}_0 -$ dim of $\tilde{\mathcal{K}}_0 = 21 <$ dim of $\tilde{\mathcal{V}}_1$, so that, in general, for arbitrary values of the breaking, the system (19) would be an incompatible one. Then its compatibility when substituting in it the values of the breaking obtained by explicit computation turns to be a nontrivial check of the correctness of the computation itself. Finally, if there is no anomaly, the breaking at order n only consists of cohomologically trivial terms and, thus by adding $\hbar^n\tilde{S}_{\rm fct}^{(n)} = -\hbar^n\tilde{X}^{(n)}$ to the previous action the breaking is canceled at the order n.

IV. DIMENSIONALLY REGULARIZED ACTION

If we want the regularized action principle of Breitenlohner and Maison to be applicable, we must define the regularized kinetic terms in the same form as the four-dimensional ones. The regularized kinetic terms are thus uniquely defined. Not so the interaction terms. For instance, the Dirac matrix part of the fermion-gauge-boson vertex has



$$\equiv \tilde\Gamma^{(0)}_{\psi\bar\psi c;\hat\Delta\ \alpha\beta}(p_1,p_2;q\equiv 0) =$$
$$= \frac{i}{2}\left[(r+2\theta)(\hat{\not p}_1+\hat{\not p}_2) - r(\hat{\not p}_1-\hat{\not p}_2)\gamma_5\right]\Big|_{\alpha\beta}$$

FIG. 2. Feynman rule of the insertion of the integrated breaking Eq. (23).

the following equivalent forms in 4 dimensions: $\gamma^\mu P_L = P_R\gamma^\mu = P_R\gamma^\mu P_L$. But these forms are not equal in the *d*-dimensional space-time of dimensional regularization because of the nonanticommutativity of γ_5. Of course, the generalization of the interaction to the dimensional regularization space is not unique, and any choice is *equally correct*. And yet, some choices will be more convenient than others.

In the case at hand it would be far more convenient to use a dimensionally regularized action which has the discrete simmetries of the four-dimensional classical action. Indeed, if the dimensionally regularized action were not *CP* invariant, we would have to enlarge the basis of the relevant spaces presented in Sec. III with *CP*-noninvariant monomials. This would make the computations very lengthy. Even with the restriction of *CP* symmetry the regularized action is not unique: there is always the freedom of adding explicit evanescent operators, i.e., proportional to $d-4$. Here we shall adopt the simplest choice available and generalize in the obvious way to *d*-dimensional space-time the BRS variations and vertices of the action (5), "barring" the boson-fermion vertex. For the latter vertex, we shall use the following regularized form:

$$\overline{\psi}[i\not\partial + A_\mu(e_L\overline{\gamma}^\mu P_L + e_R\overline{\gamma}^\mu P_R)]\psi, \tag{21}$$

i.e., the *CP*-invariant or "Hermitian" regularized form, which can be cast in the following nongauge invariant expression:

$$i\overline{\psi\not D}\psi + i\overline{\psi}\hat{\not\partial}\psi. \tag{22}$$

Hence, the regularized action S_0 we shall start with will not be BRS invariant. The regularized breaking s_dS_0, coming from the last term of Eq. (22), will read, thus,

$$\begin{aligned} s_dS_0 &= s_d\int d^dx\, i\overline{\psi}\hat\gamma^\mu\partial_\mu\psi \\ &= \int d^dx\,\frac{1}{2}c\{(r+2\theta)\partial_\mu(\overline{\psi}\hat\gamma^\mu\psi) + r(\overline{\psi}\hat\gamma^\mu\gamma_5\overleftrightarrow{\partial}_\mu\psi)\} \\ &\equiv \hat\Delta \equiv \int d^dx\,\hat\Delta(x). \end{aligned} \tag{23}$$

The Feynman rule of the insertion of this anomalous breaking is given in Fig. 2.

The breaking is an (implicit) "$(d-4)$ object," i.e., an evanescent operator or an operator which vanishes in the four-dimensional projection, and, clearly, this would be also true for any other dimensional regularization classical action we might have chosen. Other choices of regularized vertices can lead upon minimal subtraction to other values for renormalized Green functions; each value corresponding to a renormalization scheme. Hence, the different values we spoke of should be related by finite counterterms.

In order not to deal with cumbersome propagators we set $\mu^2 = \lambda v^2$ in the regularized classical. Notice that for this choice of μ^2 the starting action can be interpreted as the usual spontaneous symmetry breaking action.

V. ONE LOOP STI BREAKING IN BMHV DIMENSIONAL RENORMALIZATION

By using the action principles of Breitenlohner and Maison—which basically state that the usual formal manipulations of path integrals are allowed in the dimensionally regularized theory—it can be shown that the equation of motion holds in dimensional regularization and renormalization [3,4,5]. Therefore, the gauge fixing (9) and ghost (10) equations holds for both the regularized and MS renormalized 1PI generating functional Γ if the dimensionally regularized action of previous section is used, or if it is modified by the addition of terms independent of *B* and depending on $\overline{c}$ and K_{ϕ_2} only through the combination $\widetilde{K}_{\phi_2} = K_{\phi_2} - \rho\overline{c}$. Hence, we will restrict the possible finite counterterms of the regularized action to live in the space $\widetilde{\mathcal{V}}_0$ whose basis was given in Eq. (16). Of course, as in the previous section, we have several possible "*d*-dimensional" generalizations of a given four-dimensional finite counterterm. Two such generalizations will differ in a "*d*-dimensional" integrated evanescent operator of order $\hbar$, which modifies the value of finite four-dimensional quantities only at order $\hbar^2$. We choose the generalizations whose forms in the "*d*-dimensional" algebra of covariants are exactly the same as in Eq. (16).

In Ref. [17], again by invoking the action principles, the identity (2) was derived, and it was proved using it that at the one-loop level the breaking of the renormalized STI—its RHS—simplifies to

$$\widetilde{\Delta}^{(1)} = [N[\hat\Delta]\cdot\Gamma^R]^{(1)} + b\widetilde{S}^{(1)}_{\rm fct}, \tag{24}$$

where $N[\hat\Delta]\cdot\Gamma_R$ denotes the insertion of a normal product as defined in Refs. [4,5]: the minimally subtracted generating functional of diagrams with an insertion of the regularized operator $\hat\Delta$. Notice that after algebraic manipulations of the Feynman integrand of these diagrams with an insertion of an evanescent operator, an explicit $d-4$ factor can appear in the numerators to be canceled with the $d-4$ coming from the divergence of denominators, giving thus a local renormalized value, as expected from the general algebraic renormalization theory.

We could, of course, compute the breaking by evaluating the relevant zero and one-loop 1PI functions, inserting them in the LHS of STIs and working out the functional derivatives. But, it is clearly more efficient to compute the breaking directly using Eq. (24).

With the aid of the Bonneau identities of Ref. [4], the anomalous normal product (24), i.e., a normal product of an evanescent operator, can be decomposed in terms of some basis of standard normal products, i.e., normal products of nonevanescent operators. See Refs. [4,17] for examples. But at lowest order, this technique reads practically the same as the direct computation of the one-loop finite part of $N[\hat{\Delta}]\cdot\Gamma^R$: (i) compute the finite part of all divergent by power counting 1PI diagrams with and insertion of $\hat{\Delta}$ and any quantum or BRS external field as legs; (ii) set $\overline{g}^{\mu\nu}$ to $g^{\mu\nu}$ and $\hat{g}^{\mu\nu}$ to zero, i.e., set to zero every hatted object; (iii) find finite four-dimensional integrated operators such as the Feynman rules of its tree-level insertions match the results of (ii).

We have carried out the procedure spelled out above in a completely automatic manner by using our own MATHEMATICA™ [34] routines and the MATHEMATICA package "TRACER" [35], which manages properly and carefully the BMHV γ's algebra. The input of the programs consists of the definition of Feynman rules and the expression of the diagrams in terms of symbolic Feynman rules. For the dimensionally regularized action of Sec. IV, the one-loop contributions to the 1PI functions with a breaking insertion reads[3] [results after step (ii) for each relevant diagram are shown in Appendix C]:

$$\tilde{\Gamma}^{R(1)\mu_1}_{Ac;N[\hat{\Delta}]}(k_1)=\frac{(-6f^2v^2+k_1{}^2)k_1{}^{\mu_1}}{3}, \tag{25}$$

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AAc;N[\hat{\Delta}]}(k_1,k_2)=\frac{-4i}{3}(3\theta+r+3\theta^2 r)\epsilon(k_1,k_2\{\mu_1\},\{\mu_2\}),$$

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2\mu_3}_{AAAc;N[\hat{\Delta}]}(k_1,k_2,k_3)=\frac{2(k_1{}^{\mu_3}+k_2{}^{\mu_3}+k_3{}^{\mu_3})g^{\mu_1\mu_2}}{3}+\frac{2(k_1{}^{\mu_2}+k_2{}^{\mu_2}+k_3{}^{\mu_2})g^{\mu_1\mu_3}}{3}+\frac{2(k_1{}^{\mu_1}+k_2{}^{\mu_1}+k_3{}^{\mu_1})g^{\mu_2\mu_3}}{3},$$

$$\tilde{\Gamma}^{R(1)}_{\phi_2c;N[\hat{\Delta}]}(k_1)=\frac{4i}{3}f^2v(6f^2v^2-k_1{}^2),$$

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_2c;N[\hat{\Delta}]}(k_1,k_2)=\frac{4i}{3}f^2(18f^2v^2-3k_1{}^2-3k_1{}^2-3k_1\cdot k_2-k_2{}^2),$$

$$\tilde{\Gamma}^{R(1)}_{\phi_2\phi_2\phi_2c;N[\hat{\Delta}]}(k_1,k_2,k_3)=16if^4v,$$

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_1\phi_2c;N[\hat{\Delta}]}(k_1,k_2,k_3)=48if^4v,$$

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_1\phi_1\phi_2c;N[\hat{\Delta}]}(k_1,k_2,k_3,k_4)=48if^4,$$

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_2\phi_2\phi_2c;N[\hat{\Delta}]}(k_1,k_2,k_3,k_4)=16if^4,$$

$$\tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1c;N[\hat{\Delta}]}(k_1,k_2)=-4f^2vk_1{}^{\mu_1},$$

$$\tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1\phi_1c;N[\hat{\Delta}]}(k_1,k_2,k_3)=-4f^2k_1{}^{\mu_1},$$

$$\tilde{\Gamma}^{R(1)\mu_1}_{A\phi_2\phi_2c;N[\hat{\Delta}]}(k_1,k_2,k_3)=-4f^2[k_1{}^{\mu_1}+2(k_2{}^{\mu_1}+k_3{}^{\mu_1})],$$

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AA\phi_2c;N[\hat{\Delta}]}(k_1,k_2,k_3)=-8if^2vg^{\mu_1\mu_2},$$

[3] In order to avoid any typesetting mistake, the following results have been automatically inserted from the TEX output of the programs. The code of the programs which do all computations and generate the TEX output can be found at arXiv:hep-th in the source format of this preprint.

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AA\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3,k_4) = -8if^2 g^{\mu_1\mu_2},$$

$$\tilde{\Gamma}^{R(\underline{1})}_{\psi\psi c;N[\hat{\Delta}]}(k_1,k_2) = \frac{-\{f[3\rho r+4g^2\theta(1+\theta r)v(5+\xi')]\gamma_5\}}{6} - \frac{g^2(2\theta+r)(5+\xi')\not{k}_1}{12} - \frac{g^2(2\theta+r)(5+\xi')\not{k}_2}{12} + \frac{[-12f^2r+g^2(2\theta+r+2\theta^2 r)(5+\xi')]\not{k}_1\gamma_5}{12} + \frac{[-12f^2r+g^2(2\theta+r+2\theta^2 r)(5+\xi')]\not{k}_2\gamma_5}{12},$$

$$\tilde{\Gamma}^{R(\underline{1})}_{\psi\psi\phi_1 c;N[\hat{\Delta}]}(k_1,k_2,k_3) = \frac{-2fg^2\theta(1+\theta r)(5+\xi')\gamma_5}{3},$$

$$\tilde{\Gamma}^{R(\underline{1})}_{\psi\psi\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3) = \frac{2i}{3}fg^2\theta(\theta+r)(5+\xi')\mathbb{I},$$

where $\epsilon(k_1,k_2,\{\mu_1\},\{\mu_2\}) \equiv \epsilon_{\alpha\beta\mu_1\mu_2}k_1{}^{\alpha}k_2{}^{\beta}$ and $\mathbb{I}$ is the unit of the spinor space. Note that no significant simplification is achieved by using the standard choice of R_ξ gauge $\rho \equiv \xi v$.

Then, the coefficients $\tilde{\Delta}_j^{(1)}$ of the breaking in the basis (18) of four-dimensional integrated operators can be automatically obtained with the aid of the formulas [step (iii)]

$$\tilde{\Delta}_1^{(1)} = \tilde{\Gamma}^{R(1)}_{\phi_2 c;N[\hat{\Delta}]}(k_1\equiv 0), \quad \tilde{\Delta}_2^{(1)} = \tilde{\Gamma}^{R(1)}_{\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1\equiv 0, k_2\equiv 0),$$

$$\tilde{\Delta}_3^{(1)} = \frac{1}{3!}\tilde{\Gamma}^{R(1)}_{\phi_2{}^3 c;N[\hat{\Delta}]}, \quad \tilde{\Delta}_4^{(1)} = \frac{1}{2!}\tilde{\Gamma}^{R(1)}_{\phi_1{}^2\phi_2 c;N[\hat{\Delta}]},$$

$$\tilde{\Delta}_5^{(1)} = -\text{coeff. of } k_1{}^2 \text{ in } \tilde{\Gamma}_{\phi_2 c;N[\hat{\Delta}]}{}^{R(1)}(k_1), \quad \tilde{\Delta}_6^{(1)} = \frac{1}{3!}\tilde{\Gamma}^{R(1)}_{\phi_1{}^3 c;N[\hat{\Delta}]},$$

$$\tilde{\Delta}_7^{(1)} = \frac{1}{3!}\tilde{\Gamma}^{R(1)}_{\phi_1\phi_2{}^3 c;N[\hat{\Delta}]}, \quad \tilde{\Delta}_8^{(1)} = -\text{coeff. of } k_1{}^2 \text{ in } \tilde{\Gamma}^{R(1)}_{\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2),$$

$$\tilde{\Delta}_9^{(1)} = -\text{coeff. of } k_2{}^2 \text{ in } \tilde{\Gamma}^{R(1)}_{\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2), \quad \tilde{\Delta}_{10}^{(1)} = -\text{coeff. of } k_1\cdot k_2 \text{ in } \tilde{\Gamma}^{R(1)}_{\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2),$$

$$\tilde{\Delta}_{11}^{(1)} = -i\,\text{coeff. of } k_1{}^{\mu_1} \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{Ac;N[\hat{\Delta}]}(k_1), \quad \tilde{\Delta}_{12}^{(1)} = -i\,\text{coeff. of } k_2{}^{\mu_1} \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1 c;N[\hat{\Delta}]}(k_1,k_2),$$

$$\tilde{\Delta}_{13}^{(1)} = -\text{ coeff. of } k_1{}^{\mu_1} \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1 c;N[\hat{\Delta}]}(k_1,k_2),$$

$$\tilde{\Delta}_{14}^{(1)} = \frac{-i}{2}\text{coeff. of } k_1{}^{\mu_1} \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1^2 c;N[\hat{\Delta}]}(k_1,k_2,k_3),$$

$$\tilde{\Delta}_{15}^{(1)} = -i\,\text{coeff. of } \{k_2{}^{\mu_1},k_3{}^{\mu_1}\} \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1{}^2 c;N[\hat{\Delta}]}(k_1,k_2,k_3),$$

$$\tilde{\Delta}_{16}^{(1)} = \frac{-i}{2}\text{coeff. of } k_1{}^{\mu_1} \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{A\phi_2{}^2 c;N[\hat{\Delta}]}(k_1,k_2,k_3),$$

$$\tilde{\Delta}_{17}^{(1)} = -i\,\text{coeff. of } \{k_2{}^{\mu_1},k_3{}^{\mu_1}\} \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{A\phi_2{}^2 c;N[\hat{\Delta}]}(k_1,k_2,k_3),$$

$$\tilde{\Delta}_{18}^{(1)} = i\,\text{coeff. of } k_1{}^{\mu_1}k_1{}^2 \text{ in } \tilde{\Gamma}^{R(1)\mu_1}_{Ac;N[\hat{\Delta}]}(k_1,k_2),$$

$$\tilde{\Delta}_{19}^{(1)}=-i\ \text{coeff. of}\ \{g^{\mu_1\mu_2}k_2{}^{\mu_3},g^{\mu_1\mu_2}k_1{}^{\mu_3},g^{\mu_1\mu_3}k_1{}^{\mu_2},$$
$$g^{\mu_1\mu_2}k_3{}^{\mu_2},g^{\mu_2\mu_3}k_2{}^{\mu_1},g^{\mu_2\mu_3}k_3{}^{\mu_1}\}\quad \text{in}\ \tilde{\Gamma}^{R(1)\mu_1\mu_2\mu_3}_{AAAc;N[\hat{\Delta}]}(k_1,k_2,k_3),$$

$$\tilde{\Delta}_{20}^{(1)}=\frac{1}{2}\text{coeff. of}\ g^{\mu_1\mu_2}\ \text{in}\ \tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AA\phi_2c;N[\hat{\Delta}]},\quad \tilde{\Delta}_{21}^{(1)}=\frac{1}{2}\text{coeff. of}\ g^{\mu_1\mu_2}\ \text{in}\ \tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AA\phi_1\phi_2c;N[\hat{\Delta}]},$$

$$\tilde{\Delta}_{22}^{(1)}=-i\ \text{coeff. of}\ \{g^{\mu_1\mu_2}k_3{}^{\mu_3},g^{\mu_1\mu_3}k_2{}^{\mu_2},g^{\mu_2\mu_3}k_1{}^{\mu_1}\}\quad \text{in}\ \tilde{\Gamma}^{R(1)\mu_1\mu_2\mu_3}_{AAAc;N[\hat{\Delta}]}(k_1,k_2,k_3),$$

$$\tilde{\Delta}_{23}^{(1)}=\text{coeff. of}\ \gamma_5\ \text{in}\ \tilde{\Gamma}^{R(\underline{1})}_{\psi\psi c;N[\hat{\Delta}]}(k_1\equiv 0,k_2\equiv 0),$$

$$\tilde{\Delta}_{24}^{(1)}=-i\ \text{coeff. of}\ \{\not{k}_1P_L,\not{k}_2P_L\}\ \text{in}\ \tilde{\Gamma}^{R(\underline{1})}_{\psi\psi c;N[\hat{\Delta}]}(k_1,k_2)$$
$$=-i(\text{coeff. of}\ \not{k}_1-\text{coeff. of}\ \not{k}_1\gamma_5)\ \text{in}\ \tilde{\Gamma}^{R(\underline{1})}_{\psi\psi c;N[\hat{\Delta}]}(k_1,k_2),$$

$$\tilde{\Delta}_{25}^{(1)}=-i\ \text{coeff. of}\ \{\not{k}_1P_R,\not{k}_2P_R\}\ \text{in}\ \tilde{\Gamma}^{R(\underline{1})}_{\psi\psi c;N[\hat{\Delta}]}(k_1,k_2)$$
$$=-i(\text{coeff. of}\ \not{k}_1+\text{coeff. of}\ \not{k}_1\gamma_5)\ \text{in}\ \tilde{\Gamma}^{R(\underline{1})}_{\psi\psi c;N[\hat{\Delta}]}(k_1,k_2),$$

$$\tilde{\Delta}_{26}^{(1)}=\tilde{\Gamma}^{R(\underline{1})}_{\psi\psi\phi_2c;N[\hat{\Delta}]},\quad \tilde{\Delta}_{27}^{(1)}=\text{coeff. of}\ \gamma_5\ \text{in}\ \tilde{\Gamma}^{R(\underline{1})}_{\psi\psi\phi_1c;N[\hat{\Delta}]},$$

$$\tilde{\Delta}_{28}^{(1)}=\frac{1}{2}\text{coeff. of}\ \varepsilon^{\mu_1\mu_2\alpha\beta}k_{1\alpha}k_{2\beta}\ \text{in}\ \tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AAc;N[\hat{\Delta}]}(k_1,k_2), \tag{26}$$

where, for example, “coeff. of $\{k_2^{\mu_1},k_3^{\mu_1}\}$ in X” stands for “coefficient of $k_2^{\mu_1}$ in X or coefficient of $k_3^{\mu_1}$ in X” (that is, they must be equal).

The results, consistent when several formulas for a coefficient are possible, thus obtained read

$$(4\pi)^2\tilde{\Delta}_1^{(1)}=-8f^4v^3,\ (4\pi)^2\tilde{\Delta}_2^{(1)}=-24f^4v^2,\quad (4\pi)^2\tilde{\Delta}_3^{(1)}=\frac{-8f^4v}{3}, \tag{27}$$

$$(4\pi)^2\tilde{\Delta}_4^{(1)}=-24f^4v,\quad (4\pi)^2\tilde{\Delta}_5^{(1)}=\frac{-4f^2v}{3},\quad (4\pi)^2\tilde{\Delta}_6^{(1)}=-8f^4,$$

$$(4\pi)^2\tilde{\Delta}_7^{(1)}=\frac{-8f^4}{3},\ (4\pi)^2\tilde{\Delta}_8^{(1)}=-4f^2,\quad (4\pi)^2\tilde{\Delta}_9^{(1)}=\frac{-4f^2}{3},$$

$$(4\pi)^2\tilde{\Delta}_{10}^{(1)}=-4f^2,\quad (4\pi)^2\tilde{\Delta}_{11}^{(1)}=-2f^2v^2,\ (4\pi)^2\tilde{\Delta}_{12}^{(1)}=0,$$

$$(4\pi)^2\tilde{\Delta}_{13}^{(1)}=-4f^2v,\ (4\pi)^2\tilde{\Delta}_{14}^{(1)}=-2f^2,\quad (4\pi)^2\tilde{\Delta}_{15}^{(1)}=0,$$

$$(4\pi)^2\tilde{\Delta}_{16}^{(1)}=-2f^2,\ (4\pi)^2\tilde{\Delta}_{17}^{(1)}=-8f^2,\quad (4\pi)^2\tilde{\Delta}_{18}^{(1)}=\frac{-1}{3},$$

$$(4\pi)^2\tilde{\Delta}_{19}^{(1)}=\frac{2}{3},\ (4\pi)^2\tilde{\Delta}_{20}^{(1)}=4f^2v,\ (4\pi)^2\tilde{\Delta}_{21}^{(1)}=4f^2,$$

$$(4\pi)^2\tilde{\Delta}_{22}^{(1)}=\frac{1}{3},$$

$$(4\pi)^2\tilde{\Delta}_{23}^{(1)}=\frac{-i}{6}f[3\rho r+4g^2\theta(1+\theta r)v(5+\xi')],$$

$$(4\pi)^2\tilde{\Delta}_{24}^{(1)}=\frac{-\{[-6f^2r+g^2(2\theta+r+\theta^2r)(5+\xi')]\}}{6},$$

$$(4\pi)^2\tilde{\Delta}_{25}^{(1)}=\frac{r[-6f^2+g^2\theta^2(5+\xi')]}{6},$$

$$(4\pi)^2\tilde{\Delta}_{26}^{(1)}=\frac{-2fg^2\theta(\theta+r)(5+\xi')}{3},\quad (4\pi)^2\tilde{\Delta}_{27}^{(1)}=\frac{-2i}{3}fg^2\theta(1+\theta r)(5+\xi'),$$

$$(4\pi)^2\tilde{\Delta}_{28}^{(1)}=\frac{2(3\theta+r+3\theta^2r)}{3}.$$

This breaking is simplified a bit with the choice of gauge $\xi'\equiv-5$.

Note that if only a fermion is present of type, for example, $r=+1$, then the anomaly coefficient is not zero for any value of θ and that by adding fermions of the same type, the coefficient anomaly cannot be canceled. Fermions of both types are needed. For example, there is cancellation of the anomaly in the case of two fermions with $\theta_1=\theta_2=0$ and $r_1=+1$, $r_2=-1$ or in the case of two fermions with $\theta_1=1$, $\theta_2=-1$ and $r_1=+1$, $r_2=-1$. Remembering the definitions $e_{Rk}\equiv\theta_k$ and $e_{Lk}\equiv\theta_k+r_k$ with $r_k=\pm1$, the obtained coefficient of the anomaly can be written in the more familiar form

$$\tilde{\Delta}_{28}^{(1)}=\frac{1}{(4\pi)^2}\frac{2}{3}\sum_{k\in I\cup J}(e_{Lk}^3-e_{Rk}^3) \tag{28}$$

but the constraints (4) should never be forgotten.

VI. RESTORATION OF BRS SYMMETRY: FINITE COUNTERTERMS

We know from the algebraic theory of renormalization presented in the third section that the linear system (19) has to be compatible, but its solution is not unique. Facts which cannot be trivially deduced from Eq. (26). Using the values of the coefficients $\tilde{\Delta}_i^{(1)}$, found in the previous section, this turns to be the case and one of the solutions is

$$(4\pi)^2\tilde{x}_{0,1}^{(1)}=8f^4v^3,\ (4\pi)^2\tilde{x}_{0,2}^{(1)}=12f^4v^2,\ (4\pi)^2\tilde{x}_{0,3}^{(1)}=0, \tag{29}$$

$$(4\pi)^2\tilde{x}_{0,4}^{(1)}=8f^4v,\ (4\pi)^2\tilde{x}_{0,5}^{(1)=0,}\ (4\pi)^2\tilde{x}_{0,6}^{(1)}=2f^4,$$

$$(4\pi)^2\tilde{x}_{0,7}^{(1)}=\frac{-2f^4}{3},\ (4\pi)^2\tilde{x}_{0,8}^{(1)}=0,\ (4\pi)^2\tilde{x}_{0,9}^{(1)}=0,$$

$$(4\pi)^2\tilde{x}_{0,10}^{(1)}=\frac{2f^2}{3},\ (4\pi)^2\tilde{x}_{0,11}^{(1)}=0,\ (4\pi)^2\tilde{x}_{0,12}^{(1)}=0,$$

$$(4\pi)^2\tilde{x}_{0,13}^{(1)}=4f^2,\ (4\pi)^2\tilde{x}_{0,14}^{(1)}=f^2v^2,\ (4\pi)^2\tilde{x}_{0,15}^{(1)}=2f^2v,$$

$$(4\pi)^2\tilde{x}_{0,16}^{(1)}=f^2,\ (4\pi)^2\tilde{x}_{0,17}^{(1)}=3f^2,\ (4\pi)^2\tilde{x}_{0,18}^{(1)}=\frac{-1}{6},$$

$$(4\pi)^2\tilde{x}_{0,19}^{(1)}=0,\ (4\pi)^2\tilde{x}_{0,20}^{(1)}=\frac{-1}{12},\ (4\pi)^2\tilde{x}_{0,21}^{(1)}=0,$$

$$(4\pi)^2\tilde{x}_{0,22}^{(1)}=0,\ (4\pi)^2\tilde{x}_{0,23}^{(1)}=0,$$

$$(4\pi)^2\tilde{x}_{0,24}^{(1)}=\frac{f[3\rho r+4g^2\theta(1+\theta r)v(5+\xi')]}{6r},$$

$$(4\pi)^2\tilde{x}_{0,25}^{(1)}=0,\ (4\pi)^2\tilde{x}_{0,26}^{(1)}=0,$$

$$(4\pi)^2\tilde{x}_{0,27}^{(1)}=\frac{[-6f^2r+g^2(2\theta+r+\theta^2r)(5+\xi')]}{6},$$

$$(4\pi)^2\tilde{x}^{(1)}_{0,28}=\frac{-\{r[-6f^2+g^2\theta^2(5+\xi')]\}}{6},$$

$$(4\pi)^2\tilde{x}^{(1)}_{0,29}=\frac{2fg^2\theta(\theta+r)(5+\xi')}{3},$$

$$(4\pi)^2\tilde{x}^{(1)}_{0,30}=0,\ \ (4\pi)^2\tilde{x}^{(1)}_{0,31}=0,\ \ (4\pi)^2\tilde{x}^{(1)}_{0,32}=0.$$

Therefore, the general solution for the finite counterterms up to one-loop order reads

$$\hbar\tilde{S}^{(1)}_{\mathrm{fct}}=-\hbar\sum_{i=1}^{32}\tilde{x}^{(1)}_{0,i}\tilde{e}_i+\hbar\sum_{l=1}^{11}c^{(1)}_l\mathcal{I}_l, \tag{30}$$

with the basis $\tilde{e}_i$ being given by Eq. (16) and the symmetric terms $\mathcal{I}_l$ by Eq. (20). Therefore, the parametric family of regularized actions $S_1\equiv S_0+\hbar\tilde{S}^{(1)}_{\mathrm{fct}}$, with $\tilde{K}_{\phi_2}=K_{\phi_2}-\rho\overline{c}$, gives, by minimal subtraction in the BMHV scheme, all possible *CP*-symmetric renormalized theories, compatible with the tree-level action (5) and power counting renormalizability, and satisfying up to one-loop level both the STI (8) and the gauge-fixing equation (9).

Notice that, if $\theta\neq0$ and $\theta+r\neq0$ ($\theta=0$ or $\theta+r=0$), there is a seven- (eight-)dimensional family of solutions, or equivalently, of normalization conditions, which does not imply finite counterterms depending on BRS external fields. The restriction to this family would certainly simplify the two-loop analysis of Eq. (2). In a more general regulator independent context, the simplificatory power given by the freedom in the choice of normalization conditions have been stressed in Refs. [30,31].

Finally, note that although the starting classical action of order $\hbar^0$ was chosen to satisfy $\mu^2=\lambda v^2$ so that the monomial $\int\phi_1$ does not appear in the action, we have the freedom to impose any $O(\hbar^n)$ mean value on the field thanks to the trivial finite counterterms $\tilde{\mathcal{I}}_2$, $\tilde{\mathcal{I}}_3$ and $\tilde{\mathcal{I}}_6$. Setting that value to zero would just define one of the normalization conditions mentioned at the end of Sec. III.

VII. CONCLUSIONS

Algebraic renormalization theory has mainly been used for demonstrative purposes (however, see Refs. [30,31]). In this paper, we have shown a pedagogical example which makes manifest that the theoretical tools provided by the algebraic renormalization theory are of utmost importance in order to blindly carry out computations in a noninvariant renormalization procedure such as the BMHV scheme is for chiral gauge theories. Such noninvariant renormalization procedures seem to be unavoidable in a near future for doing trustable high-precision tests of relevant quantum field theories such as the standard model, and mastery of these techniques will be needed.

Although at first sight the method looks cumbersome for the practitioners, we want to stress that once the general expression for a noninvariant modified action have been found at some order of the perturbative expansion the automatic evaluation of renormalized diagrams satisfying the symmetries of the theory is not much more difficult than the conventional procedures, because we need to do only a minimal subtraction of all Feynman integrals obtained from the Feynman rules of the given modified action. Certainly, the γ algebra is a bit more tedious and there are more Feynman rules in the modified action than in the conventional one, but nowadays all this is perfectly admissible for the current computer codes.

The simplicity of the Abelian Higgs-Kibble model allows for explicit and order independent expressions for the possible counterterms. Thus not obscuring the main steps of the algebraic method and making it very suitable for a future study at two-loop order, this study could be easily extended to the physically relevant standard model. The arbitrariness of the choice of the regularizations of vertices and finite counterterms could be a key to simplify the computations at higher loop orders of the RHS of Eq. (2) using for example similar techniques to the ones in Ref. [13].

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APPENDIX A: MATRIX ELEMENTS OF THE LINEARIZED ST OPERATOR $\tilde{b}_0$

Using the definition (15), the $\tilde{b}$ variations of the fields are

$$\tilde{b}A_\mu=sA_\mu=\partial_\mu c,$$

$$\tilde{b}\phi_1=s\phi_1=-\phi_2 c,$$

$$\tilde{b}\phi_2=s\phi_2=(v+\phi_1)c,$$

$$\tilde{b}\psi=s\psi=ic[(\theta+r)P_L+\theta P_R]\psi,$$

$$\tilde{b}\bar{\psi}=s\bar{\psi}=i\bar{\psi}[(\theta+r)P_R+\theta P_L]c,$$

$$\tilde{b}c=0,$$

$$\tilde{b}K_{\phi_1}=\frac{\delta\tilde{\Gamma}^0}{\delta\phi_1}=\frac{\delta S_0}{\delta\phi_1}=\text{e.o.m. of }\ \phi_1$$

$$=-\Box\phi_1-(\delta_\mu A^\mu)\phi_2-2A^\mu(\delta_\mu\phi_2)+A_\mu A^\mu(v+\phi_1)+\mu^2(v+\phi_1)-\lambda[(v+\phi_1)^2+\phi_2{}^2](v+\phi_1)+\tilde{K}_{\phi_2}c-f\bar{\psi}\psi,$$

$$\tilde{b}\tilde{K}_{\phi_2}=\frac{\delta\tilde{\Gamma}^0}{\delta\phi_2}=\frac{\delta S_0}{\delta\phi_2}=\text{e.o.m. of }\ \phi_2-\rho B=-\Box\phi_2+(\partial_\mu A^\mu)(v+\phi_1)+2A^\mu(\partial_\mu\phi_1)$$

$$+A_\mu A^\mu\phi_2+\mu^2\phi_2-\lambda[(v+\phi_1)^2+\phi_2{}^2]\phi_2-K_{\phi_1}c-irf\bar{\psi}\gamma_5\psi,$$

$$\tilde{b}K_\psi=\frac{\partial\tilde{\Gamma}^0}{\delta\psi}=\text{e.o.m. of }\ \bar{\psi}=\bar{\psi}[i\overleftarrow{\partial\!\!\!/}-A\!\!\!/((\theta+r)P_L+\theta P_R)]+f[(v+\phi_1)\bar{\psi}+ir\phi_2\bar{\psi}\gamma_5]-icK_\psi[(\theta+r)P_L+\theta P_R],$$

$$\tilde{b}K_{\bar{\psi}}=\frac{\delta\tilde{\Gamma}^0}{\delta\bar{\psi}}=\text{e.o.m. of }\ \psi=[i\vec{\partial\!\!\!/}+A\!\!\!/((\theta+r)P_L+\theta P_R)]\psi-f[(v+\phi_1)\psi+ir\phi_2\gamma_5\psi]+ic[(\theta+r)P_R+\theta P_L]K_{\bar{\psi}}.$$

Therefore, applying these variations to the basis (16) of $\tilde{\mathcal{V}}_0$ and expanding the results in the basis (18) of $\tilde{\mathcal{V}}_1$, the matrix elements of this restriction of $\tilde{b}$, defined as $\tilde{b}\tilde{e}_i\equiv\tilde{b}_{0\,i}{}^j u_j$, are easily found. The first 23 rows and 20 columns of matrix $\{\tilde{b}_{0\,i}{}^j\}_{1\leqslant j\leqslant 28\ 1\leqslant i\leqslant 32}$ are

$$\begin{matrix}
-1 & 0 & 2v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -2 & 2 & 0 & 2v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1 & 0 & 4v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -3 & 2 & 0 & 0 & 2v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2v & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -4 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 4 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -v & v & 0 & -2 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -v & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & -4 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -4 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 2v & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}$$

the columns 21 to 32 are

$v(\mu^2-\lambda v^2)$	$\mu^2-\lambda v^2$	0	0	0	0	0	0	0	0	0	0
$\mu^2-3\lambda v^2$	$-2\lambda v$	$\mu^2-\lambda v^2$	0	0	0	0	0	0	0	0	0
$-(\lambda v)$	$-\lambda$	0	0	0	0	0	0	0	0	0	0
$-3\lambda v$	$-\lambda$	$-2\lambda v$	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	0
$-\lambda$	0	$-\lambda$	0	0	0	0	0	0	0	0	0
$-\lambda$	0	$-\lambda$	0	0	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0	0	0	0	0
0	0	-1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	v	0	0	0	0	0	0	0	0	0	0
0	2	0	0	0	0	0	0	0	0	0	0
0	1	v	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	2	0	0	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0	0	0	0	0
-2	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
v	1	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	$-ifr$	0	$-ir$	0	0	0	0	0	v	$-(fv)$	fv
0	0	0	0	$\theta+r$	0	-1	0	0	0	i	0
0	0	0	0	0	θ	0	-1	0	0	0	i
$-f$	0	0	0	0	0	0	0	-1	$-ir$	$-ifr$	ifr
0	0	$-ifr$	0	0	0	0	0	$-ir$	1	$-f$	f
0	0	0	0	0	0	0	0	0	0	0	0

and the rest of its elements are 0.

APPENDIX B: EXPLICIT SOLUTION OF THE ORDER ONE COHOMOLOGY OF $\tilde{b}$

Let $\tilde{\mathcal{V}}_i$ be the space of integrated Lorentz scalar *CP*-invariant polynomials in the fields ϕ_1, ϕ_2, A_μ, ψ, $\overline{\psi}$, ψ, c, K_{ϕ_1}, $\tilde{K}_{\phi_2}$, K_ψ, and $K_{\overline{\psi}}$ of maximal canonical dimension 4 and ghost number i.

We define $\tilde{\mathcal{W}}_{i+1}\equiv\tilde{b}\tilde{\mathcal{V}}_i$ and $\tilde{\mathcal{K}}_i\equiv\{\tilde{\mathcal{I}}\in\tilde{\mathcal{V}}_i/\tilde{b}\tilde{\mathcal{I}}=0\}$. Due to nilpotency of $\tilde{b}$, $\tilde{\mathcal{W}}_i\subset\tilde{\mathcal{K}}_i\subset\tilde{\mathcal{V}}_i$.

Solving explictly the cohomology of order one of $\tilde{b}$ means to find the elements of $\tilde{\mathcal{V}}_1$ which are closed, i.e., in $\tilde{\mathcal{K}}_1$ but not exact, i.e., not in $\tilde{\mathcal{W}}_1$. Those nontrivial elements of the cohomology are termed *the anomaly*.

In order to do so, we introduce a basis for $\tilde{\mathcal{V}}_2$:

$$\tilde{v}_1\equiv\int\phi_1(\Box c)c,\ \ \tilde{v}_2\equiv\int\phi_1{}^2(\Box c)c,\ \ \tilde{v}_3\equiv\int\phi_2{}^2(\Box c)c,$$

$$\tilde{v}_4\equiv\int A^\mu\phi_2(\partial_\mu c),\ \ \tilde{v}_5\equiv\int A^\mu\phi_1\phi_2(\partial_\mu c)c,\quad \tilde{v}_6\equiv\int(\partial_\nu A^\nu)A^\mu(\partial_\mu c)c,$$

$$\tilde{v}_7 \equiv \int (\partial^\nu A^\mu) A_\nu (\partial_\mu c) c, \quad \tilde{v}_8 \equiv \int (\partial^\mu A^\nu) A_\nu (\partial_\mu c) c$$

and the matrix of the restricted linear operator $\tilde{b}_1 \equiv \tilde{b}$: $\tilde{\mathcal{V}}_1 \to \tilde{\mathcal{V}}_2$ as $\tilde{b}\tilde{u}_j \equiv \tilde{b}_{1\ j}^{\ k} \tilde{v}_k$. The columns 5 to 22 of matrix $\{\tilde{b}_{1\ j}^{\ k}\}_{1 \leqslant k \leqslant 8 \, 1 \leqslant j \leqslant 28}$ are

$$\left(\begin{array}{cccccccccccccccccc}
-1 & 0 & 0 & 0 & v & -v & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & v & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)$$

and the rest of its elements are 0.

A basis of the kernel $\tilde{\mathcal{K}}_1$ of $\tilde{b}_1$ is, therefore,

$$\tilde{\mathcal{J}}_1 = \tilde{u}_{28}, \quad \tilde{\mathcal{J}}_2 = \tilde{u}_1, \quad \tilde{\mathcal{J}}_3 = \tilde{u}_2,$$

$$\tilde{\mathcal{J}}_4 = \tilde{u}_3, \quad \tilde{\mathcal{J}}_5 = \tilde{u}_4, \quad \tilde{\mathcal{J}}_6 = \tilde{u}_6,$$

$$\tilde{\mathcal{J}}_7 = \tilde{u}_7, \quad \tilde{\mathcal{J}}_8 = \tilde{u}_8, \quad \tilde{\mathcal{J}}_9 = \tilde{u}_{11},$$

$$\tilde{\mathcal{J}}_{10} = \tilde{u}_{18}, \quad \tilde{\mathcal{J}}_{11} = \tilde{u}_{23}, \quad \tilde{\mathcal{J}}_{12} = \tilde{u}_{24},$$

$$\tilde{\mathcal{J}}_{13} = \tilde{u}_{25}, \quad \tilde{\mathcal{J}}_{14} = \tilde{u}_{26}, \quad \tilde{\mathcal{J}}_{15} = \tilde{u}_{27},$$

$$\tilde{\mathcal{J}}_{16} = v\tilde{u}_5 + \tilde{u}_9, \quad \tilde{\mathcal{J}}_{17} = \tilde{u}_5 + \tilde{u}_{13}, \quad \tilde{\mathcal{J}}_{18} = 2\tilde{u}_{19} + \tilde{u}_{22},$$

$$\tilde{\mathcal{J}}_{19} = -2\tilde{u}_5 + 2\tilde{u}_{12} + \tilde{u}_{20}, \quad \tilde{\mathcal{J}}_{20} = \tilde{u}_{14} + 2\tilde{u}_{15} + \tilde{u}_{21}, \quad \tilde{\mathcal{J}}_{21} = 2v\tilde{u}_5 - 2\tilde{u}_{10} - \tilde{u}_{14} + \tilde{u}_{16},$$

$$\tilde{\mathcal{J}}_{22} = -2v\tilde{u}_5 + \tilde{u}_{10} + v\tilde{u}_{12} + \tilde{u}_{14} + \tilde{u}_{15} + \tilde{u}_{17}.$$

Note that dim of $\tilde{\mathcal{K}}_1 -$ dim of $\tilde{\mathcal{W}}_1 = 1$, so the anomaly is expanded by only one element of $\tilde{\mathcal{V}}_1$. Applying on each element of the basis of the kernel $\tilde{\mathcal{K}}_1$ a linear independence test against the set of linear independent columns of the matrix $\tilde{b}_0$, which expands the image of the operator $\tilde{b}_0$, it is inmediately found that $\tilde{\mathcal{J}}_1 = \tilde{u}_{28}$ is the anomaly, as affirmed in Sec. III.

APPENDIX C: BREAKING 1-LOOP FEYNMAN DIAGRAMS

Notation:

$$(2\pi)^4 \delta(k_1 + \cdots + k_m + k_{m+1}) \tilde{\Gamma}^{R(1)\mu_1 \cdots \mu_p}_{X_1 X_2 \cdots X_m c; N[\hat{\Delta}]}(k_1, \cdots, k_m)$$

$$= \int dx_1 \cdots dx_{m+1} e^{i(k_1 x_1 + \cdots + k_{m+1} x_{m+1})} \frac{\delta N[\hat{\Delta}] \cdot \Gamma^{R(1)}[\phi_1, \phi_2, A, \psi, \bar{\psi}, c, \bar{c}, K_{\phi_1}, K_{\phi_2}, K_\psi, K_{\bar{\psi}}]}{\delta X_{1_{\mu_1}}, (x_1) \cdots \delta X_{p_{\mu_p}}(x_p) \, \delta X_{p_{\mu_{p+1}}}(x_{p+1}) \cdots \delta X_m(x_m) \, \delta c(x_{m+1})} \Bigg|_{X \equiv 0}$$

will stand for the minimally substracted one loop 1PI functions with one insertion of the integrated breaking $N[\hat{\Delta}]$ and the fields $X_1 X_2 \cdots X_m c$ as external legs. X_i represents any field of $\phi_1, \phi_2, A, \psi, \bar{\psi}$ (all 1PI diagrams with at least a ghost or an antighost or and external BRS field are convergent by power counting, and therefore null when taking into account the

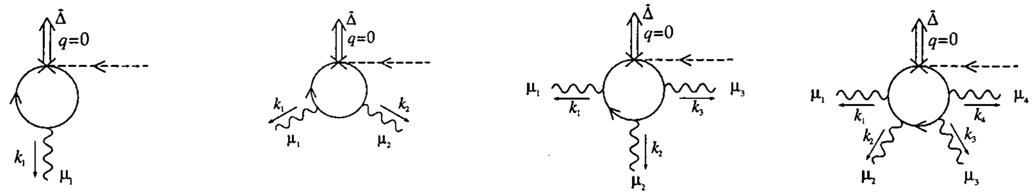

FIG. 3. Feynman diagrams with bosons needed to compute the 1PI breaking functions (C1).

insertion of the evanescent breaking operator). $\mu_1 \cdots \mu_p$ are the Lorentz indices of the corresponding bosons in $X_1 X_2 \cdots X_m$. $k_1,\ldots,k_m$ are the independent outgoing momenta of the fields $X_1 X_2 \cdots X_m$.

1. Bosonic diagrams

$$\tilde{\Gamma}^{R(1)\mu_1}_{Ac;N[\hat{\Delta}]}(k_1) = (\mathrm{i}), \tag{C1a}$$

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AAc;N[\hat{\Delta}]}(k_1,k_2) = (\mathrm{ii})_{12} + (\mathrm{ii})_{21},$$

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2\mu_3}_{AAAc;N[\hat{\Delta}]}(k_1,k_2,k_3) = (\mathrm{iii})_{123} + \text{permutations of } 123,$$

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2\mu_3\mu_4}_{AAAAc;N[\hat{\Delta}]}(k_1,k_2,k_3,k_4) = (\mathrm{iv})_{1234} + \text{permutations of } 1234.$$

The renormalized result for each diagram of Fig. 3 is

$$(4\pi)^2(\mathrm{i}) = \frac{-i}{3}(6f^2v^2 - k_1{}^2)k_1{}^{\mu_1},$$

$$(4\pi)^2(\mathrm{ii})_{12} = \frac{2(3\theta + r + 3\theta^2 r)\epsilon(k_1,k_2,\{\mu_1\},\{\mu_2\})}{3} + \frac{2i}{3}(2\theta + r)(k_1{}^{\mu_1}k_1{}^{\mu_2} - k_2{}^{\mu_1}k_2{}^{\mu_2}) - \frac{i}{3}(2\theta + r)(k_1{}^2 - k_2{}^2)g^{\mu_1\mu_2},$$

$$\begin{aligned}(4\pi)^2(\mathrm{iii})_{123} = {} & \frac{(1+9\theta^2+5\theta r+6\theta^3 r)\epsilon(k_1,\{\mu_1\},\{\mu_2\},\{\mu_3\})}{3} + \frac{(+2\theta r)\epsilon(k_2,\{\mu_1\},\{\mu_2\},\{\mu^3\})}{3} \\ & + \frac{(1+9\theta^2+5\theta r+6\theta^3 r)\epsilon(k_3,\{\mu_1\},\{\mu_2\},\{\mu_3\})}{3} + \frac{i}{3}(1+\theta^2+\theta r)k_1{}^{\mu_3}g^{\mu_1\mu_2} \\ & + \frac{i}{3}(1+2\theta^2+2\theta r)k_2{}^{\mu_3}g^{\mu_1\mu_2} + \frac{i}{3}(1+3\theta^2+3\theta r)k_3{}^{\mu_3}g^{\mu_1\mu_2} - \frac{i}{3}(1+3\theta^2+3\theta r)k_1{}^{\mu_2}g^{\mu_1\mu_3} \\ & - \frac{i}{3}(1+6\theta^2+6\theta r)k_2{}^{\mu_2}g^{\mu_1\mu_3} - \frac{i}{3}(1+3\theta^2+3\theta r)k_3{}^{\mu_2}g^{\mu_1\mu_3} + \frac{i}{3}(1+3\theta^2+3\theta r)k_1{}^{\mu_1}g^{\mu_2\mu_3} \\ & + \frac{i}{3}(1+2\theta^2+2\theta r)k_2{}^{\mu_1}g^{\mu_2\mu_3} + \frac{i}{3}(1+\theta^2+\theta r)k_3{}^{\mu_1}g^{\mu_2\mu_3},\end{aligned}$$

$$(4\pi)^2(\mathrm{iv})_{1234} = \frac{-2\theta(1+6\theta^2+4\theta r+3\theta^3 r)\epsilon(\{\mu_1\},\{\mu_2\},\{\mu_3\},\{\mu_4\})}{3}. \tag{C1b}$$

Note that although the four boson diagram is divergent by power counting, the total result for the assocciated 1PI function must be zero due to *CP* invariance of the regularized action and the dimensional renormalization procedure. As a check of the automated programs and of the preservation of discrete *CP* symmetry by the renormalization scheme, we have computed explicitly the value of the diagram (iv) of Fig. 3, which is not zero, and, as can easily seen in last equation, the total result after suming all permutations turns to be the expected result zero. That is, the discrete *CP* sym-

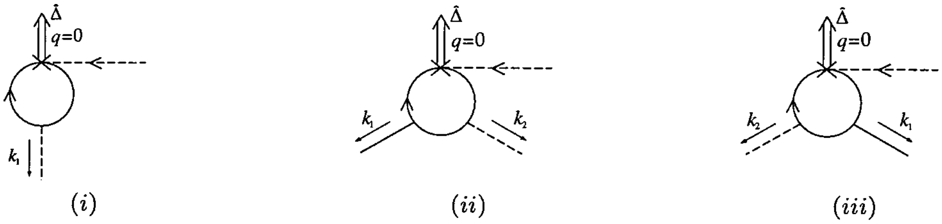

FIG. 4. Feynman diagrams linear or quadratic in scalar fields needed to compute the 1PI breaking functions (C2).

metry is reflected in perturbative dimensional renormalization as a cancellation between permuted diagrams.

2. Diagrams linear and quadractic in scalar fields

$$\tilde{\Gamma}^{R(1)}_{\phi_2 c;N[\hat{\Delta}]}(k_1) = (\mathrm{i}),$$

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2) = (\mathrm{ii}) + (\mathrm{iii}). \tag{C2a}$$

The renormalized result for each diagram of Fig. 4 is

$$(4\pi)^2(\mathrm{i}) = \frac{4f^2v(-6f^2v^2+k_1{}^2)}{3}, \tag{C2b}$$

$$(4\pi)^2(\mathrm{ii}) = \frac{2\,f^2(-18f^2v^2+3k_1{}^2+3k_1\cdot k_2+k_2{}^2)}{3},$$

$$(4\pi)^2(\mathrm{iii}) = \frac{2\,f^2(-18f^2v^2+3k_1{}^2+3k_1\cdot k_2+k_2{}^2)}{3}.$$

3. Diagrams with three scalar fields

$$\tilde{\Gamma}^{R(1)}_{\phi_2\phi_2\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3) = (\mathrm{i})_{123} + \text{permut. of } 123,$$

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3) = (\mathrm{ii})_{123} + (\mathrm{iii})_{123} + (\mathrm{iv})_{123} + \text{permut. of } 12, \tag{C3a}$$

where, due to the locality and dimensionality of breaking terms, all the permutations should be obviously equal.

The renormalized result for each diagram of Fig. 5 is

$$(4\pi)^2(\mathrm{i})_{123} = \frac{-8f^4v}{3},$$

$$(4\pi)^2(\mathrm{ii})_{123} = -8f^4v,$$

$$(4\pi)^2(\mathrm{iii})_{123} = -8f^4v,$$

$$(4\pi)^2(\mathrm{iv})_{123} = -8f^4v. \tag{C3b}$$

4. Diagrams with four scalar fields

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_1\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3,k_4) = (\mathrm{i})_{1234} + (\mathrm{ii})_{1234} + (\mathrm{iii})_{1234} + (\mathrm{iv})_{1234} + \text{permut. of } 123, \tag{C4a}$$

$$\tilde{\Gamma}^{R(1)}_{\phi_1\phi_2\phi_2\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3,k_4) = (\mathrm{vi})_{1234} + (\mathrm{vii})_{1234} + (\mathrm{viii})_{1234} + \text{permut. of } 234,$$

where, again, all the permutations must be the same.

The renormalized result for each diagram of Fig. 6 is

$$(4\pi)^2(\mathrm{i})_{1234} = -2f^4,$$

$$(4\pi)^2(\mathrm{ii})_{1234} = -2f^4,$$

$$(4\pi)^2(\mathrm{iii})_{1234} = -2f^4,$$

$$(4\pi)^2(\mathrm{iv})_{1234} = -2f^4,$$

$$(4\pi)^2(\mathrm{v})_{1234} = \frac{-10f^4}{3},$$

$$(4\pi)^2(\mathrm{vi})_{1234} = 2f^4,$$

$$(4\pi)^2(\mathrm{vii})_{1234} = 2f^4,$$

$$(4\pi)^2(\mathrm{viii})_{1234} = \frac{-10f^4}{3}. \tag{C4b}$$

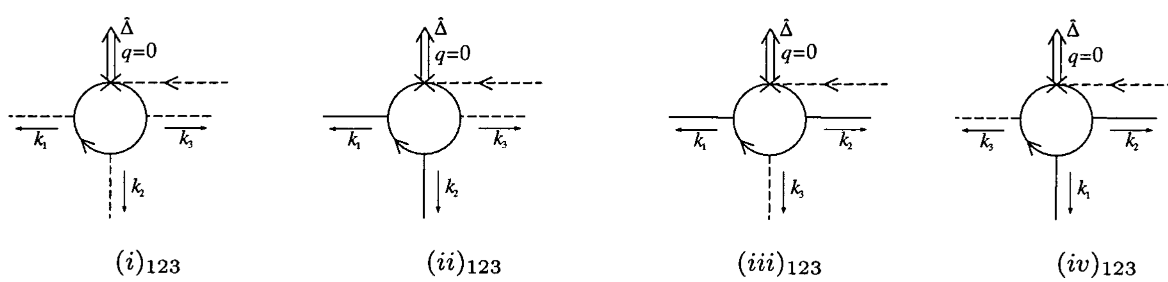

FIG. 5. Feynman diagrams with three scalar fields needed to compute the 1PI breaking functions (C3).

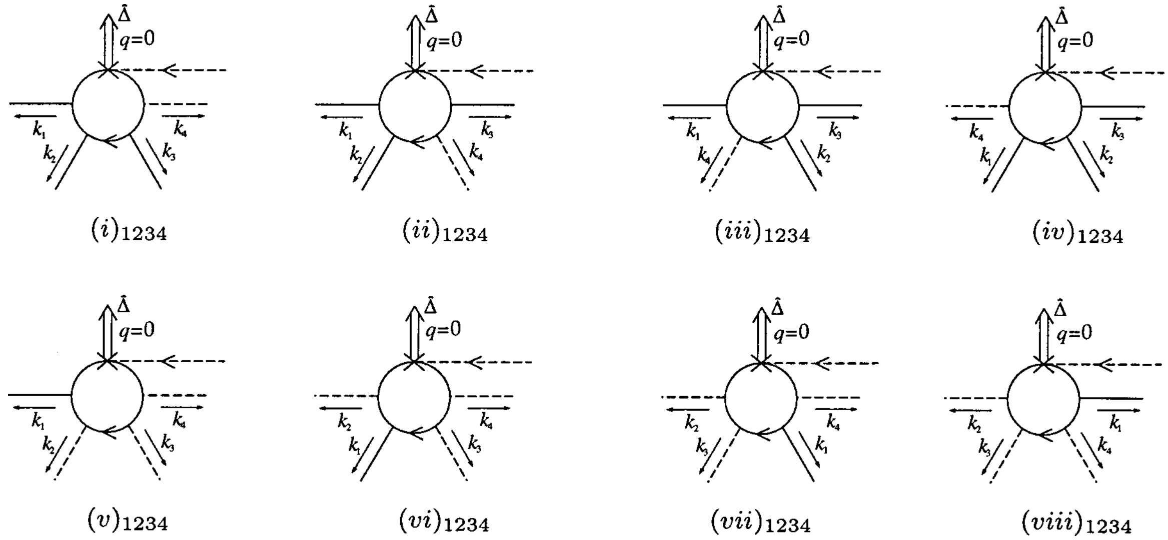

FIG. 6. Feynman diagrams with four scalar fields needed to compute the 1PI breaking functions (C4).

5. Diagrams with two or three boson and scalar fields

$$\tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1 c;N[\hat{\Delta}]}(k_1,k_2)=(\mathrm{i})+(\mathrm{ii}), \tag{C5a}$$

$$\tilde{\Gamma}^{R(1)\mu_1}_{A\phi_1\phi_1 c;N[\hat{\Delta}]}(k_1,k_2,k_3)=(\mathrm{iii})_{123}+(\mathrm{iv})_{123}+(\mathrm{v})_{123} + \text{permut. of } 23,$$

$$\tilde{\Gamma}^{R(1)\mu_1}_{A\phi_2\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3)=(\mathrm{vi})_{123}+(\mathrm{vii})_{123}+(\mathrm{viii})_{123} + \text{permut. of } 23,$$

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AA\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3)=(\mathrm{ix})_{123}+(\mathrm{x})_{123}+(\mathrm{xi})_{123} + \text{permut. of } 12.$$

The renormalized result for each diagram of Fig. 7 is

$$(4\pi)^2(\mathrm{i})=-2if^2 v k_1{}^{\mu_1}, \tag{C5b}$$

$$(4\pi)^2(\mathrm{ii})=-2if^2 v k_1{}^{\mu_1},$$

$$(4\pi)^2(\mathrm{iii})_{123}=-if^2(k_1{}^{\mu_1}+k_3{}^{\mu_1}),$$

$$(4\pi)^2(\mathrm{iv})_{123}=if^2(k_2{}^{\mu_1}+k_3{}^{\mu_1}),$$

$$(4\pi)^2(\mathrm{v})_{123}=-if^2(k_1{}^{\mu_1}+k_2{}^{\mu_1}),$$

$$(4\pi)^2(\mathrm{vi})_{123}=\frac{-i}{3}f^2(3k_1{}^{\mu_1}+4k_2{}^{\mu_1}+5k_3{}^{\mu_1}),$$

$$(4\pi)^2(\mathrm{vii})_{123}=-if^2(k_2{}^{\mu_1}+k_3{}^{\mu_1}),$$

$$(4\pi)^2(\mathrm{viii})_{123}=\frac{-i}{3}f^2(3k_1{}^{\mu_1}+5k_2{}^{\mu_1}+4k_3{}^{\mu_1}),$$

$$(4\pi)^2(\mathrm{ix})_{123}=\frac{4f^2(1+\theta^2+\theta r)v g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{x})_{123}=\frac{-4f^2(-1+2\theta^2+2\theta r)v g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{xi})_{123}=\frac{4f^2(1+\theta^2+\theta r)v g^{\mu_1\mu_2}}{3}.$$

6. Diagrams with four boson and scalar fields

$$\tilde{\Gamma}^{R(1)\mu_1\mu_2}_{AA\phi_1\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3,k_4)=(\mathrm{i})_{1234}+(\mathrm{ii})_{1234}+(\mathrm{iii})_{1234} +(\mathrm{iv})_{1234}+(\mathrm{v})_{1234}+(\mathrm{vi})_{1234} +(\mathrm{vii})_{1234}+(\mathrm{viii})_{1234} +(\mathrm{ix})_{1234}+(\mathrm{xi})_{1234}+(\mathrm{xii})_{1234} +\text{permut. of } 12. \tag{C6a}$$

The renormalized result for each diagram of Fig. 8 is

$$(4\pi)^2(\mathrm{i})_{1234}=2f^2\theta(\theta+r)g^{\mu_1\mu_2},$$

$$(4\pi)^2(\mathrm{ii})_{1234}=\frac{2f^2(1+\theta^2+\theta r)g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{iii})_{1234}=-2f^2\theta(\theta+r)g^{\mu_1\mu_2},$$

$$(4\pi)^2(\mathrm{iv})_{1234}=\frac{2f^2(1+\theta^2+\theta r)g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{v})_{1234}=-2f^2\theta(\theta+r)g^{\mu_1\mu_2},$$

$$(4\pi)^2(\mathrm{vi})_{1234}=-2f^2\theta(\theta+r)g^{\mu_1\mu_2},$$

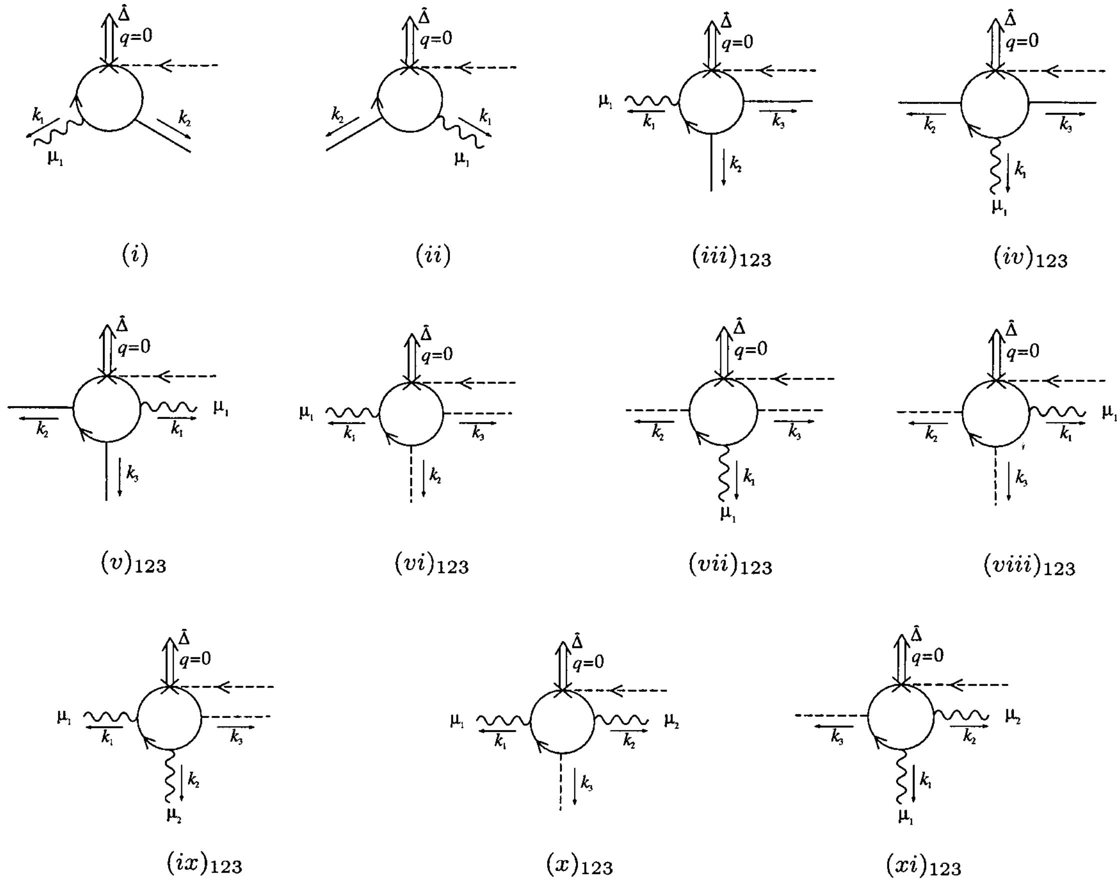

FIG. 7. Feynman diagrams with two or three boson and scalar fields needed to compute the 1PI breaking functions (C5).

$$(4\pi)^2(\mathrm{vii})_{1234}=\frac{2f^2(1+\theta^2+\theta r)g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{viii})_{1234}=\frac{2f^2(1+\theta^2+\theta r)g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{ix})_{1234}=\frac{2f^2(1+\theta^2+\theta r)g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{x})_{1234}=-2f^2\theta(\theta+r)g^{\mu_1\mu_2},$$

$$(4\pi)^2(\mathrm{xi})_{1234}=\frac{2f^2(1+\theta^2+\theta r)g^{\mu_1\mu_2}}{3},$$

$$(4\pi)^2(\mathrm{xii})_{1234}=2f^2\theta(\theta+r)g^{\mu_1\mu_2}. \tag{C6b}$$

7. Diagrams with fermion fields

$$\tilde{\Gamma}^{R(1)}_{\psi\bar{\psi}c;N[\hat{\Delta}]}(k_1,k_2)=(\mathrm{i})+(\mathrm{ii})+(\mathrm{iii})+(\mathrm{iv})+(\mathrm{v})+(\mathrm{vi})+(\mathrm{vii}). \tag{C7a}$$

The renormalized result for each diagram of Fig. 9 is

$$(4\pi)^2(\mathrm{i})=\frac{-i}{2}f^2r(2fv\gamma_5+\not{k}_1\gamma_5+\not{k}_2\gamma_5),$$

$$(4\pi)^2(\mathrm{ii})=\frac{i}{2}f^2r(2fv\gamma_5-\not{k}_1\gamma_5-\not{k}_2\gamma_5),$$

$$(4\pi)^2(\mathrm{iii})=\frac{-2i}{3}fg^2\theta(1+\theta r)v(5+\xi')\gamma_5-\frac{i}{12}g^2(2\theta+r)(5+\xi')\not{k}_1-\frac{i}{12}g^2(2\theta+r)(5+\xi')\not{k}_2+\frac{i}{12}g^2(2\theta+r+2\theta^2r)(5+\xi')\not{k}_1\gamma_5+\frac{i}{12}g^2(2\theta+r+2\theta^2r)(5+\xi')\not{k}_2\gamma_5,$$

$$(4\pi)^2(\mathrm{iv})=\frac{i}{4}f\rho[(2\theta+r)\mathbb{I}-r\gamma_5],$$

$$(4\pi)^2(\mathrm{v})=\frac{-i}{4}f\rho[(2\theta+r)\mathbb{I}+r\gamma_5],$$

$$(4\pi)^2(\mathrm{vi})=0,$$

$$(4\pi)^2(\mathrm{vii})=0. \tag{C7b}$$

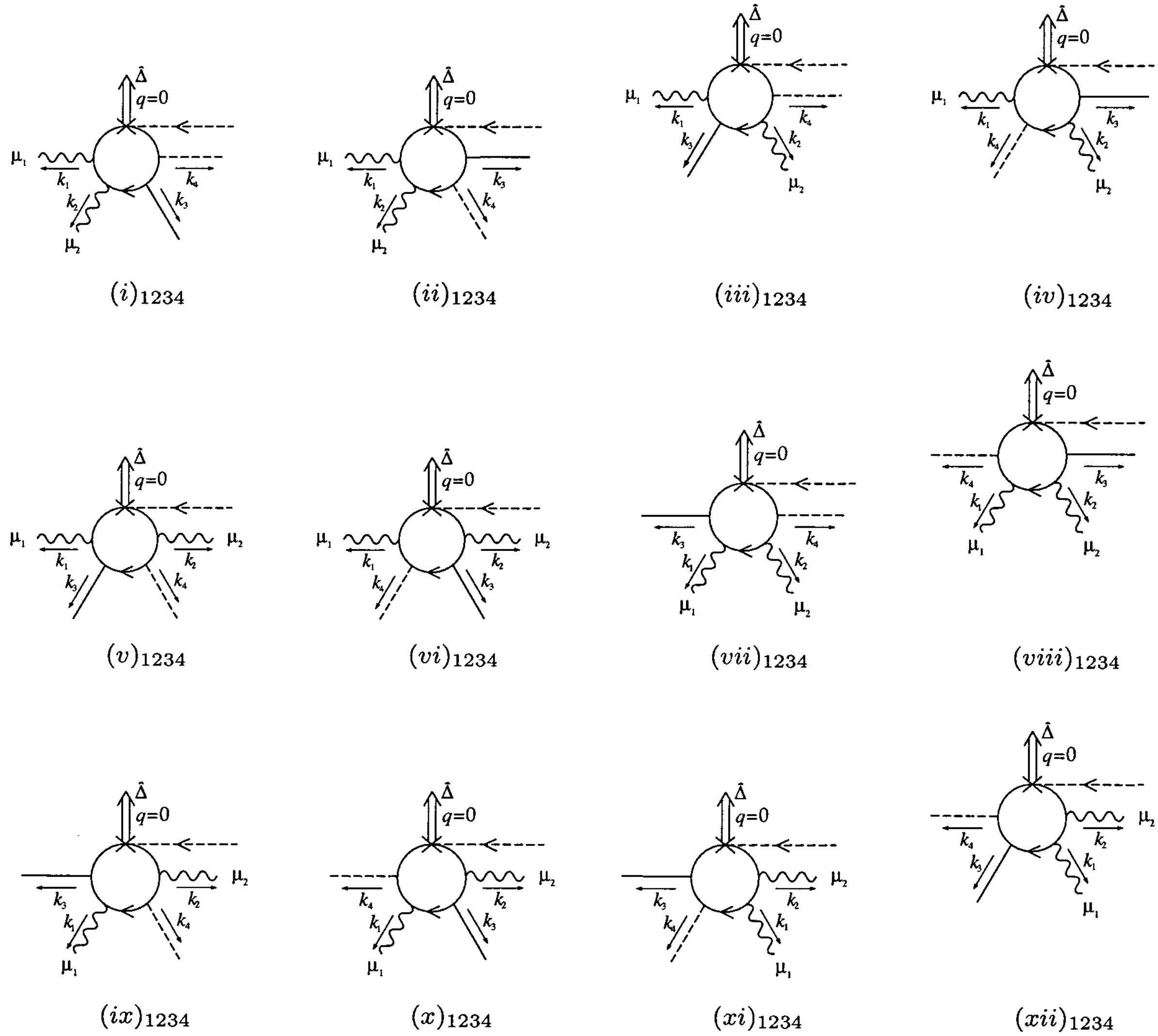

FIG. 8. Feynman diagrams with four boson and scalar fields needed to compute the 1PI breaking functions (C6).

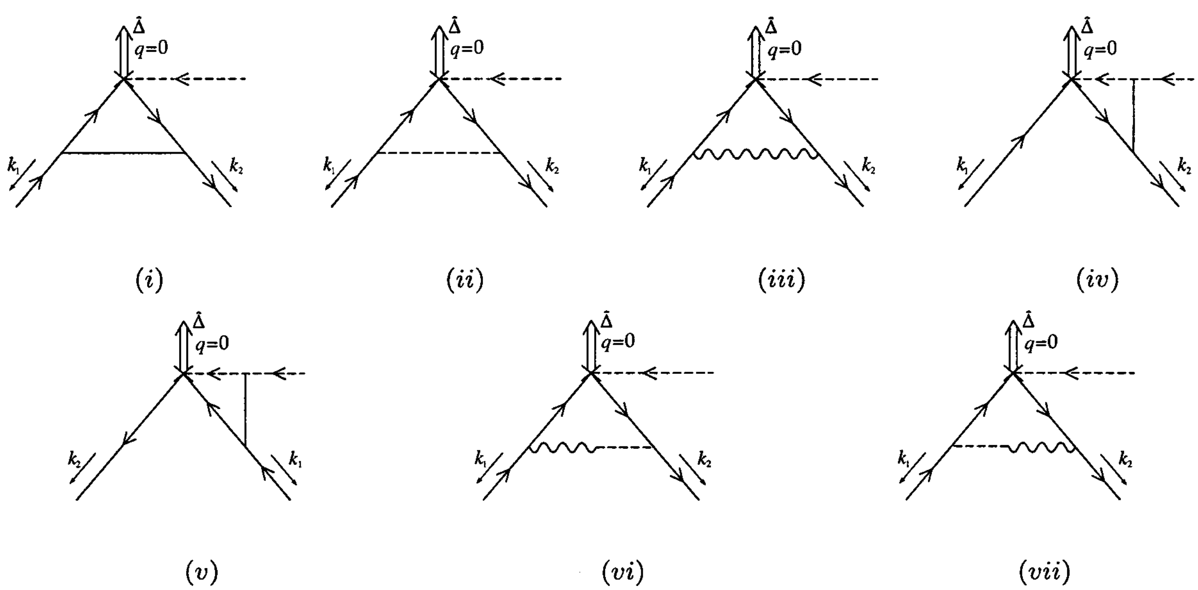

FIG. 9. Feynman diagrams with fermion fields needed to compute the 1PI breaking functions (C7).

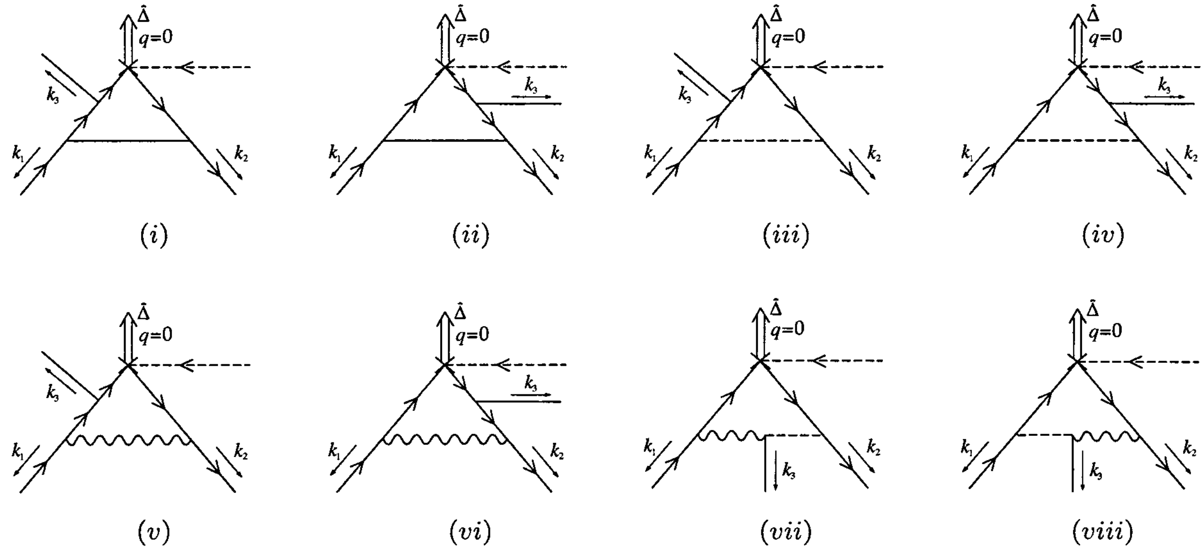

FIG. 10. Feynman diagrams with one scalar and fermion fields needed to compute the 1PI breaking functions (C8).

8. Diagrams with ϕ_1 and fermion fields

$$\tilde{\Gamma}^{R(1)}_{\psi\psi\phi_1 c;N[\hat{\Delta}]}(k_1,k_2,k_3) = (\mathrm{i})+(\mathrm{ii})+(\mathrm{iii})+(\mathrm{iv})+(\mathrm{v}) + (\mathrm{vi})+(\mathrm{vii})+(\mathrm{viii}). \quad \text{(C8a)}$$

Note that there are other possible diagrams at one loop with the same external legs, but due to power counting they are convergent.

The renormalized result for each diagram of Fig. 10 is

$$(4\pi)^2(\mathrm{i}) = \frac{-i}{2} f^3 r \gamma_5, \quad \text{(C8b)}$$

$$(4\pi)^2(\mathrm{ii}) = \frac{-i}{2} f^3 r \gamma_5,$$

$$(4\pi)^2(\mathrm{iii}) = \frac{i}{2} f^3 r \gamma_5,$$

$$(4\pi)^2(\mathrm{iv}) = \frac{i}{2} f^3 r \gamma_5,$$

$$(4\pi)^2(\mathrm{v}) = \frac{-i}{3} f g^2 \theta(1+\theta r)(5+\xi')\gamma_5,$$

$$(4\pi)^2(\mathrm{vi}) = \frac{-i}{3} f g^2 \theta(1+\theta r)(5+\xi')\gamma f_5,$$

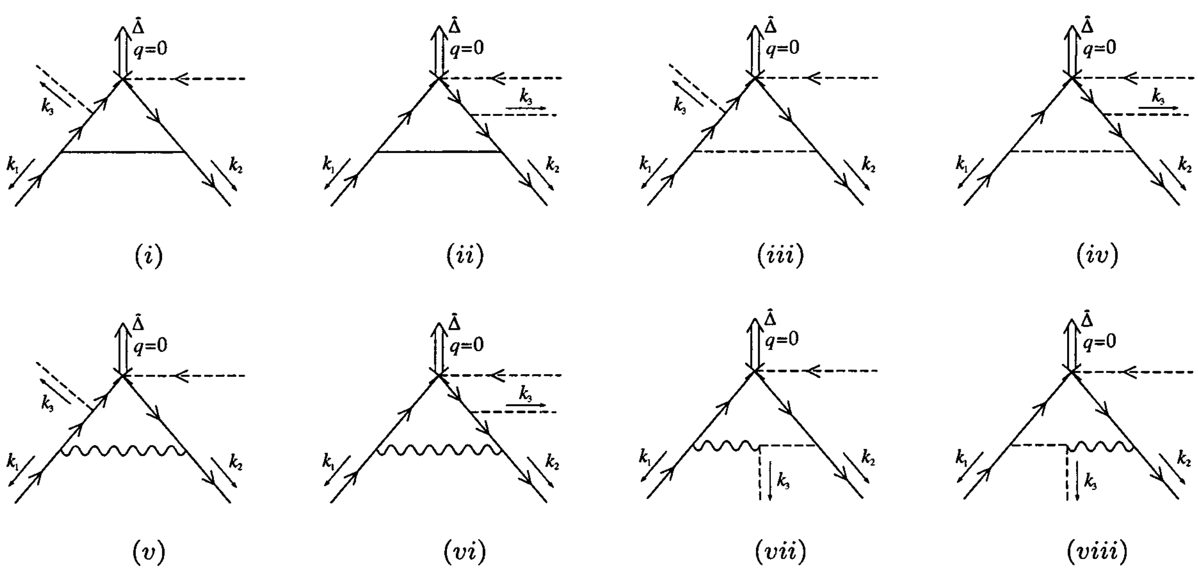

FIG. 11. Feynman diagrams with one scalar and fermion fields needed to compute the 1PI breaking functions (C9).

$$(4\pi)^2(\mathrm{vii})=0,$$

$$(4\pi)^2(\mathrm{viii})=0.$$

9. Diagrams with ϕ_2 and fermion fields

$$\tilde{\Gamma}^{R(1)}_{\psi\psi\phi_2 c;N[\hat{\Delta}]}(k_1,k_2,k_3)=(\mathrm{i})+(\mathrm{ii})+(\mathrm{iii})+(\mathrm{iv})+(\mathrm{v})+(\mathrm{vi})+(\mathrm{vii})+(\mathrm{viii}). \quad \text{(C9a)}$$

Again there are other convergent diagrams at one loop with the same external legs.

The renormalized result for each diagram of Fig. 11 is

$$(4\pi)^2(\mathrm{i})=\frac{f^3\mathbb{I}}{2}, \quad \text{(C9b)}$$

$$(4\pi)^2(\mathrm{ii})=\frac{f^3\mathbb{I}}{2},$$

$$(4\pi)^2(\mathrm{iii})=\frac{-(f^3\mathbb{I})}{2},$$

$$(4\pi)^2(\mathrm{iv})=\frac{-(f^3\mathbb{I})}{2},$$

$$(4\pi)^2(\mathrm{v})=\frac{-[fg^2\theta(\theta+r)(5+\xi')\mathbb{I}]}{3},$$

$$(4\pi)^2(\mathrm{vi})=\frac{-[fg^2\theta(\theta+r)(5+\xi')\mathbb{I}]}{3},$$

$$(4\pi)^2(\mathrm{vii})=0,$$

$$(4\pi)^2(\mathrm{viii})=0.$$

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